

# Summarizing Graphs at Multiple Scales: New Trends



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# Roadmap

- 1:30-1:45pm Introduction [Jilles]
- 1:45-2:50pm Network-level Summaries [Francesco]
- 2:55-3:20pm Multi-network Summaries [Danai]
- 3:20-3:40pm ——— *break* ———
- 3:40-4:05pm Multi-network Summaries [Danai]
-  4:10-4:40pm Node-level Summaries [Jilles]
- 4:40-4:50pm Conclusion [Jilles]

# Part III: Local Summarization



Jilles Vreeken

# Why do we want a summary?

We want to gain **insight** in the structure of the data

- capturing the **key aspects** of the data,
- in **easily interpretable** terms,
- **without redundancy**

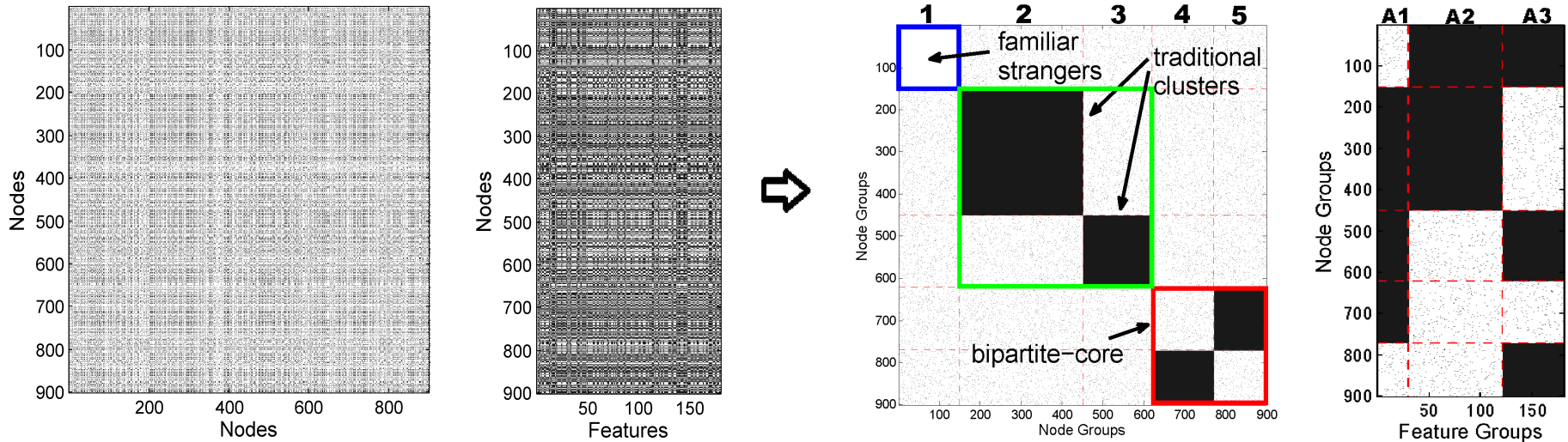
The techniques we've seen so far aim at this

- but, do they really deliver? in all interesting cases?

All deliver **one** single summary for all of the data

- what do we lose by explaining all the data at once?

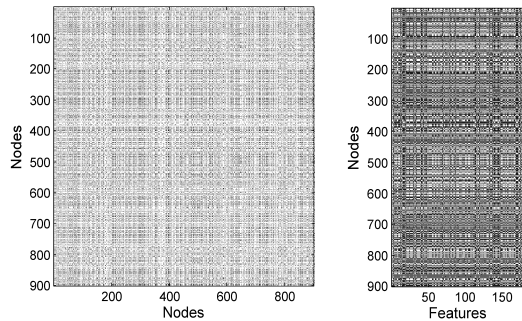
# Nodes with 'Descriptions'



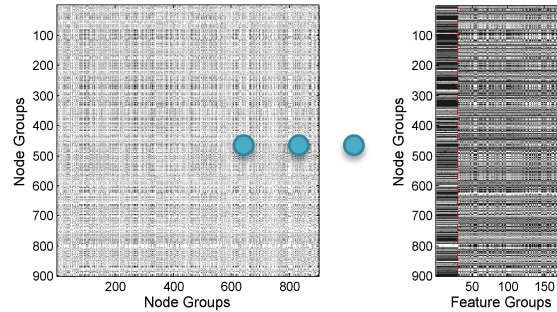
## Parameter-free Identification of Cohesive Subgroups in Large Attributed Graphs (Akoglu et al 2012)

- find joint-partition of adjacency matrix **and** feature matrices
- feature-matrix grid cells can be interpreted as 'descriptions', e.g. 'people with features A1 and A2 but not A3 know each other well', 'people who buy A1, A3, but not A2 all know each other'

