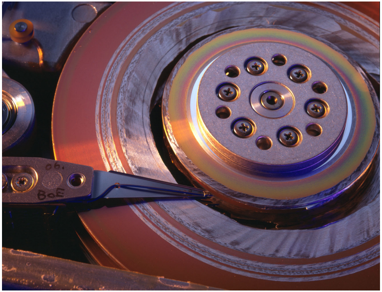
Lossy Trapdoor Functions and Their Applications

Chris Peikert   Brent Waters

SRI International

On Losing Information



On Losing Information

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2.3 MB → 0.4 MB

On Losing Information

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Lossy object *indistinguishable from original*

This Talk

1. Trapdoor functions without factoring: discrete log & lattices

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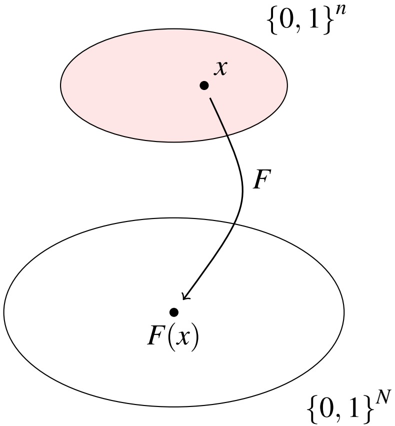
2. Black-box chosen-ciphertext security via randomness recovery

3. A new general primitive: Lossy Trapdoor Functions

Public Key Cryptography

1-1 Trapdoor Functions

$$(F, F^{-1}) \leftarrow S$$



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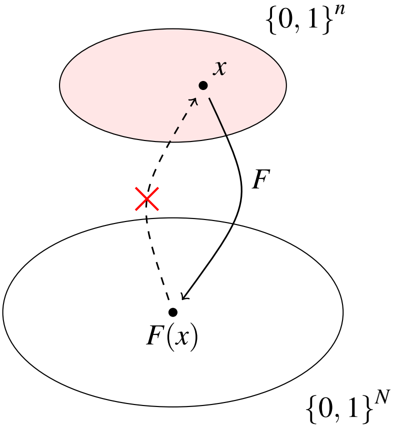

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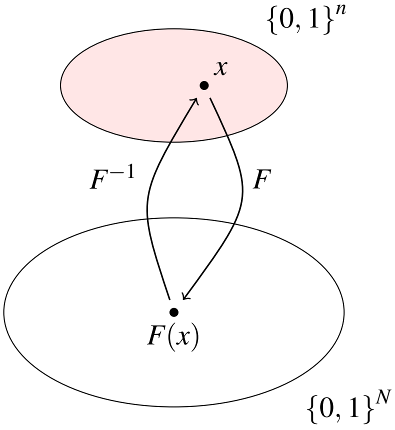

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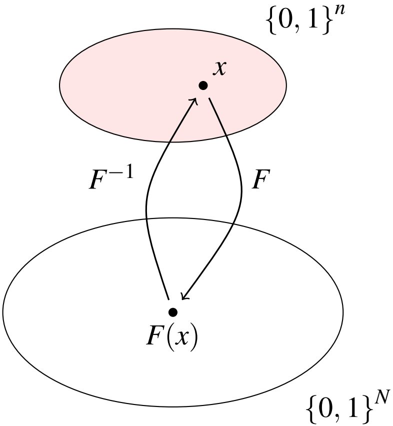
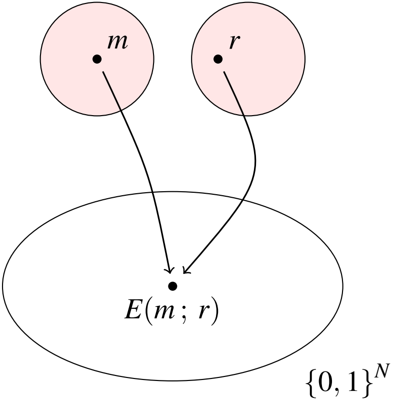


