

# Lecture 21: Color

# Announcements

- PS9 and PS10 deadlines extended to April 21
- Graduate grading policy:  
<https://rackham.umich.edu/covid-19/>

- All graduate students enrolled in Rackham programs will receive a grade of either “Satisfactory” (“S”) or “No Record Covid” (“NRC”) for coursework graded on the letter system. A grade of S will earn course credit. The minimum grade for S will be B-, consistent with grading practices in Rackham programs. Language will be added to transcripts explaining the university’s policy for the winter 2020 term.
- Until July 1, 2020, students will be allowed to review their course grade and convert grades of S and NRC into letter grades if they wish.
- A grade of S that is converted into a letter grade will earn credit and be calculated into the GPA. A grade of NRC that is converted into a letter grade of C- or above will also earn credit and be used to calculate the GPA. A letter grade below a C- will not earn credit but will be used to calculate the GPA.
- A flexible withdrawal policy will also be instituted. Students will have until April 21 to withdraw from a course and not have the course appear on their transcript.
- Individual Rackham programs may have additional degree requirements for academic performance, such as in required or core classes. Students should consult with their academic program for further information in these cases.
- Instructors may still choose to issue an Incomplete (I) and approve an extension with a deadline for completion of final work due in the course. An incomplete will not be permanently recorded on the transcript and will be converted to a letter grade when the instructor submits the final grade.



# Today

- Color physics
- Color perception
- Color constancy

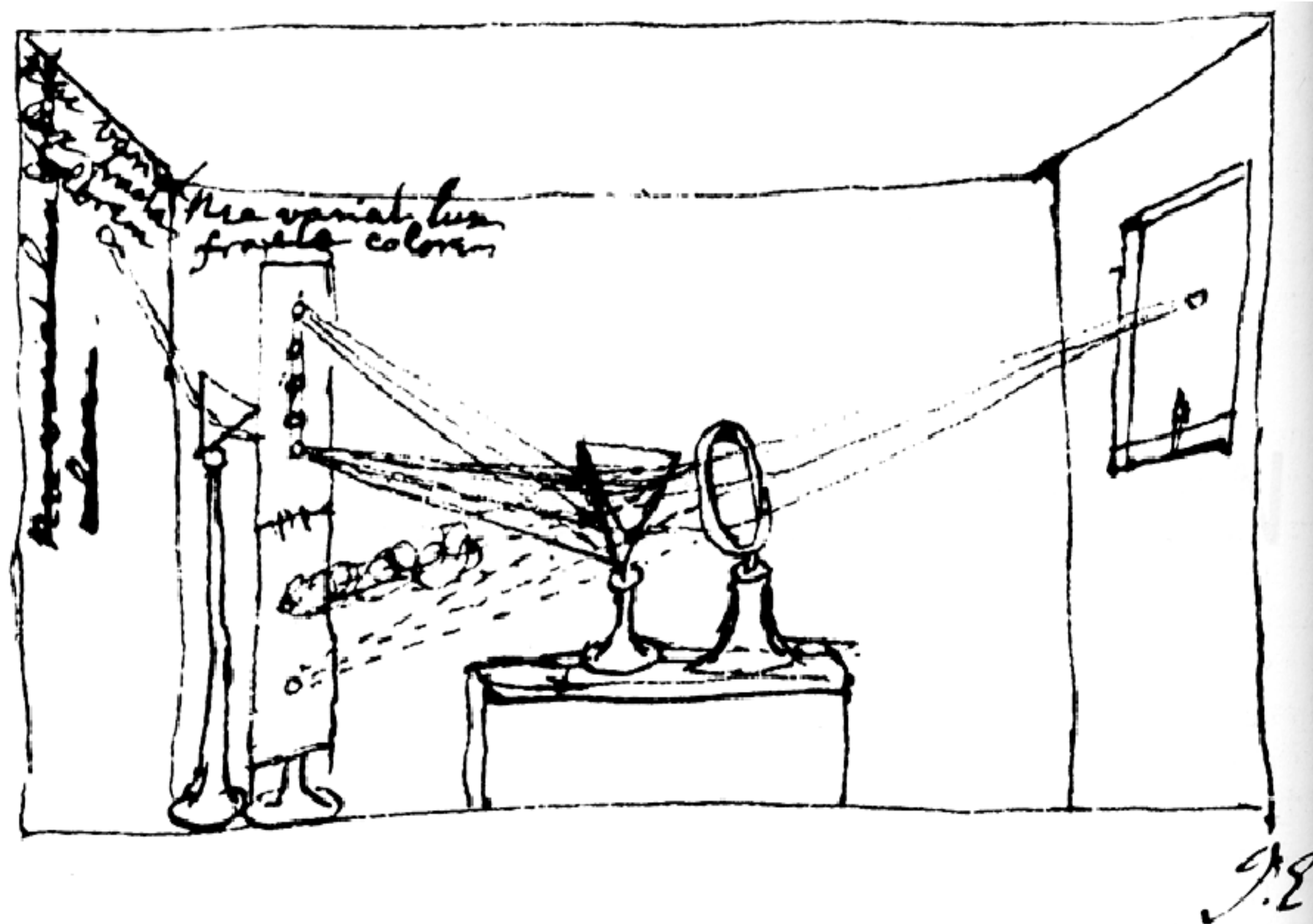
# Why does a visual system need color? (an incomplete list...)

- To tell what food is edible.
- To distinguish material changes from shading changes.
- To group parts of one object together in a scene.
- To find people's skin.
- Check whether a person's appearance looks normal/healthy.





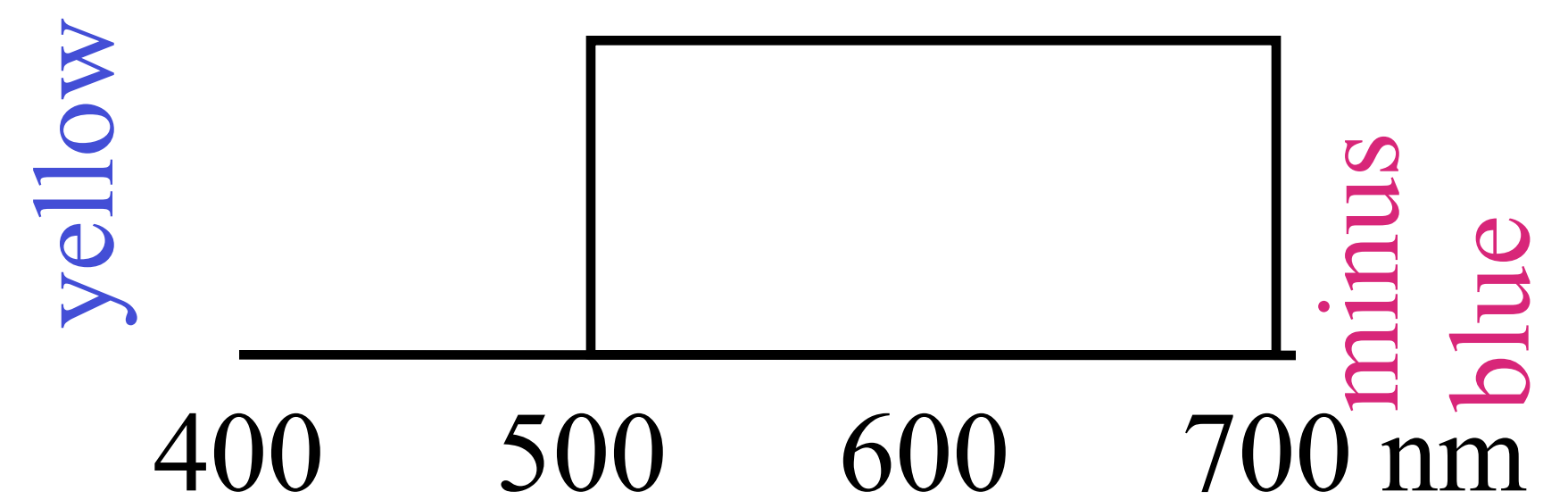
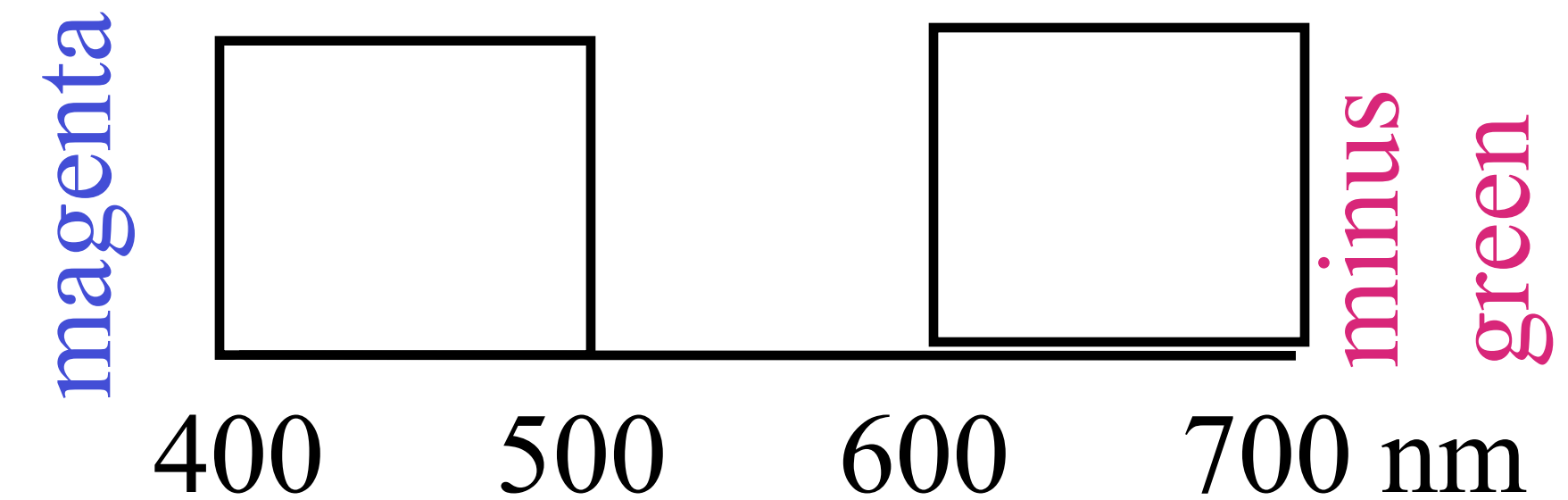
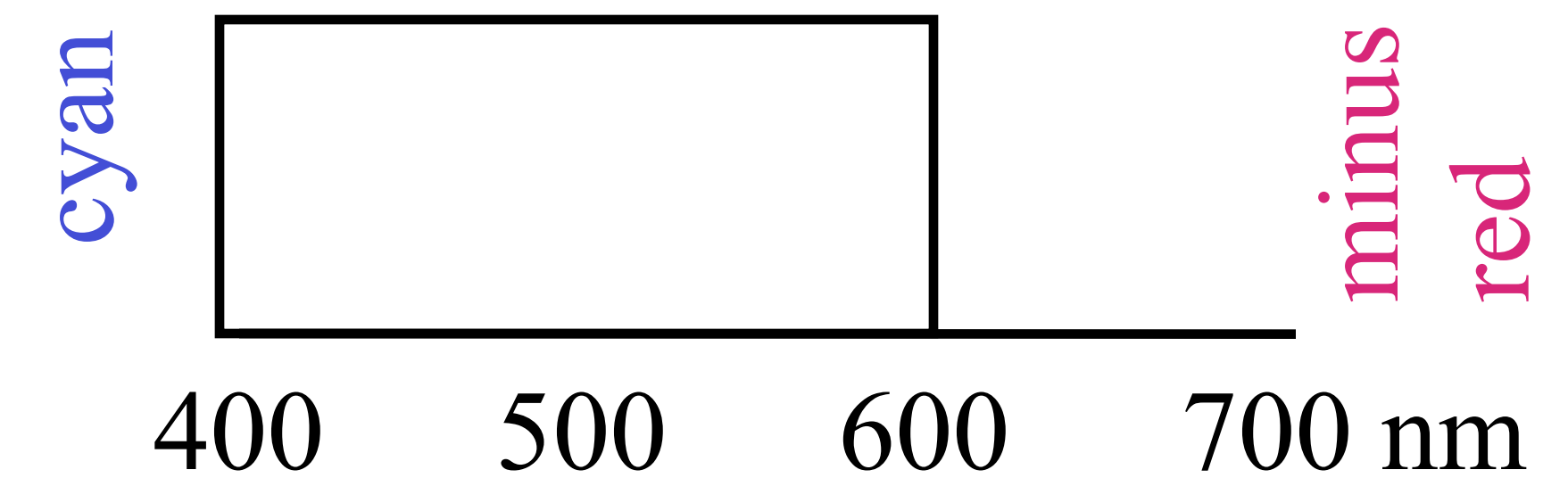
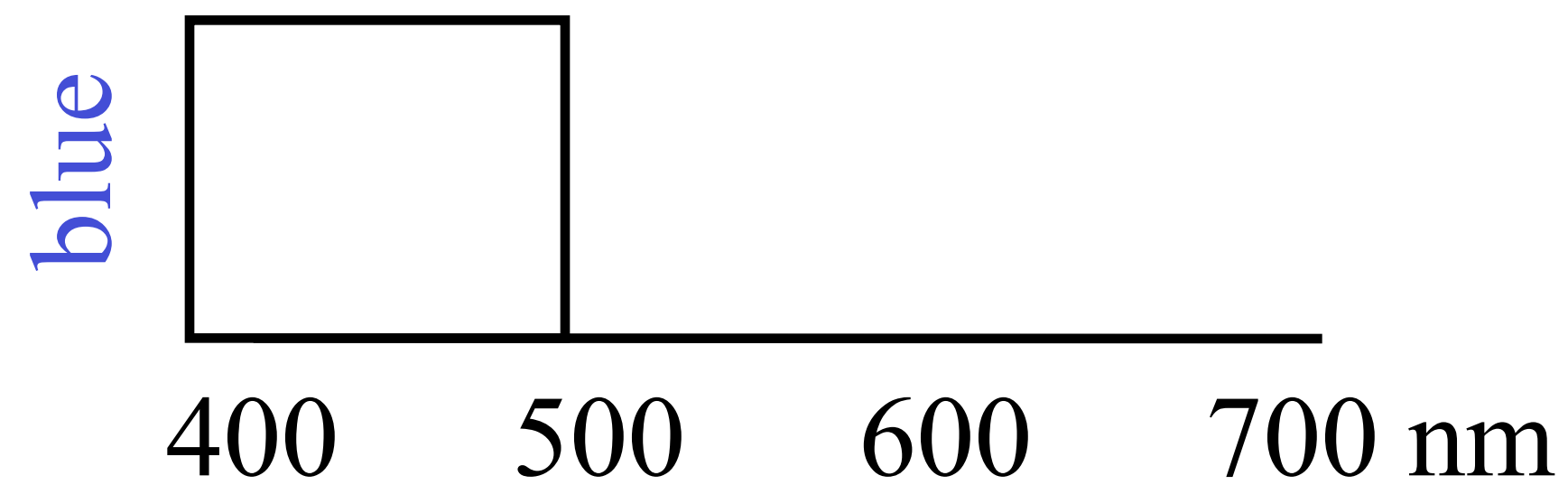
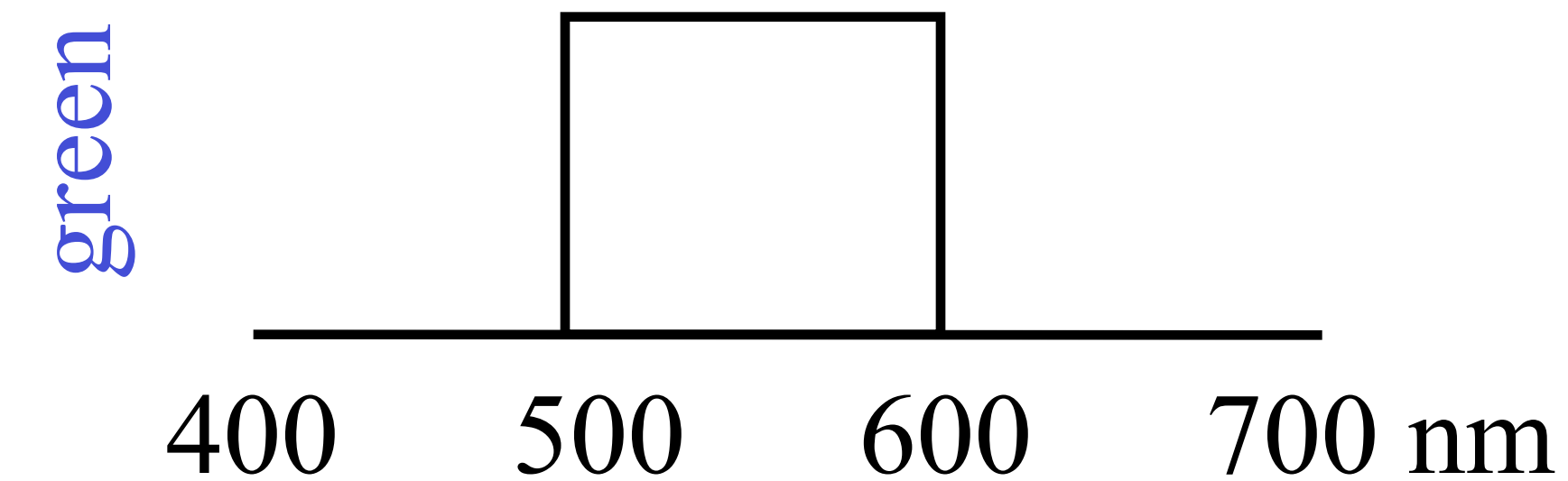
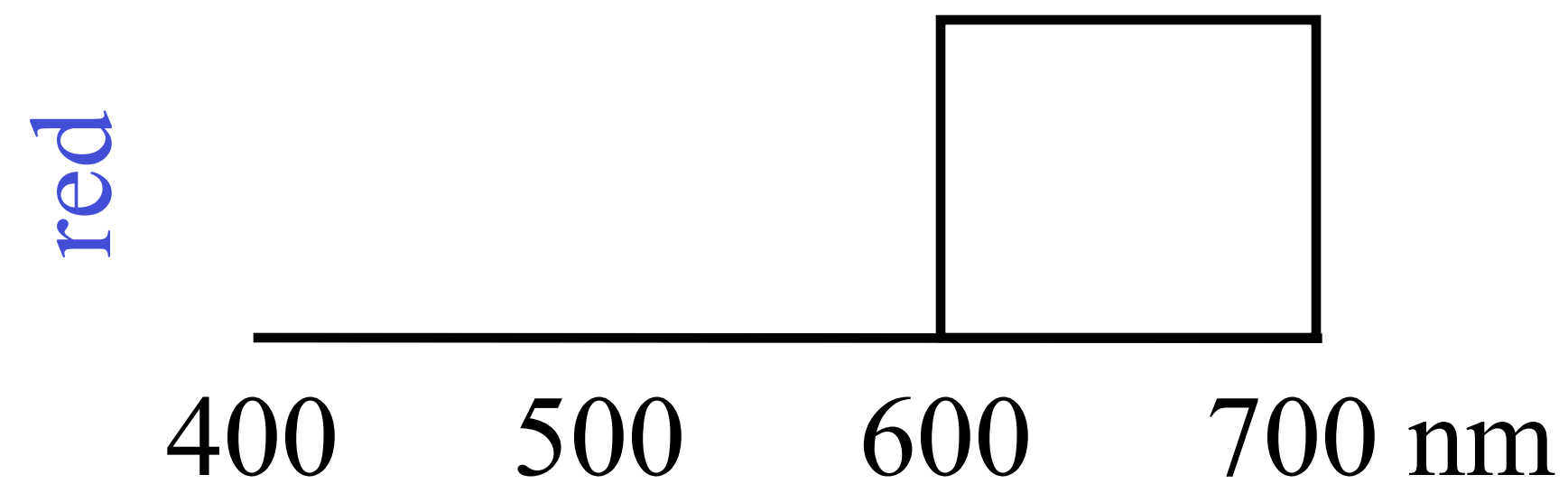
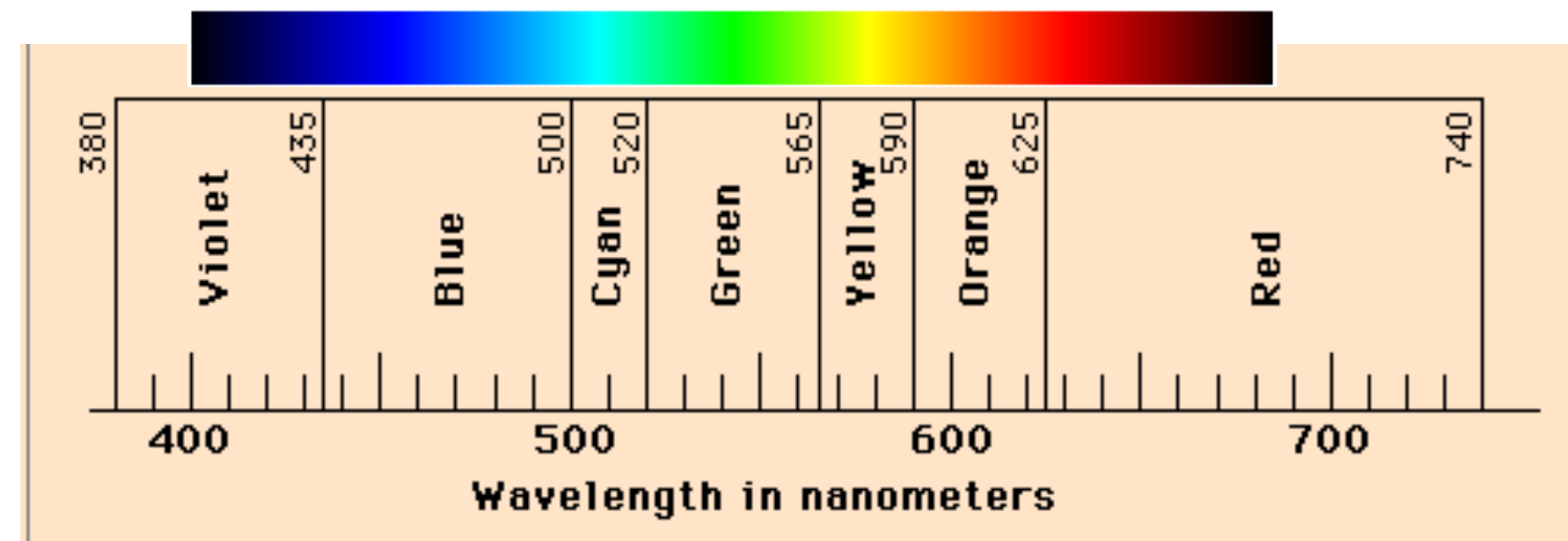
# Color



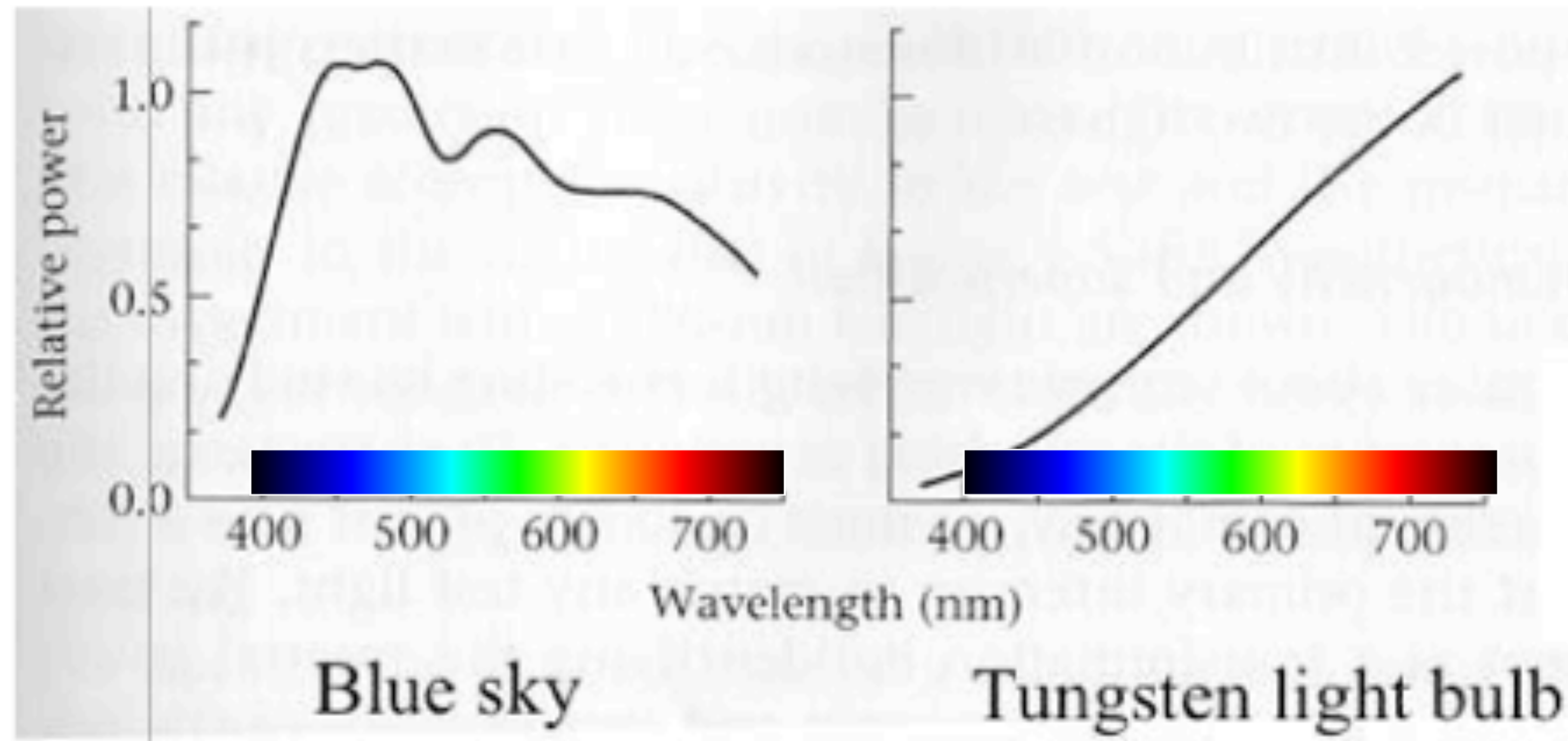
4.1 NEWTON'S SUMMARY DRAWING of his experiments with light. Using a point source of light and a prism, Newton separated sunlight into its fundamental components. By reconverging the rays, he also showed that the decomposition is reversible.

From Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

# Color names for cartoon spectra



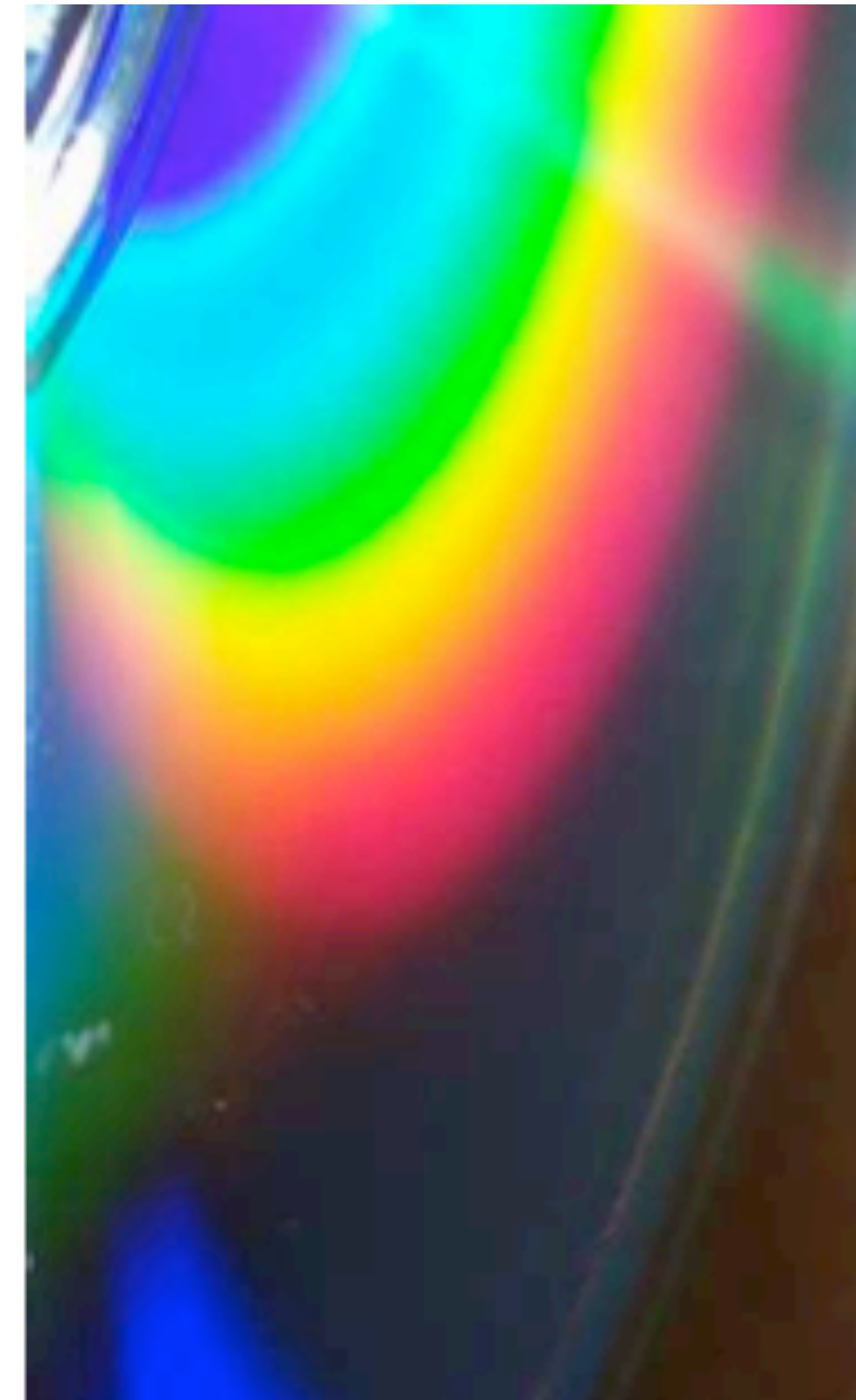
# Spectrum



- Light characterized by **spectrum**: amount of energy in each wavelength.
- This is a distribution. One value per wavelength.

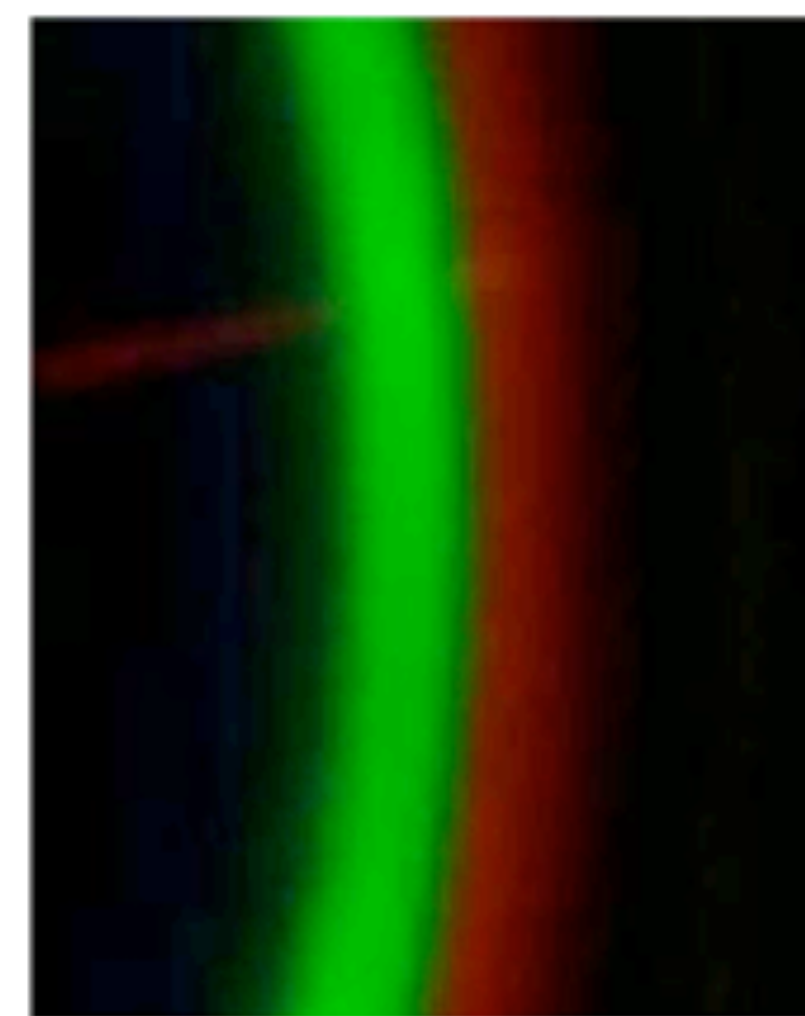


# Homemade spectrograph





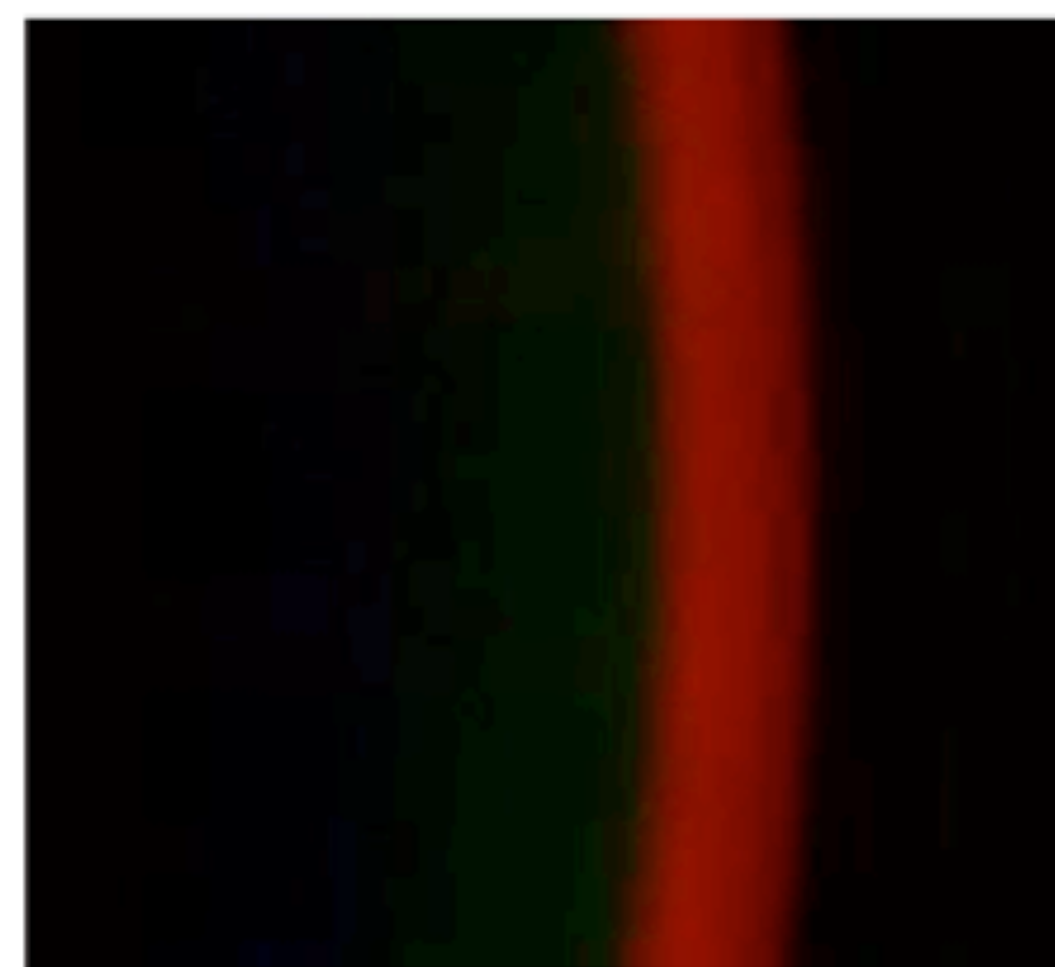
(a)



(b)



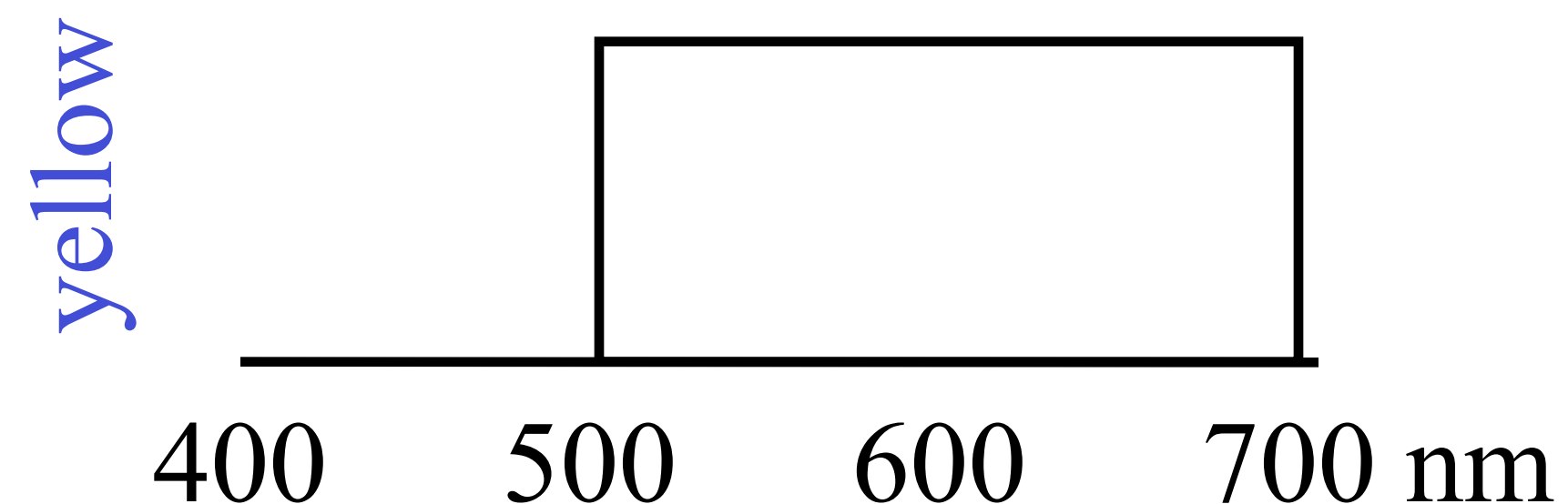
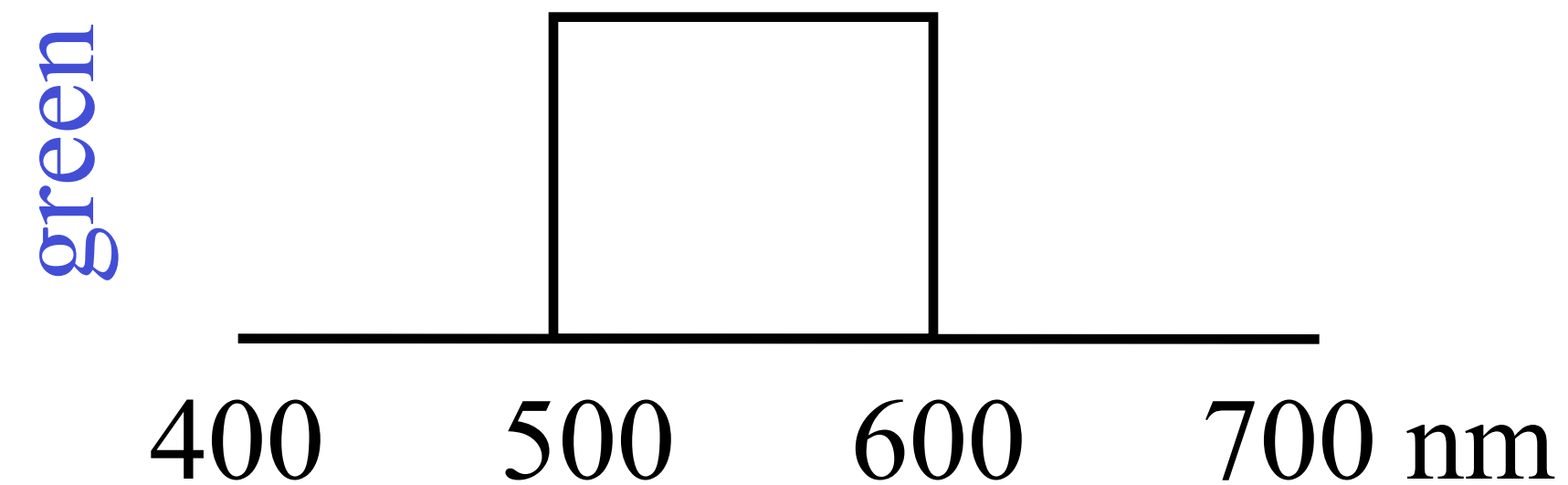
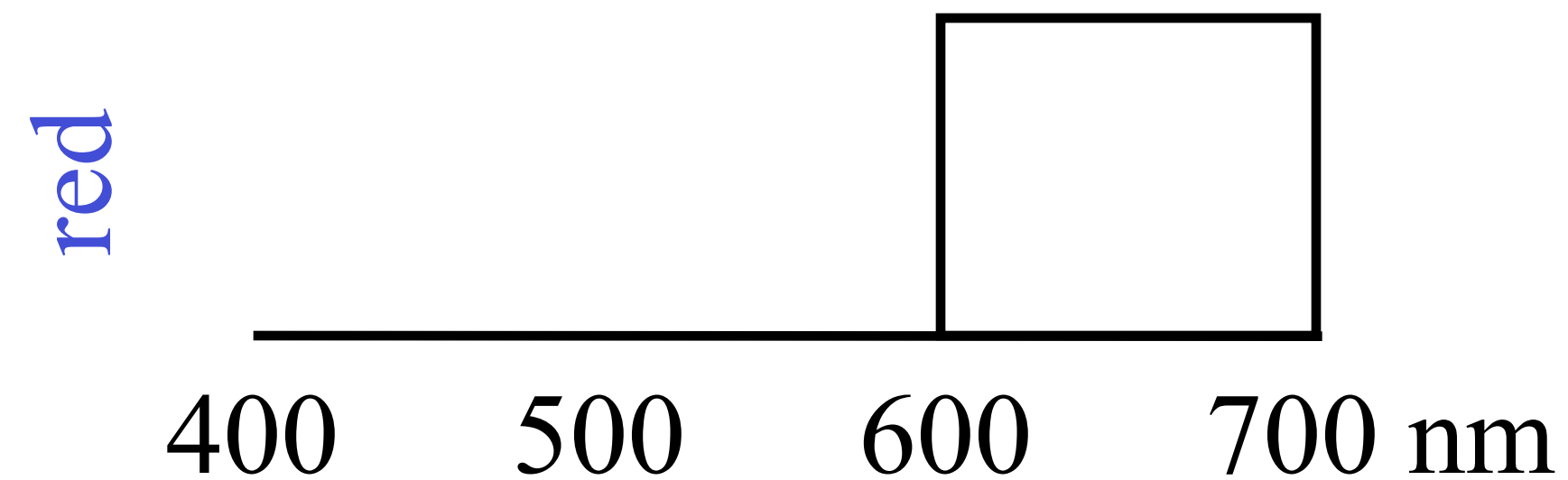
(c)



(d)



# Additive color mixing



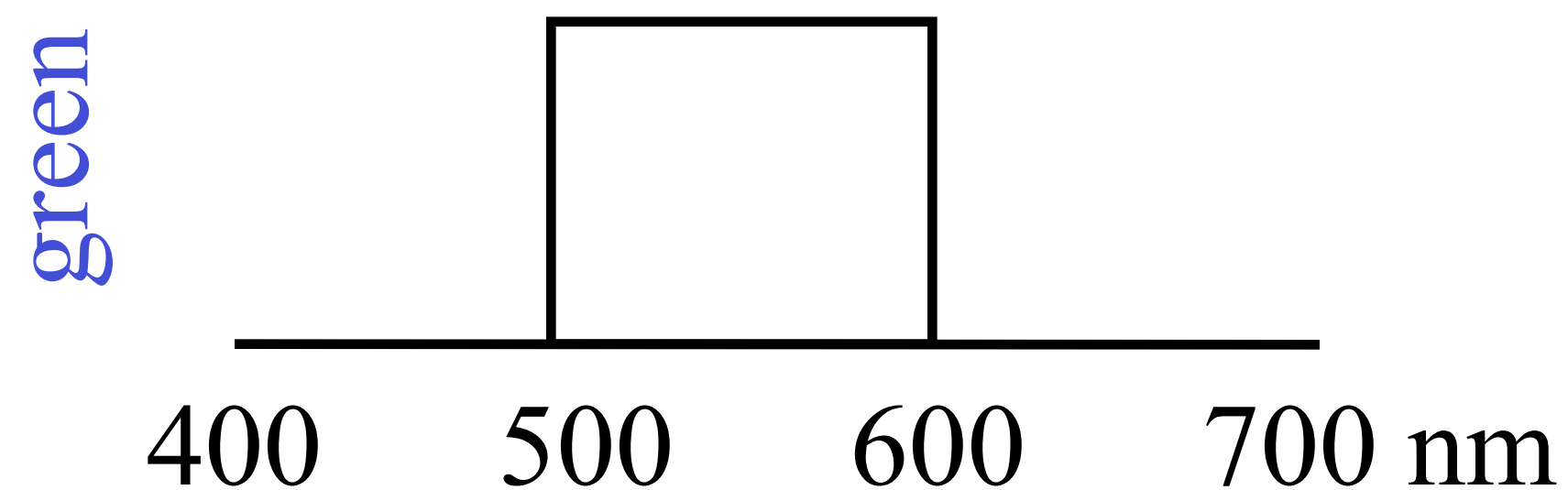
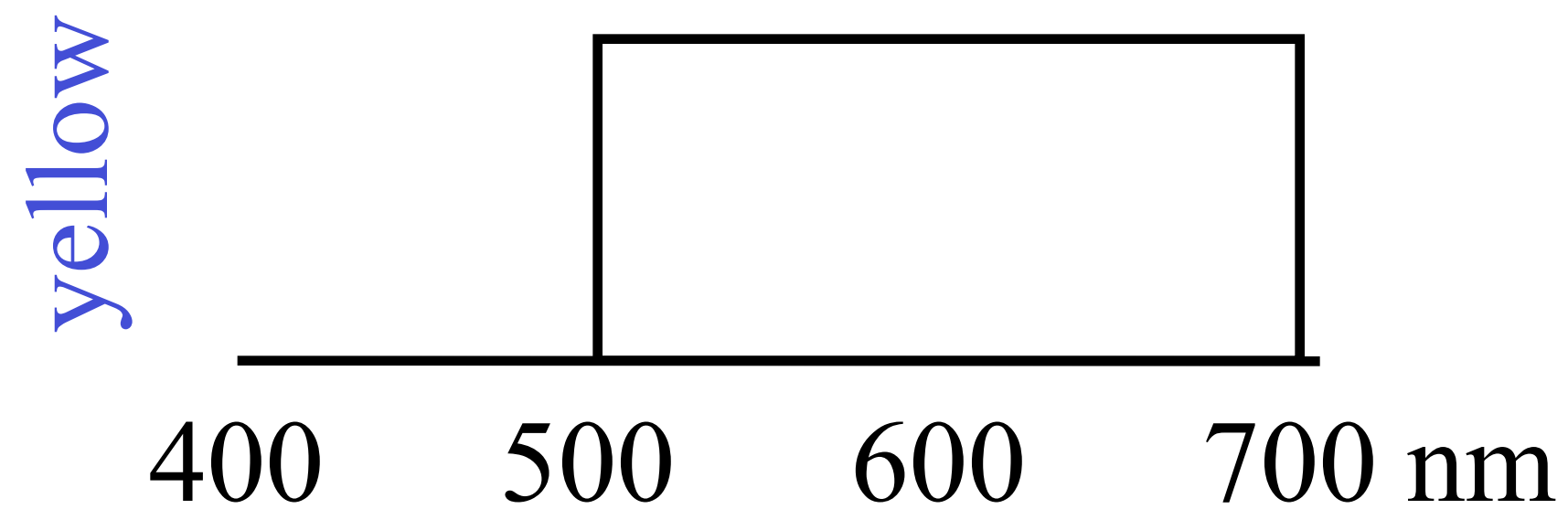
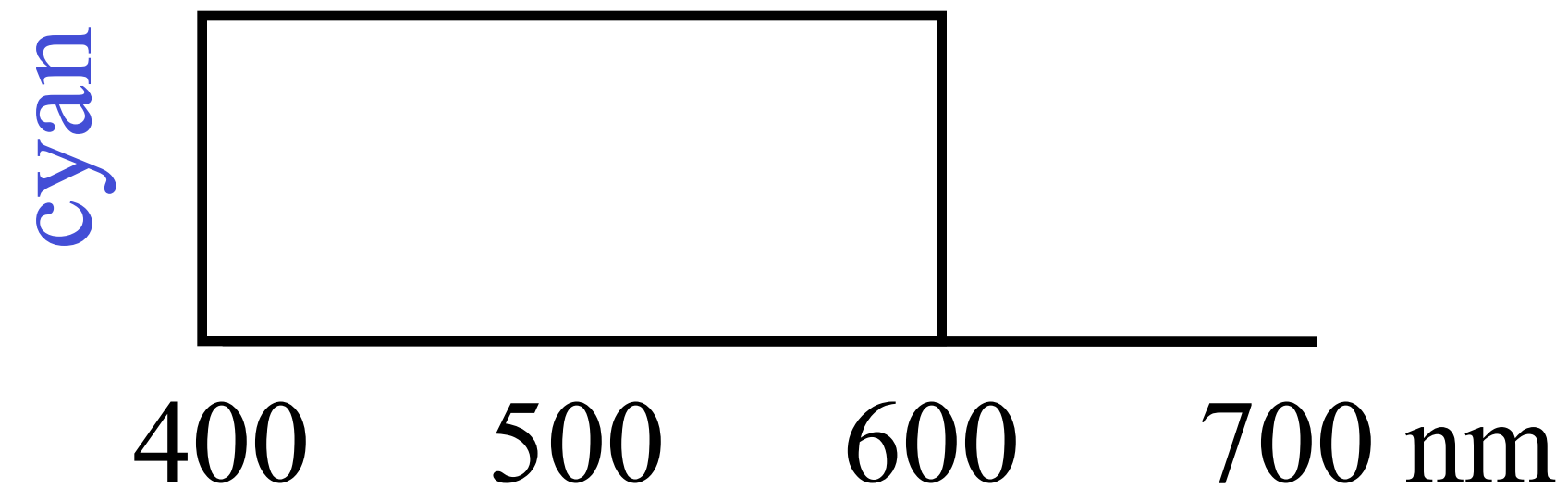
When colors combine by *adding* the color spectra. Example color displays that follow this mixing rule: CRT phosphors, multiple projectors aimed at a screen.

Red and green make...

Yellow!



