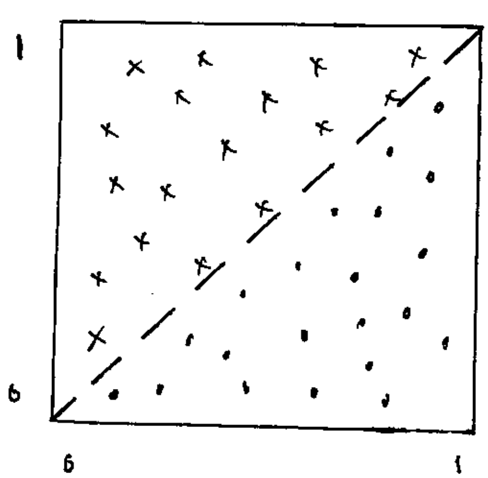
ENSEMBLE METHODS

The idea behind ensemble methods is to generate several classifiers $f_1, \dots, f_T$ using a variety of methods, and to combine them into a single classifier that performs better than any individual classifier.

Let's look at an ...

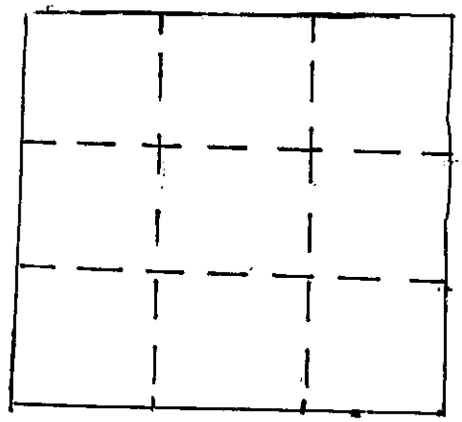
Example Averaged Shifted Histograms

Suppose we observe two dimensional data, $x_i \in [0,1]^2$.



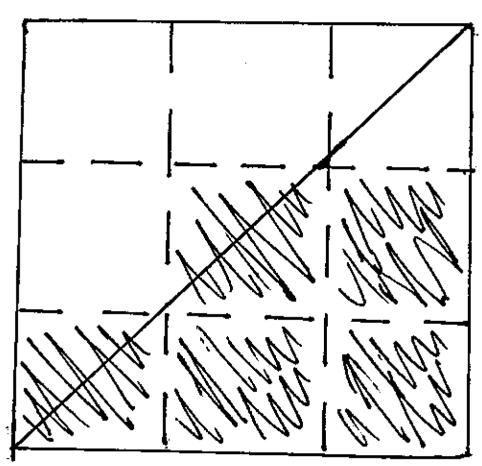
Bayes error $= 0$

A very basic (and not recommended!) classifier is a <u>histogram</u> rule:



- assign the same label to patterns x
  A in the same ________.
- determine label by __________ ________.

As you can imagine, this classifier will not perform very well.



Let's generate a whole bunch of equally

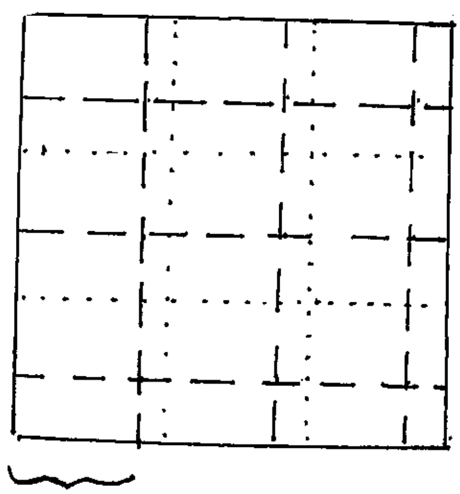
(B) ____________ classifiers as follows.

For $t = 1, \ldots, T$

- generate $\varepsilon_t^{(1)}, \varepsilon_t^{(2)} \in [0\ \tfrac{1}{3})$

  ____________ ______ __________ .

- shift the histogram by $[\varepsilon_t^{(1)}, \varepsilon_t^{(2)}]^T$
  and construct f_t based on the shifted partition.