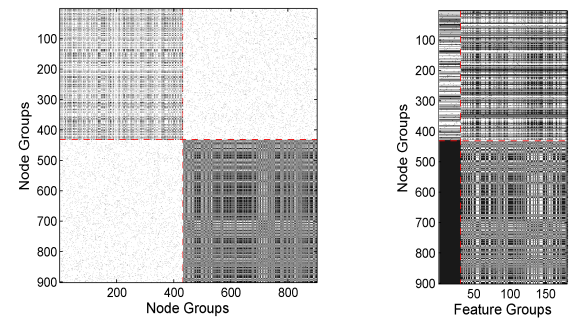
# Nodes with 'Descriptions'



$$k = 1, l = 1$$



$$k = 1, l = 2$$



$$k = 2, l = 2$$

Iteratively add a split on either features or nodes

- after each split, re-arrange nodes and features s.t. **sum of entropies** over each **induced grid cell** is minimized
- stop when MDL determines splitting does not provide sufficient gain

# Globally not-quite Optimal

Globally summarizing gives an overview, but

- we are not always equally interested in all the data

Moreover, by optimizing a single global objective,

- **choices** made in how we summarize **one part** of the data have an effect on **other parts** of the data
- subgraph  $G'$  may be easy to explain, but its **locally optimal** summary **may not fit** the **global** summary well
- **globally optimal** often means **locally suboptimal!**

# Local Summaries

Why not mine **local** summaries?

- node groups with **exceptional connectivity** that come with **easy to interpret descriptions**

For example,

**'people who watch cat videos**      **often interact'**



By not having to care about all the data all the time

- we obtain locally optimal and actionable summaries
- easy to interpret, allowing for **alternate explanations**



# Subgroup Discovery in Graphs

## Subgroup Discovery

- given data  $D$  and a language  $\mathcal{L}$ ,  
find those expressions  $\sigma \in \mathcal{L}$ ,  
such that for a score  $s : D \rightarrow \mathbb{R}$   
we have high  $|s(D) - s(\sigma(D))|$

# Subgroup Discovery in Graphs

## Subgraph Discovery

- given a graph  $G(V, E)$ ,  
language  $\mathcal{L}$  of **expressions over nodes** (and/or edges),  
e.g. '**nodes with *cat\_video = yes***'  
and a **score  $s$**  over subgraphs,  
e.g. '**average number of edges per node**'
- find those expressions  $\sigma \in \mathcal{L}$ ,  
such that the **score** over induced subgraph  $G_\sigma$  is high,  
e.g. stands out from the **score** over the whole graph  $G$

Easily understandable, actionable, **local** summaries

# Discovering Subgroups in Graphs

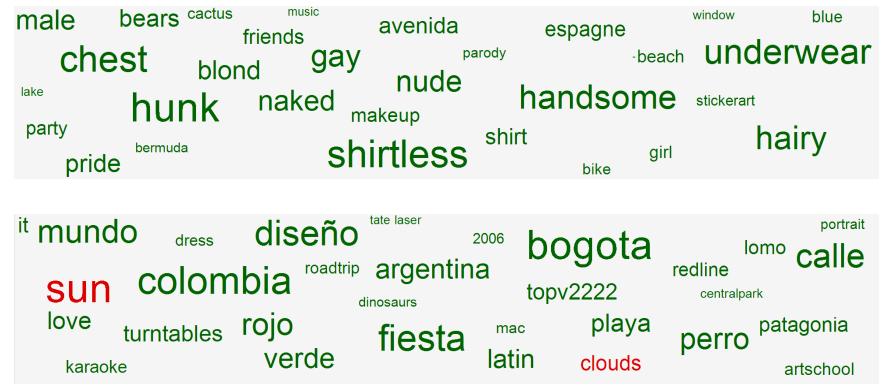
## Key challenges

- define **score  $s$** 
  - ✧ existing measures mainly consider **density**
    - useful scores are often non-monotone, non-submodular, etc
- define a **language  $\mathcal{L}$** 
  - ✧ existing languages consider
    - explicit node attributes (**cat-video = yes**)
    - implicit node attributes (**in-degree > 3**)
- efficient **algorithm** to search over  $2^{\mathcal{L}}$ 
  - ✧ **beam search** often used as greedy heuristic without guarantees
  - ✧ exact search is possible using **branch-and-bound** if we have an efficiently computable tight optimistic estimator  $\bar{s}$

# Example Subgroup Discovery

Description	#nodes
80s	519
Girl Groups <b>AND</b> 80s	215
Atmospheric	171
Synth-Pop	122

Descriptive Communities  
found in last.fm



Descriptive Communities  
found in Flickr

# References – Local Summaries

Arno Knobbe. **Multi-Relational Data Mining**. ISBN 978-1-58603-661-4, IOS Press, 2005.

Henrik Grosskreutz, Stefan Rüping and Stefan Wrobel. **Tight Optimistic Estimates for Fast Subgroup Discovery**. In Proceedings of the European Conference on Machine Learning and Knowledge Discoveryr in Databases (ECML PKDD), pages 440-456, Springer, 2008.

Martin Atzmueller and Folke Mitzlaff. **Efficient Descriptive Community Mining**. In *Proceedings of 24<sup>th</sup> International FLAIRS Conference*, AAAI, 2011.

