ECE 598 Computational Power Systems

Online optimization for Volt/VAr Control

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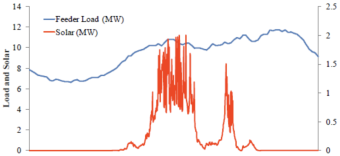
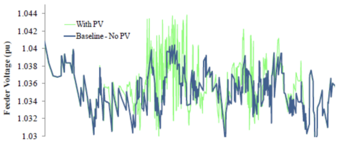
Why voltage control?

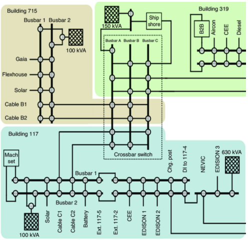
- **Voltage drops** due to both active and reactive loads (significant line resistances)
- **Over-voltage**: Reduced light bulb life and electronic devices
- **Under-voltage**: lower illumination, heating devices (e.g., water heaters) operate slower, higher starting currents on motors and overheating
- Voltage fluctuations and transformer overloads due to solar and other **DERs**



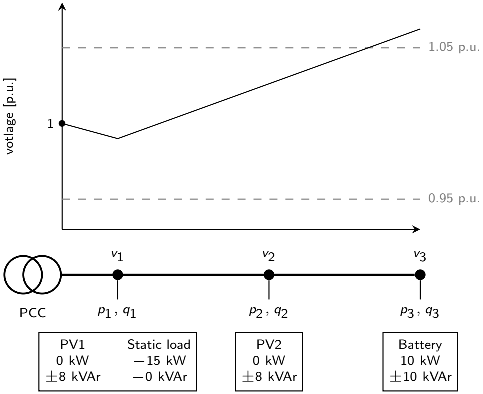


https://www.esig.energy/wiki-main-page/time-series-power-flow-analysis-for-distribution-connected-pv-generation/

DTU-Risø



- Diverse energy mix: wind turbines, photovoltaic (PV) plants, diesel generators
- Energy storage: 15 kW/120 kWh vanadium redox flow battery
- Flexible grid: autonomous and grid-connected modes, combinations thereof.



- 3-bus distribution feeder[1]: 1 static load and 3 inverter-interfaced devices
- The battery is set to inject 10 kW to cause over-voltage at the end of the feeder
- Goal: device reactive control strategy for inverters to keep voltage within limits

[1] Data from L. Ortmann *et al.* Experimental validation of feedback optimization in power distribution grids. 2020

Designing controller

- For simplicity, we do not control active power injection (only reactive)
- We continuously take measurements throughout the grid
- Reactive power injections of inverters obey the rule

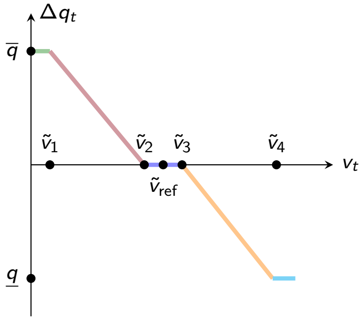
$$\mathbf{q}_t = \mathbf{q}_{t-1} + \eta \Delta \mathbf{q}_t$$

  where $\eta > 0$ is a gain (constant) and $\Delta \mathbf{q}_t$ is the external signal which depends on measurements and prompts inverters to change reactive power injection

- How to select $\Delta \mathbf{q}_t$ to steer voltages to admissible range of $0.95 - 1.05$ p.u.?
  - Voltage droop control
  - AC-OPF-based controlr
  - Feedback optimization-based control

Voltage droop control

$$\Delta q_t(v_t) = \begin{cases} \overline{q} & , v_t < v_1 \\ \overline{q}\frac{\tilde{v}_2 - v_t}{\tilde{v}_2 - \tilde{v}_1} & , \tilde{v}_1 \leqslant v_t < \tilde{v}_2 \\ 0 & , \tilde{v}_2 \leqslant v_t < \tilde{v}_3 \\ \underline{q}\frac{v_t - \tilde{v}_3}{\tilde{v}_4 - \tilde{v}_3} & , \tilde{v}_3 \leqslant v_t < \tilde{v}_4 \\ \underline{q} & , v_t \geqslant \tilde{v}_4 \end{cases}$$