Public Key Encryption

$$(E, D) \leftarrow S$$



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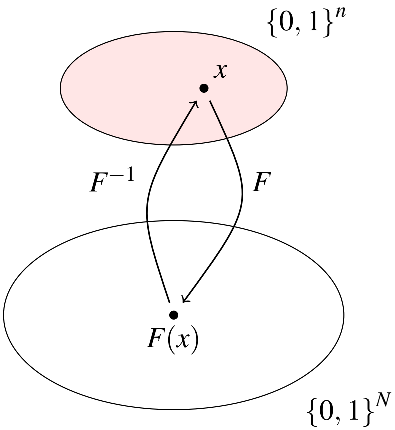

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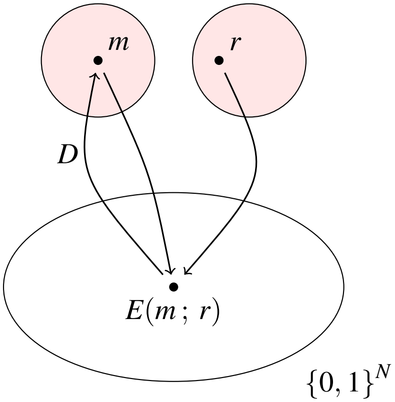

Realizing Public Key Crypto

	Factoring	Discrete log	Lattices
PKE	✔ [RSA,. . .]	✔ [ElGamal]	✔ [AD,R1,R2]
CCA	✔ [DDN,. . . ,CS2]	✔ [CS1]	??
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Lattice-Based Crypto:

- Simple & parallelizable
- Resist *quantum* algorithms (so far)
- Security from *worst-case* assumptions [Ajtai,. . .]

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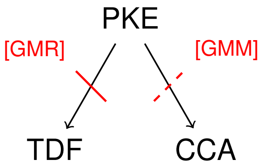
Black-Box Separations:



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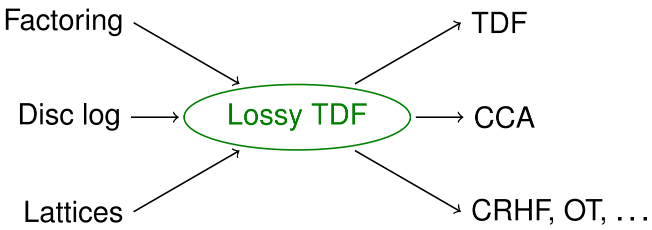
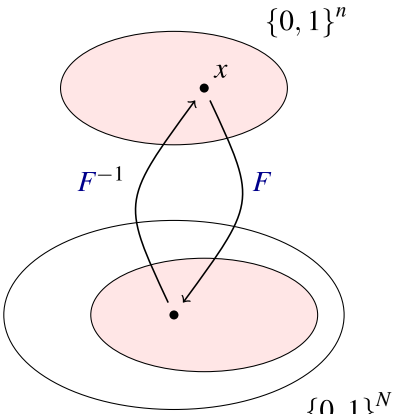


Lossy Trapdoor Functions

$(F, F^{-1}) \leftarrow S_{\text{inj}}$



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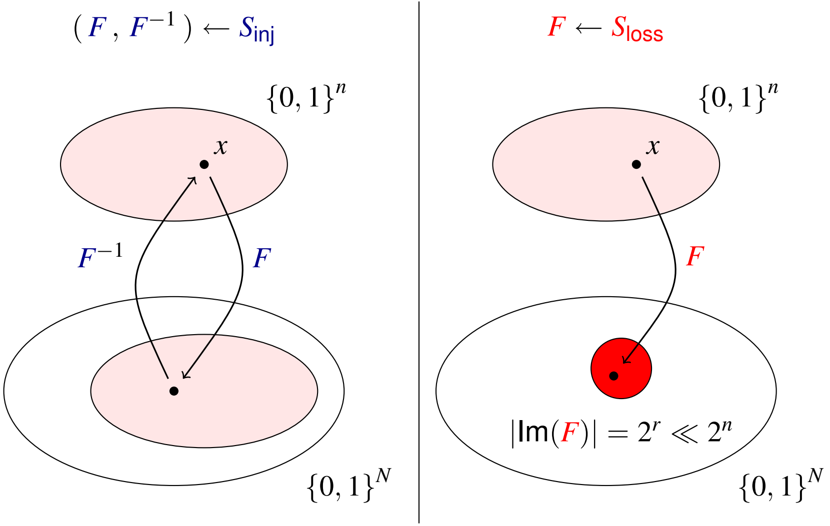

