Lattice Cryptography for the Internet

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Post-Quantum Cryptography
2 October 2014

Lattice-Based Cryptography

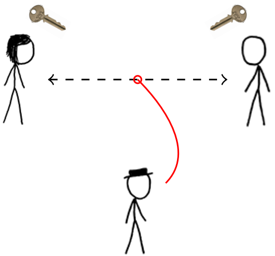
$y = g^x \bmod p$

$N = p \cdot q$

$m^e \bmod N$

$e(g^a, g^b)$

$\Longrightarrow$



(Images courtesy xkcd.org)

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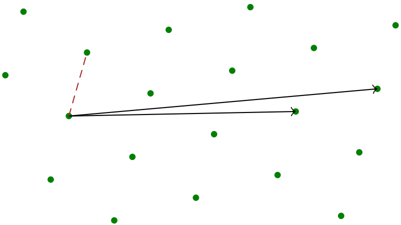

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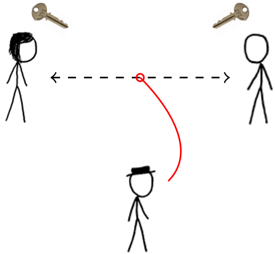

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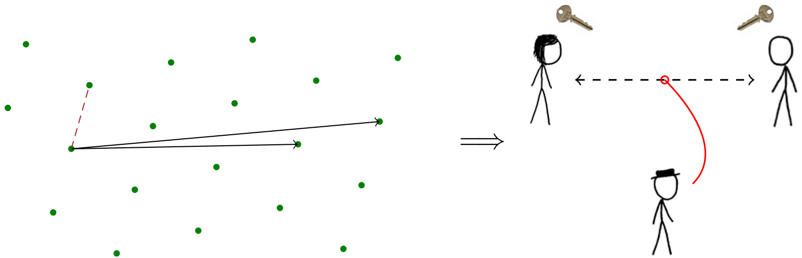

Amazing!

- Simple, efficient, and highly parallel crypto schemes
- Resists attacks by quantum algorithms (so far)
- Security from worst-case complexity assumptions
- Solves "holy grail" problems in crypto: FHE, obfuscation, ...

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Meanwhile, in the Real World...

- The vast majority of (public-key) crypto used in practice: signatures and key exchange/transport, over the Internet.

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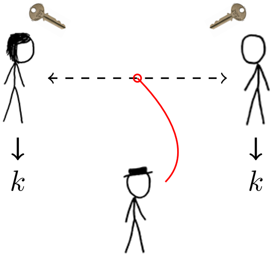
  - ⋆ Follow-up [BCNS'14]: TLS/SSL suite (in C) using these components, with estimated > 128 bit security: practical!

Authenticated Key Exchange



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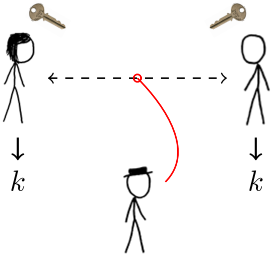

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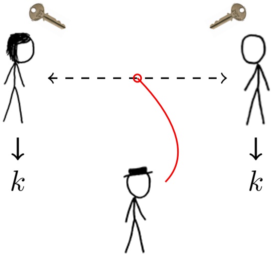

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- Adversary controls network, session initiation, may corrupt parties.
- Many intricate models and definitions, offering strong guarantees. [BR'93,BR'95,Kra'96,BCK'98,Sho'99,CK'01-02,LMQ'03,Kra'05,. . .]

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- Bottom line: minor changes to protocol design should make standardization and implementation easier.