# Subtractive color mixing



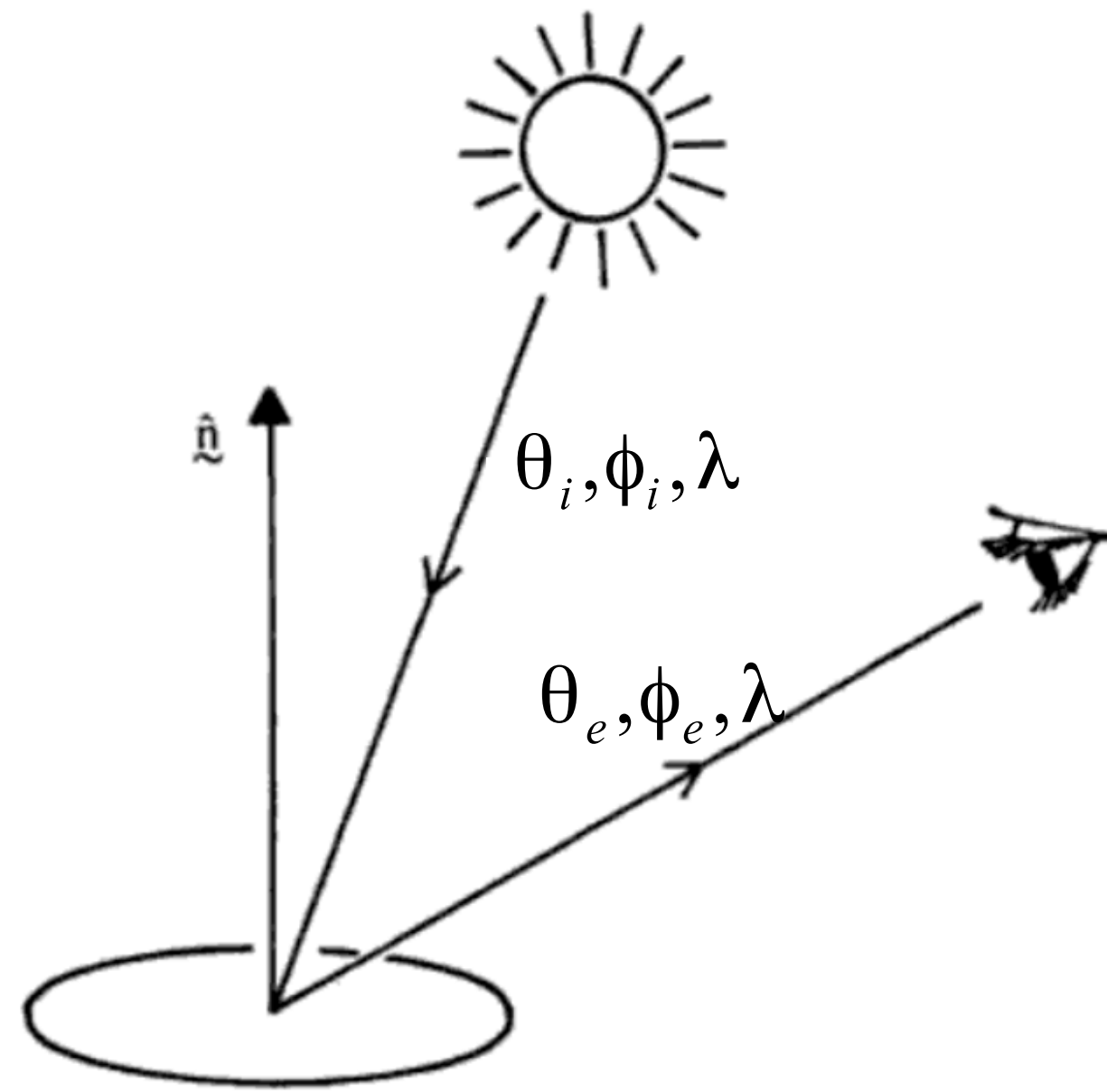
When colors combine by *multiplying* the color spectra. Examples that follow this mixing rule: most photographic films, paint, cascaded optical filters, crayons.

Cyan and yellow (in crayons, called “blue” and yellow) make...

Green!

How does light interact with surfaces?

# The interaction of light with surfaces



**Figure 10-7.** The bidirectional reflectance distribution function is the ratio of the radiance of the surface patch as viewed from the direction  $(\theta_e, \phi_e)$  to the irradiance resulting from illumination from the direction  $(\theta_i, \phi_i)$ .

Spectral radiance: power in a specified direction, per unit area, per unit solid angle, per unit wavelength.

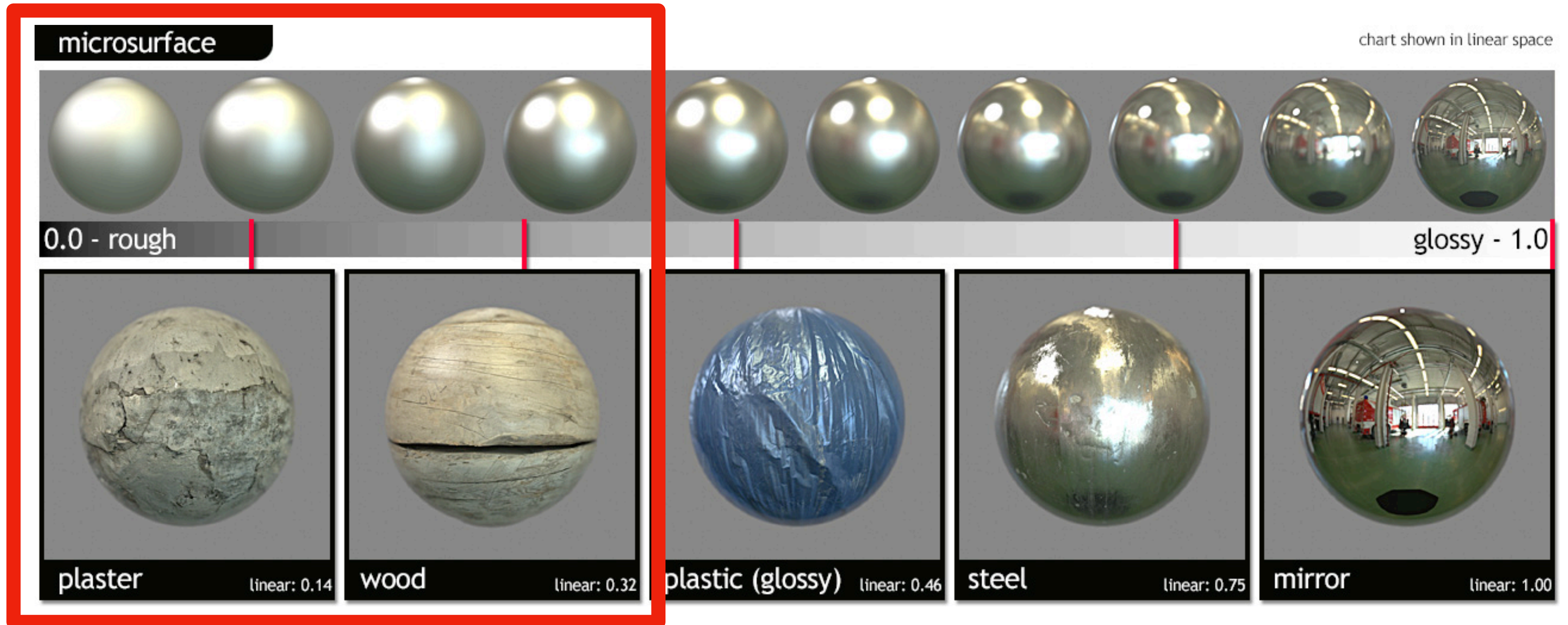
$$BRDF = f(\theta_i, \phi_i, \theta_e, \phi_e, \lambda) = \frac{L(\theta_e, \phi_e, \lambda)}{E(\theta_i, \phi_i, \lambda)}$$

Spectral irradiance: incident power per unit area, per unit wavelength

[Horn, 1986]



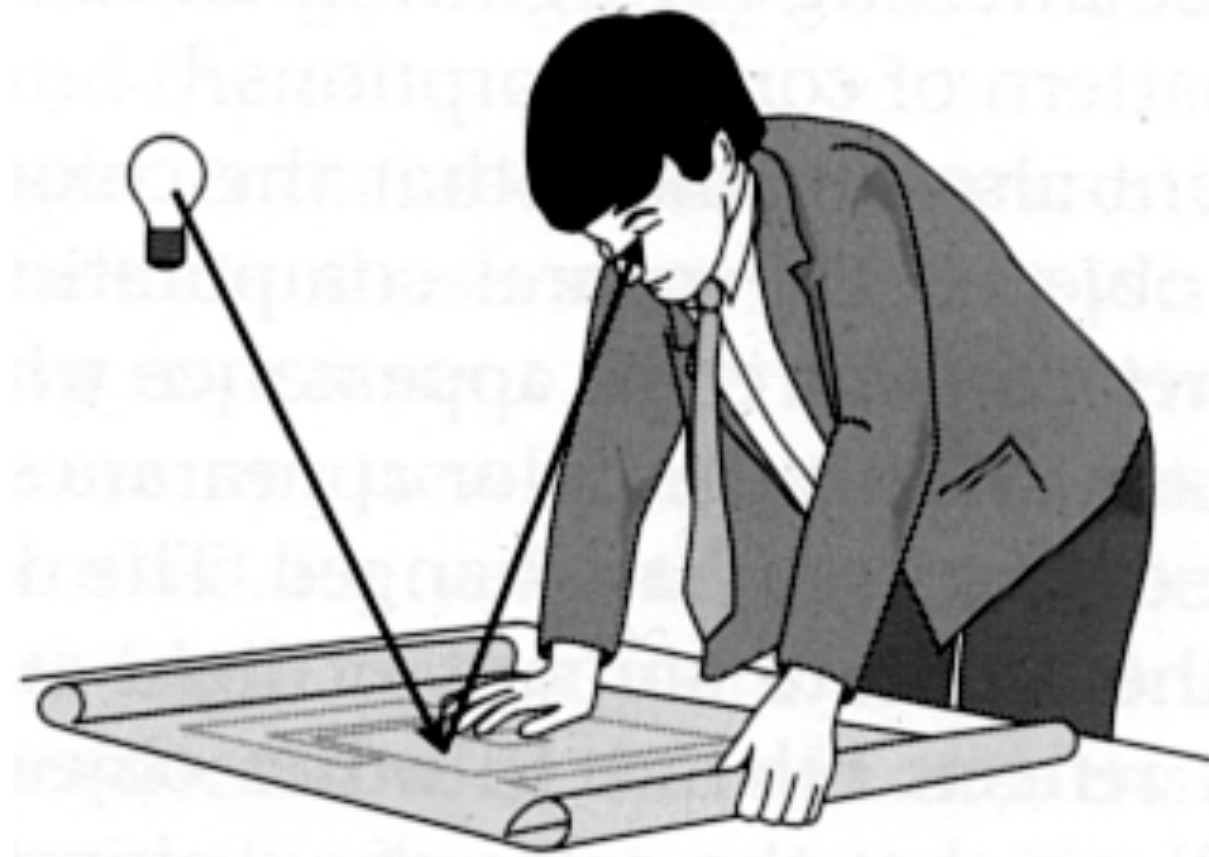
# Effect of BRDF on sphere rendering



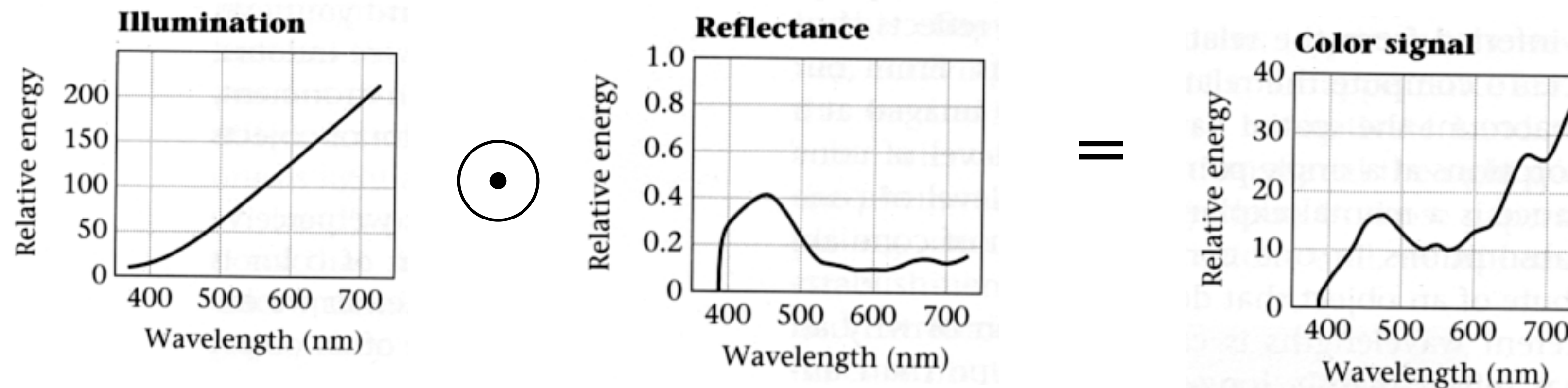
Diffuse reflection



# Simplified rendering models: BRDF $\rightarrow$ reflectance



For diffuse reflections, we replace the BRDF calculation with a wavelength-by-wavelength scalar multiplication (i.e. pointwise multiplication)



# Today

- Color physics
- Color perception
- Color constancy

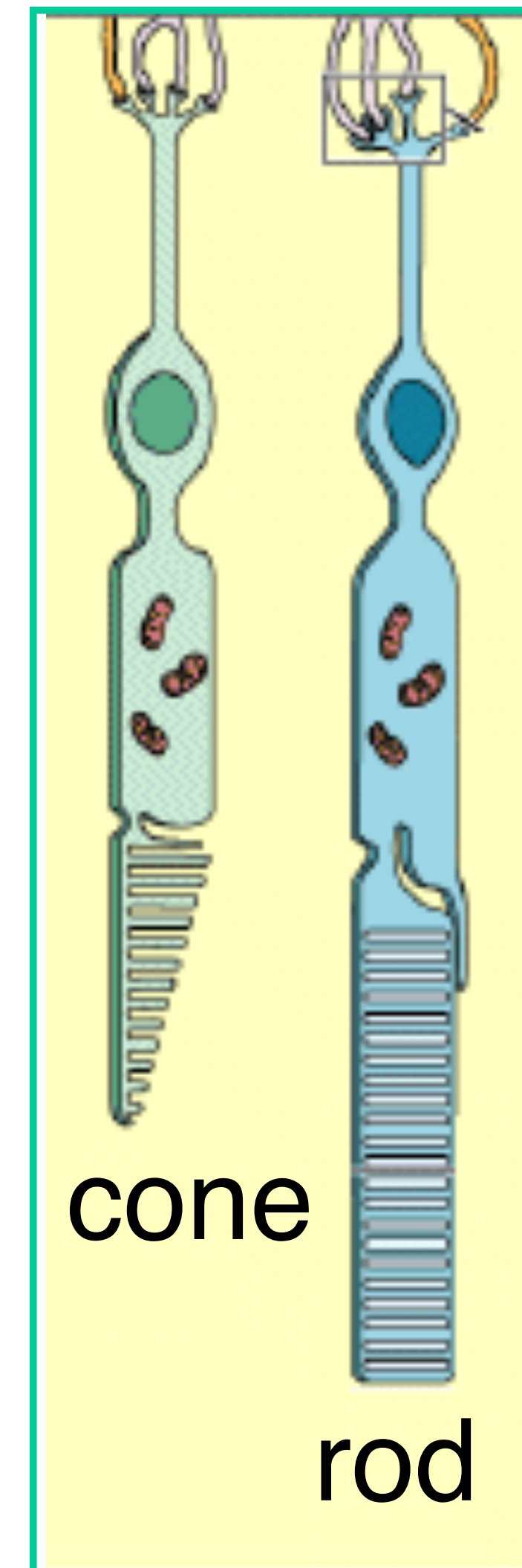
# Two types of light-sensitive receptors

## **C**ones

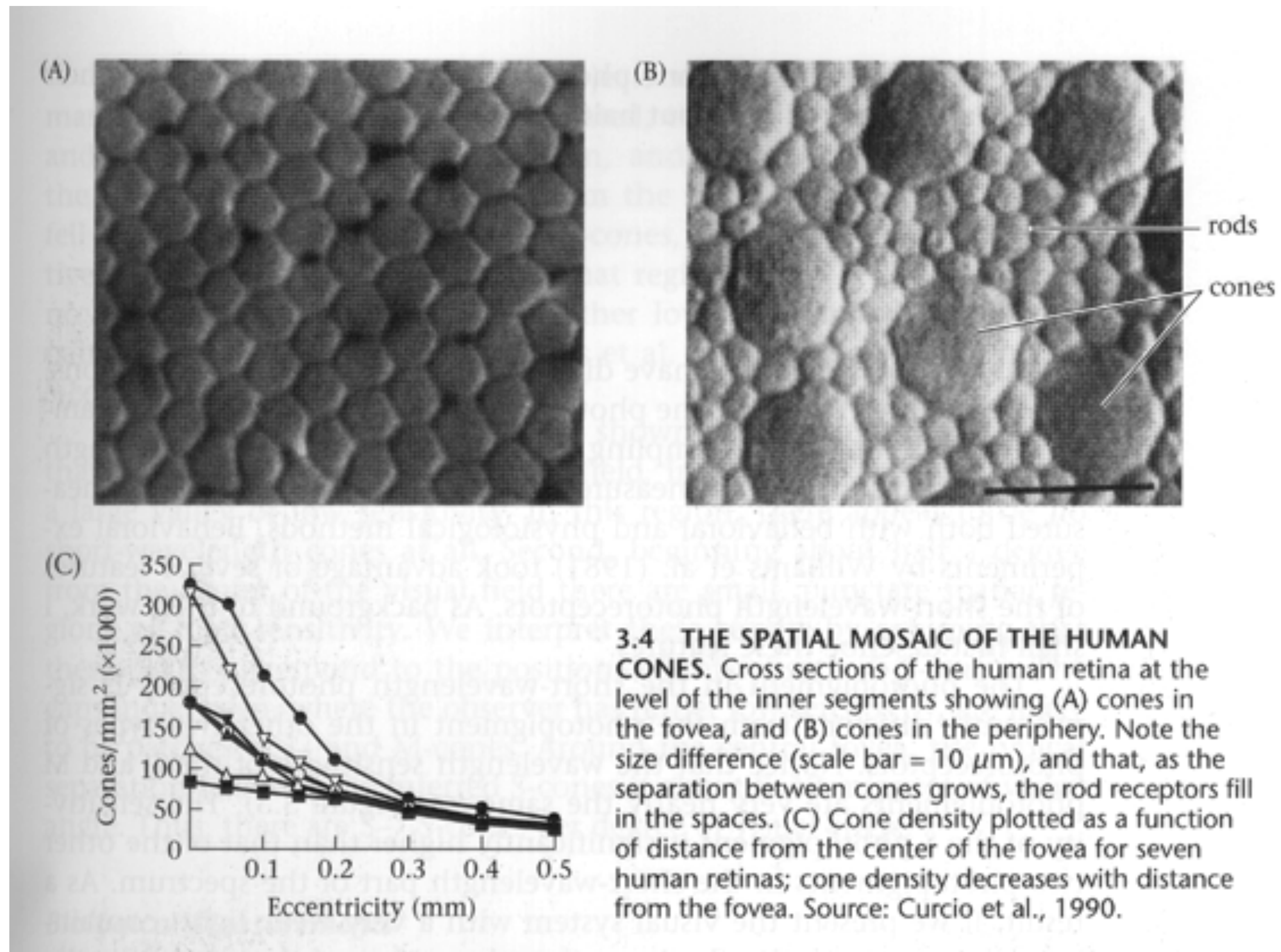
cone-shaped  
less sensitive  
operate in high light  
color vision