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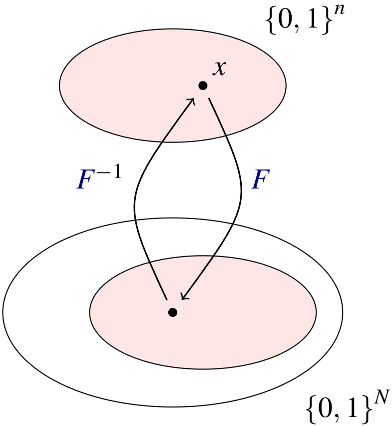

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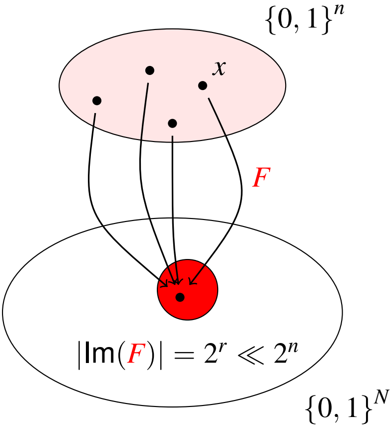

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$$F \overset{c}{\approx} F$$

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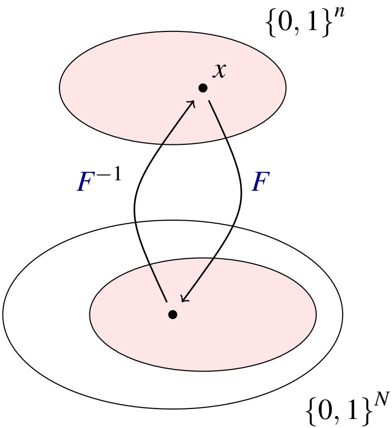

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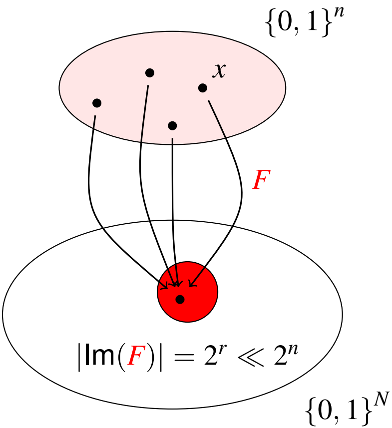

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Theorem

- S_{inj} generates 1-1 trapdoor functions (F, F^{-1}).

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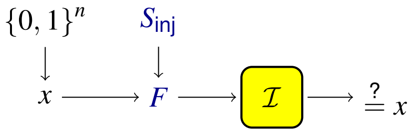

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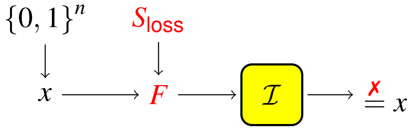

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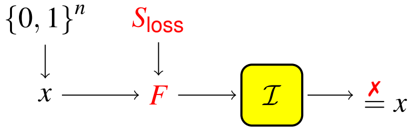

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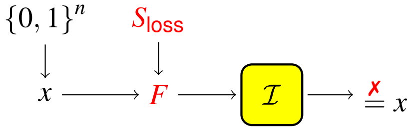

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Main Technique

- Swapping F with F yields *statistically secure* system.