$\varepsilon_t^{(2)}$

$\varepsilon_t^{(1)}$

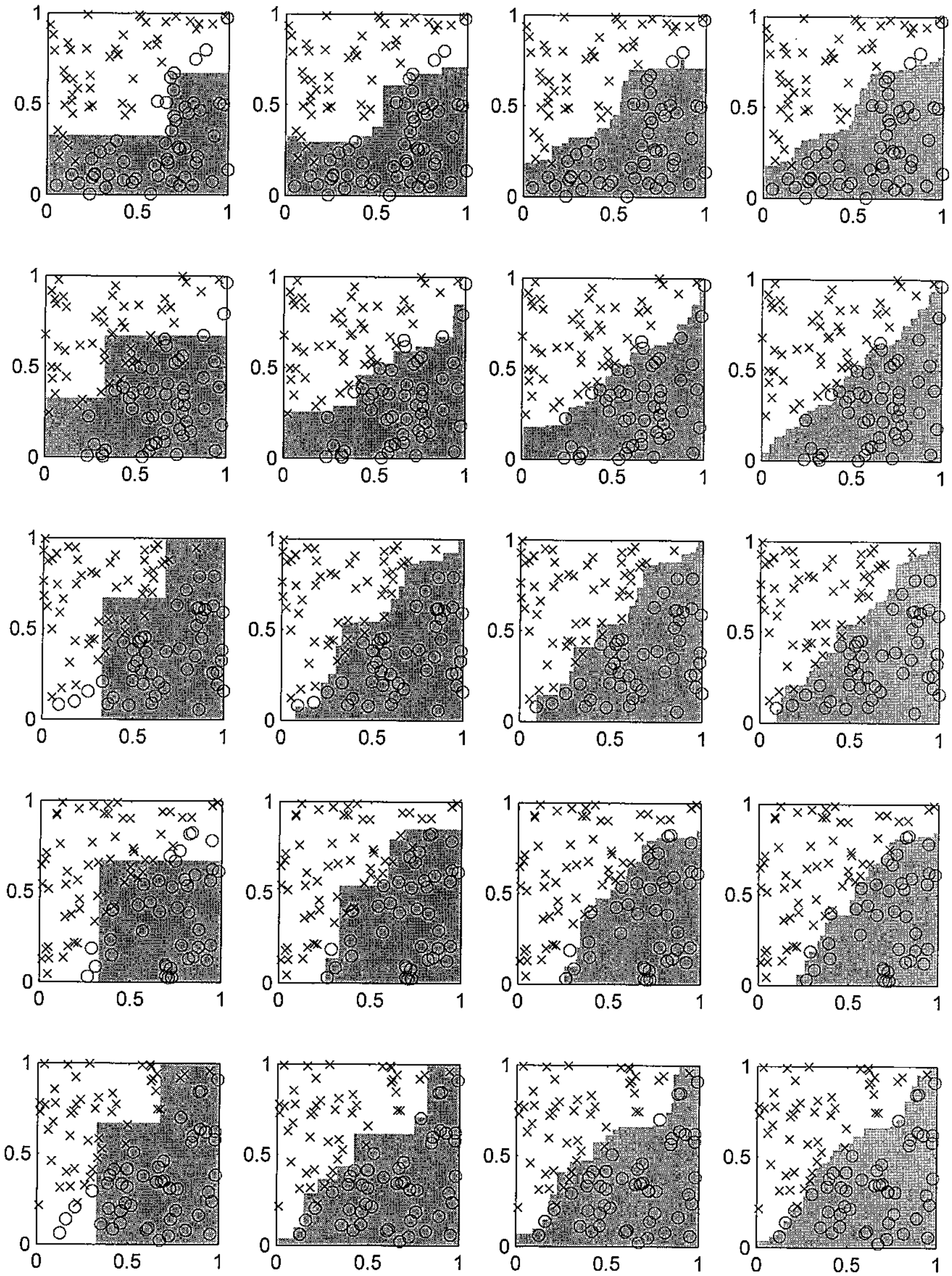
Now define the <u>ensemble classifier</u>

(C) $f(x) =$

This classifier is remarkably effective.