- Piecewise linear control law complying with recent grid codes[2]
- Linear response to voltage at inverter's bus with some deadband.
- Critical points $\tilde{v}$ can be changed to tune inverter's response
- Limitation: response to only local voltage measurements can be insufficient

[2] IEEE 1547–2018, Standard for interconnection and interoperability of distributed energy resources with associated electric power systems interfaces

Voltage control using AC-OPF model

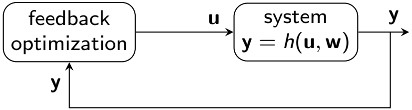
$$
\begin{aligned}
\mathbf{q}_t^\star = \underset{\mathbf{v},\mathbf{q}_t}{\operatorname{argmin}} \quad & \frac{1}{2}(\mathbf{q}_t - \mathbf{q}_{t-1})^\top \mathbf{C}(\mathbf{q}_t - \mathbf{q}_{t-1}) \\
\text{subject to} \quad & \mathbf{v}_t = v_0\mathbf{1} + \mathbf{R}\mathbf{p}_t + \mathbf{X}(\widehat{\mathbf{q}}_t + \mathbf{q}_t) \\
& \underline{\mathbf{v}} \leqslant \mathbf{v}_t \leqslant \overline{\mathbf{v}} \\
& \underline{\mathbf{q}} \leqslant \mathbf{q}_t \leqslant \overline{\mathbf{q}}
\end{aligned}
$$

- Instead of voltage, it requires active $\mathbf{p}_t$ and reactive $\mathbf{q}_t$ power measurements
- It also requires the full knowledge of the grid model in terms of $\mathbf{R}$ and $\mathbf{X}$
- Computes the least-cost reactive power injection change:

$$\mathbf{q}_t = \mathbf{q}_{t-1} + \eta(\mathbf{q}_t^\star - \mathbf{q}_{t-1})$$

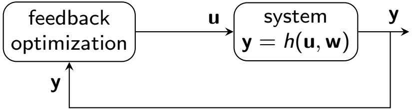
- Compared to voltage droop control, does $\mathbf{q}_t^\star$ depend only on local measurements?
- What data do we need to know to implement such control strategy in practice?

Feedback optimization



- $\mathbf{u}$ set-point (e.g., reactive power injection by inverters)
- $\mathbf{y}$ output (e.g., voltage measurements across the grid)
- $\mathbf{w}$ uncontrollable input (e.g., PV or wind active power generation)
- $h(\mathbf{u}, \mathbf{w})$ map from inputs to outputs (e.g., power flow equations)

Feedback optimization



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Feedback optimization:

$$
\begin{aligned}
\underset{\mathbf{u}}{\text{minimize}} \quad & f(\mathbf{u}) && \text{cost of actuation effort} \\
\text{subject to} \quad & g(\mathbf{y}) \leqslant 0 && \text{constraint on the output } \mathbf{y} = h(\mathbf{u}, \mathbf{w}) \\
& \mathbf{u} \in \mathcal{U} && \text{actuation bounds}
\end{aligned}
$$

- Assumption: output $\mathbf{y}$ is measured in real-time, exogenous input $\mathbf{w}$ is unknown
- Real-time measurements are used to iteratively adjust the set-points $\mathbf{u}$
- Reduced model information: h is unknown, but ∂h can be estimated
- The closed-loop system converges to the solutions of the optimization problem