Leman Akoglu, Hanghang Tong, Brendan Meeder, and Christos Faloutsos. **PICS: Parameter-free identification of cohesive subgroups in large attributes graphs**. In *Proceedings of the SIAM Conference on Data Mining (SDM)*, pages 439-450, SIAM, 2012.

Simon Pool, Francesco Bonchi, and Matthijs van Leeuwen. **Description-Driven Community Detection**. In *ACM Trans. Intell. Syst. Technol.* 5(2), 28:1-28, ACM, 2014.

Martin Atzmueller, Stephan Dörfel, and Folke Mitzlaff. **Description-oriented community detection using exhaustive subgroup discovery**. In *Info. Sci.* 329, pages 965-984, 2016.

# Explain me this...

We do not always care about summarize graph  $G$  entirely

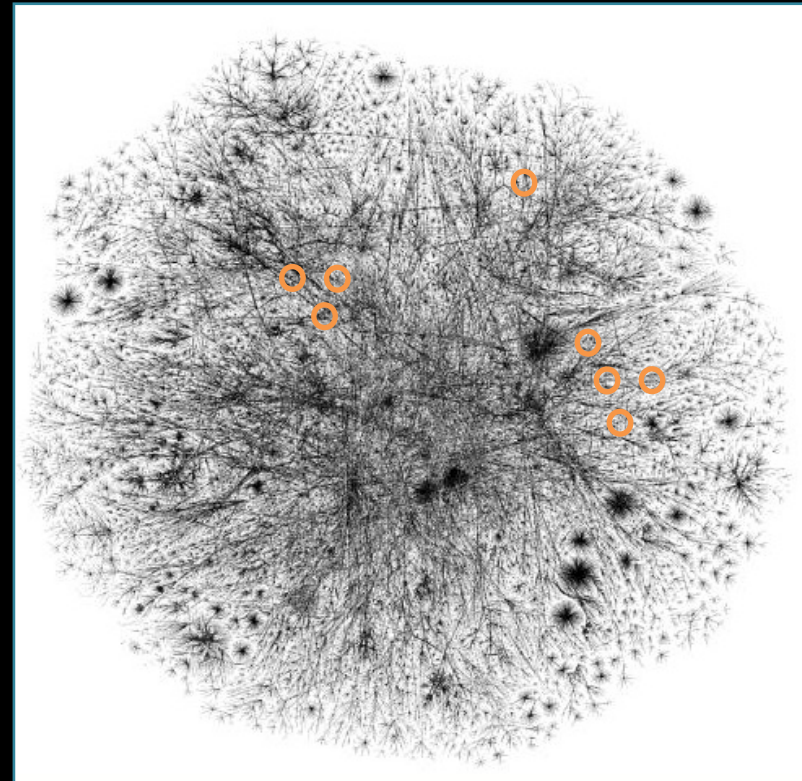
- how to explain nodes  $S \subseteq V$  marked by external process?

What can  $G$  explain about  $S$  ?

- are  $S$  close by each other?
- are  $S$  segregated?
- how many groups do they form?

How can we connect  $S$  using  $G$ ?

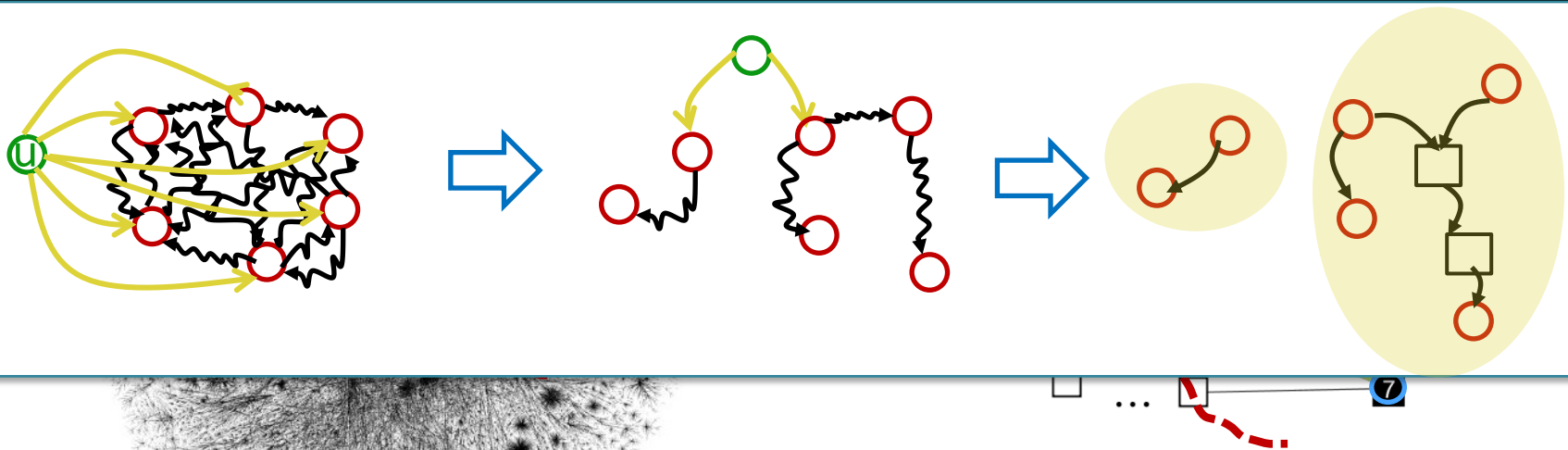
- with “simple” paths
- using “good” connectors



