ECE 598: Computational Power Systems **Winter 2025**

Lecture 7: Feedback Optimization for Volt/VAr Control

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Note: *LaTeX template courtesy of UC Berkeley EECS dept.*

7.1 Background and motivation

Both optimization and control are useful tools for operating the grid in a desirable way. So far, we have been using optimization; choosing values for decision variables in order to minimize some objective function, like cost for example. Control, however, involves continuously updating an input variable. The goal of control is the steer a dynamic system to some desirable range. We will use optmizaiton as a tool to solve a control problem.

A common control problem in power distribution systems is voltage control. This is because in distribution systems, there are large voltage drops due to both active and reactive loads and significant line resistances. We also want to prevent over and under voltages at the loads; over voltage can result in reduced light bulb and electronic device life, and under voltage can result in reduced illumination, heating devices operating slower, and higher starting currents on motors, which can lead to overheating. Voltage fluctuations and transformer overloads are more common now as well due to solar and other DERs.

We show different methods of voltage control on DTU-Risϕ as an example system. It has the following characteristics:

- 3-bus distribution feeder[1]: 1 static load and 3 inverter-interfaced devices
- The battery is set to inject 10 kW to cause over-voltage at the end of the feeder

The goal is to create a device reactive control strategy for inverters to keep voltage within limits. The control design a) does not consider controlling active power injection, b) involves continuously taking measurements throughout the grid, and c) reactive power injections obey the rule:

$$\mathbf{q}_t = \mathbf{q}_{t-1} + \eta \Delta \mathbf{q}_t$$

where $\eta > 0$ is a gain (constant) and $\Delta \mathbf{q}_t$ is the external signal which depends on measurements and prompts inverters to change reactive power injection. η controls how fast you can change the reactive power injection. We will explore three different methods to select $\Delta \mathbf{q}_t$ to steer voltages to admissible range of $0.95 - 1.05$ p.u.:

- Voltage droop control
- AC-OPF-based controller
- Feedback optimization-based control

[1]Data from L. Ortmann *et al.* Experimental validation of feedback optimization in power distribution grids. 2020

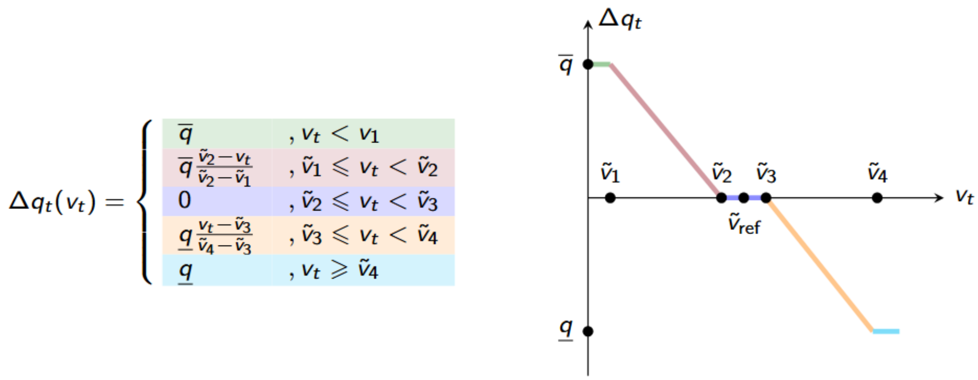

Figure 7.1: Voltage droop control.

7.2 Droop control

Droop control is where each bus reacts to voltage variations locally by consuming or supplying reactive power according to a piecewise linear control law, shown by Figure 7.1. Consuming reactive power lowers the voltage, while supplying reactive power raises the voltage. Each bus only requires local voltage measurements to enact the law. Critical points $\tilde{v}$ can be changed to tune inverter's response; however, response to only local voltage measurements can be insufficient.