Feedback optimization principle - part I

$$\begin{array}{rlr} \underset{\mathbf{u}}{\text{minimize}} & f(\mathbf{u}) & \text{cost of actuation effort} \\ \text{subject to} & g(\mathbf{y}) \leqslant 0 & \text{constraint on the output } \mathbf{y} = h(\mathbf{u}, \mathbf{w}) \\ & \mathbf{u} \in \mathcal{U} & \text{actuation bounds} \end{array}$$

- Use measurements y instead of the model $h(\mathbf{u}, \mathbf{w})$
- Dualize constraint on the output

$$\mathcal{L}(\mathbf{u}, \boldsymbol{\lambda}) = f(\mathbf{u}) + \boldsymbol{\lambda}^\top g(h(\mathbf{u}, \mathbf{w}))$$

- Instead of optimization above, solve

$$\max_{\boldsymbol{\lambda} \geqslant 0} \phi(\boldsymbol{\lambda})$$

  where

$$\phi(\boldsymbol{\lambda}) = \min_{u \in \mathcal{U}} \mathcal{L}(\mathbf{u}, \boldsymbol{\lambda})$$

  is the dual function

- How to solve this optimization?

Feedback optimization principle - part II

- Gradient ascent with a fix step size

$$\boldsymbol{\lambda}_{t+1} = [\boldsymbol{\lambda}_t + \rho \nabla_{\boldsymbol{\lambda}} \phi(\boldsymbol{\lambda})]_{\geqslant \mathbf{0}}$$

  where $\rho > 0$ is a tuning parameter

- $\nabla_{\boldsymbol{\lambda}} \phi(\boldsymbol{\lambda}) = g(h(\mathbf{u}, \mathbf{w}))$ – gradient is given by violation of the dualized constraint

- Do we need to know model h to compute constraint violation? No, only $\mathbf{y}$!

$$\boldsymbol{\lambda}_{t+1} = [\boldsymbol{\lambda}_t + \rho g(\mathbf{y}_t)]_{\geqslant \mathbf{0}}$$

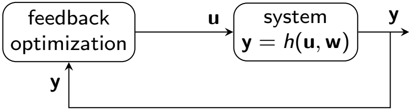
  $\boldsymbol{\lambda}$ integrates constraint violation with step ρ (integral part of a PI-controller)

- Using $\boldsymbol{\lambda}_{t+1}$, update set-points by solving

$$\begin{aligned} \boldsymbol{u}_{t+1} &= \arg\min_{\mathbf{u}} \mathcal{L}(\mathbf{u}, \boldsymbol{\lambda}_{t+1}) \\ &\arg\min_{\mathbf{u}} f(\mathbf{u}) + \boldsymbol{\lambda}_{t+1}^{\top} g(\mathbf{y}_t) \end{aligned}$$

  and apply them to the system

Feedback optimization algorithm



For $t = 0, 1, \ldots, +\infty$ do

Step 1: Measure system output $\mathbf{y}_t$
Step 2: Update duals $\boldsymbol{\lambda}_{t+1} = [\boldsymbol{\lambda}_t + \rho g(\mathbf{y}_t)]_{\geqslant \mathbf{0}}$
Step 3: Update set-points $\boldsymbol{u}_{t+1} = \arg\min_{\mathbf{u}} f(\mathbf{u}) + \boldsymbol{\lambda}_{t+1}^\top g(\mathbf{y}_t)$
Step 4: Apply set-points to the system

Practical feedback optimization for voltage control

- We take voltage measurements $\widehat{\mathbf{v}}$ from unknown PF equations $\mathbf{v}(\mathbf{q}, \mathbf{w})$
- Model-free AC optimal power flow problem:

$$
\begin{array}{llll}
\underset{\mathbf{q}}{\text{minimize}} & \frac{1}{2}\mathbf{q}^\top \mathbf{C}\mathbf{q} & & \text{quadratic cost of actuation } (\mathbf{M} \succeq \mathbf{0}) \\
\text{subject to} & \underline{\mathbf{v}} \leqslant \widehat{\mathbf{v}} \leqslant \overline{\mathbf{v}} : & \underline{\boldsymbol{\lambda}}, \overline{\boldsymbol{\lambda}} & \text{linear constraints on } \widehat{\mathbf{v}} = \mathbf{v}(\mathbf{q}, \mathbf{w}) \\
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- The reactive set points are updated by solving for some fixed

