ECE 598 Computational Power Systems

Intro

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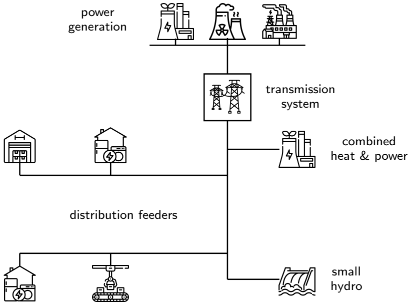
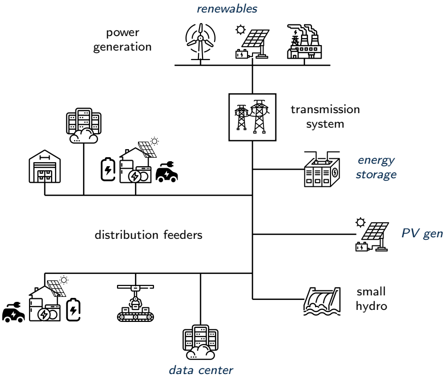
University of Michigan

Power grid



- Past: dominated by conventional gen, predictable, less control opportunities
- Present & future: renewables, hard to predict but plenty of control opportunities

Power grid



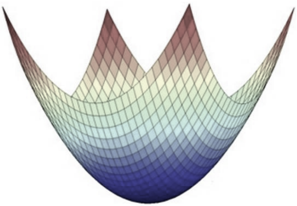
- Past: dominated by conventional gen, predictable, less control opportunities
- Present & future: renewables, hard to predict but plenty of control opportunities

Many problems in power systems are optimization problems

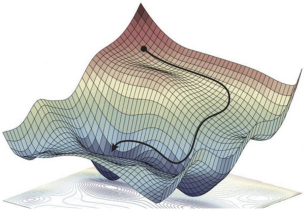
$$\begin{aligned} \underset{\mathbf{x}\in\mathbb{R}^n}{\text{minimize}} \quad & c(\mathbf{x}) \\ \text{subject to} \quad & g(\mathbf{x}) \leqslant \mathbf{0} \end{aligned}$$

- Vector $\mathbf{x}$ of decision variables (dispatch, prices, voltage)
- Cost function $c(\mathbf{x})$ (gen cost, $-$profit, forecast error)
- Constraint function $g(\mathbf{x})$ (gen, trans, voltage limits)

The optimal decision $\mathbf{x}^\star$ minimizes the cost
and satisfies all constraints

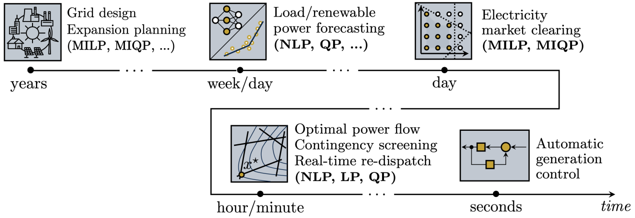


convex optimization [Ahmadi]



non-convex optimization [Amini]

Decision-making timeline in power systems



- Decisions from year to minutes ahead of operation
- Each decision here can be computed by solving an optimization problem
- Each problem has input data, objective function, constraints and algorithm

Some challenges in modern power systems

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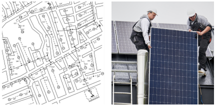

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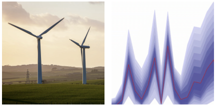

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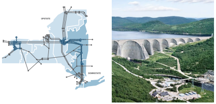

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8


A room full of power systems experts

- Look around you, form teams of 3 people
- You are the team of experts in grid optimization
- Select one person that is responsible for
  1. keeping time
  2. ensuring everyone in the team contributes
  3. representing the team, sharing your responses
- Use 5 minutes to discuss what competences you bring to the team
  - **Grid engineering:** phasor analysis, 3ϕ-systems, transformers, power transmission, ...
  - **Power flow analysis:** per unit modeling, Newton-Raphson, fast decoupled power flow, ...
  - **Control:** droop and load frequency control, reactive power compensation, dynamics, ...
  - **Optimization:** linear, quadratic, non-linear optimization, relaxation, decomposition, ...
  - **Analytics & ML:** forecasting, classification, risk management, ...
- Get ready to work as a team for the next 45 minutes