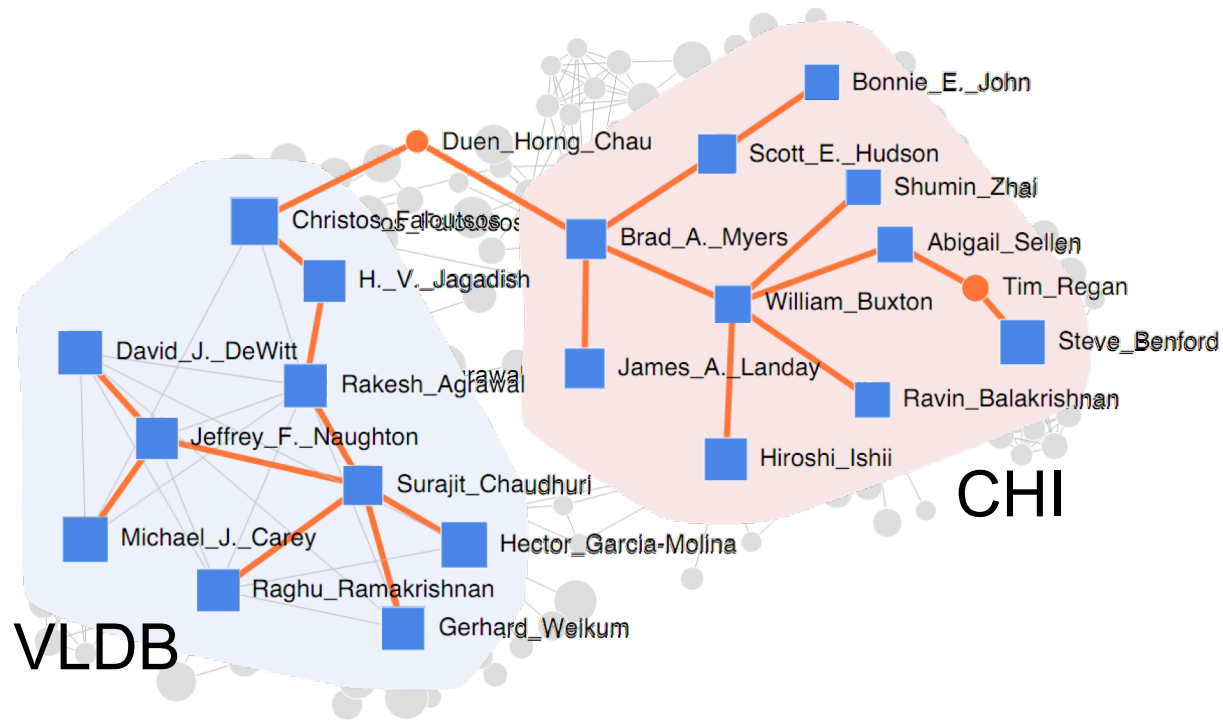
# Simple Connection Pathways

Main idea: use the network structure to explain  $S$

- partition  $S$  into **groups** of nodes, such that:
  - ✧ “**simple**” **paths** connect the nodes in each **group**, nodes in different groups are “**not easily reachable**”



# Example in Co-Authorship Graph





# Subjectively Interesting

For **dense** graphs, there are no 'simple' paths

- plain path simplicity by MDL does not work (well)

However, some paths are more **expected**

- for example, paths between **recently active nodes**
- we can express  $\Pr(\text{path})$  using such **external** information

And mine most informative SteinerTree incrementally

- iteratively add  $edge$  with highest  $\frac{InfContent(edge)}{DescLength(edge)}$

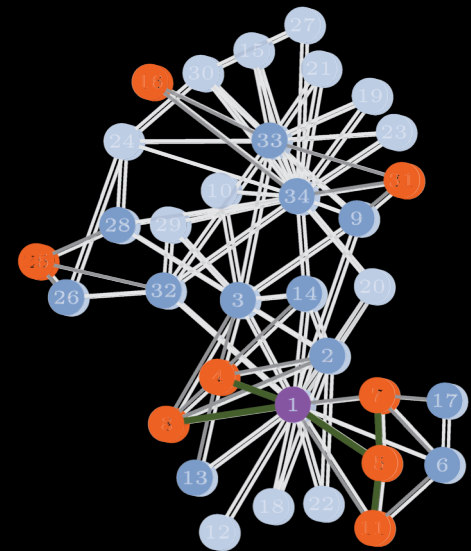
# Bump Hunting in the Dark

Why explain **all** query nodes  $S \subseteq V$ ?

- why not **as many as possible** with one connected subgraph?