$$\mathcal{L}(\mathbf{q}, \underline{\boldsymbol{\lambda}}, \overline{\boldsymbol{\lambda}}) = \frac{1}{2}\mathbf{q}^\top \mathbf{C}\mathbf{q} + \overline{\boldsymbol{\lambda}}^\top (\mathbf{v}(\mathbf{q}, \mathbf{w}) - \overline{\mathbf{v}}) + \underline{\boldsymbol{\lambda}}^\top (\underline{\mathbf{v}} - \mathbf{v}(\mathbf{q}, \mathbf{w}))$$

$$\implies \nabla_{\mathbf{q}} \mathcal{L}(\mathbf{q}, \underline{\boldsymbol{\lambda}}, \overline{\boldsymbol{\lambda}}) = \mathbf{C}\mathbf{q} + \frac{\partial \mathbf{v}(\mathbf{q}, \mathbf{w})}{\partial \mathbf{q}}^\top (\overline{\boldsymbol{\lambda}} - \underline{\boldsymbol{\lambda}}) = \mathbf{0}$$

$$\implies \mathbf{q} = \mathbf{C}^{-1} \frac{\partial \mathbf{v}(\mathbf{q}, \mathbf{w})}{\partial \mathbf{q}}^\top (\underline{\boldsymbol{\lambda}} - \overline{\boldsymbol{\lambda}}), \quad \text{followed by projection on } [\underline{\mathbf{q}}, \overline{\mathbf{q}}]$$

- How to approximate the sensitivity of voltages to reactive power injections?

Practical feedback optimization for voltage control

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- Model-free AC optimal power flow problem:

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$$\mathcal{L}(\mathbf{q}, \underline{\boldsymbol{\lambda}}, \overline{\boldsymbol{\lambda}}) = \frac{1}{2}\mathbf{q}^\top \mathbf{C}\mathbf{q} + \overline{\boldsymbol{\lambda}}^\top (\mathbf{v}(\mathbf{q}, \mathbf{w}) - \overline{\mathbf{v}}) + \underline{\boldsymbol{\lambda}}^\top (\underline{\mathbf{v}} - \mathbf{v}(\mathbf{q}, \mathbf{w}))$$

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- How to approximate the sensitivity of voltages to reactive power injections?
- From LinDistFlow model, $\frac{\partial \mathbf{v}(\mathbf{q}, \mathbf{w})}{\partial \mathbf{q}} = \mathbf{X}$ – reduced bus reactance matrix.

Feedback optimization algorithm for voltage control

For $t = 0, 1, \ldots, +\infty$ do

Step 1: Measure voltage $\widehat{\mathbf{v}}_t$

Step 2: Update duals

$$\underline{\boldsymbol{\lambda}}_{t+1} = [\underline{\boldsymbol{\lambda}}_t + \rho(\underline{\mathbf{v}} - \widehat{\mathbf{v}}_t)]_{\geqslant \mathbf{0}}$$

$$\overline{\boldsymbol{\lambda}}_{t+1} = [\overline{\boldsymbol{\lambda}}_t + \rho(\widehat{\mathbf{v}}_t - \overline{\mathbf{v}})]_{\geqslant \mathbf{0}}$$

Step 3: Update reactive set-points

$$\tilde{\boldsymbol{q}}_{t+1} = \mathbf{C}^{-1}\mathbf{X}^\top(\underline{\boldsymbol{\lambda}}_{t+1} - \overline{\boldsymbol{\lambda}}_{t+1})$$

$$\boldsymbol{q}_{t+1} = \max\{\underline{\mathbf{q}}, \min\{\tilde{\boldsymbol{q}}_{t+1}, \overline{\mathbf{q}}\}\}$$

Step 4: Apply set-points to inverters

Feedback optimization algorithm for voltage control

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$$\boldsymbol{q}_{t+1} = \max\{\underline{\mathbf{q}}, \min\{\tilde{\boldsymbol{q}}_{t+1}, \overline{\mathbf{q}}\}\}$$

Step 4: Apply set-points to inverters

What if the problem is infeasible?