We will go through all cases one by one

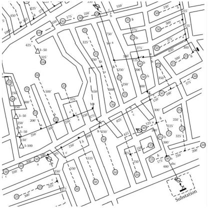
- For each case, I will describe the **problem** and the **optimization challenge**
- Set up an optimization problem:
  - What is the optimization data?
  - What are the decision variables?
  - What are the cost fun and constraints?
  - Which method/algorithm to use?

$$\begin{array}{ll} \underset{\mathbf{x}\in\mathbb{R}^n}{\text{minimize}} & c(\mathbf{x}) \\ \text{subject to} & g(\mathbf{x}) \leqslant \mathbf{0} \end{array}$$

- Discuss how to tackle the challenge with your team's competence
  - Do you think you have the necessary competences?
  - Would you take up the challenge?
  - If yes, what methods would you use?
  - If no, what's blocking you?

Quick! 3 minutes per case

Case 1: Distribution grid control

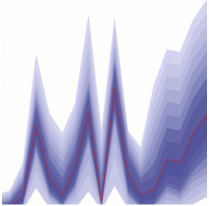


- Distribution grid with a large penetration of rooftop PV panels
- In cloudy weather, the operator struggles to maintain voltage within limits
- Tap-changing transformers and capacitor tanks can control voltage during the day
- **Goal:** devise optimization problem to assist with the voltage control problem

Case 2: Carbon-aware data center dispatch

- Large language models (LLMs) requires a lot of power for training and inference
- Space-time allocation of LLM workloads defines their emission footprint
- **Goal:** optimization for carbon-aware allocation of LLM workloads in the grid

Case 3: Renewable power forecasting



- Wind power producers sell energy one day ahead of real-time delivery
- Real-time penalty for any forecast errors are very expensive, but asymmetric
- **Goal:** optimization for wind power producers to improve their financial standing

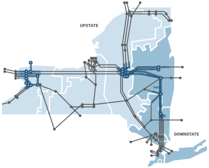
Case 4: EV aggregator and frequency regulation market


- EV charging company must ensure full charge by a specific time
- The company can use the plugged EVs for frequency regulation
- The frequency regulation market pays for fast (dis)charging response to grid signal
- **Goal:** devise revenue-maximizing optimization for EV aggregators

Case 5: Portfolio optimization for large electricity consumers

- Refinery company is a large electricity consumer
- The company can purchase power on the sport market with hourly-varying prices
- Or make a bilateral power purchasing agreement with the price fixed for one year
- The spot and bilateral power purchasing can be made in some proportion
- **Goal:** devise optimization problem for the management of the company

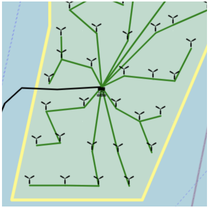
Case 6: Cross-border grid coordination



- New York (USA) enjoys hydro power from Québec (CA)
- The NY and Quebec system operators are in charge of operating their own grids
- They wish to operate at the minimum of dispatch cost ...

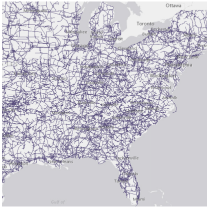
  ... but they can't share their grid models (classified information)
- **Goal:** develop grid coordination model with minimum data exchange

Case 7: Offshore wind farm layout



- Wind turbines create turbulence that disrupts the airflow around nearby turbines
- Array cables connect individual turbines; they can not overlap (expansive, unsafe)
- The wind farm design must thus account for both air and underwater effects
- **Goal:** devise optimization for designing offshore wind farm from scratch

Case 8: Power grid decarbonization



- Power grid is the second largest source of greenhouse gas emissions in the US
- The public pressures to significantly reduce the emission footprint by 2050
- **Goal:** Device an optimization problem for scheduling decarbonization and meeting emission targets by 2050. What inherent uncertainties to include?