Find **connected** subgraph  $G'$  with **many** nodes **in  $S$**  and **few** nodes **not in  $S$**

- i.e.  $G' \subseteq G$  is connected, with high  $|S \cap V'|$ , and low  $|V' \setminus S|$
- this is known as **discrepancy maximization**: NP-hard on graphs
- note the relation to subgroup discovery!



# Bump Hunting in the Dark

Find connected  $G'$  with high  $|S \cap V'|$  and low  $|V' \setminus S|$

- NP-hard, no known approximation algorithms

If graph  $G$  is a **tree** it's **easy**, but if it's a **graph** it's **hard**

- **main idea** find a tree  $G_T \subseteq G$ , then find  $G'$  on  $G_T$
- linear time heuristics to find  $G_T$  based on **spanning trees**
- variants for **full graph access**, and for **local expansion**

Key open questions

- **weighing** scheme, **expansion** strategy, **stopping** criteria
- and, how to expand to other, more refined **measures**

# Minimally Inefficient

**Connectedness** of  $G'$  restricts usefulness

- instead, find that set of nodes  $C \subseteq V \setminus S$  such that induced subgraph  $G' = G[S \cup C]$  is **cohesive**

**Cohesiveness** relates to **reachability**

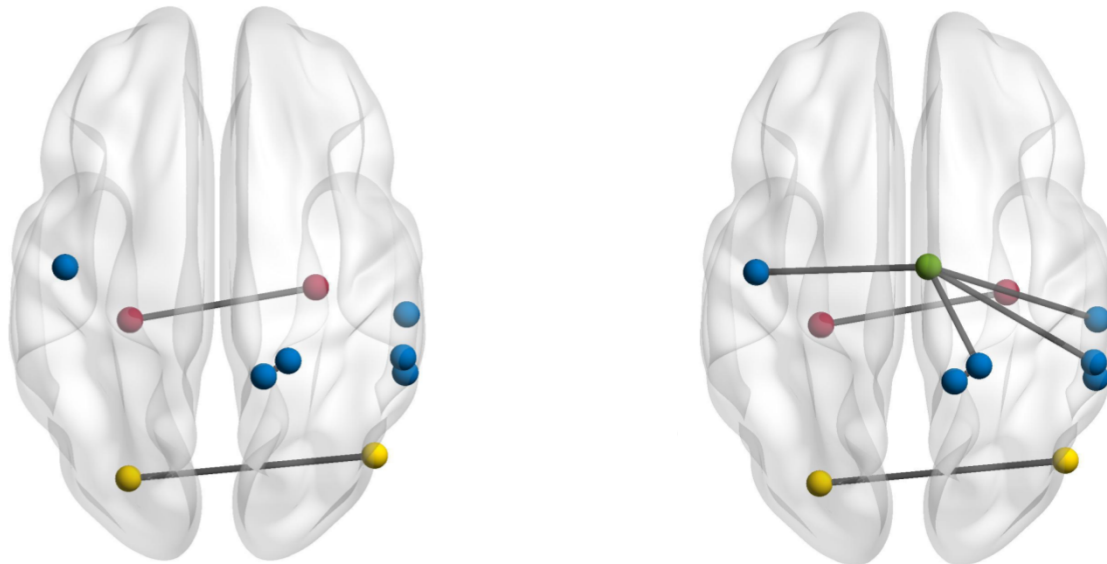
- if  $G'$  is not connected, **shortest path** may be **infinite**
- **efficiency** of a graph defined as

$$\varepsilon(G) = \frac{1}{|V|(|V| - 1)} \sum_{\substack{u, v \in V \\ u \neq v}} \frac{1}{d_G(u, v)}$$

# Minimally Inefficient

## Minimum Inefficiency Subgraph problem

- find those nodes  $C \subseteq V \setminus S$  such that induced subgraph  $G' = G[S \cup C]$  is **minimally inefficient**
- NP-hard, not known to be approximable: **greedy heuristic**



# References – Connected Nodes

Leman Akoglu, Jilles Vreeken, Hanghang Tong, Nikolaj Tatti, and Christos Faloutsos. **Mining Connection Pathways for Marked Nodes in Large Graphs**. In *Proceedings of the 13th SIAM International Conference on Data Mining (SDM)*, pages 37-45, SIAM, 2013.

Stephan Seufert, Klaus Berberich, Srikanta J. Bedathur, Sarath Kumar Kondreddi, Patrick Ernst, and Gerhard Weikum. **ESPRESSO: Explaining Relationships between Entity Sets**. In *Proceedings of the 25th ACM International on Conference on Information and Knowledge Management (CIKM)*, pages 1311-1320, ACM, 2016.

Aristides Gionis, Michael Mathioudakis, and Antti Ukkonen. **Bump hunting in the dark: Local discrepancy maximization on graphs**. *IEEE Trans. Knowl. Data Eng.*, 29(3):529–542, IEEE, 2017.

Florian Adriaens, Jefrey Lijffijt, and Tijn De Bie. **Subjectively interesting connecting trees**. In *Proceedings of the European Conference on Machine Learning and Knowledge Discovery in Databases (ECML PKDD)*, pages 53–69, Springer, 2017.