- Duals keep integrating (windup)
- Solution $\rightarrow$ anti-windup. For example, for the dual of the lower voltage limit:

$$\underline{\boldsymbol{\lambda}}_{t+1} = \begin{cases} \underline{\boldsymbol{\lambda}}_t & \text{, if } \underline{\mathbf{v}} - \widehat{\mathbf{v}}_t > 0 \text{ and } \mathbf{q} = \overline{\mathbf{q}} \\ \underline{\boldsymbol{\lambda}}_t + \rho(\underline{\mathbf{v}} - \widehat{\mathbf{v}}_t) & \text{, if otherwise} \end{cases}$$

Summary

- Volt/VAr control to maintain normal operation of appliances in distribution grids

- Three Volt/VAr control strategies:
  - Droop control (current industry standard), inefficient due to its “local” nature
  - OPF-based control (ideal, yet impractical due to unrealistic knowledge assumptions)
  - Feedback optimization-based control (acts on measurements, middle-ground solution)

- Feedback optimization-based control leverages Lagrange duality to replace unknown model (OPF equations) with measurements

- Only needs the gradient of the PF equations. Hence, it is robust to model mismatch (e.g., reactance estimation errors, as reactance does not change in sign)

Resources

- Ortmann, L., Hauswirth, A., Caduff, I., Dörfler, F., & Bolognani, S. (2020). Experimental validation of feedback optimization in power distribution grids. Electric Power Systems Research, 189, 106782.

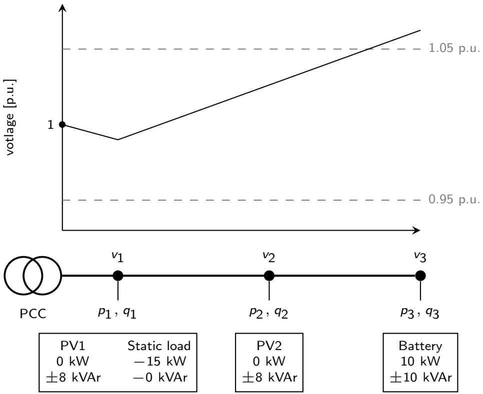
- Bolognani, S., Carli, R., Cavraro, G., & Zampieri, S. (2014). Distributed reactive power feedback control for voltage regulation and loss minimization. IEEE Transactions on Automatic Control, 60(4), 966-981.

- Dall'Anese, E., & Simonetto, A. (2016). Optimal power flow pursuit. IEEE Transactions on Smart Grid, 9(2), 942-952.

Tutorial time

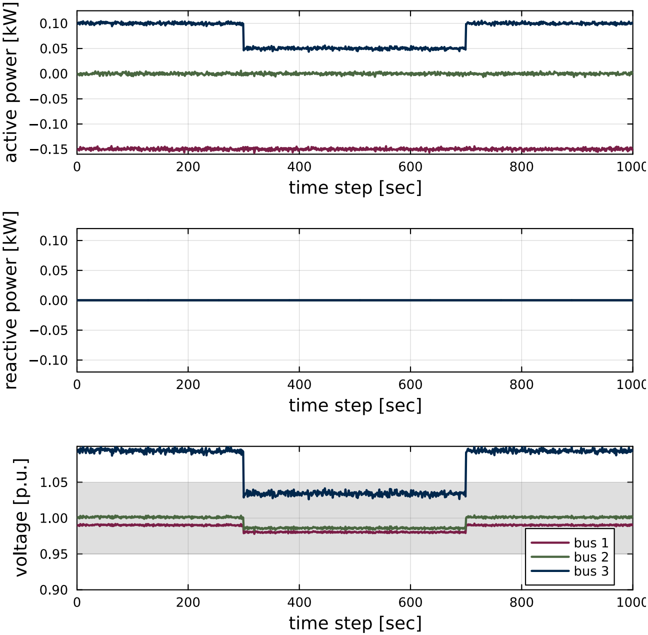
- Look around you and form teams of 2 people (1 min)
- Pick one person to code; the other one guides
- Work in pairs for the whole tutorial session