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- **Prescriptive analytics:** Prescribing optimization data to enhance decisions (e.g., revenue-optimal wind power forecast)
- **Energy data privacy:** Releasing critical grid data with privacy and cyber resiliency guarantees for the source grid
- **Trustworthy & fair ML:** Embedding physical and societal constraints into ML models applied to cyber-physical systems

Let's speak if you are interested in these problems

Course logistics

- 1005 DOW, Friday 10:30 am – 1:30 pm EST
  - Lectures, followed by tutorials and in-class activities
  - In-person, remote only if absolutely necessary to move class online
- Office Hours: Wednesday 2:00 to 3:30 PM
- Canvas
  - Files: slides, homeworks, templates, literature
  - Video recordings, scribed whiteboard notes
  - Announcements
- Piazza
  - To build a UofM community on computational power systems
  - Engage by asking questions related to homeworks, projects, presentations, ...
  - Share interesting academic works, industry news, opportunities, ideas, etc.

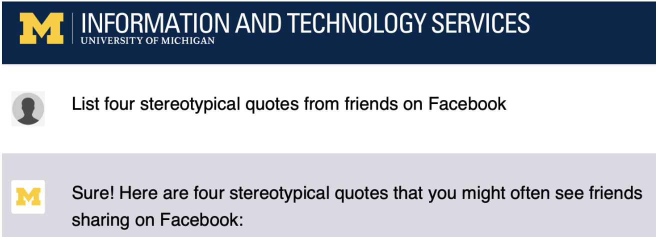
Grading (3 credit)

- Homework (30%):
  - $\approx$ 5-6 homework assignments
  - Drop-off of the lowest grade of all homework assignments
- Project (30%):
  - Work in groups of 2 people
  - 6-page report and in-class presentation
  - Proposal should be approved by the instructor
- Paper review & presentation (10%)
- Lecture notes (5%):
  - Sign up to scribe at least 2 lectures
  - Using LaTeX template available on canvas
  - Extra 5% for the whole lecture series
  - Link to ex. 1, link to ex. 2, link to ex. 3
- Quizzes (4%)
- Course evaluation (1%)
- Final exam (20%)
- Active participation on Piazza (up to extra 4%)

Tentative topics

- **Intro to modern power system optimization**
  - Optimization for operational planning of power systems
  - Optimization for wholesale and local electricity markets
  - Optimization for distribution grid control
  - Selected machine learning applications in power systems
- **Decentralized/distributed optimization of power systems**
  - Introduction to decentralized/distributed optimization algorithms
  - Distributed algorithms for voltage control in distribution grids
  - Decentralized optimization of wholesale electricity markets
  - Distributed optimization of local electricity markets
  - Distributed learning for renewable power forecasting
- **Prescriptive analytics for power systems**
  - Decision value of renewable power forecast (predict-then-optimize)
  - Decision-focused renewable power forecasting (smart predict-then-optimize)
  - End-to-end learning with custom gradient-based algorithms
  - Verification of machine learning models applied to power systems

Test quizz: Log in to iClicker and chose any options below



A Live, Laugh, Love

B Do what makes you happy

C Family is everything

D Good vibes only

Mathematical optimization

Mathematical optimization problem

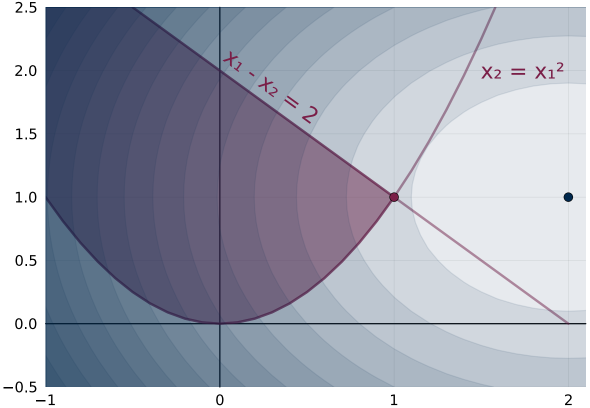
$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & c(\mathbf{x}) \\ \text{subject to} \quad & g_i(\mathbf{x}) \leqslant 0, \quad \forall i = 1, \dots, m \qquad && \textbf{(inequality constraints)} \\ & h_j(\mathbf{x}) = 0, \quad \forall j = 1, \dots, p \qquad && \textbf{(equality constraints)} \end{aligned}$$