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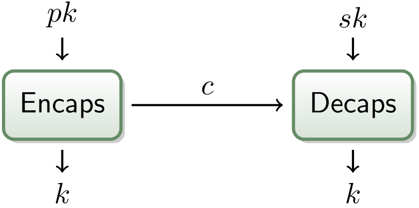

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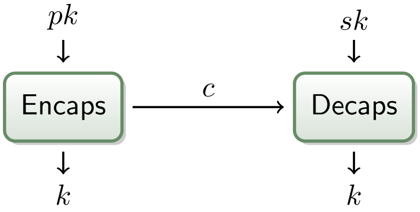

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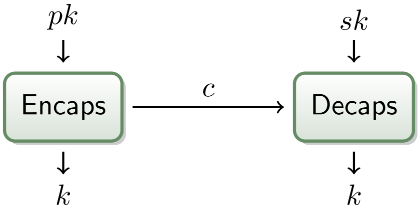

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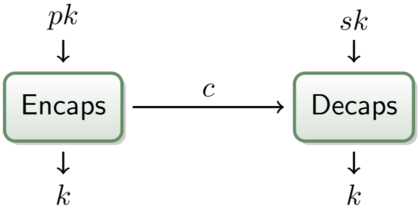

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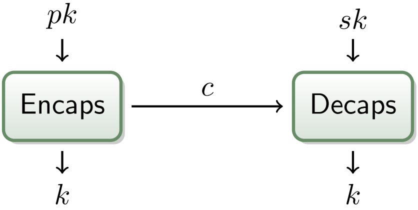

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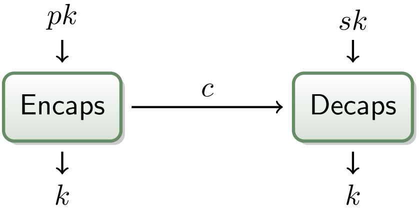

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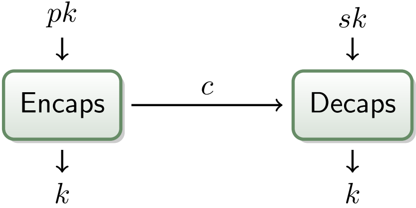

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5. FO transforms [FO'99a,b]: from any CPA-secure encryption (in ROM)

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    - Prefer [FO'99b] because it maintains plaintext length
    - Subtlety: RO yields random coins for encryption (Gaussian sampling)

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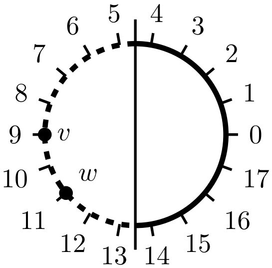

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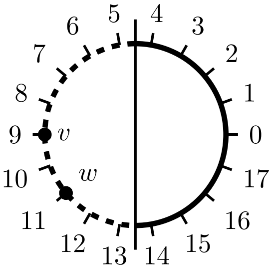

- Problem: $\log q$ factor overhead per msg bit (plus ctext 'preamble').

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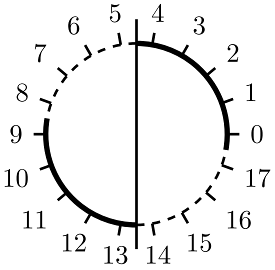

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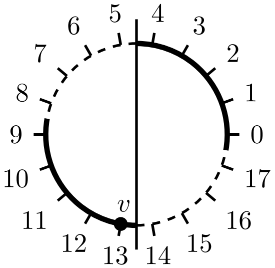

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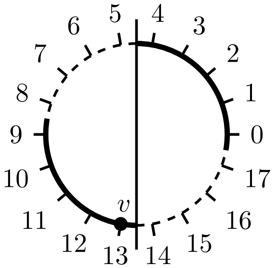

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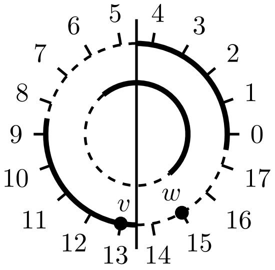

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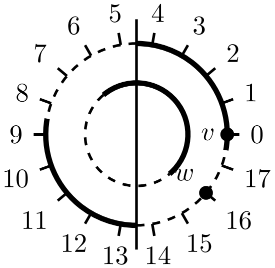

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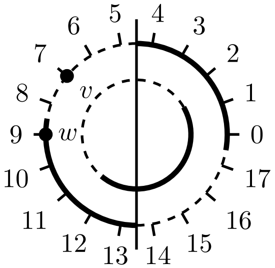

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- Passively secure under ring-LWE [LPR'10].
- Generalizes (tightly!) to any cyclotomic ring using tools from [LPR'13].

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