ECE 598: Computational Power Systems **Winter 2025**

Lecture 8: Online optimization for economic dispatch

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8.1 Introduction to Online Optimization in Power Systems

In this lecture, we discuss the application of online optimization methods to various challenges in power systems. Specifically, we focus on real-time economic re-dispatch and voltage control in distribution networks. Online optimization plays a critical role in enabling real-time decisions and adjustments to maintain system stability, efficiency, and reliability.

8.1.1 Real-Time Economic Re-Dispatch in Power Systems

Real-time economic re-dispatch is crucial in adapting the generation schedule to deviations from day-ahead forecasts. It helps system operators adjust the dispatch of power generation to account for unexpected changes in demand or system conditions. This can be thought of as an online optimization problem, where adjustments need to be made as new measurements and forecasts are received.

The optimization problem can be formulated as:

$$\underset{r_{\text{up}}, r_{\text{dw}}}{\text{minimize}} \quad \|p_{\text{da}} + r_{\text{up}} - r_{\text{dw}}\|_C^2 + c_u^T r_{\text{up}} - c_d^T r_{\text{dw}}$$

subject to:

$$1^T r_{\text{up}} = \Delta d_{\text{up}}, \quad 1^T r_{\text{dw}} = \Delta d_{\text{dw}}$$

$$0 \leq r_{\text{up}} \leq \overline{r}_{\text{up}}, \quad 0 \leq r_{\text{dw}} \leq \overline{r}_{\text{dw}}$$

Where: - p_{da} is the day-ahead scheduled power, - r_{up} and r_{dw} are the upward and downward regulation powers, respectively, - C is the cost matrix for the regulation (as a subscript), - c_u and c_d are the upward and downward regulation costs, - Δd_{up} and Δd_{dw} represent the upward and downward re-dispatch requirements.

System operators solve this optimization every 5 to 15 minutes based on updated measurements, ensuring that generation follows the fluctuations in demand while minimizing costs.

8.1.1.1 Asymmetric Regulation Costs

In most practical scenarios, the regulation costs for upward and downward adjustments are not equal. For instance, upward regulation (increasing generation) typically incurs higher costs than downward regulation (decreasing generation). The cost vector c_u represents the cost associated with upward regulation, and c_d represents the cost associated with downward regulation.

Asymmetric costs can lead to interesting trade-offs in optimization, where the system may prioritize downward regulation to reduce overall costs, especially if the cost of upward regulation is particularly high.

8.1.1.2 Pricing of Real-Time Re-Dispatch

Once the optimization problem has been solved, the system operator determines the prices for upward and downward regulation. These prices reflect the marginal cost of maintaining system balance. They are critical for compensating the generators that provide regulation services. The optimization problem for pricing is:

$$\underset{r_{\text{up}},r_{\text{dw}}}{\text{minimize}} \quad \|p_{\text{da}} + r_{\text{up}} - r_{\text{dw}}\|_C^2 + c_u^T r_{\text{up}} - c_d^T r_{\text{dw}}$$

subject to the same constraints as before. The solutions provide the dual prices λ_{up} and λ_{dw}, which represent the payments to generators providing upward and downward regulation services.

8.1.1.3 Expanding the Optimization Expression

The optimization problem involves the term $\|p_{\text{da}} + r_{\text{up}} - r_{\text{dw}}\|_C^2$, which represents the squared norm of the vector $p_{\text{da}} + r_{\text{up}} - r_{\text{dw}}$, weighted by the matrix C. The notation $\|\cdot\|_C$ refers to the weighted Euclidean norm (or ℓ_2-norm) using the matrix C.

We can expand this expression as follows:

$$\|p_{\text{da}} + r_{\text{up}} - r_{\text{dw}}\|_C^2 = (p_{\text{da}} + r_{\text{up}} - r_{\text{dw}})^T C (p_{\text{da}} + r_{\text{up}} - r_{\text{dw}})$$

This represents the inner product of the vector $p_{\text{da}} + r_{\text{up}} - r_{\text{dw}}$ with itself, weighted by the matrix C.

8.1.2 Online Feedback Optimization for Voltage Control

In addition to economic re-dispatch, online optimization techniques are also used for voltage control in distribution networks, particularly when dealing with inverters and distributed energy resources (DERs). The goal is to maintain the voltage levels within acceptable limits despite variations in power generation, consumption, and system conditions.