7.3 OPF-based control

Voltage control can be improved by taking measurements of real and reactive power injections throughout the grid and knowing the grid resistance and reactance. The least cost reactive power injection change is:

$$\mathbf{q}_t = \mathbf{q}_{t-1} + \eta(\mathbf{q}_t^\star - \mathbf{q}_{t-1})$$

where:

$$\begin{aligned} \mathbf{q}_t^\star = \underset{\mathbf{v},\mathbf{q}_t}{\text{argmin}} \quad & \frac{1}{2}(\mathbf{q}_t - \mathbf{q}_{t-1})^\top \mathbf{C}(\mathbf{q}_t - \mathbf{q}_{t-1}) \\ \text{subject to} \quad & \mathbf{v}_t = v_0\mathbf{1} + \mathbf{R}\mathbf{p}_t + \mathbf{X}(\widehat{\mathbf{q}}_t + \mathbf{q}_t) \\ & \underline{\mathbf{v}} \leqslant \mathbf{v}_t \leqslant \overline{\mathbf{v}} \\ & \underline{\mathbf{q}} \leqslant \mathbf{q}_t \leqslant \overline{\mathbf{q}} \end{aligned}$$

The downside of this control method is that it requires full knowledge of the grid parameters and power injections.

7.4 Feedback optimization

Feedback optimization combines the benefits of droop control (locally-based) and OPF-based control (ability for inverters to help each other) and avoids the detriments (no ability to help each other and need for full knowledge of the grid, respectively). It assumes the decision makers (those who set the reactive power injection of the inverters) don't know h(**u**,**w**), the function that maps the inputs to the outputs; instead, the algorithm uses output measurements (y) to update (u). In this way, feedback optimization only relies on grid measurements, not full knowledge of the grid.

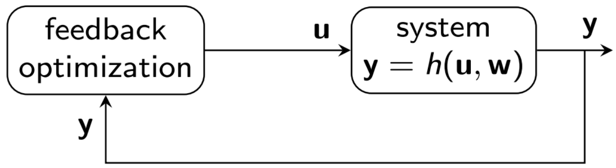

Figure 7.2: Feedback System

Variable	General Definition	Voltage Control Example
u	Set-point	Reactive power injection by inverters
y	Output	Voltage measurements across the grid
w	Uncontrollable input	PV or wind active power generation
h(**u**,**w**)	Function that maps from inputs to outputs	Power flow equations

Table 7.1: Variable Definitions

We work from the perspective of the inverter (the "feedback optimization" box in FIG 7.2). To us, y is a measurement taken in real time. These real-time measurements are used to iteratively adjust the set-points **u**. We have no knowledge of the exogenous input w or h(u,w). Since we don't know the power flow equations, h(u,w), we have reduced model information; however, we can estimate δh from historic measurement and set-point data. A closed-loop system converges to the solution of the optimization results from the OPF-based control problem.

The optimization formulation of the voltage control problem using the feedback optimization variables is:

$$\min_{\mathbf{u}} \quad f(\mathbf{u}) \qquad\qquad \text{cost of actuation effort} \tag{7.1}$$

$$\text{subject to} \quad g(\mathbf{y}) \leq 0 \qquad \text{dual: } \lambda \quad \text{constraint on the output of } y = h(u,w) \tag{7.2}$$

$$\mathbf{u} \in \mathcal{U} \qquad\qquad \text{actuation bounds} \tag{7.3}$$

We can't solve this formulation because we have no knowledge of h(**u**,**w**). To get to a formulation we can solve, we need to reformulate this problem so we only need to know the gradient of h(**u**,**w**), not h(**u**,**w**) itself. We do that by using the dual and strong duality.

We start by dualizing the constraint on the output to get a partial Lagrangian.

$$L(\mathbf{u},\lambda) = f(\mathbf{u}) + \lambda^T g(h(\mathbf{u},\mathbf{w}))$$

We then solve the dual:

$$\max_{\lambda \geq 0} \phi(\lambda)$$

where

$$\phi(\lambda) = \min_{u \in \mathcal{U}} \mathcal{L}(u,\lambda)$$

is the dual function.

To solve the dual, we use the gradient ascent algorithm with a fixed step size. Gradient ascent is similar to gradient descent, which we worked with previously; it's just maximizing instead of minimizing.

$$\lambda_{t+1} = [\lambda_t + \rho \nabla_\lambda \phi(\lambda)]_{\geq 0}$$

Where $\rho > 0$ is a tuning parameter.

$\nabla_\lambda \phi(\lambda) = g(h(\mathbf{u,w}))$ is the gradient given by violation of the dualized constraint. We don't need to know the model to compute a constraint violation, only the measurement y.

$$\lambda_{t+1} = [\lambda_t + \rho g(\mathbf{y}_t)]_{\geq 0}$$

λ integrates the constraint violation with step ρ (integral part of a PI-controller).