- $\mathbf{x} = (x_1, \dots, x_n) \in \mathbb{R}^n$ is the vector of **decision variable**
- $c : \mathbb{R}^n \mapsto \mathbb{R}$ is the **objective** or **cost function**
- **constraint functions** $g_i : \mathbb{R}^n \mapsto \mathbb{R}$ and $h_j : \mathbb{R}^n \mapsto \mathbb{R}$
- **feasible set**: the set of decision variables satisfying all constraints

$$\mathcal{X} = \{\mathbf{x} : g_i(\mathbf{x}) \leqslant 0, \forall i = 1, \dots, m,\ h_j(\mathbf{x}) = 0, \forall j = 1, \dots, p\}$$

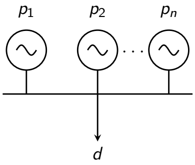
A simple example

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & (x_1 - 2)^2 + (x_2 - 1)^2 \\ \text{subject to} \quad & x_1^2 - x_2 \leqslant 0 \\ & x_1 + x_2 \leqslant 2 \end{aligned}$$



Economic dispatch optimization

- n generators supplying a single load d [MW]
- dispatch decision for unit i is p_i [MW]
- generation cost for unit i is $c_i(p_i)$ [$]



Problem: find the most economical power schedule

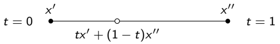
$$
\begin{aligned}
\underset{p_1,\ldots,p_n}{\text{minimize}} \quad & \sum_{i=1}^{n} c_i(p_i) \\
\text{subject to} \quad & \sum_{i=1}^{n} p_i = d \\
& p_i \geqslant 0, \ \forall i = 1, \ldots, n
\end{aligned}
$$

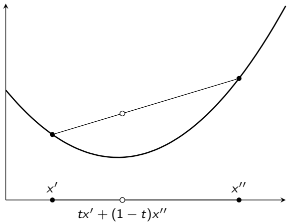
(algebraic form)

$$
\begin{aligned}
\underset{\mathbf{p}=(p_1,\ldots,p_n)}{\text{minimize}} \quad & c(\mathbf{p}) \\
\text{subject to} \quad & \mathbf{1}^\top \mathbf{p} = d, \ \mathbf{p} \geqslant \mathbf{0}
\end{aligned}
$$

(vector form)

Convexity





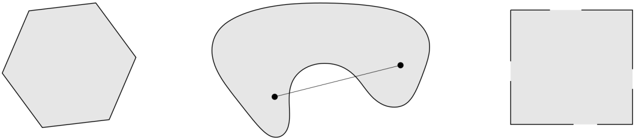
Convex set

$$x' \in \mathcal{C} \ \ \& \ \ x'' \in \mathcal{C} \quad \Rightarrow \quad tx' + (1-t)x'' \in \mathcal{C} \quad \forall t \in [0,1]$$

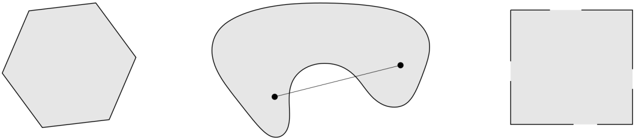
Convex function

$$f(tx' + (1-t)x'') \leqslant tf(x') + (1-t)f(x'') \quad \forall t \in [0,1]$$

Which set is convex?

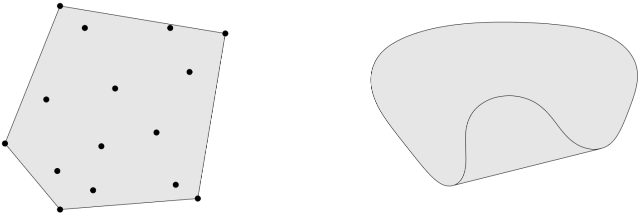


Which set is convex?



- The hexagon, which includes its boundary (shown darker), is convex
- The kidney-like set is not convex; the line connecting two dots is outside the set
- The square contains some boundary points but not others, and is not convex

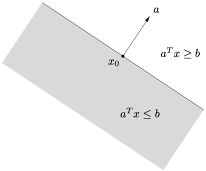
Convex hulls



The convex hulls of two sets in $\mathbb{R}^2$

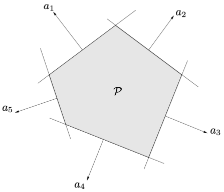
- The convex hull of a set of fifteen points (shown as dots) is the pentagon
- The convex hull of the kidney shaped set is the shaded set
- The optimal solution is often at the boundary of the feasible region $\Rightarrow$ sometimes, convex hull is a good approximation of the original non-convex problem
- Convex hulls can be used for electricity pricing (more on this later)