The optimization problem for voltage control can be written as:

$$\underset{q}{\text{minimize}} \quad \frac{1}{2} q^T C q$$

subject to voltage constraints:

$$\underline{v} \leq v(q, w) \leq \overline{v}$$

and actuation bounds:

$$\underline{q} \leq q \leq \overline{q}$$

Where: - q represents the reactive power set-points, - C is the cost matrix for the reactive power adjustments, - $v(q, w)$ is the voltage at each bus as a function of the reactive power injections and weights (with the new notation w), - $\overline{v}$ and $\underline{v}$ are the upper and lower voltage limits at each bus, with the overline and underline notation, - $\overline{q}$ and $\underline{q}$ are the upper and lower bounds for the reactive power set-points.

This problem is solved periodically, and the results are used to update the set-points for reactive power injections from various DERs, such as batteries and solar inverters.

8.1.2.1 Dual Variables in Voltage Control

Voltage control requires the use of dual variables to enforce the voltage constraints. These duals represent the price associated with maintaining voltage within the specified limits. The optimization problem can be augmented with duals to handle these constraints, and the dual variables are updated based on the voltage measurements from the inverters.

The reactive power set-points are updated by solving a saddle point optimization problem:

$$L(q, \underline{\lambda}, \overline{\lambda}) = \frac{1}{2} q^T C q + \overline{\lambda}^T (v(q, w) - \overline{v}) + \underline{\lambda}^T (\underline{v} - v(q, w))$$

Where λ is the vector of dual variables corresponding to the voltage constraints. The solution to this optimization problem provides the updated reactive power set-points.

8.1.2.2 Handling Measurement and Control in Real-Time

One of the key challenges in online voltage control is the need for real-time measurements and communication. Voltage measurements are taken from inverters and other devices in the grid, and these measurements are used to update the dual variables and the reactive power set-points. This process must happen quickly to ensure that voltage levels remain within acceptable limits, especially as new renewable energy sources are added to the grid.

8.2 Algorithm Implementation: Mixed-Saddle Flow for Online Optimization

The mixed-saddle flow algorithm is used to solve both economic re-dispatch and voltage control problems. The basic idea is to iteratively update the decision variables and dual prices using gradient-based methods. The algorithm involves the following steps:

8.2.1 Economic Re-Dispatch Using Mixed-Saddle Flow

The optimization problem for economic re-dispatch can be formulated as a mixed-saddle flow problem. The dual price updates for the system imbalance $\delta(t)$ are given by:

$$\lambda(t+1) = \lambda(t) + \eta\delta(t)$$

Where $\delta(t)$ represents the total imbalance in the system, and η is the step size. The regulation set-points for upward and downward regulation are updated using projected gradient descent:

$$r_{\text{up}}(t+1) = P[0, \overline{r}_{\text{up}}]\left[r_{\text{up}}(t) - \eta\left(2C(p_{\text{da}} + r_{\text{up}}(t) - r_{\text{dw}}(t)) + c_{\text{up}} - \lambda(t+1)1\right)\right]$$

$$r_{\text{dw}}(t+1) = P[0, \overline{r}_{\text{dw}}]\left[r_{\text{dw}}(t) + \eta\left(2C(p_{\text{da}} + r_{\text{up}}(t) - r_{\text{dw}}(t)) + c_{\text{dw}} - \lambda(t+1)1\right)\right]$$

Where $P[0, r]$ is the projection operator that ensures the regulation set-points stay within bounds.

8.2.2 Voltage Control Using Mixed-Saddle Flow

For voltage control, the algorithm works similarly by updating dual variables $\lambda(t)$ and reactive power set-points $q(t)$ using gradient descent and projections. The steps are as follows:

1. Measure voltage $v(t)$. 2. Update the dual variables $\lambda(t+1) = \lambda(t) + \eta(v(t) - v_{\text{ref}})$. 3. Update the reactive power set-points $q(t+1) = \text{project}(C^{-1}X^T(\lambda(t+1) - \lambda(t+1)))$.

This ensures that the voltage remains within the desired range while minimizing the cost associated with reactive power adjustments.

8.3 Discussion: Practical Considerations and Challenges

While online optimization techniques provide a powerful tool for real-time system management, they also come with certain challenges. These include:

- **Communication Overhead:** Real-time optimization requires continuous measurement and communication of system variables (e.g., voltage, imbalance). Efficient communication protocols must be in place to handle this without overloading the system.
- **Algorithm Stability and Convergence:** The choice of step size η plays a crucial role in ensuring that the algorithm converges quickly and stably. Too large a step size can lead to oscillations, while too small can result in slow convergence.
- **Feasibility Guarantees:** The feasibility of the optimization solutions is not always guaranteed, especially when the system is under stress (e.g., during generator failures or extreme demand spikes). Feasibility guarantees need to be established, and contingency management strategies must be in place.

These issues highlight the importance of careful implementation and tuning of online optimization algorithms in real-world power systems.