## **Rods**

rod-shaped  
highly sensitive  
operate at night  
gray-scale vision

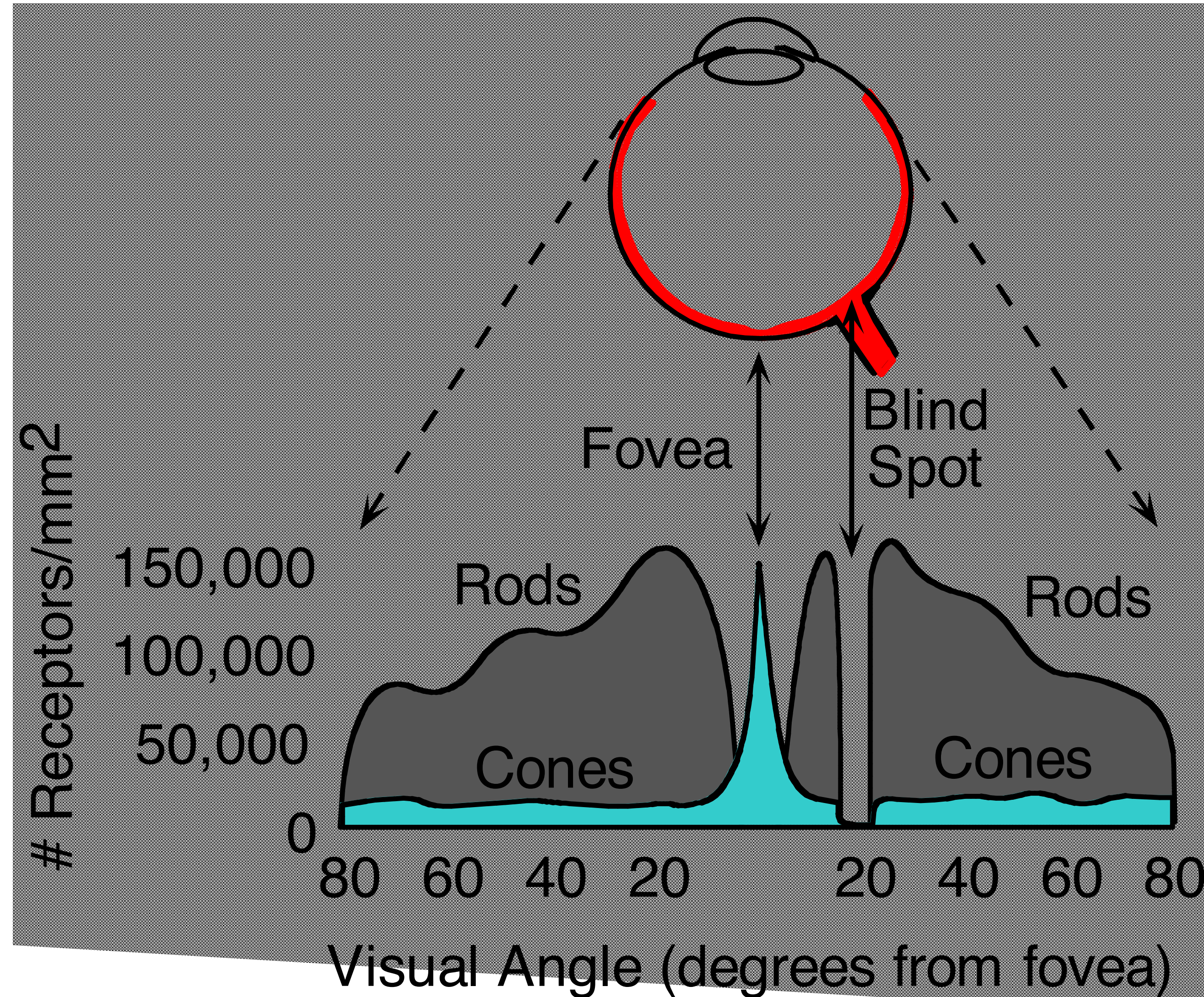


# Human photoreceptors



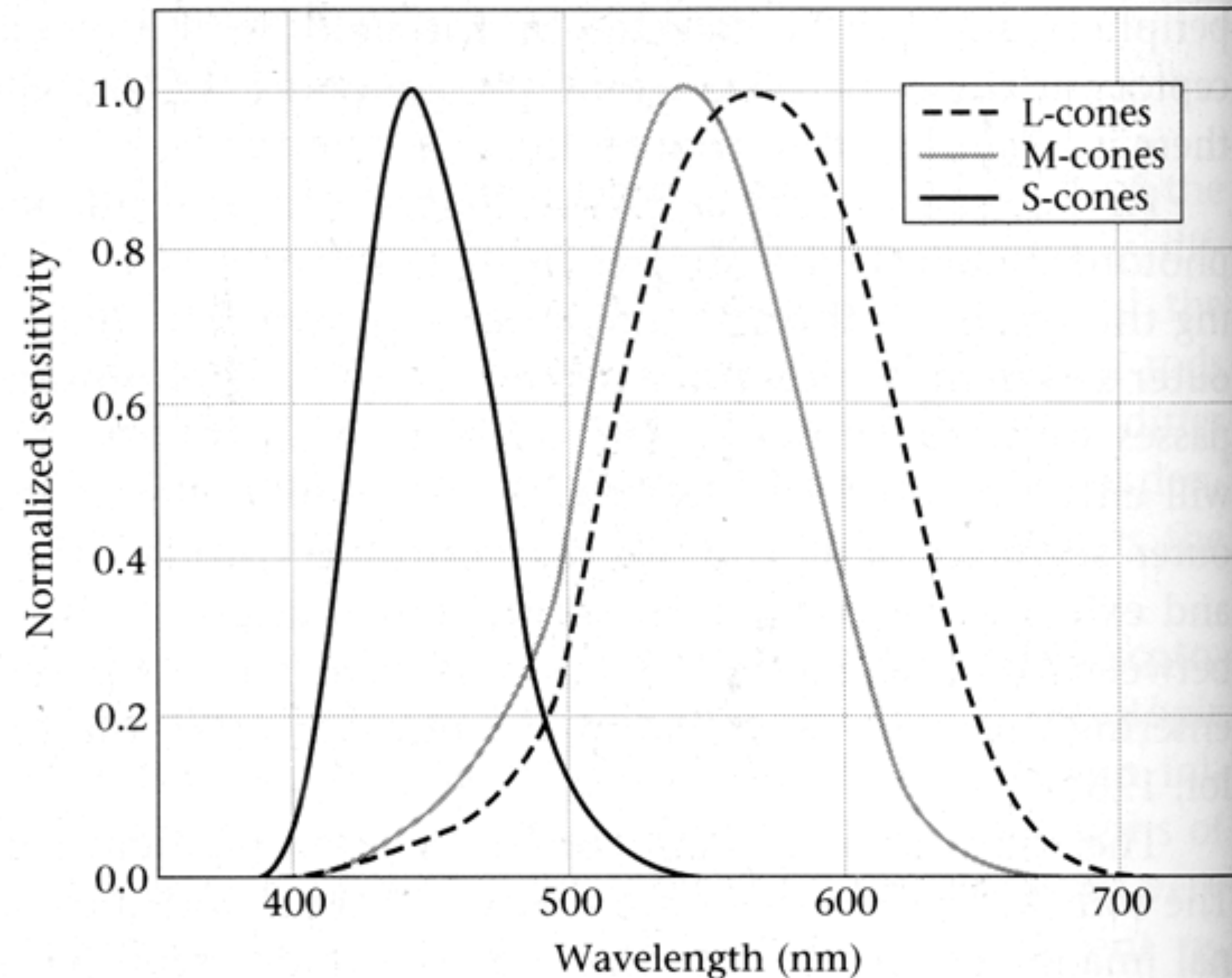


# Distribution of Rods and Cones

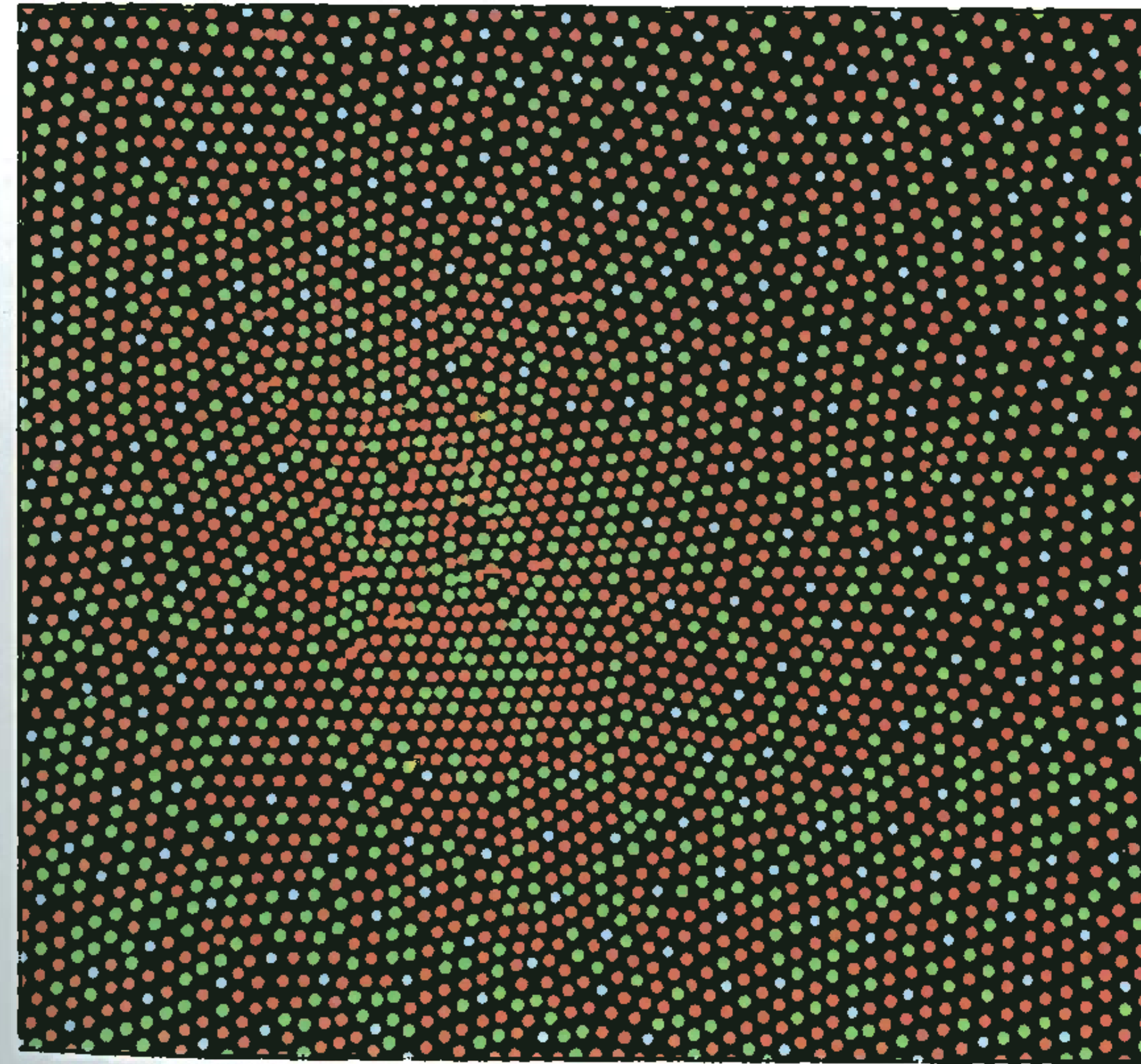


# Human eye photoreceptor spectral sensitivities

**3.3 SPECTRAL SENSITIVITIES OF THE L-, M-, AND S-CONES** in the human eye. The measurements are based on a light source at the cornea, so that the wavelength loss due to the cornea, lens, and other inert pigments of the eye plays a role in determining the sensitivity. Source: Stockman and MacLeod, 1993.



# Cone distribution



**Figure 1-5.** A representation of the retinal photoreceptor mosaic artificially colored to represent the relative proportions of L (red), M (green), and S (blue) cones in the human retina. Modeled after Williams et al. (1991).

From [Fairchild, “Human Color Vision”, 2005]



# What artifacts does this lead to?

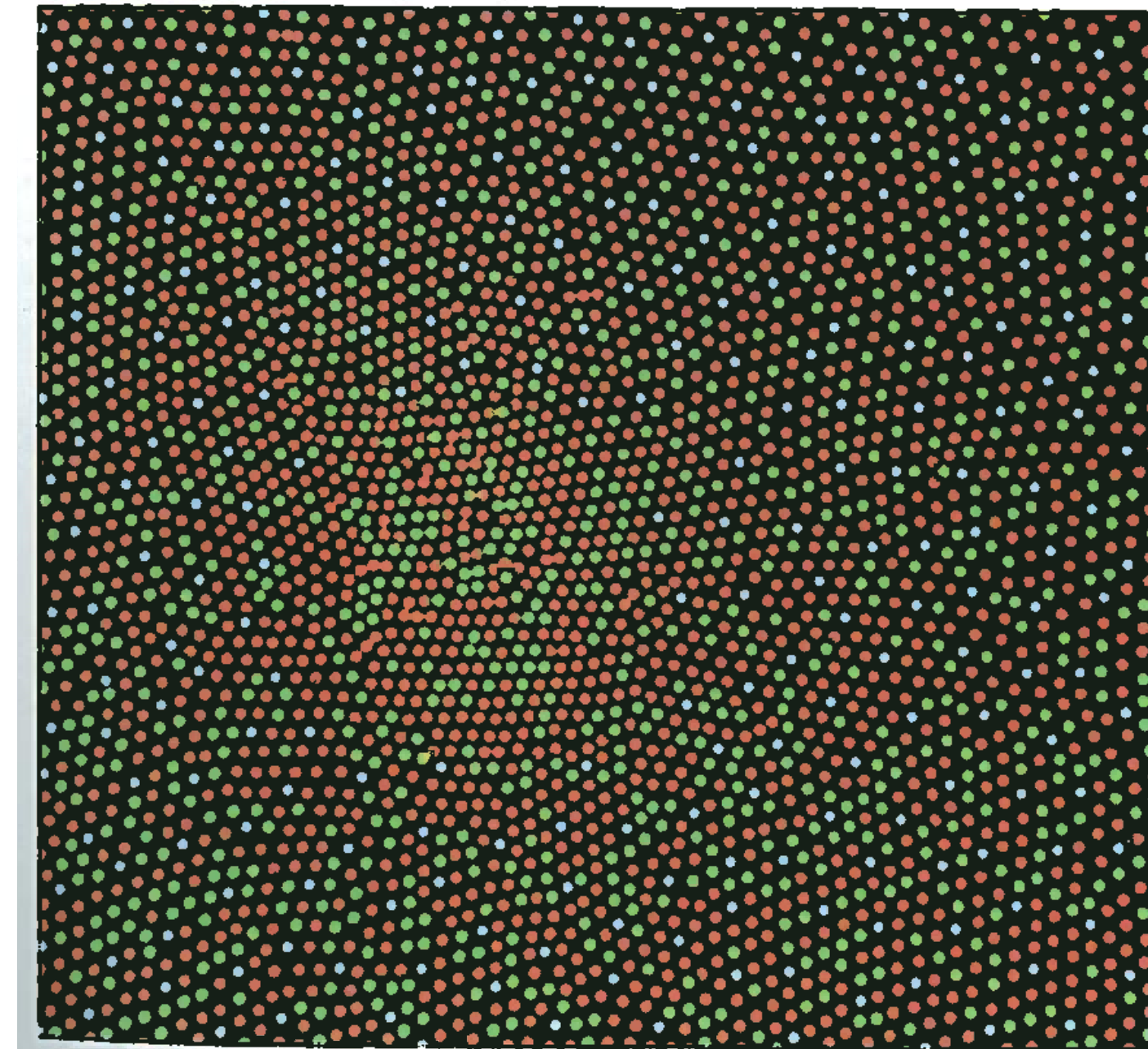
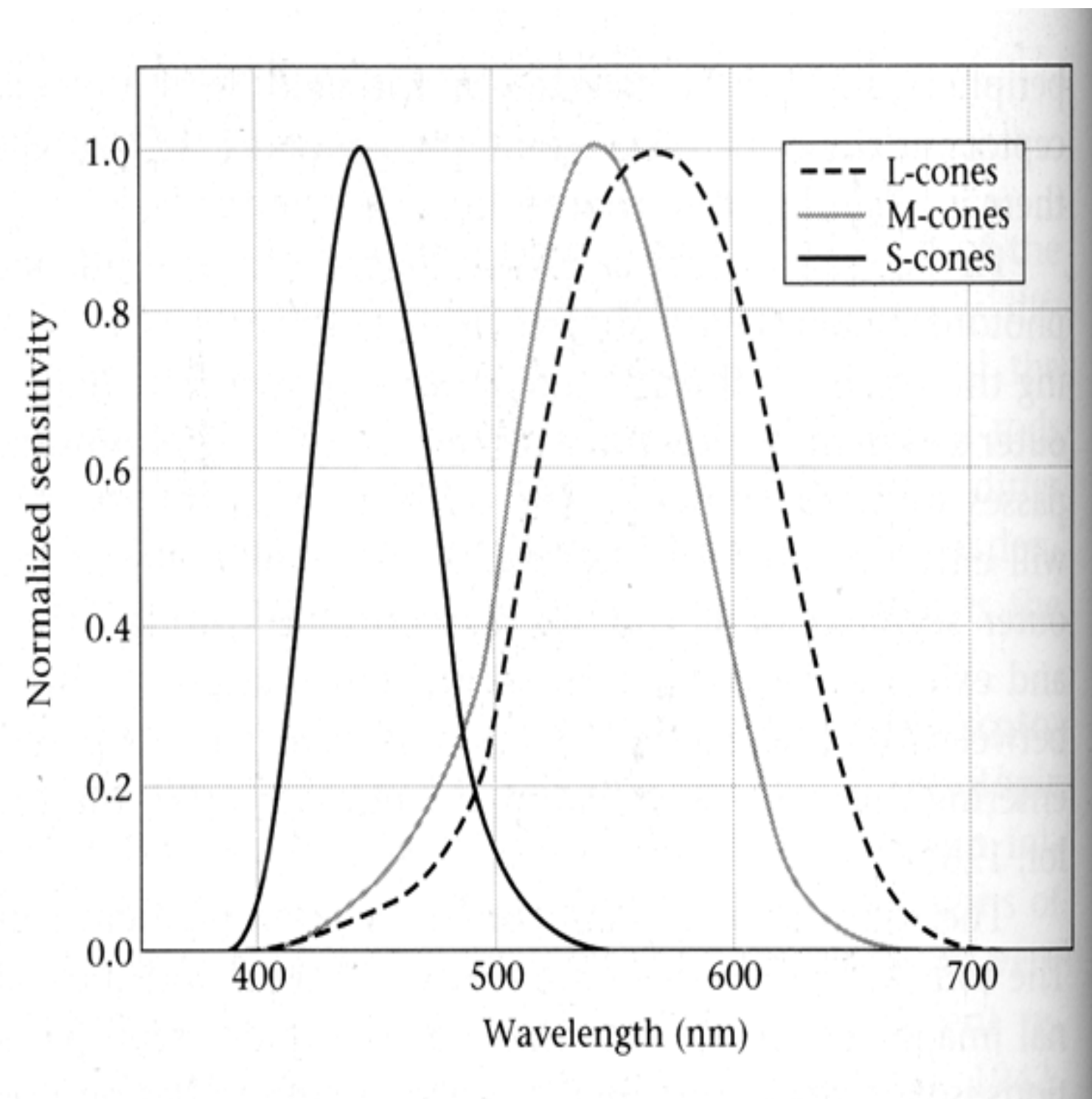
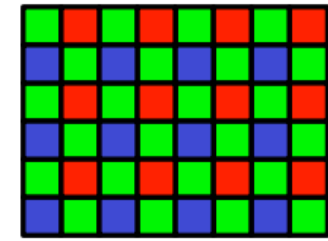


Figure 1-5. A representation of the retinal photoreceptor mosaic artificially colored to represent the relative proportions of L (red), M (green), and S (blue) cones in the human retina. Modeled after Williams et al. (1991).

- Color spatial artifacts.
- Under-sampling in spectra. Only three receptors!

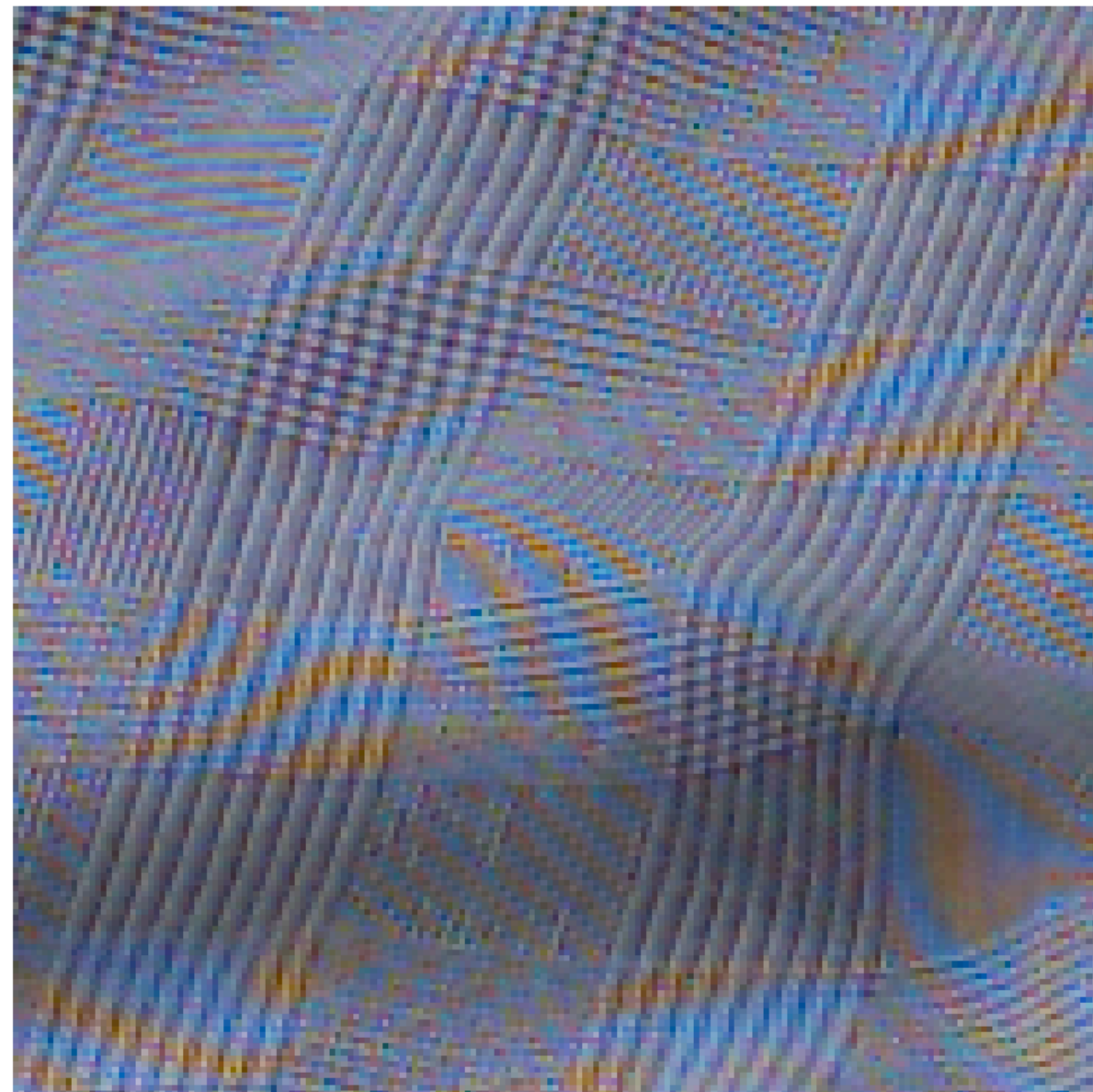


# Spatially offset color sampling in cameras



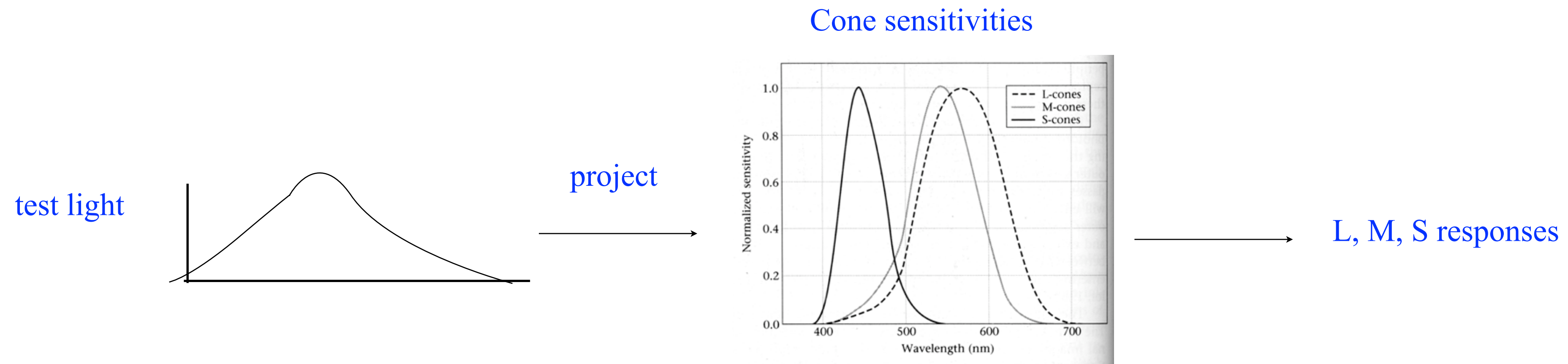
Sensor sampling pattern

Color fringes





# How we sense light spectra



Biophysics: integrate the response over all wavelengths, weighted by the photosensor's sensitivity at each wavelength.

Mathematically: take dot product of input spectrum with the cone sensitivity basis vectors. Project the high-dimensional test light into a 3D space.

$$R = C t$$

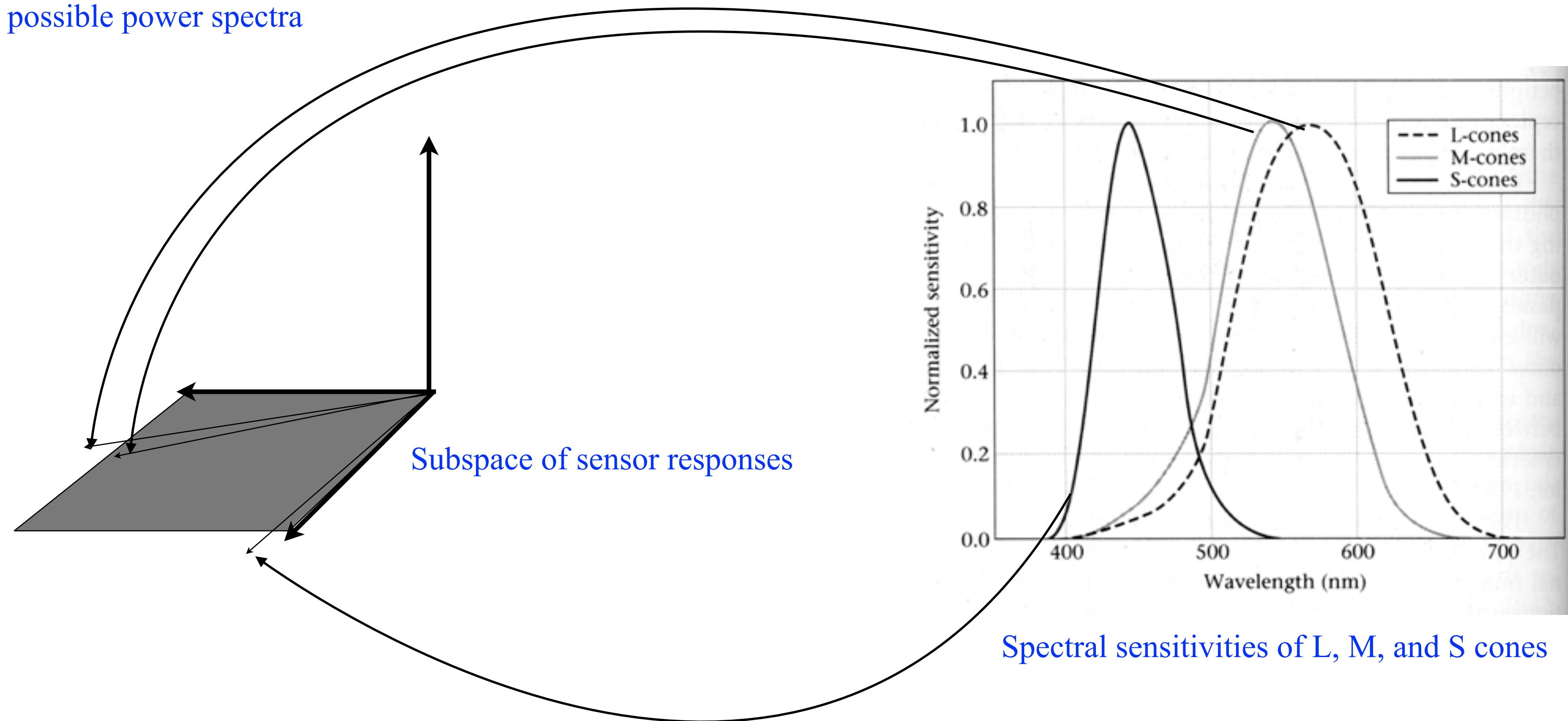
Diagram illustrating the mathematical relationship:

cone responses  $R$  = cone sensitivities  $C$  \* input spectrum  $t$

The diagram shows a vertical teal bar labeled  $R$  (cone responses) equals a teal rectangle labeled  $C$  (cone sensitivities) multiplied by a vertical teal bar labeled  $t$  (input spectrum).