Natali Ruchansky, Francesco Bonchi, David Garcia-Soriano, Francesco Gullo, and Nicolas Kourtellis. **To Be Connected, or Not to Be Connected: That is the Minimum Inefficiency Subgraph Problem**. In *Proceedings of the 26<sup>th</sup> ACM Conference on Information and Knowledge Management (CIKM)*, pages 879-888, ACM, 2017.

# A Picture Says More Than...

Ideally a summary is **easy to understand**

- most solutions provide pretty **bad** presentation
- and, no support for **exploration** of the **summary space**

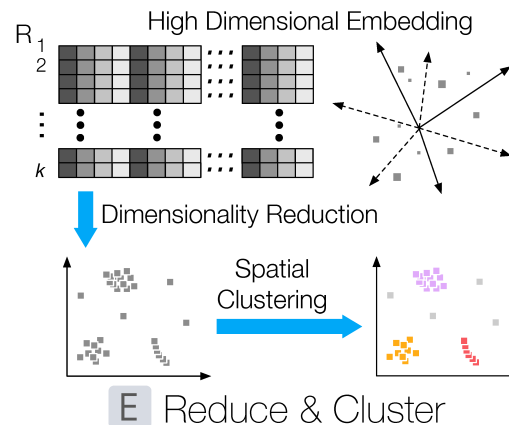
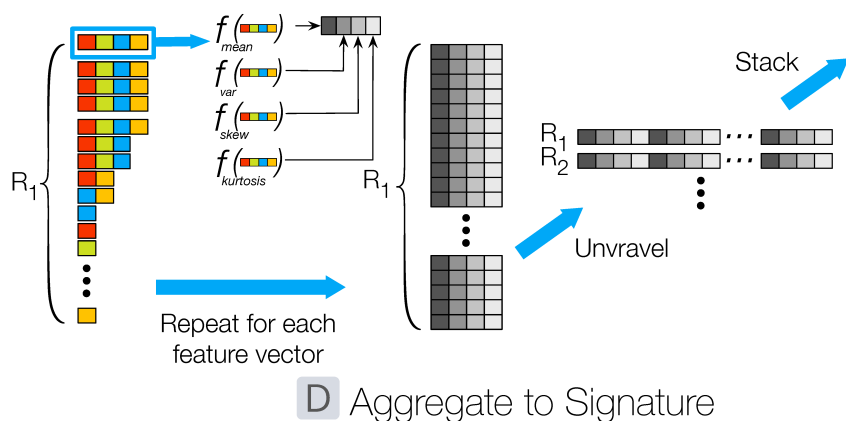
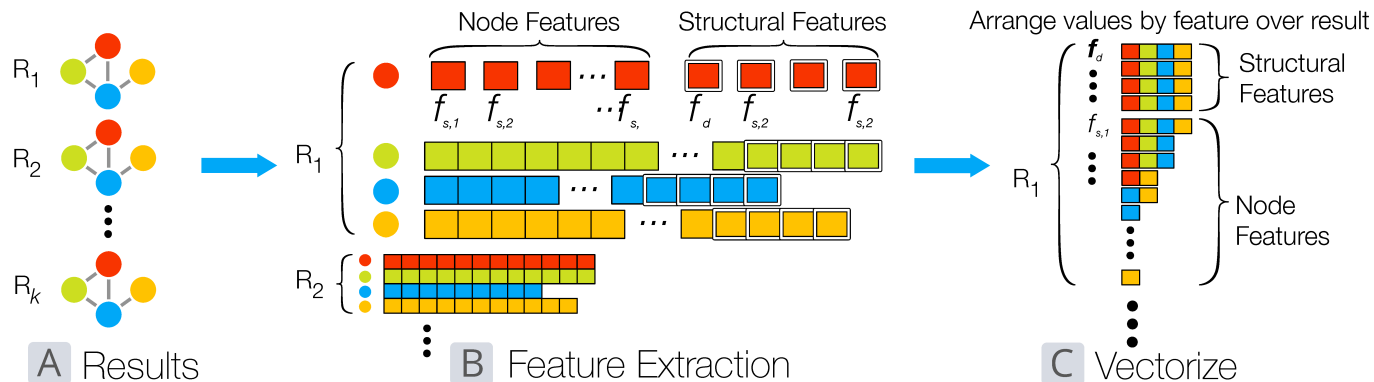
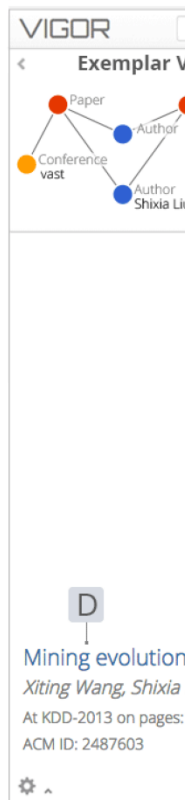
Key difficulties

- how should we present a summary?
- how can we interact with it?