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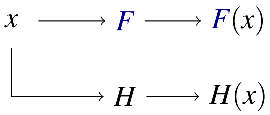

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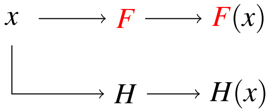

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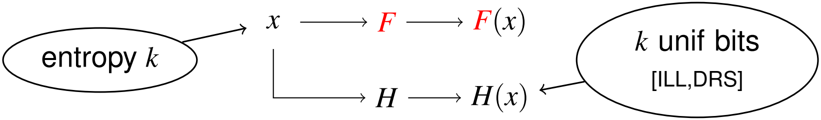

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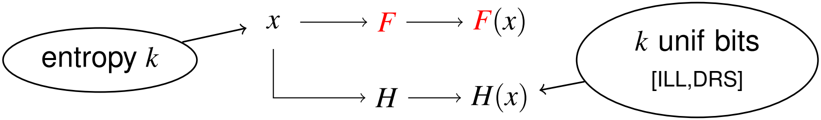

- Public key (F, H), secret key F^{-1}.

  Encrypt $m \in \{0,1\}^k$ as $(F(x),\ m \oplus H(x))$.

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Why It Matters

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Technical Difficulty

- Verify ciphertext is “well-formed”
- Usually via zero-knowledge proof
- Our approach: recover randomness

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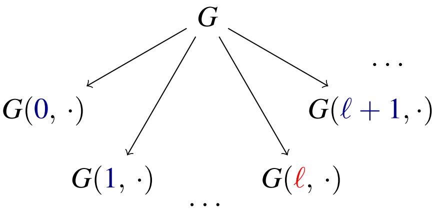

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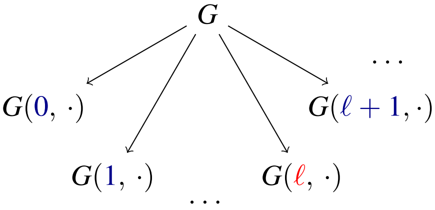
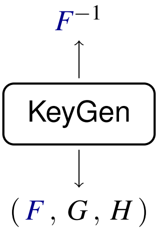


- Lossy TDFs $\Leftrightarrow$ all-but-one TDFs.

Lossy TDFs ⇒ CCA-Secure Encryption



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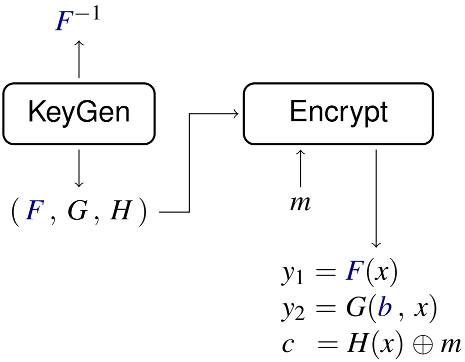

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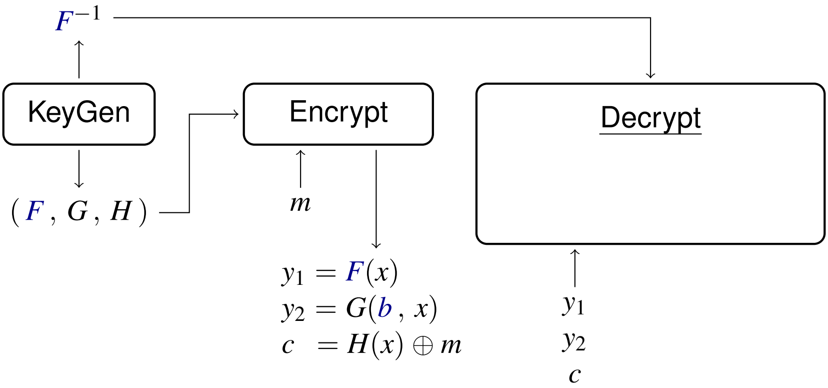