# Cone response curves as basis vectors in a 3D subspace of light power spectra

3D depiction of the high-dimensional space of all possible power spectra





UNITED STATES DEPARTMENT OF AGRICULTURE

# COLOR STANDARDS

*for*

FROZEN

FRENCH FRIED POTATOES



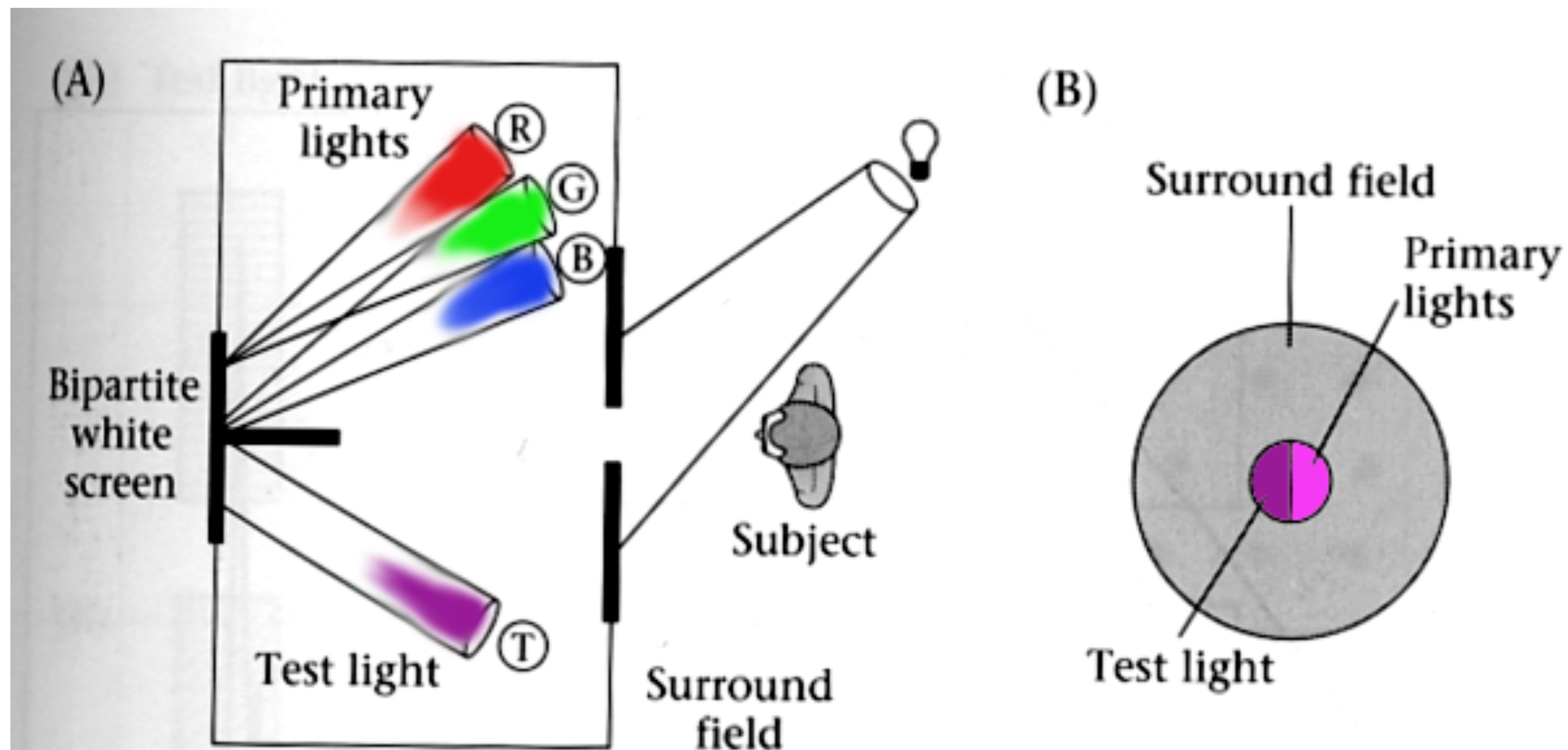
FOURTH EDITION, 1988  
© 1988 KOLLMORDEN CORPORATION

MUNSELL COLOR  
BALTIMORE, MARYLAND  
64-1



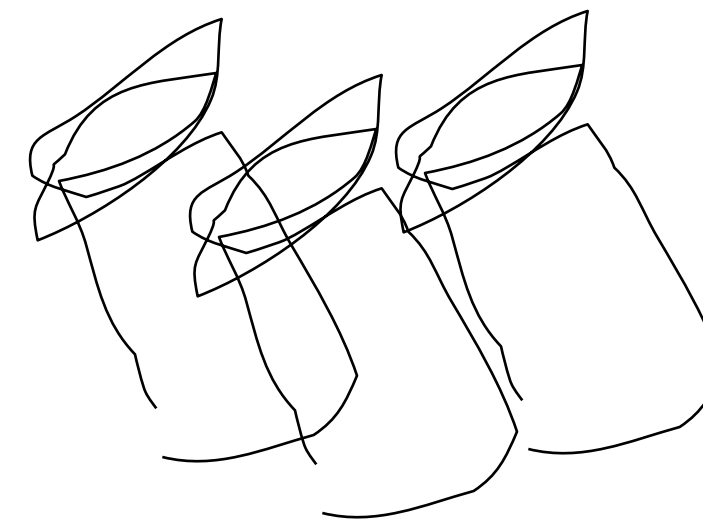
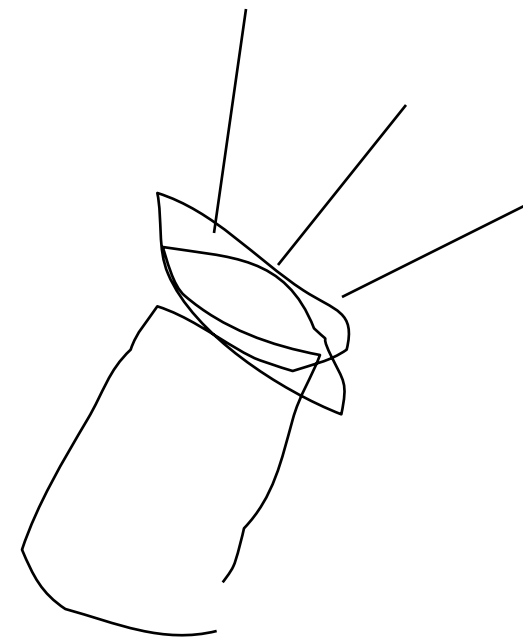
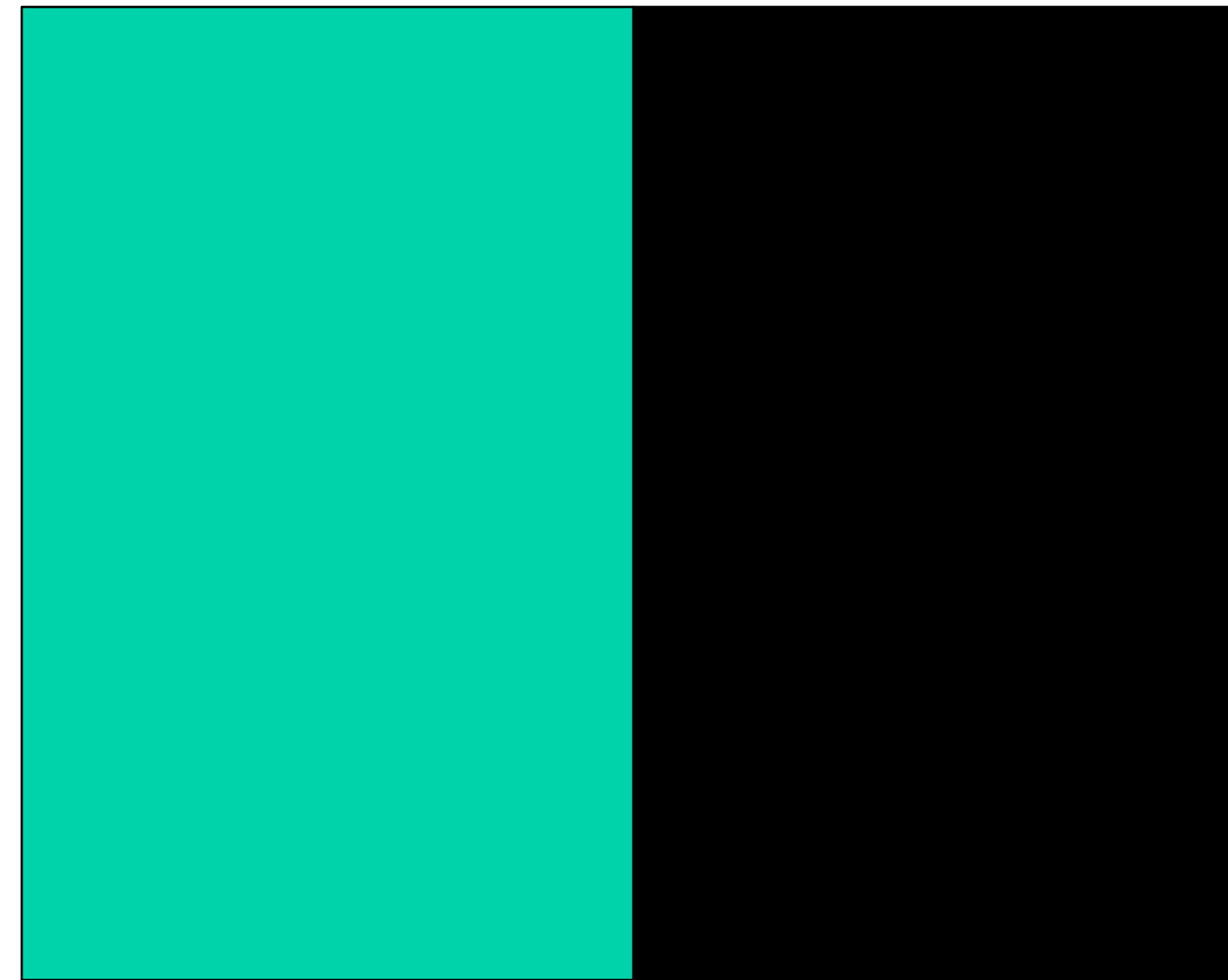


# Color matching experiment

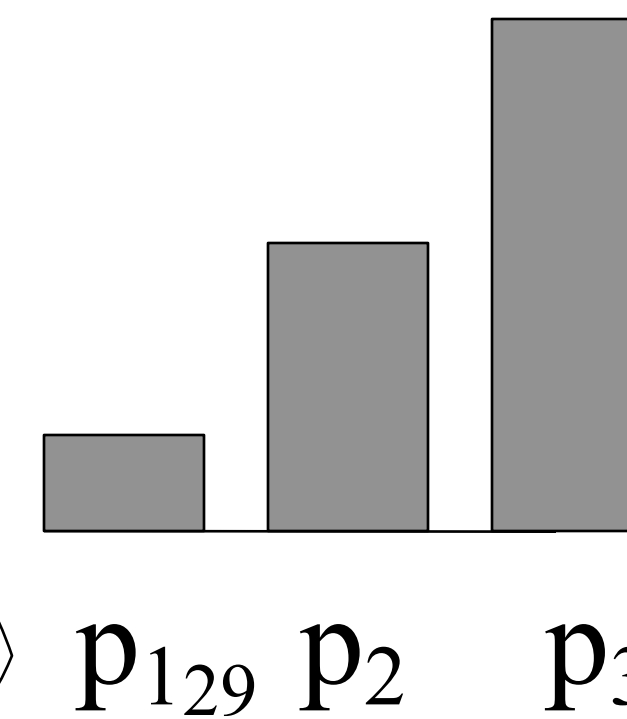
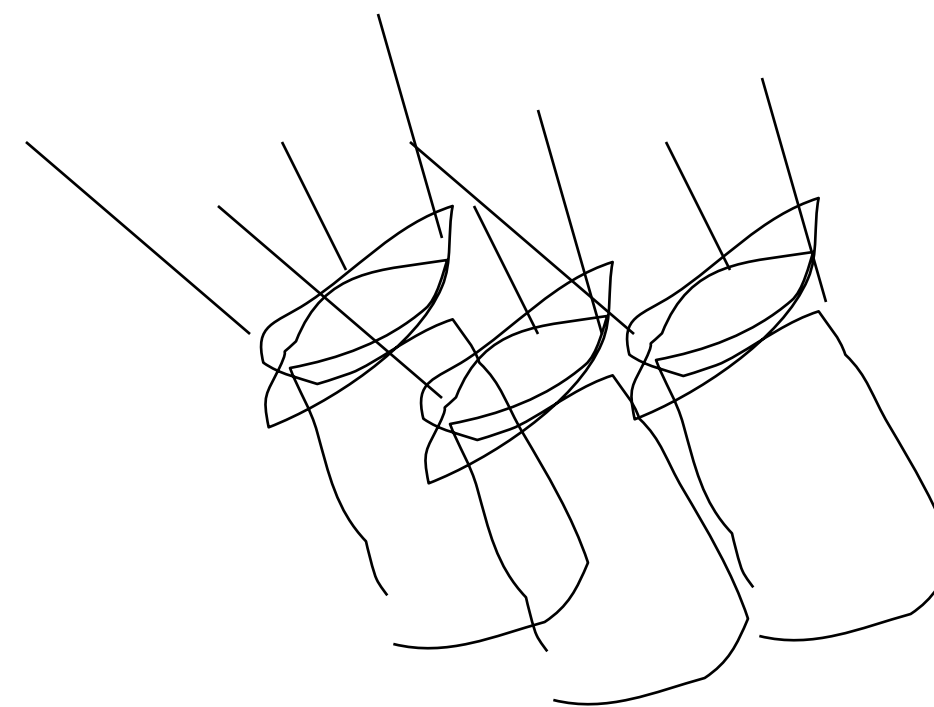
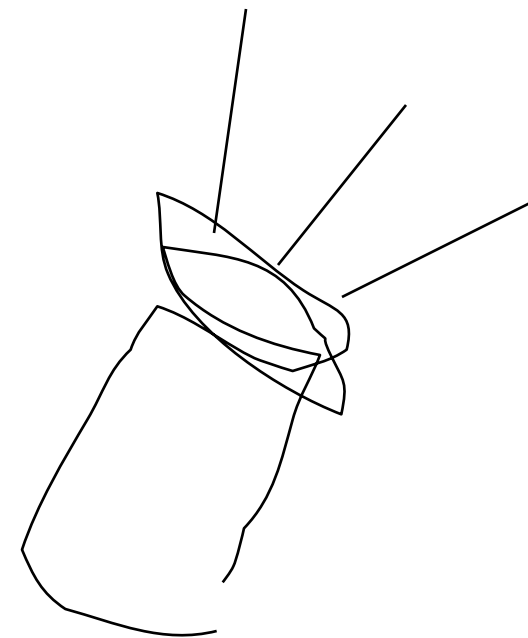
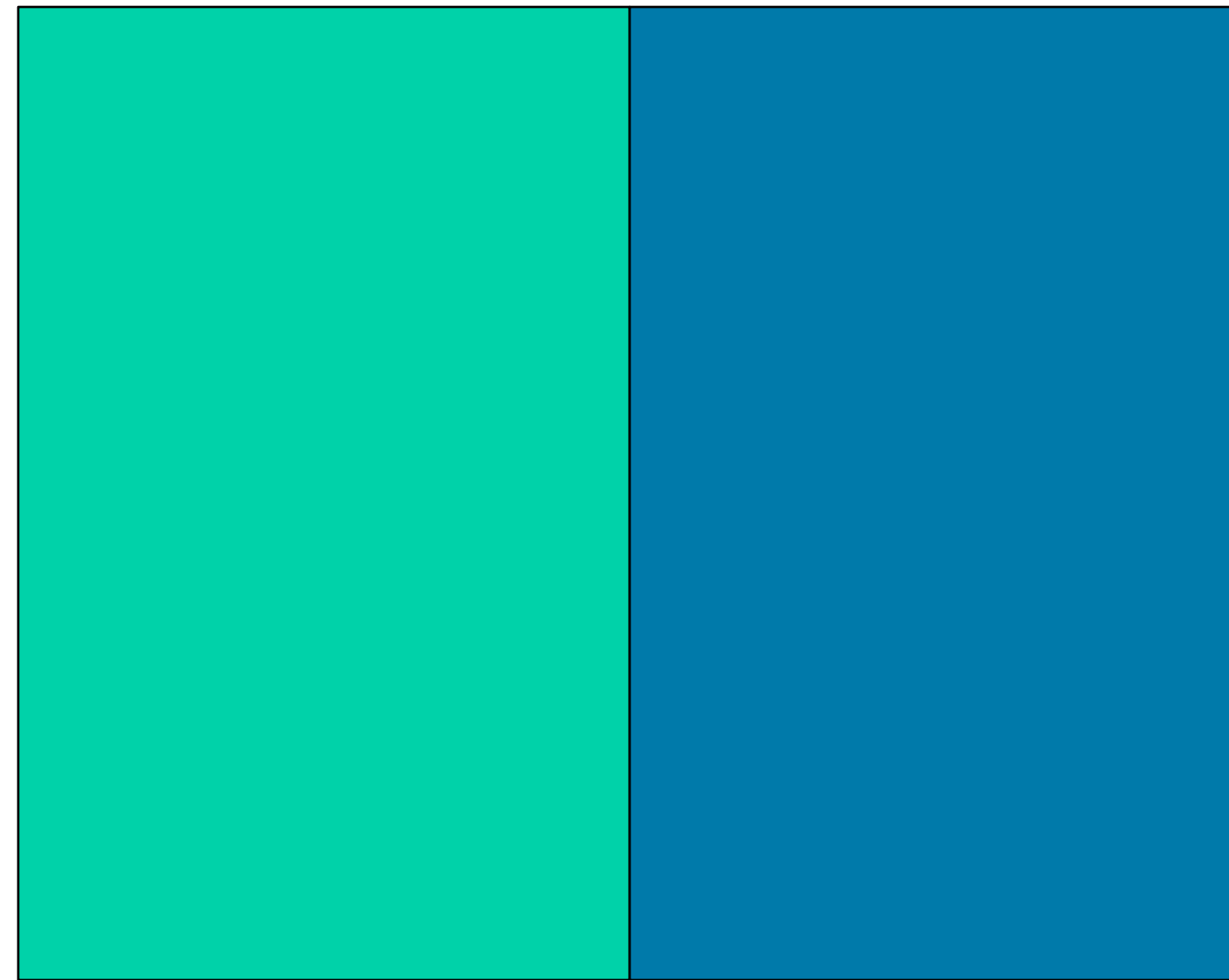


**4.10 THE COLOR-MATCHING EXPERIMENT.** The observer views a bipartite field and adjusts the intensities of the three primary lights to match the appearance of the test light. (A) A top view of the experimental apparatus. (B) The appearance of the stimuli to the observer. After Judd and Wyszecki, 1975.

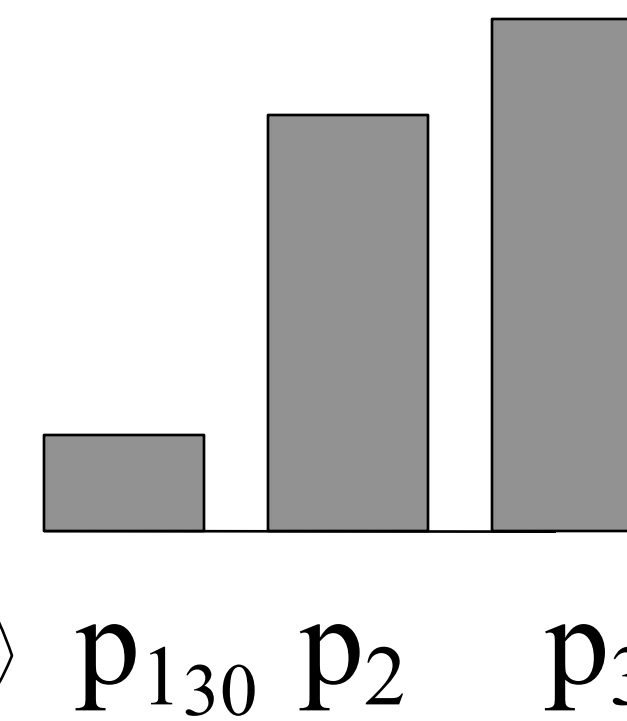
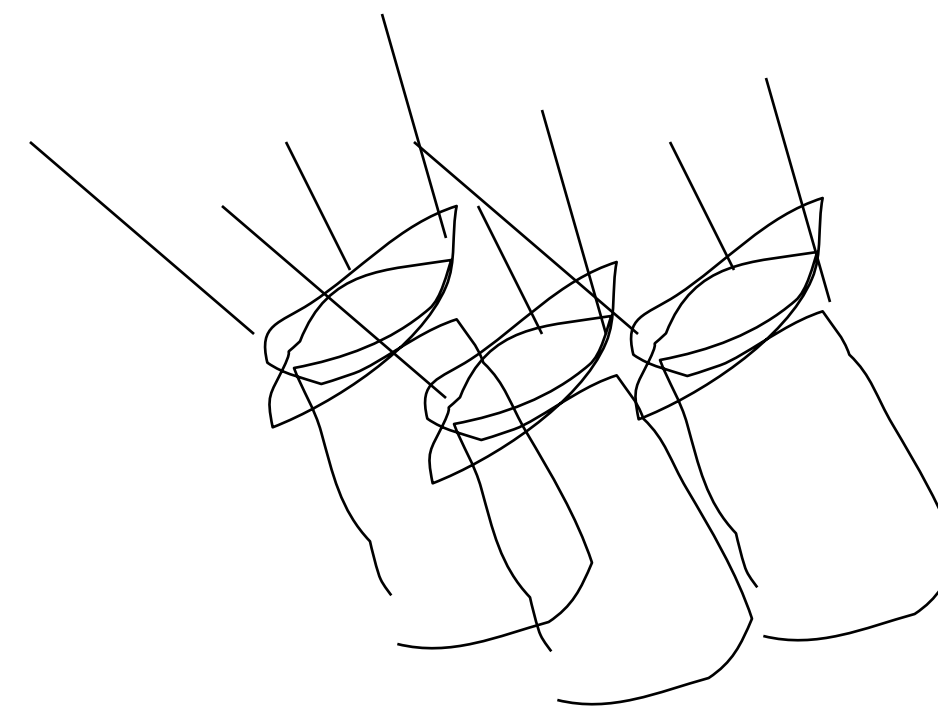
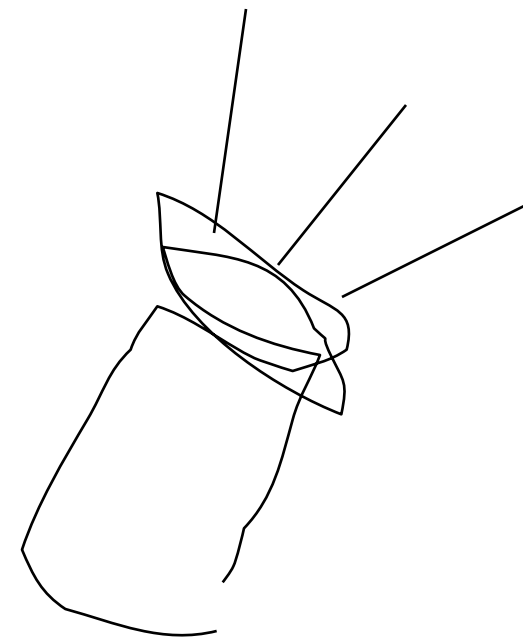
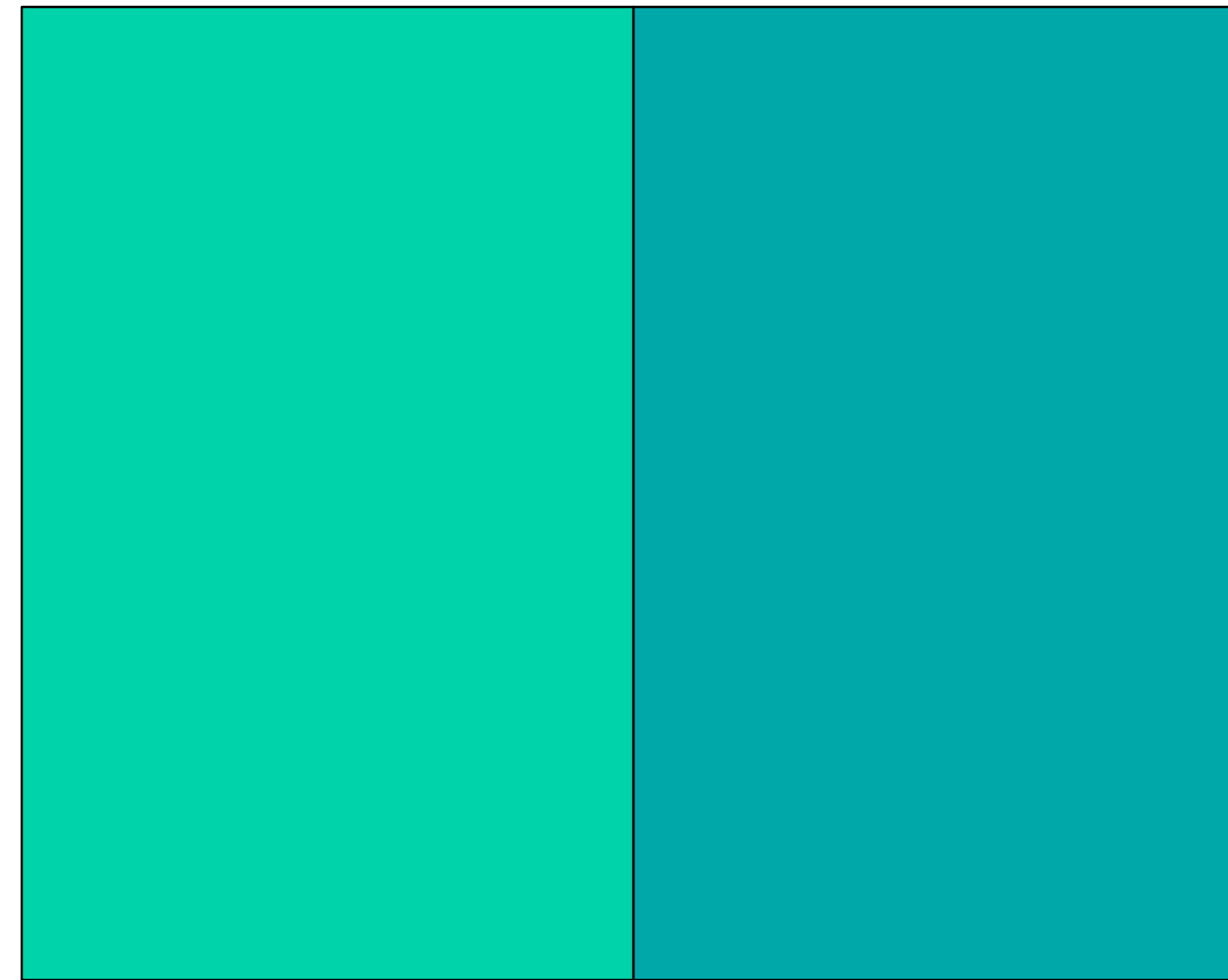
# Color matching experiment