# of votes = 1 5 11 21

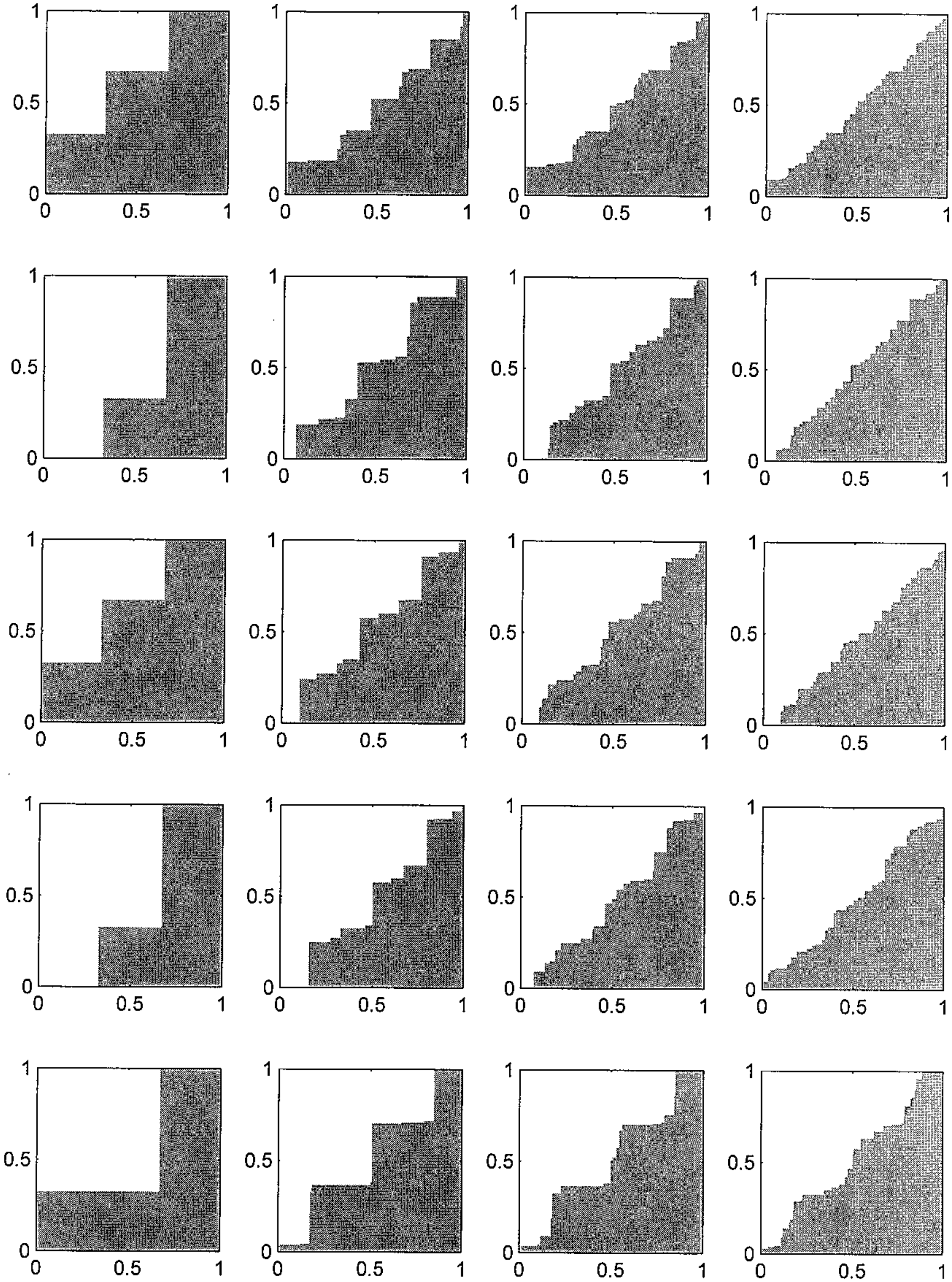
5 realizations of data



n= 100 points

# of votes = 1 5 11 21

5 realizations of data



n = 1000 points

Performance

Fix $x \in [0,1]^2$. Let $f^*(x)$ = correct label.

For any $t = 1, \dots, T$ we have

$$\Pr\{f_t(x) \neq f^*(x)\}$$

with respect to choice of $\varepsilon_t^{(1)}, \varepsilon_t^{(2)}$

$$= \Pr\left\{\begin{array}{l}\text{cell containing } x \text{ has } < \tfrac{1}{2} \\ \text{its area in same class as } x\end{array}\right\}$$

$$=: p(x) < \frac{1}{2}$$ [unless x is on the Bayes decision boundary]

Introduce the variable $Z_x \sim \text{binom}(T, p(x))$.

Then

$$\Pr\{f(x) \neq f^*(x)\}$$

$$= \Pr\{Z_x > \tfrac{T}{2}\}$$

$$= \Pr\{Z_x > T \cdot p(x) + T(\tfrac{1}{2} - p(x))\}$$

$\boxed{\text{Chernoff's bound}}$

$$\leq e^{-T(\frac{1}{2} - p(x))^2} \to 0 \quad \text{as } T \to \infty$$

This simple example illustrates two important properties of ensemble rules:

1. Combining classifiers that are

(D) ________ and ________ to

form a classifier that is

________ and ________.

2. Increased ________.

Definition: A classifier (or model) is stable if small changes in the training data do not result in large changes to the final classifier.

Our primary example of an unstable

(E) classifier is a ________ ________.

On the downside, we lose ________.

Bagging

(F) Bagging stands for __________ ______________.

Fix $B \geq 1$. Let I_b be a subset of $\{1, 2, \ldots, n\}$ of size n, obtained by sampling with replacement.

Suppose we have adopted a specific learning strategy (e.g., decision trees, LDA) and set

$$f_b =$$

The bagging classifier is

$$f(x) =$$

Random forests

Random forests are ensemble methods that combine decision trees and some kind of randomization or resampling.

In addition to bagging, the most notable other random forest grows a large number of trees using a greedy procedure such that, at each node, the split is selected from a ________ ________ of ________.

Among other advantages, this allows the application of trees to very ______ __________ data.

$\boxed{\text{Key}}$ A. cell, majority vote

B. poor, uniformly at random

C. majority vote over $f_1(x), \ldots, f_T(x)$

D. simple, poor ; complex, accurate ; stability

E. decision tree ; interpretability

F. Bootstrap aggregation,

$f_b(x)$ = classifier based on $\{(x_i, y_i)\}_{i \in I_b}$

$f(x)$ = majority vote over $f_b(x)$, $b = 1, \ldots, B$

G. random subset of features ; high dimensional