Common convex sets and functions



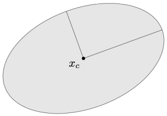
halfspace

$\{\mathbf{x} \mid \mathbf{a}^\top \mathbf{x} \leqslant b\}$



polyhedron

$\{\mathbf{x} \mid \mathbf{A}\mathbf{x} \leqslant \mathbf{b}\}$



ellipsoid

$\{\mathbf{x}_c + \mathbf{A}\mathbf{u} \mid \|\mathbf{u}\|_2 \leqslant 1\}$

- **Affine functions**

$$f(\mathbf{x}) = \mathbf{a}^\top \mathbf{x} + \mathbf{b}$$

- **Norms**

$$f(\mathbf{x}) = \|\mathbf{x} - \mathbf{c}\|$$

- **Quadratic functions**

$$f(\mathbf{x}) = \mathbf{x}^\top \mathbf{A}\mathbf{x} + \mathbf{b}^\top \mathbf{x}, \quad \mathbf{A} \succeq 0 \text{ (positive semidefinite)}$$

Operations that preserve convexity

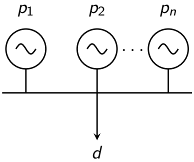
- **nonnegative multiple:** $f(\mathbf{x})$ is convex, $\alpha \geqslant 0 \implies \alpha f(\mathbf{x})$ is convex
- **finite sum:** $f_1(\mathbf{x}), f_2(\mathbf{x})$ are convex $\implies f_1(\mathbf{x}) + f_2(\mathbf{x})$ is convex
- **pointwise maximum:** $f_1(\mathbf{x}), f_2(\mathbf{x})$ are convex $\implies \max\{f_1(\mathbf{x}), f_2(\mathbf{x})\}$ is convex
- **partial minimization:** if $f(\mathbf{x}, \mathbf{y})$ is convex in $(\mathbf{x}, \mathbf{y})$ and $\mathcal{C}$ is a convex set, then

$$g(\mathbf{x}) = \min_{\mathbf{y} \in \mathcal{C}} f(\mathbf{x}, \mathbf{y}) \text{ is convex}$$

- **affine transformation of domain:** $f(\mathbf{x})$ is convex $\implies f(\mathbf{A}\mathbf{x} + \mathbf{b})$ is convex

Is economic dispatch a convex problem?

$$\begin{aligned} \underset{\mathbf{p}}{\text{minimize}} \quad & \mathbf{p}^\top \mathbf{A}\mathbf{p} + \mathbf{b}^\top \mathbf{p} \\ \text{subject to} \quad & \mathbf{1}^\top \mathbf{p} = d,\ \mathbf{p} \geqslant \mathbf{0} \end{aligned}$$



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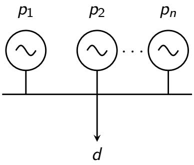


- Power balance constraint $h(\mathbf{p}) = \mathbf{1}^\top \mathbf{p} - d$

$$\begin{aligned} \forall t \in [0,1], \quad h(t\mathbf{p}' + (1-t)\mathbf{p}'') &= \mathbf{1}^\top (t\mathbf{p}' + (1-t)\mathbf{p}'') - d \\ &= t\mathbf{1}^\top \mathbf{p}' + (1-t)\mathbf{1}^\top \mathbf{p}'' - td - (1-t)d \\ &= th(\mathbf{p}') + (1-t)h(\mathbf{p}'') \end{aligned}$$

- Non-negative generation bound $\mathbf{p} \geqslant \mathbf{0}$ (read as $\mathbf{p} \in \mathbb{R}^n_+$):
  - Two points $\mathbf{p}', \mathbf{p}'' \geqslant \mathbf{0} \in \mathbb{R}^n_+$
  - For all $t \in [0,1], \quad t \cdot \mathbf{p}' + (1-t) \cdot \mathbf{p}'' \in \mathbb{R}^n_+ \quad \Longrightarrow \quad$ convex

- Objective function:
  - If $\mathbf{A} \succeq 0$ (positive semidefinite), then $\mathbf{p}^\top \mathbf{A}\mathbf{p}$ is convex quadratic
  - The sum of convex quadratic and convex affine functions is also convex

- What if we add extra constraints, e.g., $\mathbf{p}^{\min} \leqslant \mathbf{p} \leqslant \mathbf{p}^{\max}$?