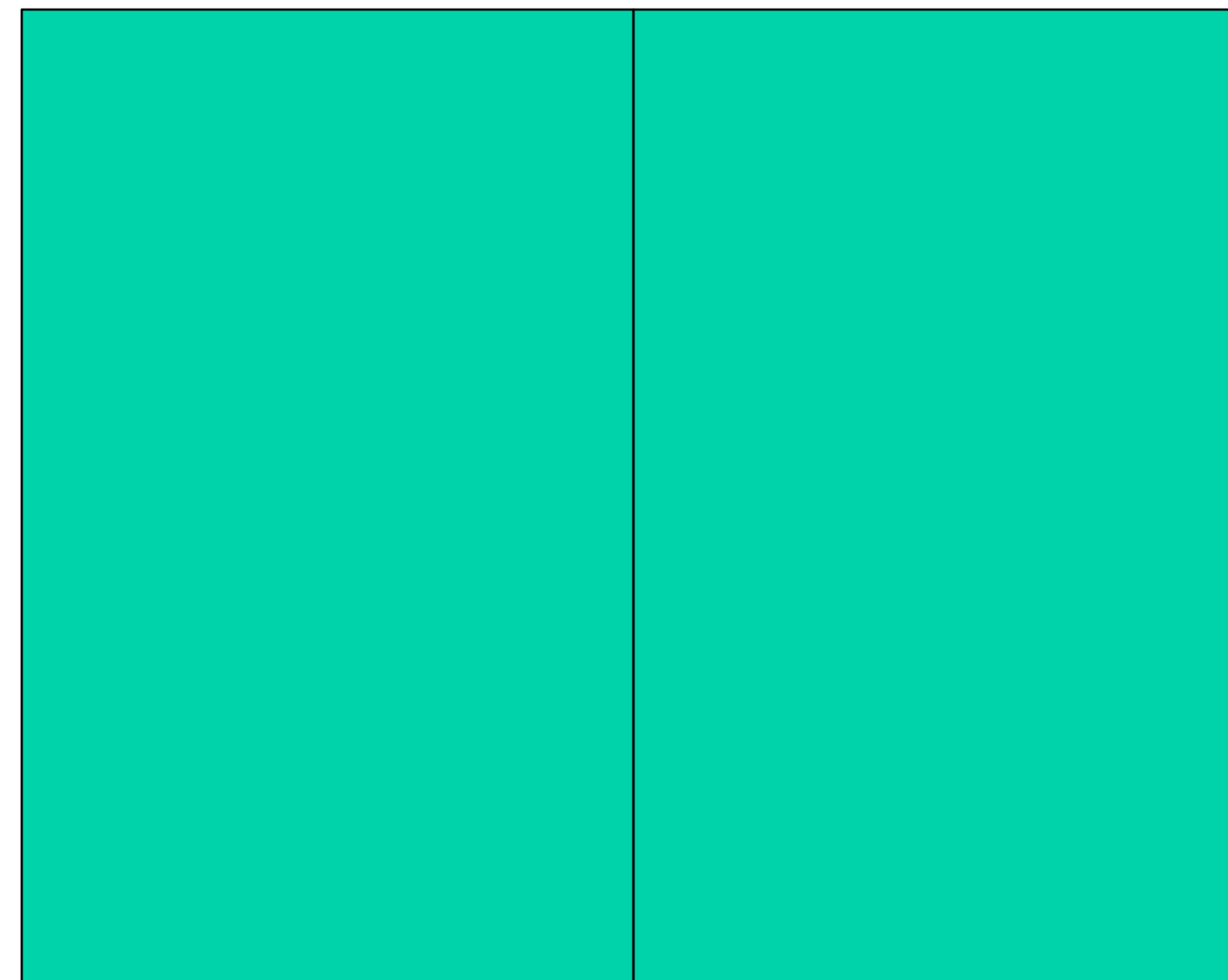
# Color matching experiment



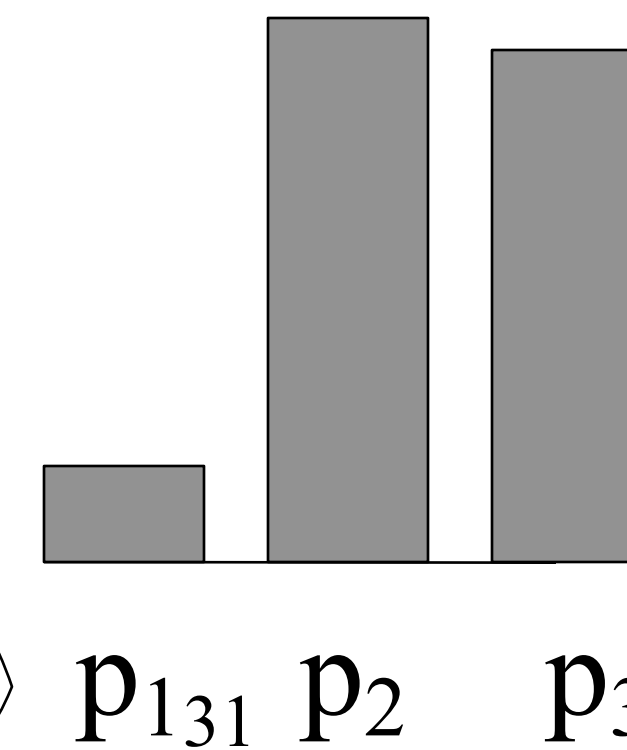
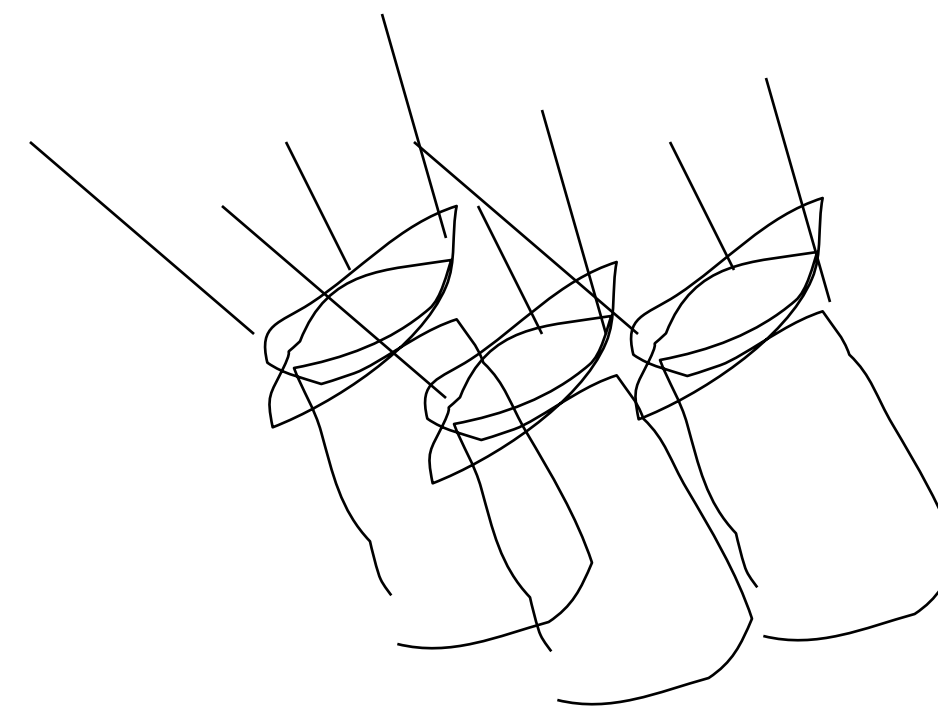
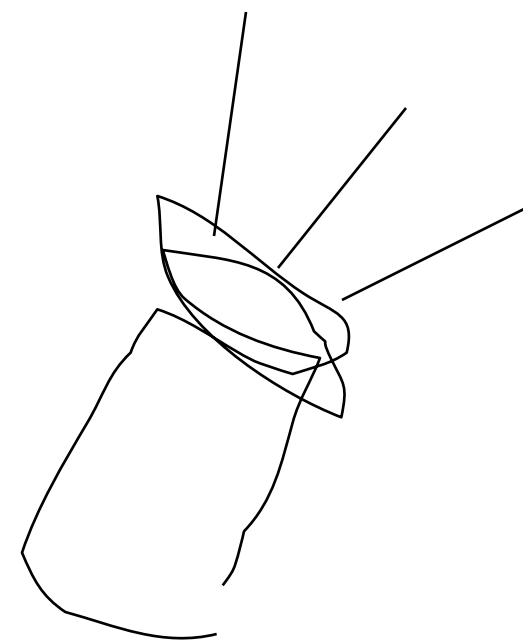
# Color matching experiment



# Color matching experiment

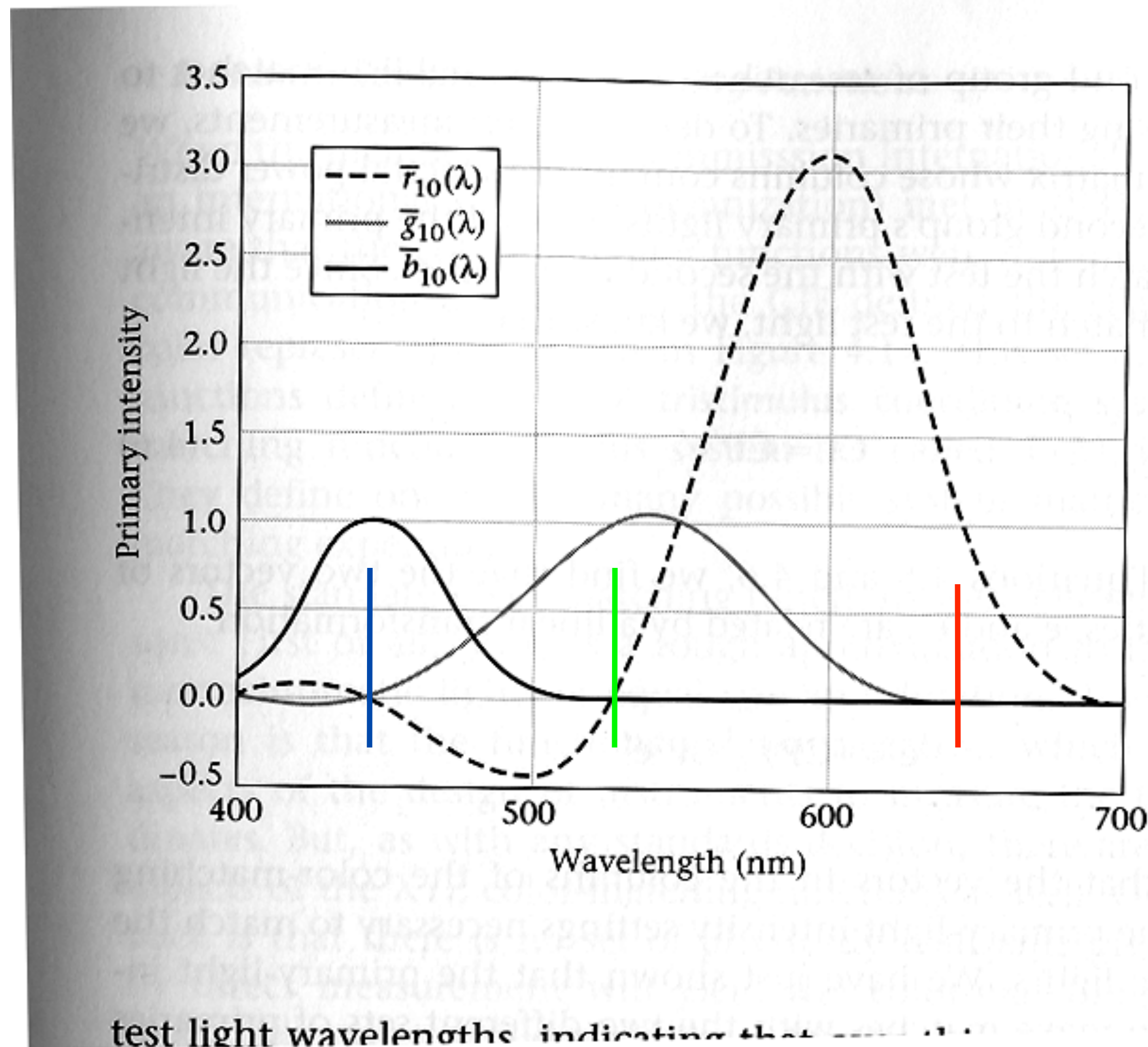


The primary color  
amounts needed  
for a match





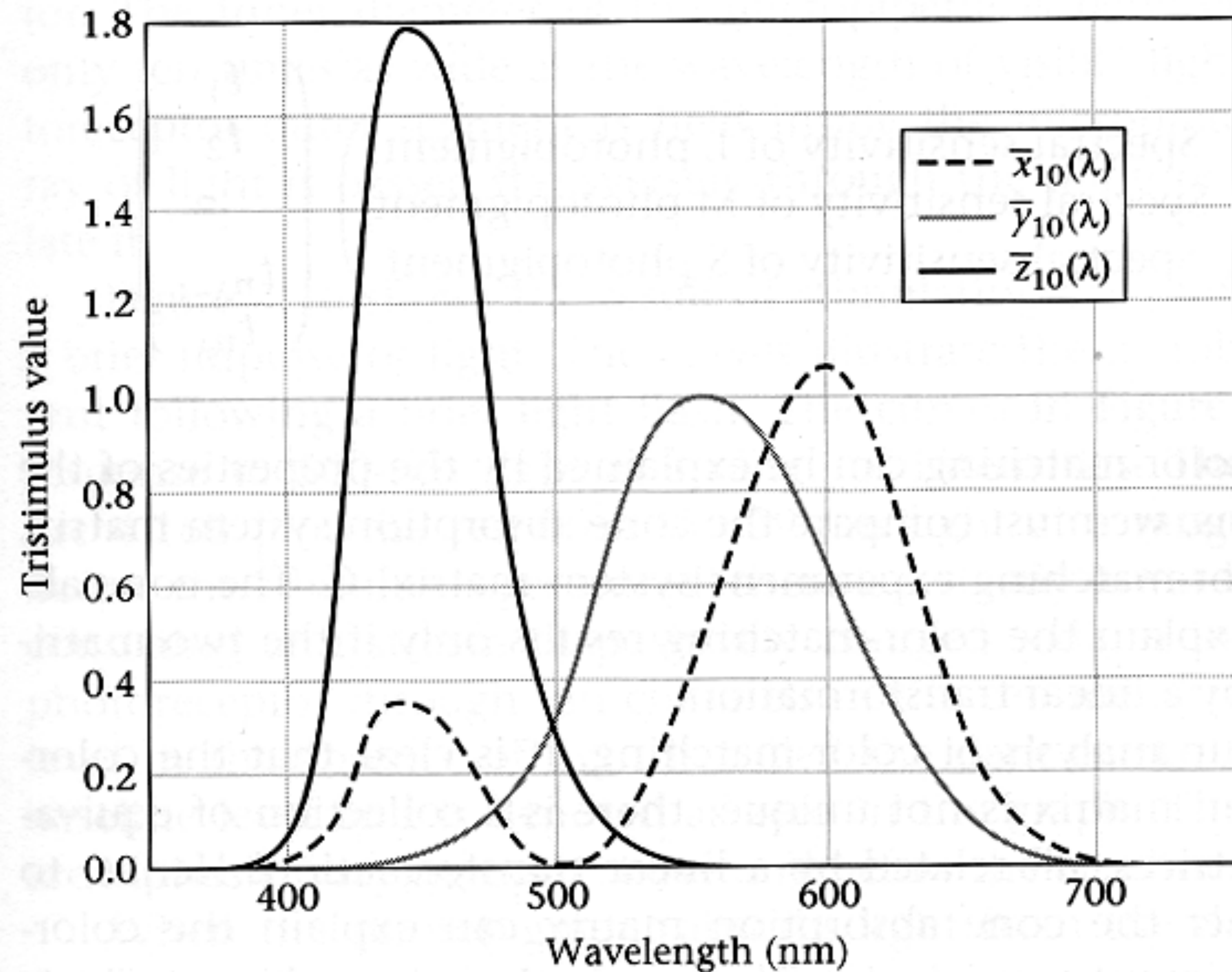
“Color matching functions” let us find other basis vectors for the eye response subspace of light power spectra



■  $p_1 = 645.2 \text{ nm}$   
■  $p_2 = 525.3 \text{ nm}$   
■  $p_3 = 444.4 \text{ nm}$

**4.13 THE COLOR-MATCHING FUNCTIONS ARE THE ROWS OF THE COLOR-MATCHING SYSTEM MATRIX.** The functions measured by Stiles and Burch (1959) using a 10-degree bipartite field and primary lights at the wavelengths 645.2 nm, 525.3 nm, and 444.4 nm with unit radiant power are shown. The three functions in this figure are called  $\bar{r}_{10}(\lambda)$ ,  $\bar{g}_{10}(\lambda)$ , and  $\bar{b}_{10}(\lambda)$ .

# Other color matching functions



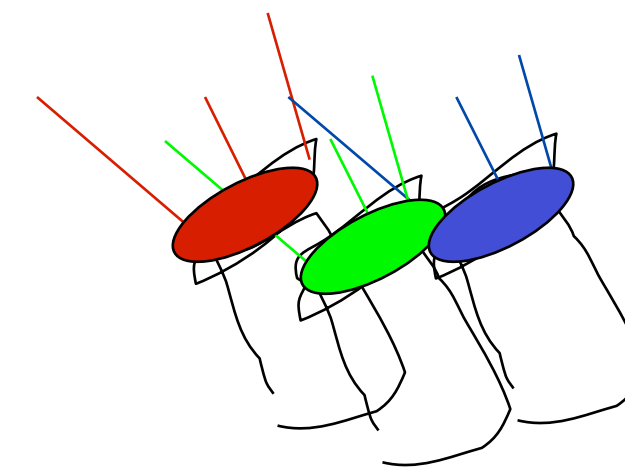
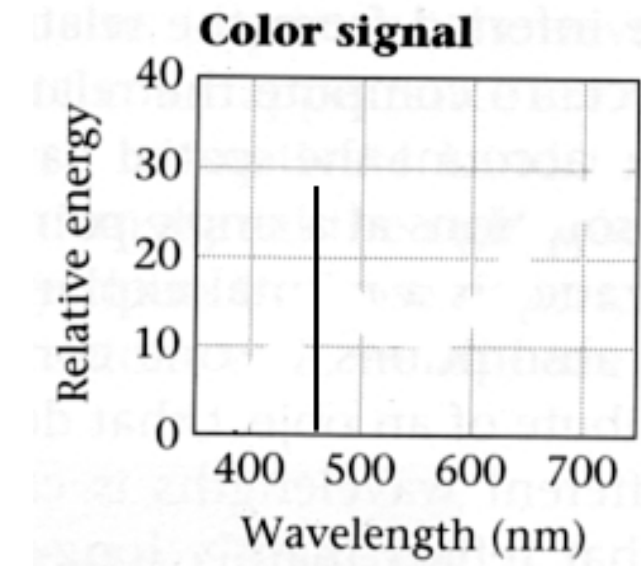
**4.14 THE XYZ STANDARD COLOR-MATCHING FUNCTIONS.** In 1931 the CIE standardized a set of color-matching functions for image interchange. These color-matching functions are called  $\bar{x}(\lambda)$ ,  $\bar{y}(\lambda)$ , and  $\bar{z}(\lambda)$ . Industrial applications commonly describe the color properties of a light source using the three primary intensities needed to match the light source that can be computed from the XYZ color-matching functions.



# Using the color matching functions to predict the primary match to a new spectral signal

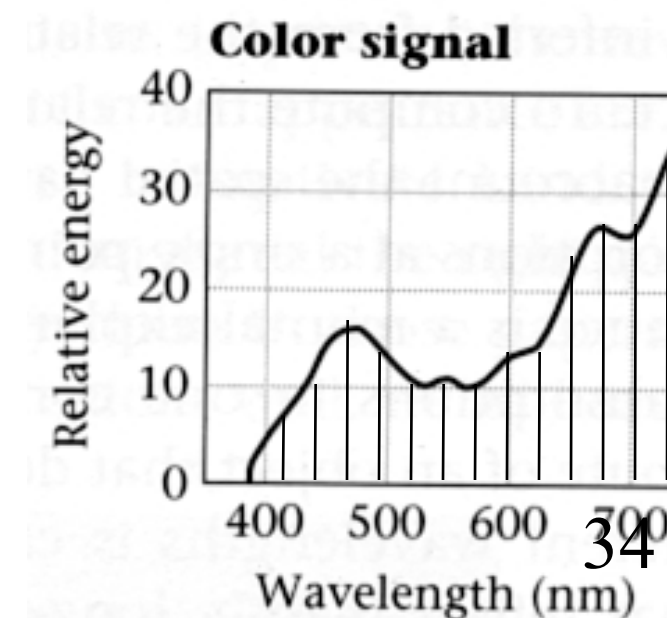
We know that a monochromatic light of wavelength  $\lambda_i$  will be matched by the amounts  $c_1(\lambda_i), c_2(\lambda_i), c_3(\lambda_i)$

of each primary.



And any spectral signal can be thought of as a linear combination of very many monochromatic lights, with the linear coefficient given by the spectral power at each wavelength.

$$\vec{t} = \begin{pmatrix} t(\lambda_1) \\ \vdots \\ t(\lambda_N) \end{pmatrix}$$





Using the color matching functions to predict the primary match to a new spectral signal

Store the color matching functions in the rows of the matrix,  $C$

$$C = \begin{pmatrix} c_1(\lambda_1) & \cdots & c_1(\lambda_N) \\ c_2(\lambda_1) & \cdots & c_2(\lambda_N) \\ c_3(\lambda_1) & \cdots & c_3(\lambda_N) \end{pmatrix}$$

Let the new spectral signal be described by the vector  $t$ .

$$\vec{t} = \begin{pmatrix} t(\lambda_1) \\ \vdots \\ t(\lambda_N) \end{pmatrix}$$

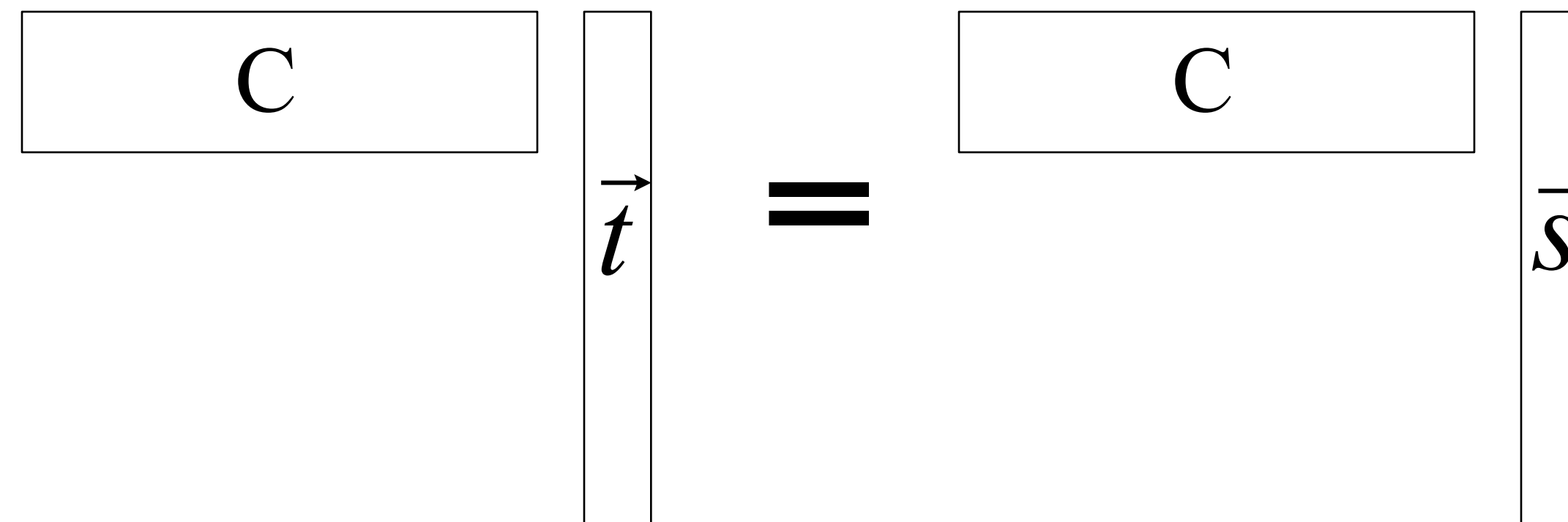
# Color metamerism: different spectra looking the same color

Two spectra,  $t$  and  $s$ , perceptually match when

$$C\vec{t} = C\vec{s}$$

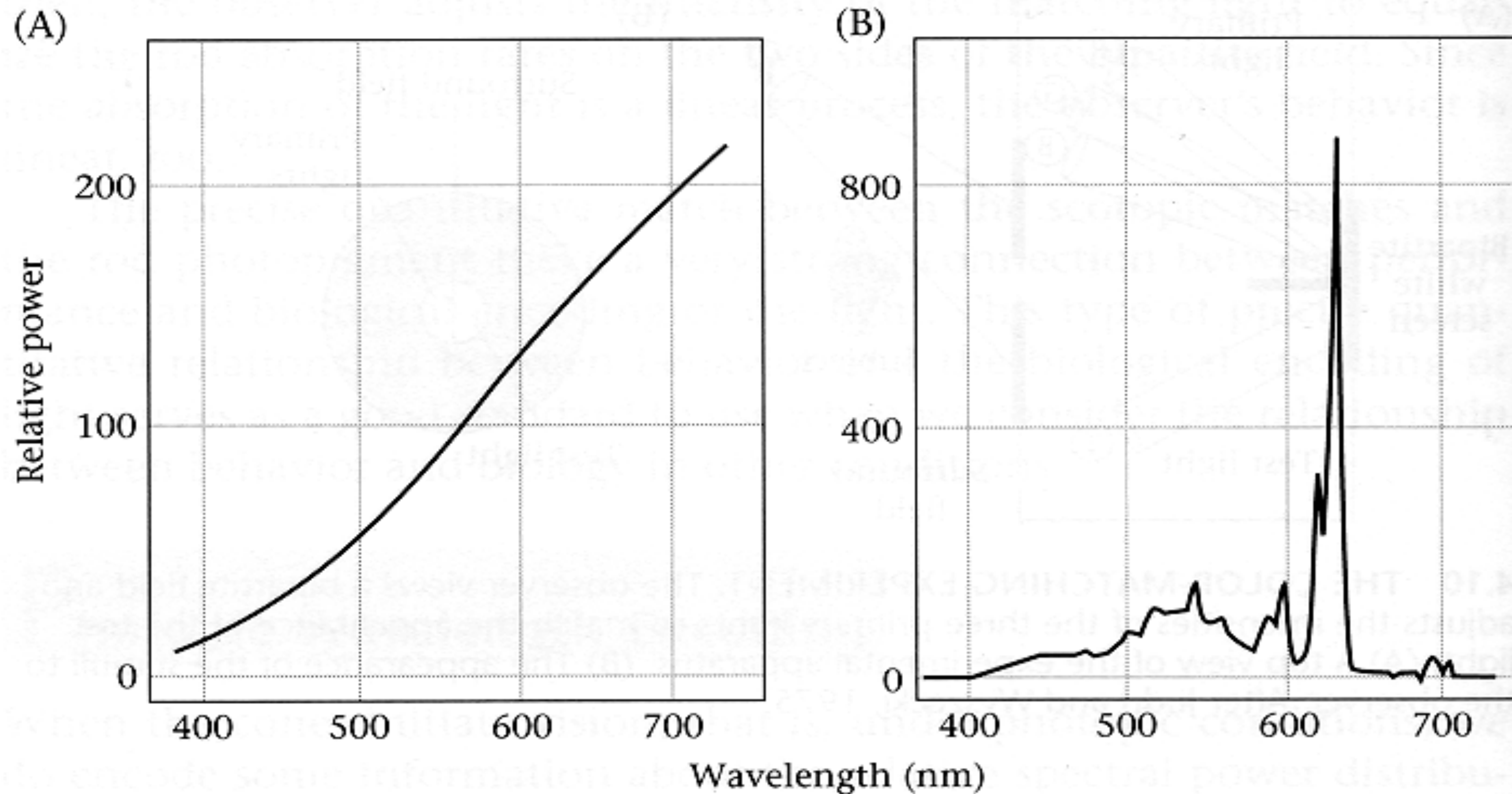
where  $C$  are the color matching functions for some set of primaries.