Only few visual summary exploration tools exist

- and out of those, we only cover two

# Interactive Exploration of Graph Query Results



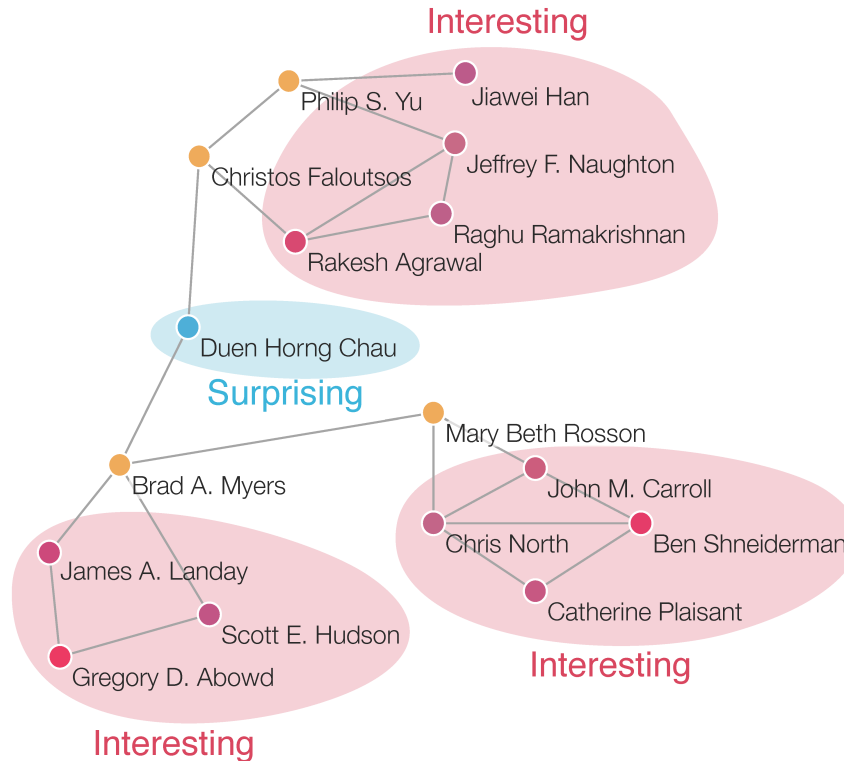
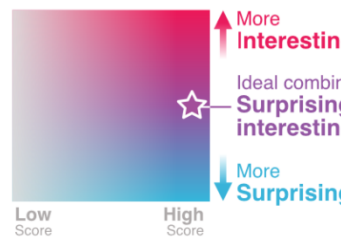


# Graph Exploration

## A. The FACETS UI 1.

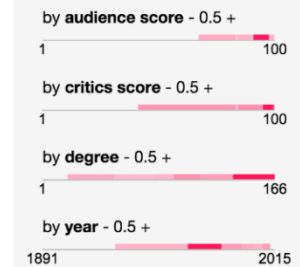
* name	criti...	audie...	degree
Brazil	89	98	76
Twelve Monkeys (12...	84	88	52
Primer	79	73	21
Breathless (A bout...	89	96	19
Fahrenheit 451	69	81	9
Miss Muerte (Miss ...	73	-1	4
The Element of Crime	71	77	3
Alphaville	80	82	18
Die Hard 3: With a...	83	52	34
TRON	66	70	47

## B. Node Color Encoding

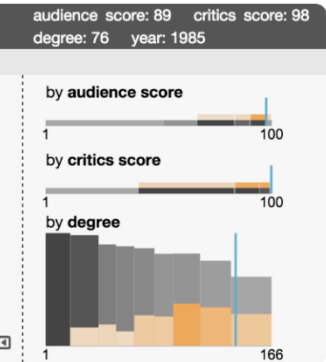


## 4. User Profile View

### User Profile



S...



## 3. Neighborhood Summary

# References – Visual Exploration


Bryan Perozzi and Leman Akoglu. **Discovering communities and anomalies in attributed graphs: Interactive visual exploration and summarization**. In *ACM Trans. Knowl. Disc. Data Min*, 12(2):24:1–24:40, ACM, 2018.

Robert Pienta, Fred Hohman, Alex Endert, Acar Tamersoy, Kevin A. Roundy, Christopher S. Gates, Shamkant B. Navathe, and Duen Horng Chau. **VIGOR: interactive visual exploration of graph query results**. In *IEEE Trans. Vis. Comput. Graph.*, 24(1):215–225, IEEE, 2018.

Robert Pienta, Minsuk Kahng, Zhiyuan Lin, Jilles Vreeken, Partha P. Talukdar, James Abello, Ganesh Parameswaran, and Duen Horng Chau. **FACETS: adaptive local exploration of large graphs**. In *Proceedings of the SIAM International Conference on Data Mining (SDM)*, pages 597–605, 2017.

Boxin Du, Si Zhang, Nan Cao, and Hanghang Tong. 2017. **FIRST: Fast Interactive Attributed Subgraph Matching**. In *Proceedings of the 23rd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD)*, pages 1447–1456, ACM, 2017.