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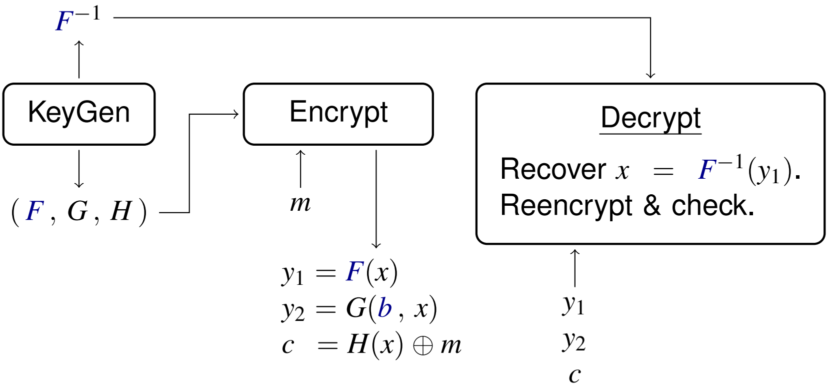

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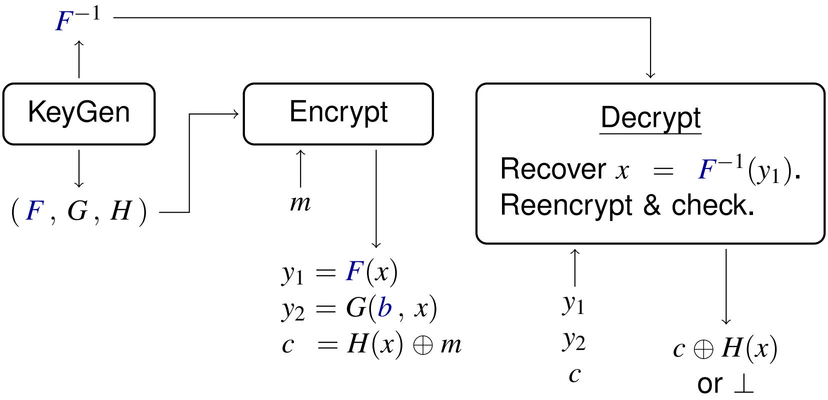

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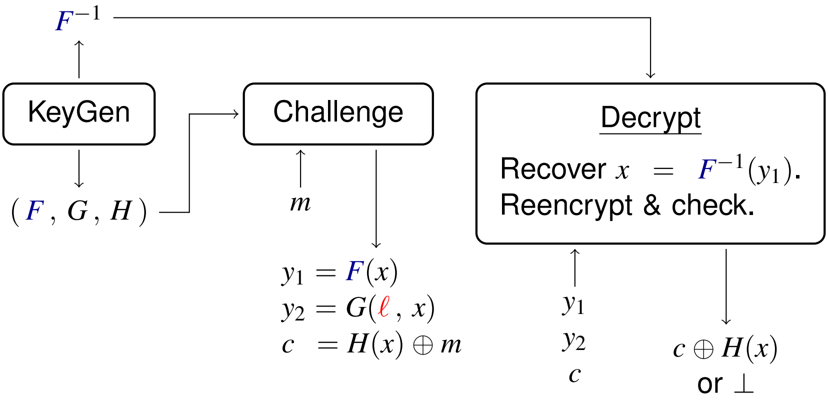

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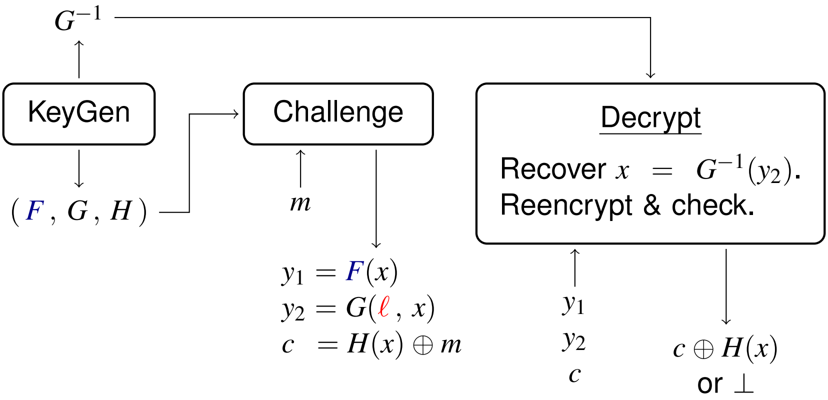