Graphically,



# Metameric lights

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

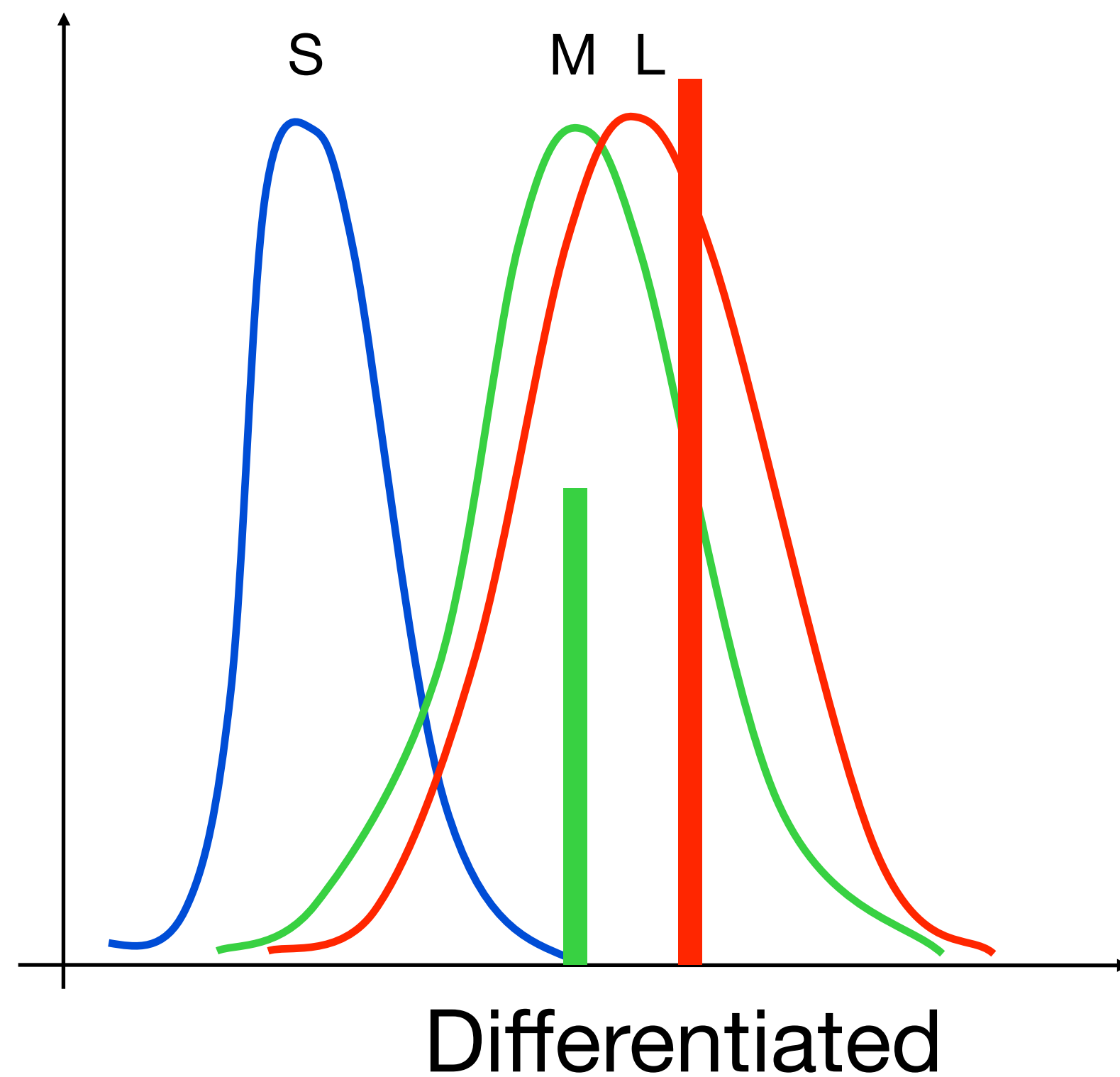


**4.11 METAMERIC LIGHTS.** Two lights with these spectral power distributions appear identical to most observers and are called metamers. (A) An approximation to the spectral power distribution of a tungsten bulb. (B) The spectral power distribution of light emitted from a conventional television monitor whose three phosphor intensities were set to match the light in panel A in appearance.

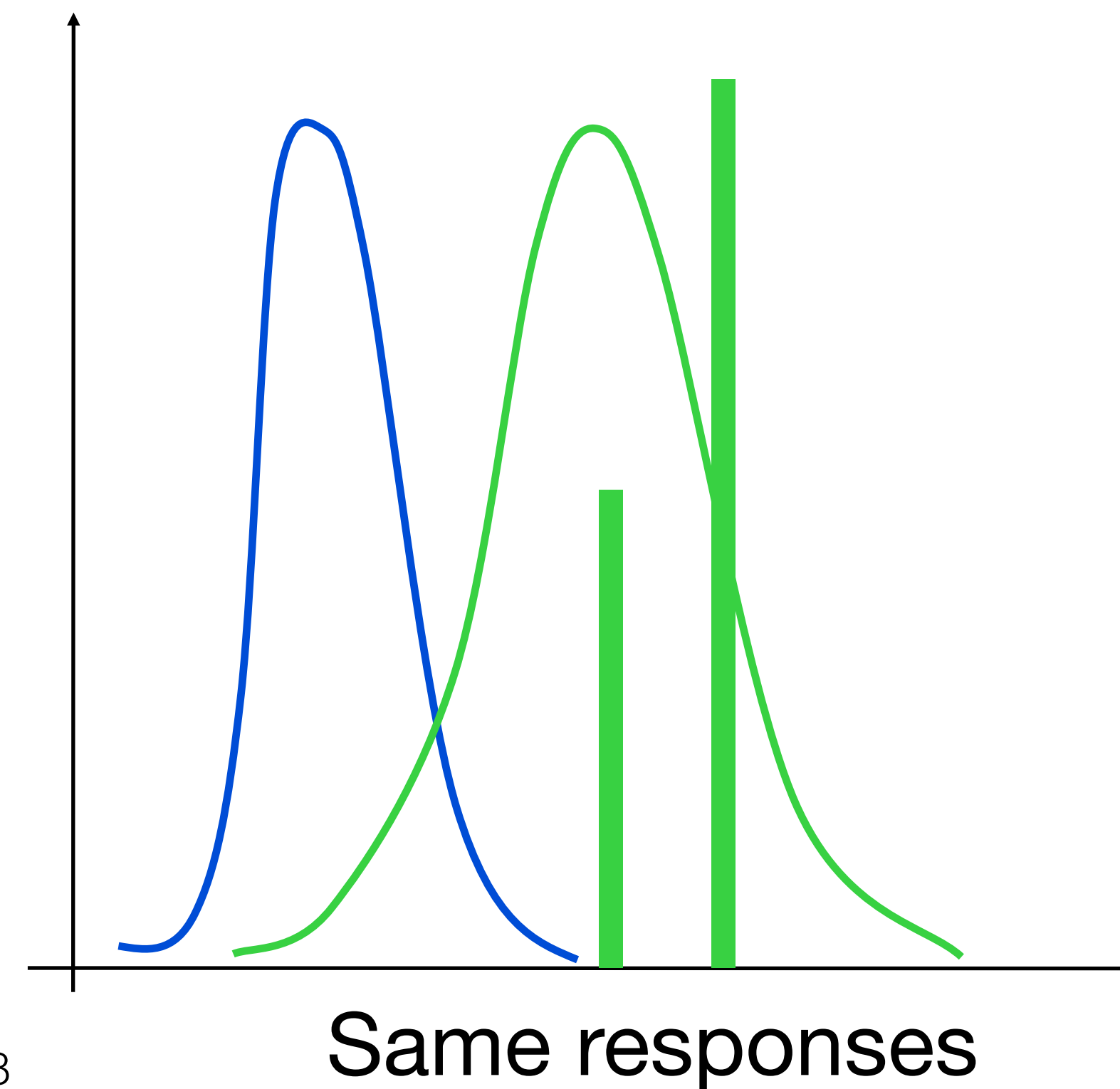


# Color blindness

- Classical case: 1 type of cone is missing (e.g. red)
- Makes it impossible to distinguish some spectra
- There are also tetrachromats, who have 4 cones!



38



Source: F. Durand

# Lab color components



L



a



b

A rotation of the color coordinates into directions that are more perceptually meaningful:  
L: luminance,  
a: red-green,  
b: blue-yellow

How sensitive are we to spatial frequency in these dimensions?



# Blurring the “L” component



original



processed



L



a



b



# Blurring the “a” component



original



processed



L



a



b



# Blurring the “b” component



original



processed



L



a



b

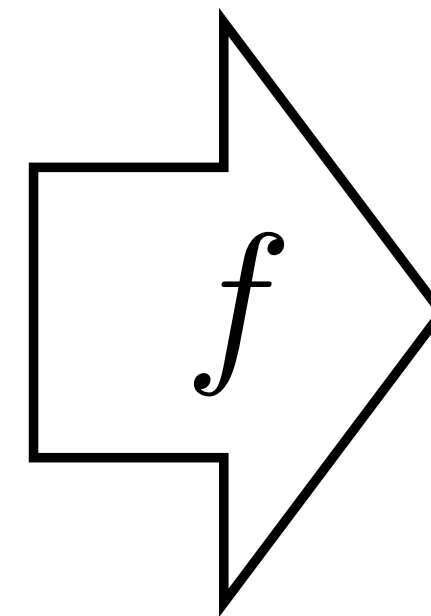
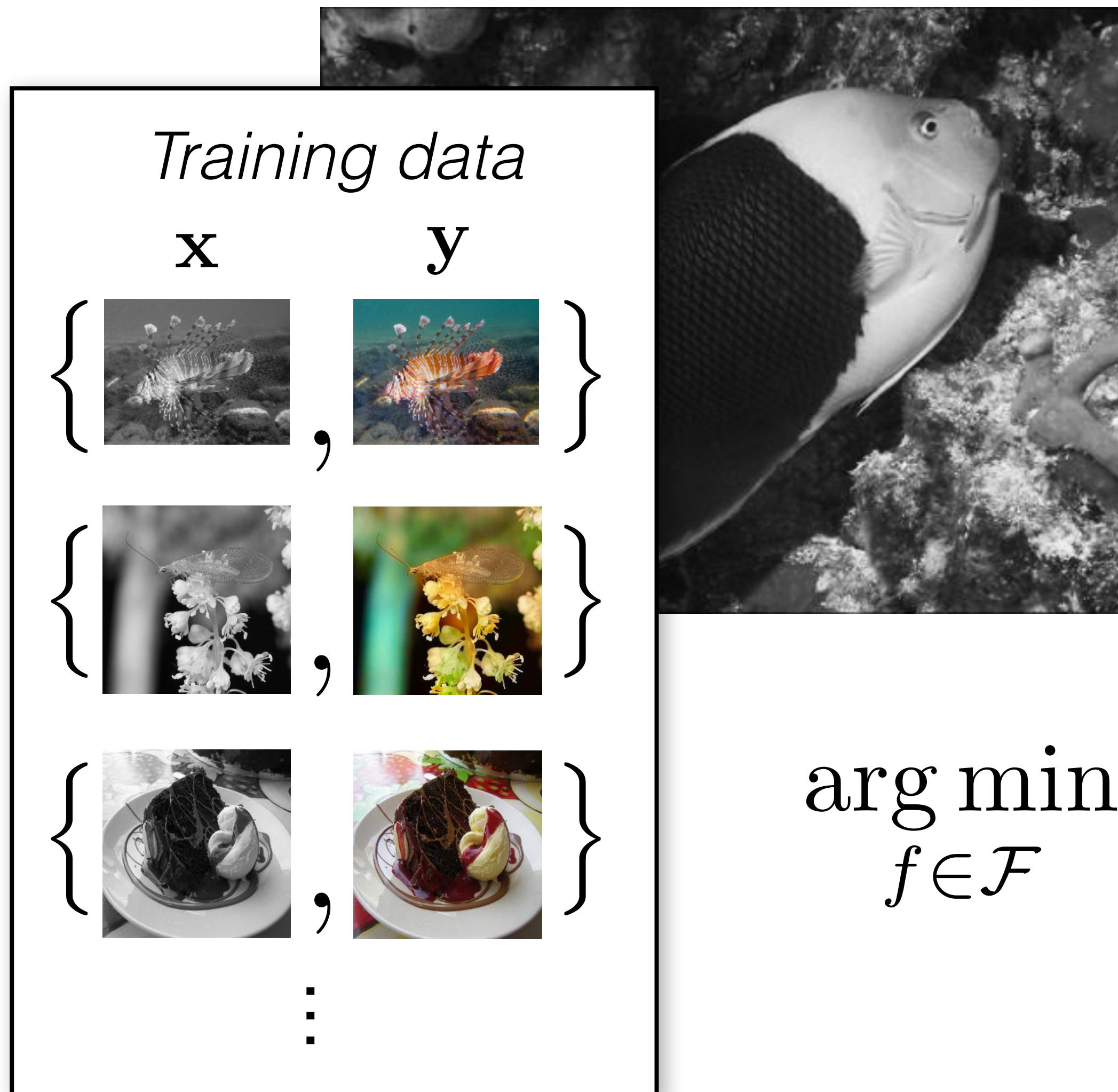
- Suggests that we are less sensitive to spatial position of color!
- Easy way to compress images (used in JPEG).



# Application: image colorization

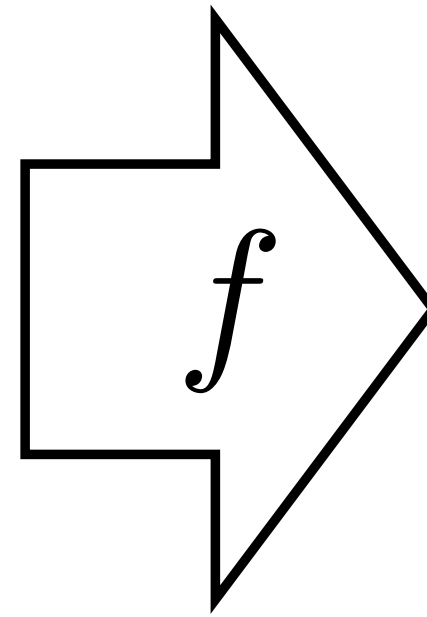
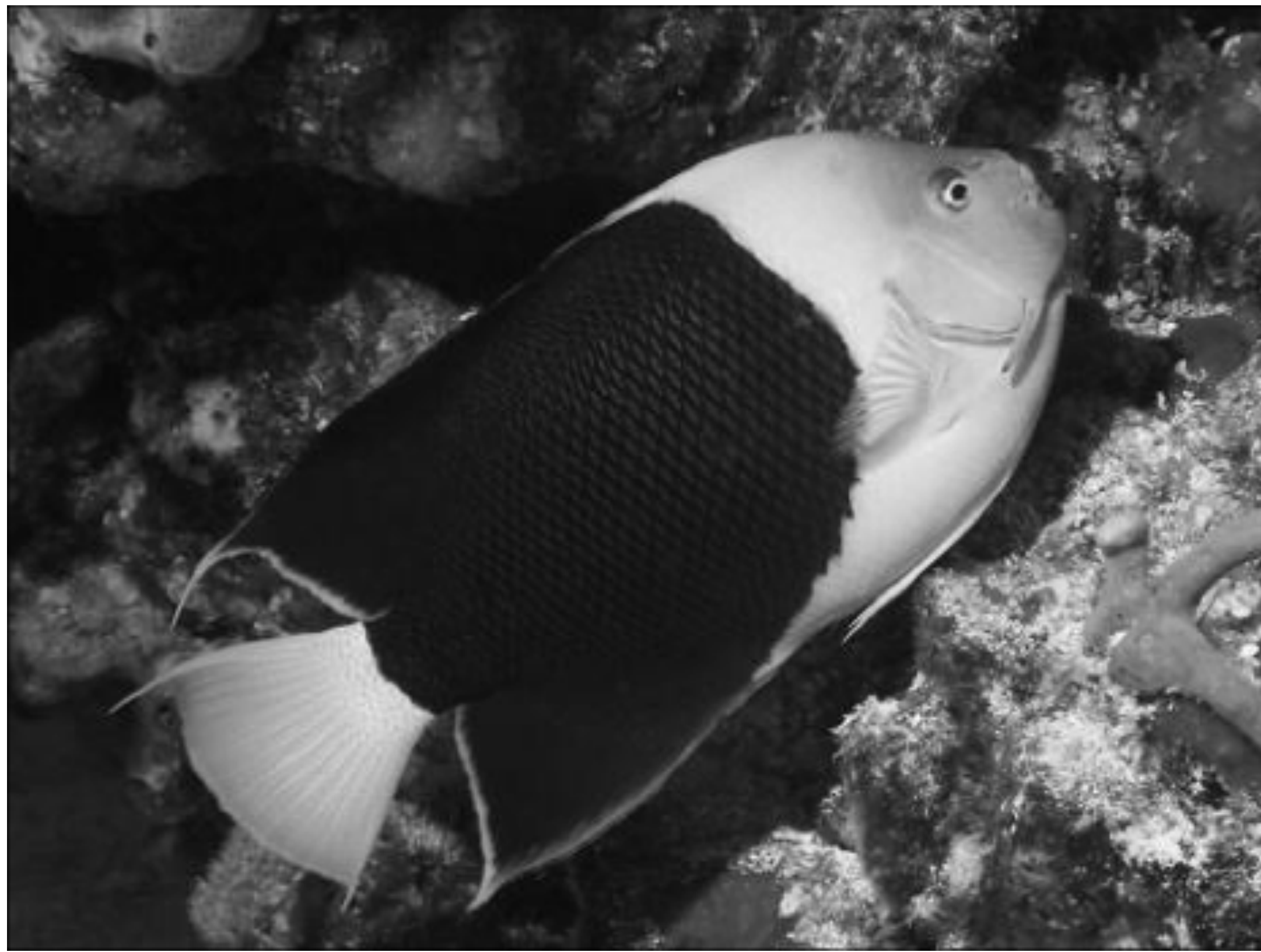
Input  $\mathbf{x}$

Output  $\mathbf{y}$



$$\arg \min_{f \in \mathcal{F}} \sum_{i=1}^N \mathcal{L}(f(\mathbf{x}_i), \mathbf{y}_i)$$





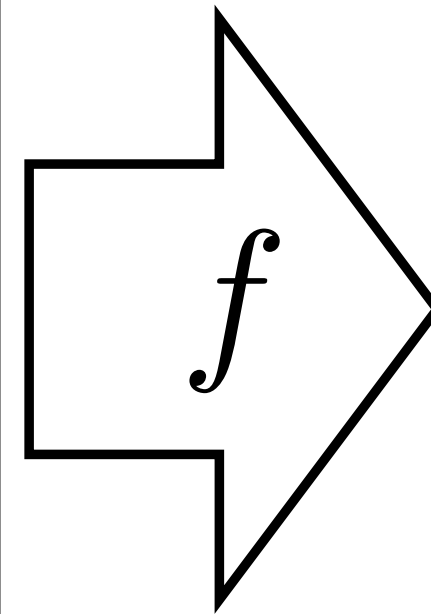
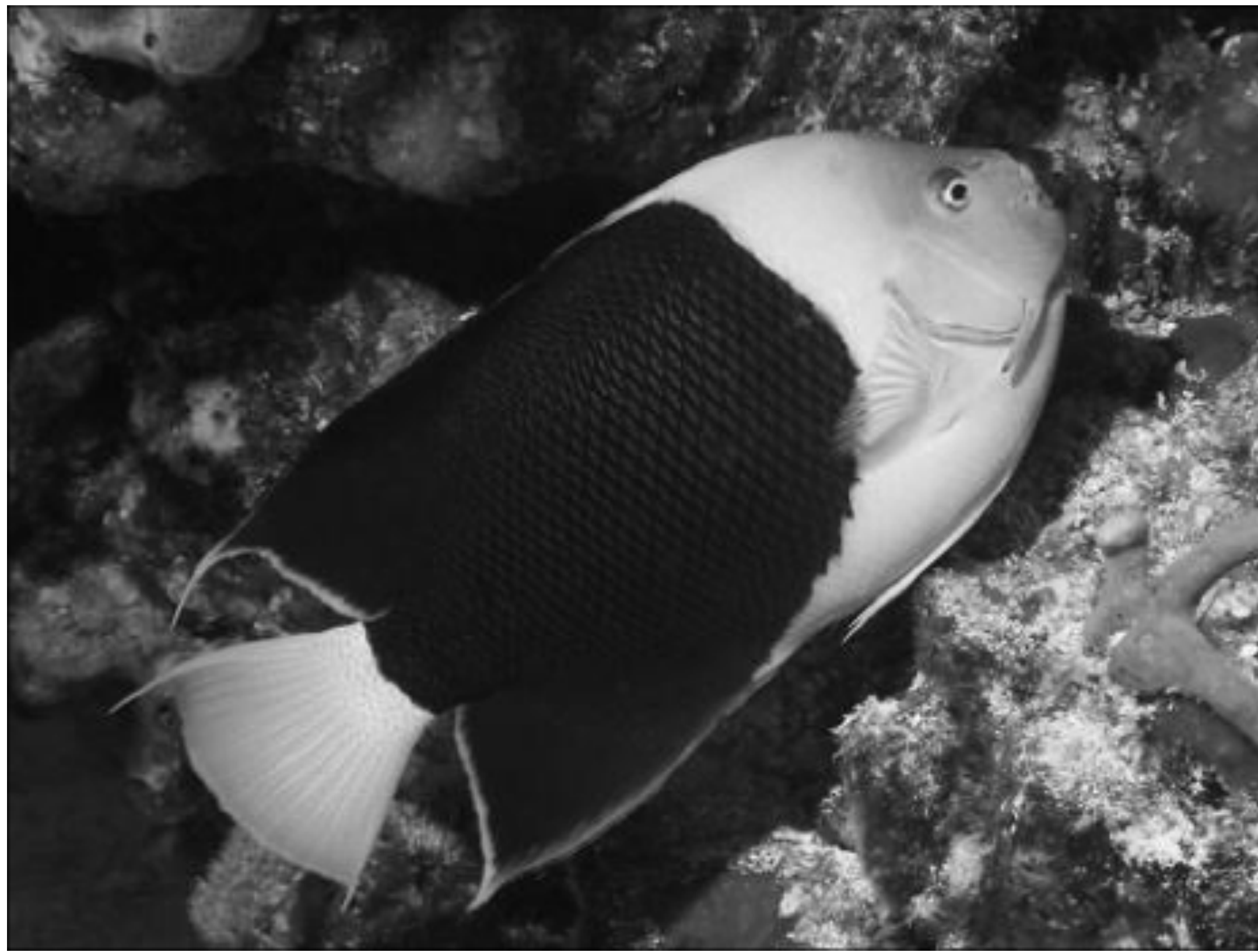
Grayscale image: **L channel**