Tutorial: Risø

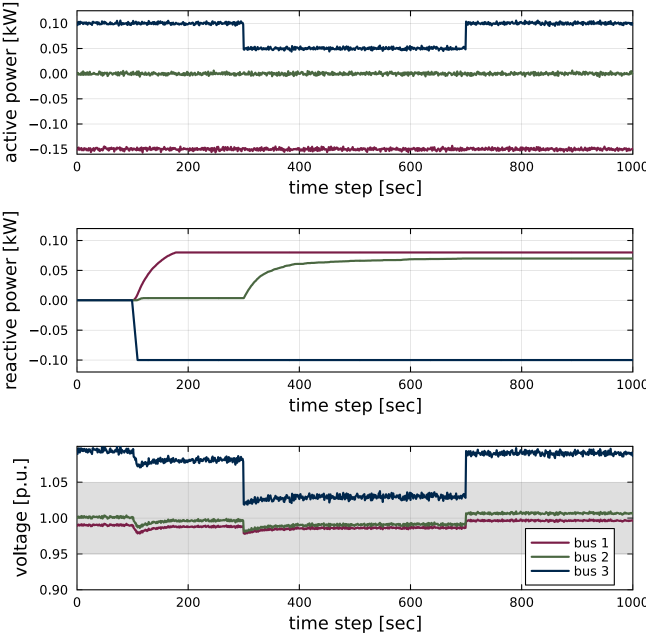


- Simulation horizon of 1,000 seconds
- At 300's sec, battery starts injecting 50% less active power
- At 700's sec, battery restores active power at 10 kW
- Implement droop and OPF-based voltage control starting from 100's sec
- Use `tutorial_6_Volt/VAr_control` to start the tutorial

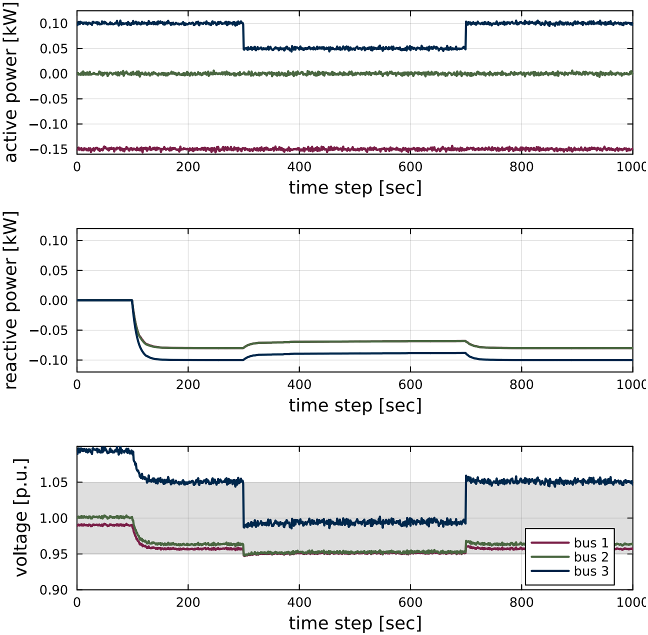
Expected results: no voltage control



Expected results: droop control



Expected results: OPF-based control



Expected results: Online feedback optimization-based control