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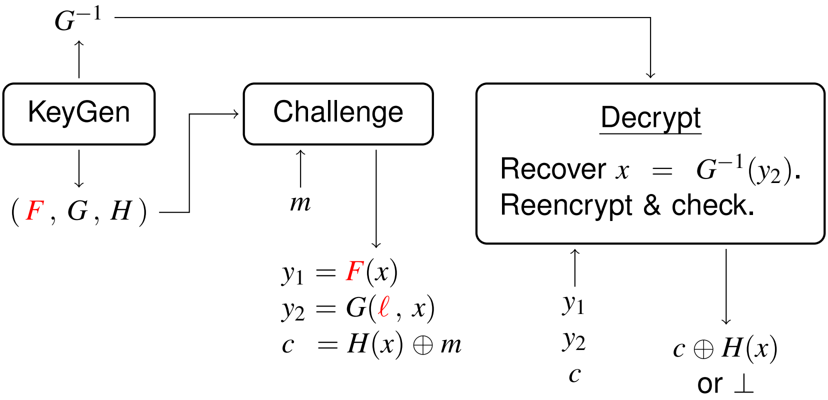

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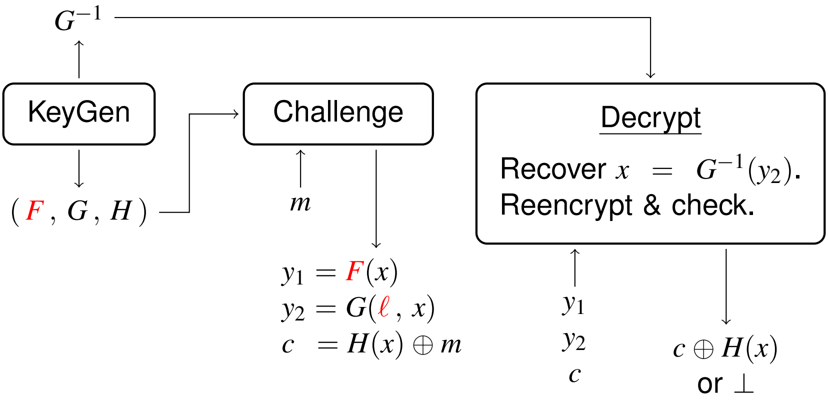

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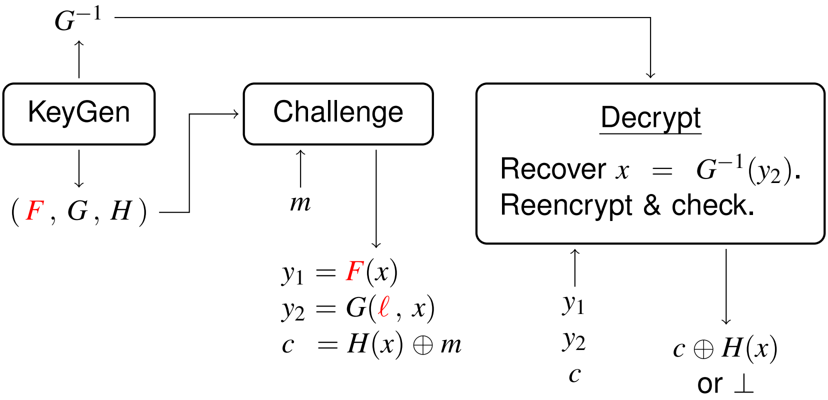

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- (One-time signature for CCA2 security. [DolevDworkNaor])

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- Randomness in each 0 leaks information!

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- Just need $n > |R|$ for lossiness.

Concrete Assumptions

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   - Additive homomorphism in ElGamal: message in the exponent
   - Reusing randomness [NaorReingold,Kurosawa,. . .]

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   - Additive homomorphism in ElGamal: message in the exponent
   - Reusing randomness [NaorReingold,Kurosawa,. . .]
2. Learning With Errors (LWE) on lattices [Regev]
   - Bounded homomorphism
   - Reuse most randomness — but not the error terms

Future Directions

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- Other indistinguishable properties of "huge" objects?