# Roadmap

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- 1:45-2:50pm Network-level Summaries [Francesco]
- 2:55-3:20pm Multi-network Summaries [Danai]
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- 4:10-4:40pm Node-level Summaries [Jilles]
-  4:40-4:50pm Conclusion [Jilles]

# What Have We Seen

## Summarization of Single Networks

- a lot of work done, in many different angles

## Summarization of Multiple Networks

- not so much work done, big open problems

## Summarization of Sets of Nodes

- very little work done, very interesting challenges

# Single Networks

Single network summarization is **challenging**

- *how to decide what is important? Lossy or lossless? What is the goal of the summary? How to keep things tractable?*

Main focus: **unattributed undirected** networks

- simple problem is already hard enough, covers many settings

There exists, but only very **limited work** on

- attributed, directed, or signed networks

Big challenges, huge opportunities!

# Multiple Networks

- Multi-network summarization is more **challenging** than network-level summarization
  - ✧ *How to reduce re-computations? pick the right temporal granularity? handle node additions / deletions? make methods scale to multiple networks?*
- Main focus: **temporal** networks
  - ✧ Applying static methods on snapshots is not sufficient
  - ✧ Different models: static snapshots / tensor, graph stream
- Very **limited work** on
  - ✧ attributed temporal networks
  - ✧ multiple disparate networks
- “**One size does not fit all**”! We should be thinking about tailored summaries: domain-specific, personalized, query-driven etc.

Big challenges, huge opportunities!

# Nodes

Taking a **local** rather than a **global** perspective

- descriptions of subgraphs, much easier to understand
- no global choices that affect locally optimality!

**Surprisingly little** work done

- discovering **explainable** subgraphs
- **explaining** node sets
- **interactive** exploration and interaction

Big challenges, huge opportunities!

# Open Research Problems



Richer data (attributed, temporal, spatio-temporal, multilayer)

Real-time graph summarization (streaming, incremental)

Summary maintenance

Evaluation (which metrics?)

Automated insight extraction (explanation, storytelling)



# Open Research Problems



Scalable, high quality attribute-aware summaries

Application-driven (domain-dependent) summarization

Summarization of uncertain graphs

Summarizing a set of graphs

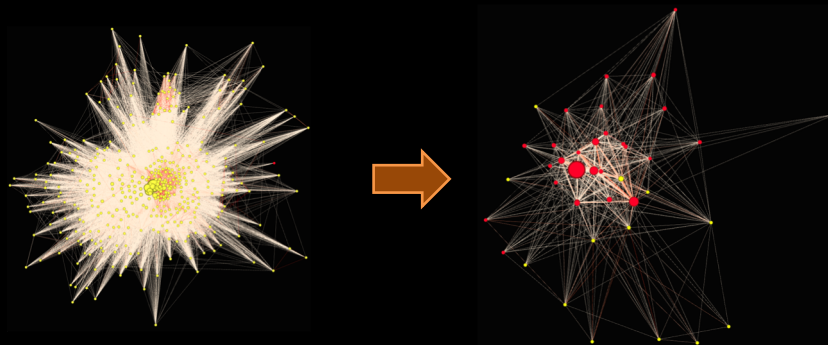
Differential summaries on massive networks

# Conclusions

Graph summarization is **important** and has **impact**:

- reduction of data volume + storage
- speedup of algorithms + queries
- interactive analysis
- noise elimination (patterns)

There is **a lot of potential** for **high-impact contributions!**

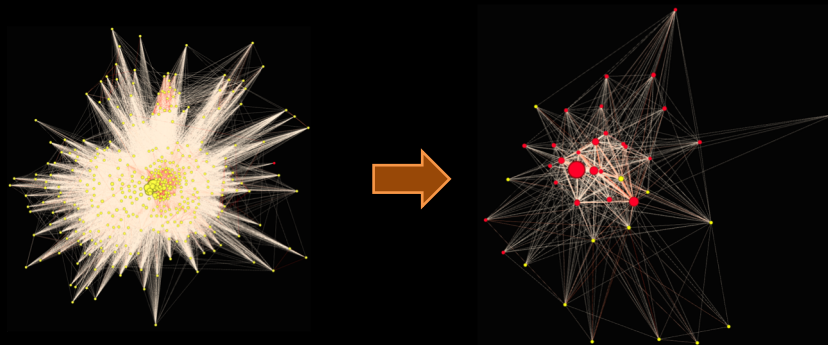


# Conclusions

## Graph summarization

- no unique approach or notion
- no widely accepted objective function
- no standard evaluation measure or benchmark
- highly domain and application dependent

There is **a lot of potential** for **high-impact contributions!**



## Resources

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D. Koutra “Summarizing Large-Scale Graph Data: Algorithms, Applications & Open Challenges” (SDM 2017 - tutorial)

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A. Khan, S.S. Bhowmick, F. Bonchi “Summarizing Static and Dynamic Big Graphs” (VLDB 2017 - tutorial)

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Liu, Safavi, Dighe, Koutra “Graph Summarization Methods and Applications: A Survey” (ACM Comp. Surv. 2018)