Using λ_{t+1}, we update the set points by solving:

$$\mathbf{u}_{t+1} = \arg\min_{\mathbf{u}} \mathcal{L}(\mathbf{u}, \lambda_{t+1}) \tag{7.4}$$

$$= \arg\min_{\mathbf{u}} f(\mathbf{u}) + \lambda_{t+1}^T g(\mathbf{y}_t) \tag{7.5}$$

and apply them to the system.

7.4.1 Feedback Optimization Algorithm

In summary:

For $t = 0, 1, ..., +\infty$:

1. Measure system output y_t
2. Update duals $\lambda_{t+1} = [\lambda_t + \rho g(\mathbf{y}_t)]_{\geq 0}$
3. Update set points $\mathbf{u}_{t+1} = \arg\min_{\mathbf{u}} f(\mathbf{u}) + \lambda_{t+1}^T g(\mathbf{y}_t)$
4. Apply set points to the system

Unlike other primal-dual algorithms we have learned, this algorithm doesn't "converge"; it runs continuously as the system does, since its goal is to keep the voltage at the nodes within the acceptable range.

7.4.2 Practical Feedback Optimization for Voltage Control

We now apply this algorithm to our case of interest: voltage control. We take voltage measurements $\hat{v}$ from unknown power flow equations $v(q, w)$. The model-free formulation of AC optimal power flow is:

$$\min_q \quad \frac{1}{2} q^T C q \qquad \text{quadratic cost of actuation } (M \geq 0) \tag{7.6}$$

$$\text{subject to} \quad \underline{v} \leq \hat{v} \leq \overline{v} \qquad \underline{\lambda}, \overline{\lambda} \quad \text{linear constraints on } \hat{v} = v(q, w) \tag{7.7}$$

$$\underline{q} \leq q \leq \overline{q} \qquad \text{linear actuation bounds} \tag{7.8}$$

The reactive set points are updated by solving the following setup:

$$\mathcal{L}(q, \underline{\lambda}, \overline{\lambda}) = \frac{1}{2} q^T C q + \overline{\lambda}^T (v(q.w) - \overline{v}) + \underline{\lambda}^T (\underline{v} - v(q, w))$$

$$\nabla_q \mathcal{L}(q, \underline{\lambda}, \overline{\lambda}) = Cq + \frac{\partial v(q, w)}{\partial q}^T (\overline{\lambda} - \underline{\lambda}) = 0$$

$$q = C^{-1} \frac{\partial v(q,w)}{\partial q}^T (\overline{\lambda} - \underline{\lambda})$$

The sensitivity of the voltages to reactive power injections is approximated from historical data. From the linear distributed flow model, the partial with respect to q is the reduced bus reactance matrix.

$$\frac{\partial v(q,w)}{\partial q} = X$$

We don't know this matrix, since we don't have comprehensive information about the system, so we estimate it based on historical measurements. However, we *do* always know the direction, because X is always positive. While our estimate isn't perfect, we're at least guaranteed to go in the right direction.

7.4.2.1 Feedback Optimization Algorithm for Voltage Control

For $t = 0, 1... + \infty$:

1. Measure voltage $\hat{v}_t$
2. Update duals
   - $\underline{\lambda}_{t+1} = [\underline{\lambda}_t + \rho(\underline{v} - \hat{v}_t]_{\geq 0}$
   - $\overline{\lambda}_{t+1} = [\overline{\lambda}_t + \rho(\hat{v}_t - \overline{v}]_{\geq 0}$
3. Update reactive set points
   - $\tilde{q}_{t+1} = C^{-1} X^T (\overline{\lambda}_{t+1} - \underline{\lambda}_{t+1})$
   - $q_{t+1} = \max\{\underline{q}, \min\{\tilde{q}, \overline{q}\}\}$
4. Apply set points to inverters

What happens if the problem is infeasible? The duals keep integrating, a phenomena referred to as "windup." The solution to this potential issue is "anti-windup." For example, for the dual of the lower voltage limit, you would have:

$$\underline{\lambda}_{t+1} = \begin{cases} \underline{\lambda_t} & \underline{v} - \hat{v}_t > 0 \text{ and } q = \overline{q} \\ \underline{\lambda_t} + \rho(\underline{v} - \hat{v}_t) & \text{otherwise} \end{cases}$$