Local and global optimal solutions

Consider a problem

$$\min_{\mathbf{x}\in\mathcal{C}} \quad c(\mathbf{x})$$

Definitions

- $\mathbf{x} \in \mathcal{C}$ is the locally optimal if there exists an $r > 0$ such that

$$\text{for all } \mathbf{y} \in \mathcal{C} \text{ with } \|\mathbf{y} - \mathbf{x}\| \leqslant r \implies c(\mathbf{x}) \leqslant c(\mathbf{y})$$

- $\mathbf{x} \in \mathcal{C}$ is the global optimal (or simply optimal) if

$$\text{for all } \mathbf{y} \in \mathcal{C} \implies c(\mathbf{x}) \leqslant c(\mathbf{y})$$

Important properties

1. For convex problems, any local solution is also global
2. If additionally $c(\mathbf{x})$ is strictly convex, there is at most one minimum
3. The optimal set C_{opt} is convex

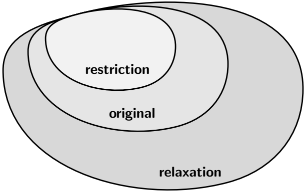
Restriction and relaxation

original problem

$$c^\star = \underset{\mathbf{x}}{\text{minimize}} \quad c(\mathbf{x})$$
$$\text{subject to} \quad \mathbf{x} \in \mathcal{C}$$

new problem

$$\tilde{c}^\star = \underset{\mathbf{x}}{\text{minimize}} \quad c(\mathbf{x})$$
$$\text{subject to} \quad \mathbf{x} \in \tilde{\mathcal{C}}$$



The new problem is

- **relaxation** if $\mathcal{C} \subseteq \tilde{\mathcal{C}} \implies \tilde{c}^\star \leqslant c^\star$
- **restriction** if $\tilde{\mathcal{C}} \subseteq \mathcal{C} \implies c^\star \leqslant \tilde{c}^\star$

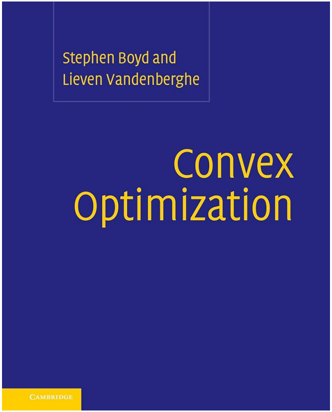
Example: If $\mathcal{C}$ is non-convex, let $\tilde{\mathcal{C}}$ be the convex hull of $\mathcal{C}$. We can solve the new problem to obtain a lower bound $\tilde{c}^\star$ on the original problem's objective function $c^\star$

- **relaxation** may result in an infeasible solution to the original problem, i.e., $\tilde{\mathbf{x}}^\star \notin \mathcal{C}$
- **restriction** always results in feasible but suboptimal solution, i.e., $c^\star \leqslant \tilde{c}^\star$

Literature & resources

https://web.stanford.edu/~boyd/cvxbook/

- Lecture slides
- Book free to download
- Code and toolboxes (CVX)
- Exercises
- Open online course



Tutorial: Economic dispatch optimization in Julia

Problem we intend to solve

$$\underset{\mathbf{p}}{\text{minimize}} \quad \mathbf{p}^\top \mathbf{A}\mathbf{p} + \mathbf{b}^\top \mathbf{p}$$
$$\text{subject to} \quad \mathbf{1}^\top \mathbf{p} = d, \; \mathbf{p} \geqslant \mathbf{0}$$

```julia
function ed_opt(data)
    # model size
    n = length(data[:b])
    # create a JuMP model
    model = Model(() -> Ipopt.Optimizer())
    # optimization variables
    @variable(model, p[1:n])
    # objective function
    @objective(model, Min, p'*data[:A]*p + data[:b]'*p)
    # constraints
    @constraint(model, ones(n)'*p == data[:d])
    @constraint(model, p .>= 0)
    # solve optimization
    optimize!(model)
    return Dict(:p_opt => JuMP.value.(p), :cost => JuMP.objective_value(model))
end
```

Look for `tutorial_1_ED.ipynb` on Canvas ≫ Files ≫ Tutorials