$$\mathbf{x} \in \mathbb{R}^{H \times W \times 1}$$

Color information: **ab channels**

$$\mathbf{y} \in \mathbb{R}^{H \times W \times 2}$$





Grayscale image: **L channel**

$$\mathbf{x} \in \mathbb{R}^{H \times W \times 1}$$

Color information: **ab channels**

$$\mathbf{y} \in \mathbb{R}^{H \times W \times 2}$$



# Choosing loss and representation

Input



Output



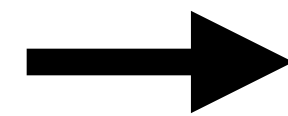
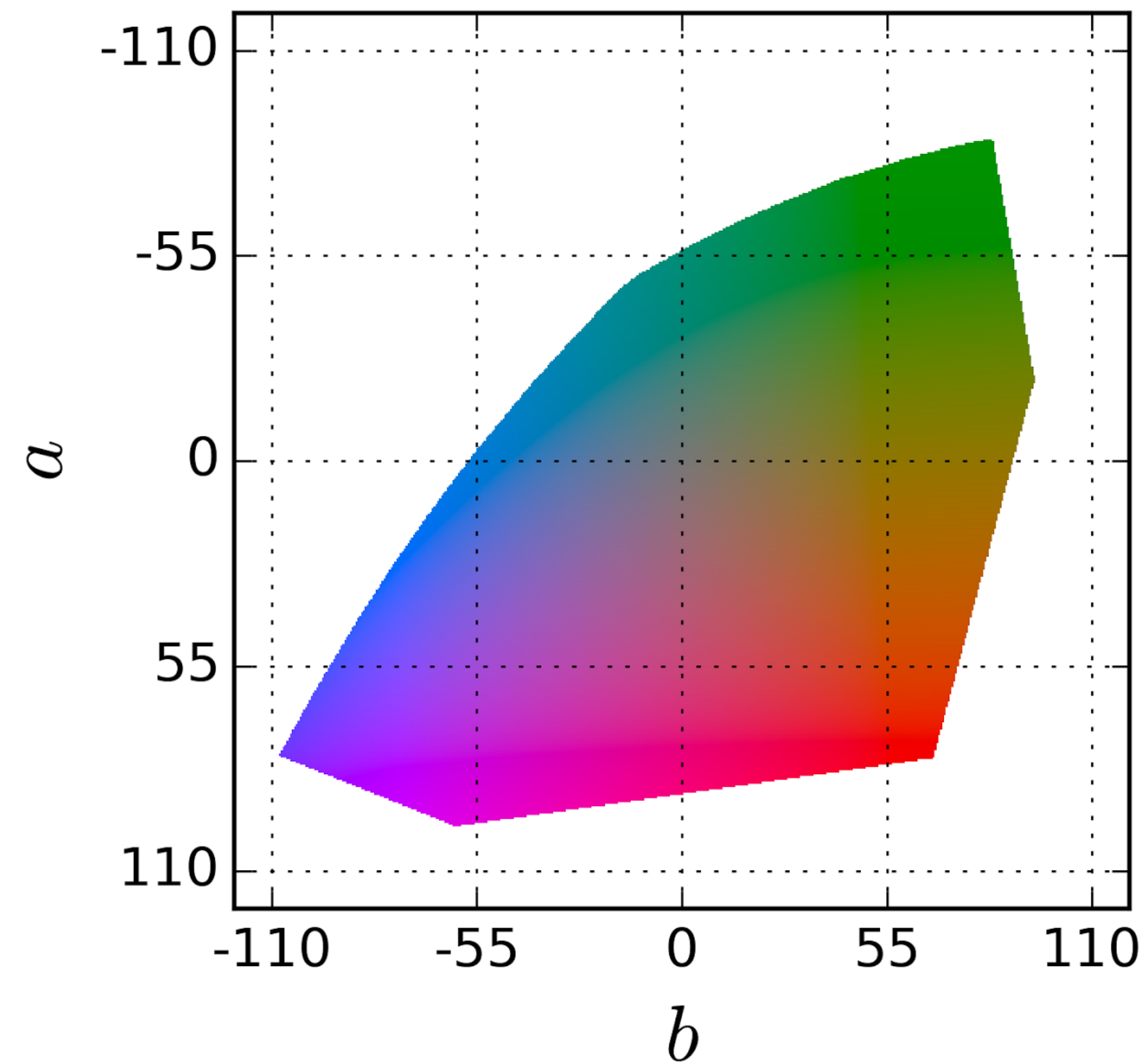
Ground truth



$$\mathcal{L}(f(\mathbf{x}), \mathbf{y}) = \|f(\mathbf{x}) - \mathbf{y}\|_2^2$$

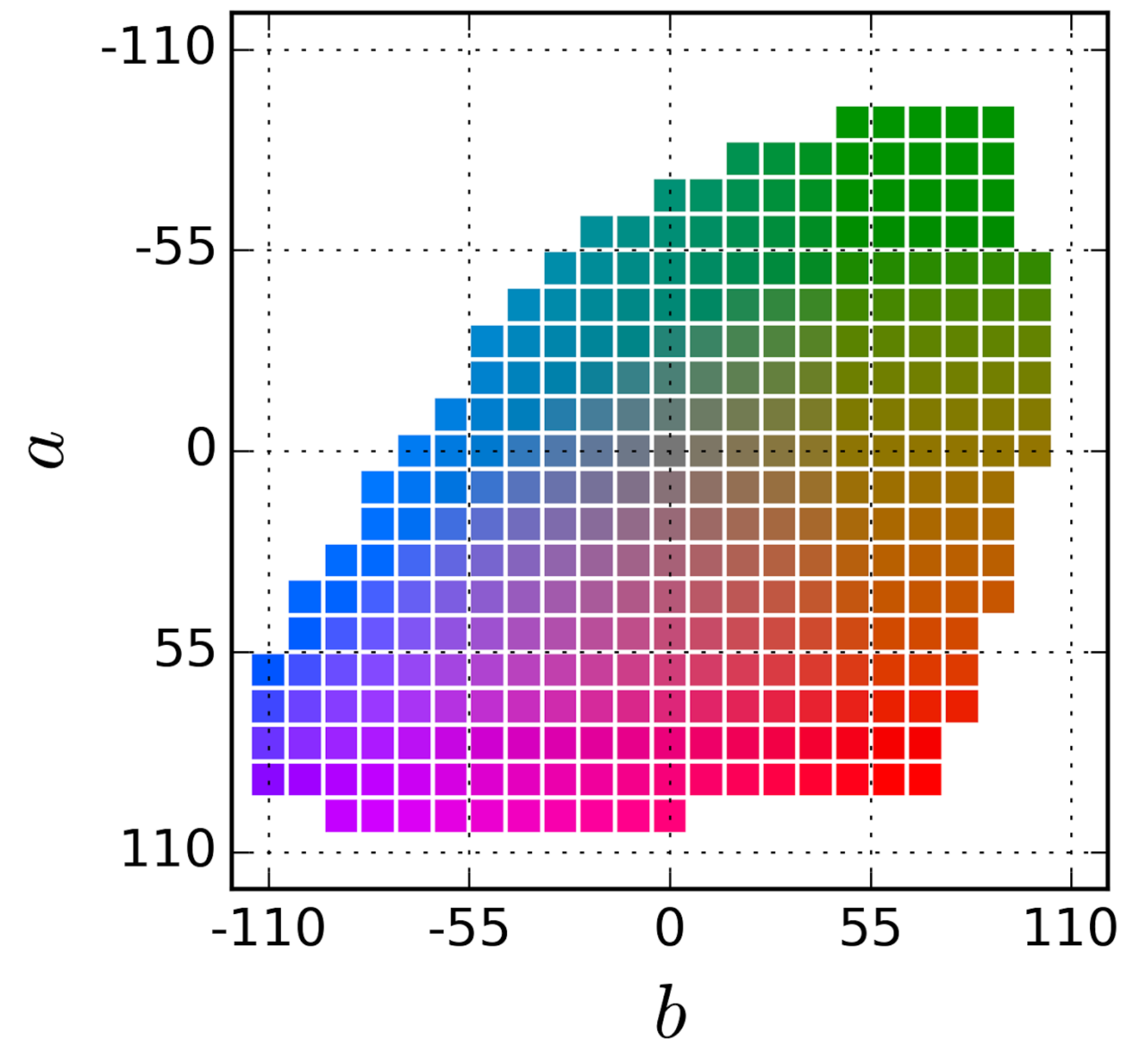


$$\mathbf{y} \in \mathbb{R}^{H \times W \times 2}$$



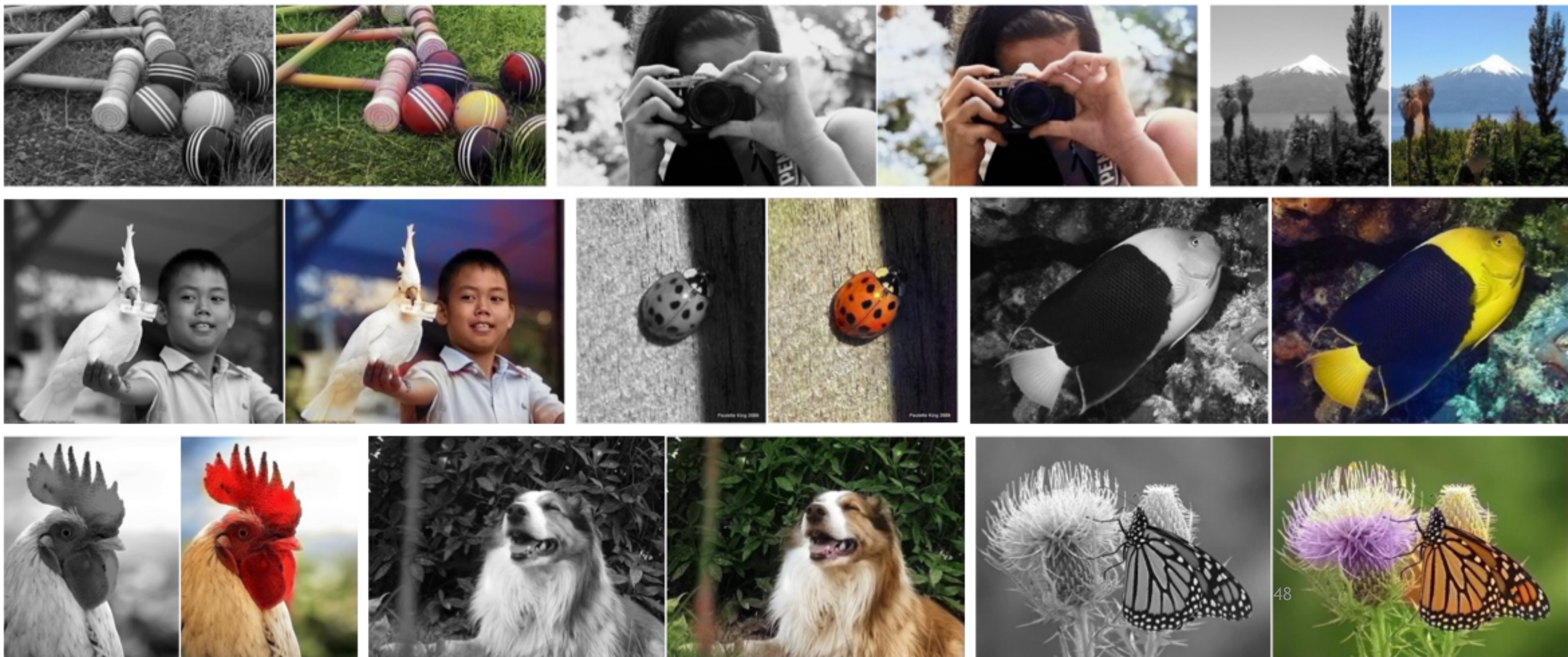
one-hot representation of  $K$  discrete classes

$$\mathbf{y} \in \mathbb{R}^{H \times W \times K}$$





# Success cases







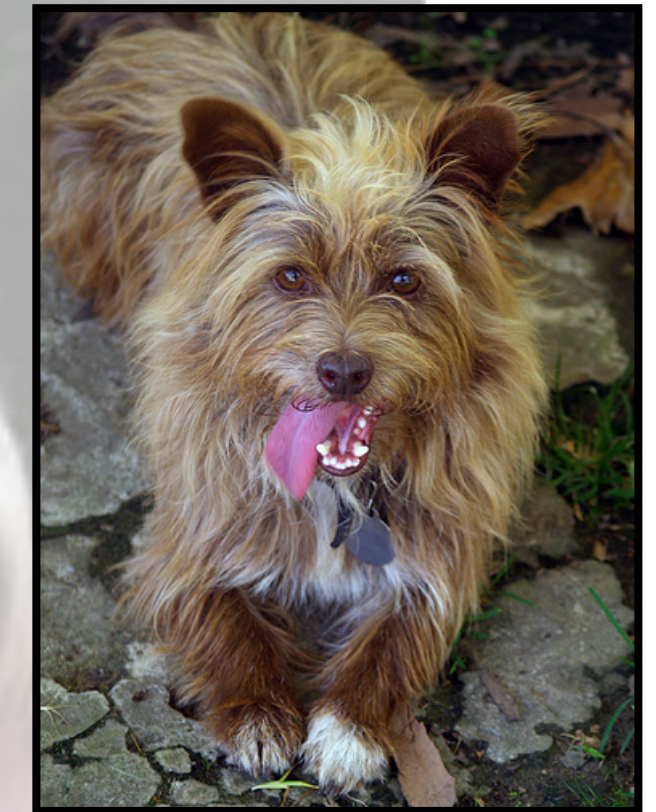
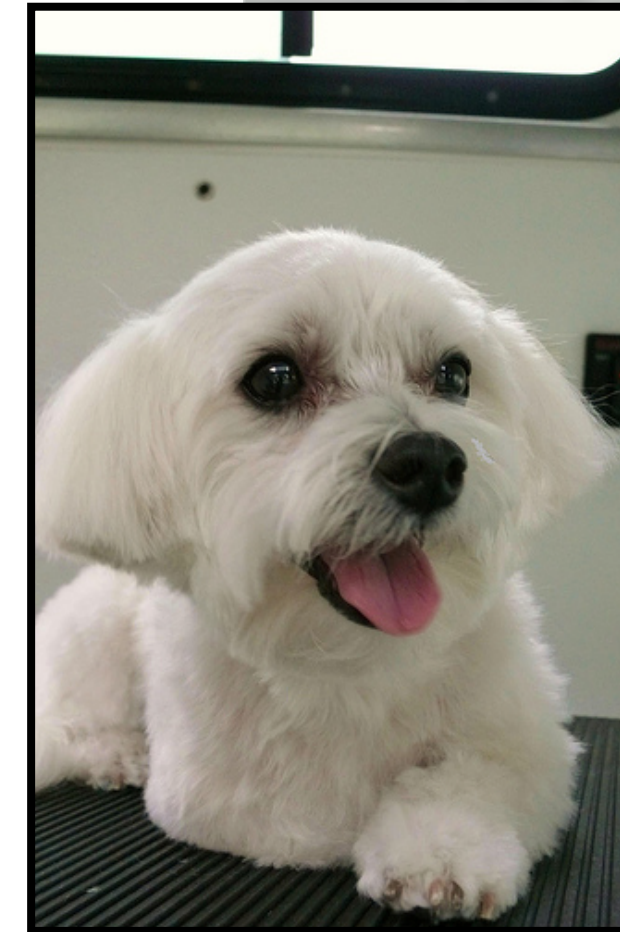


# Instructive failure





# Instructive failure







[from Reddit /u/SherySantucci]

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[Zhang et al., ECCV 2016]





[Recolorized by Reddit ColorizeBot]

53

[Zhang et al., ECCV 2016]





Photo taken by  
Reddit /u/  
Timteroo,  
Mural from street  
artist Eduardo  
Kobra

[Zhang et al., ECCV 2016]





Recolorized by  
Reddit  
ColorizeBot

[Zhang et al., ECCV 2016]

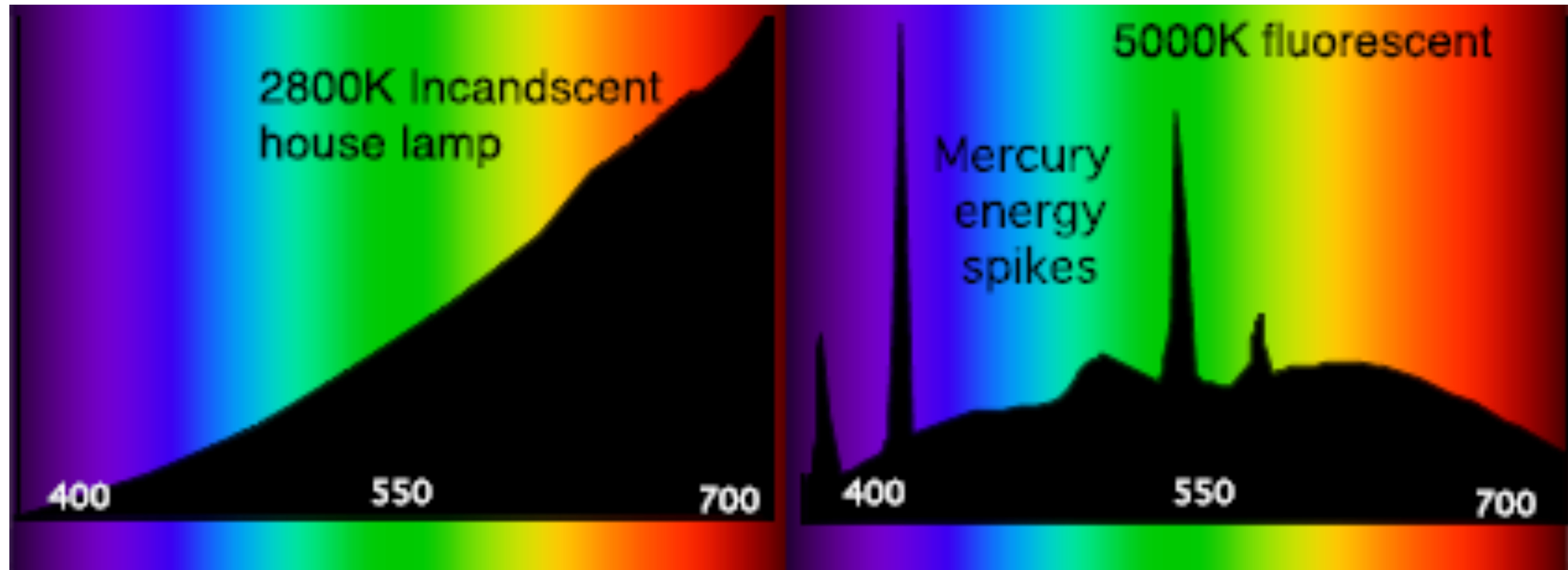


# Today

- Color physics
- Color perception
- Color constancy



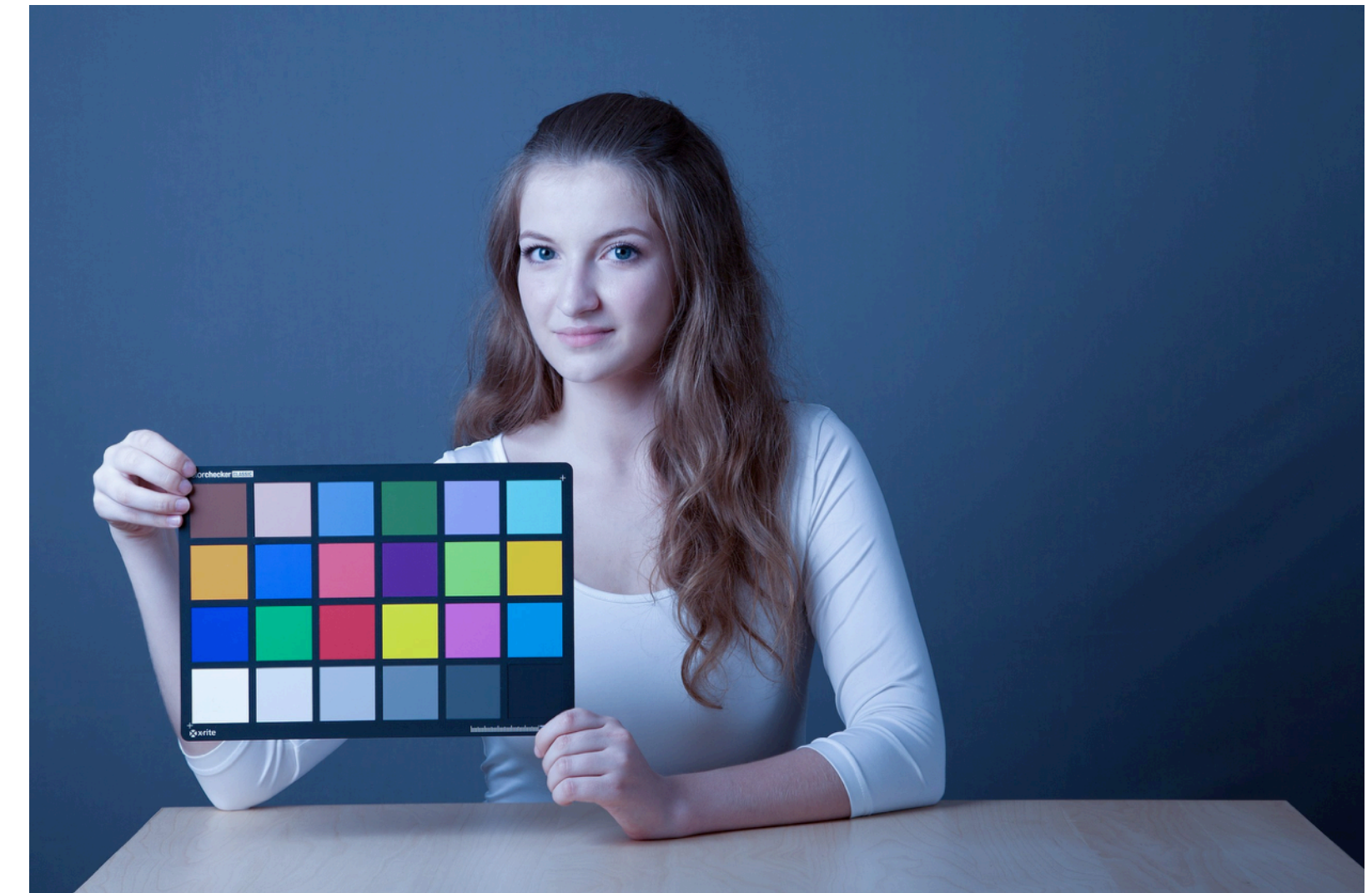
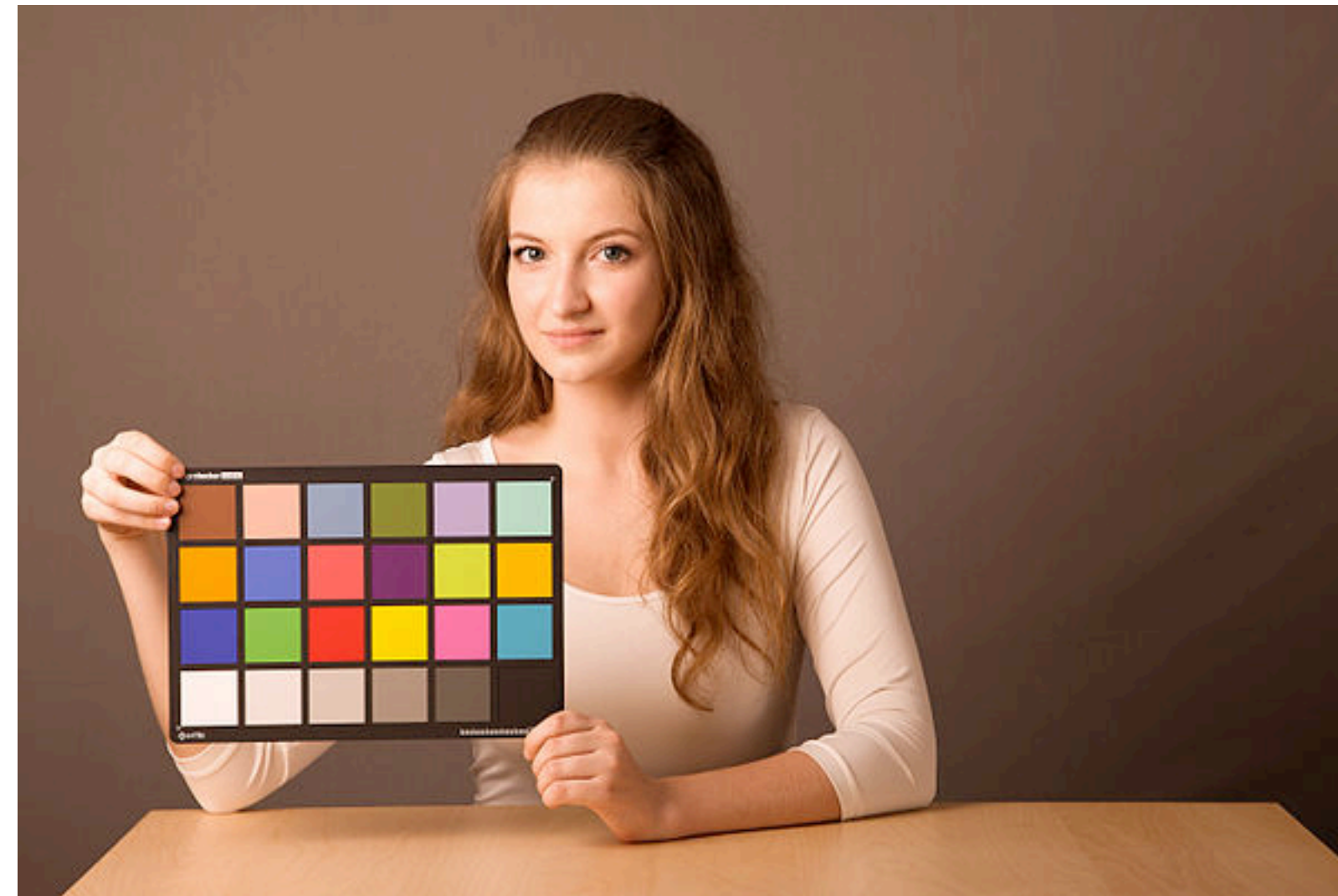
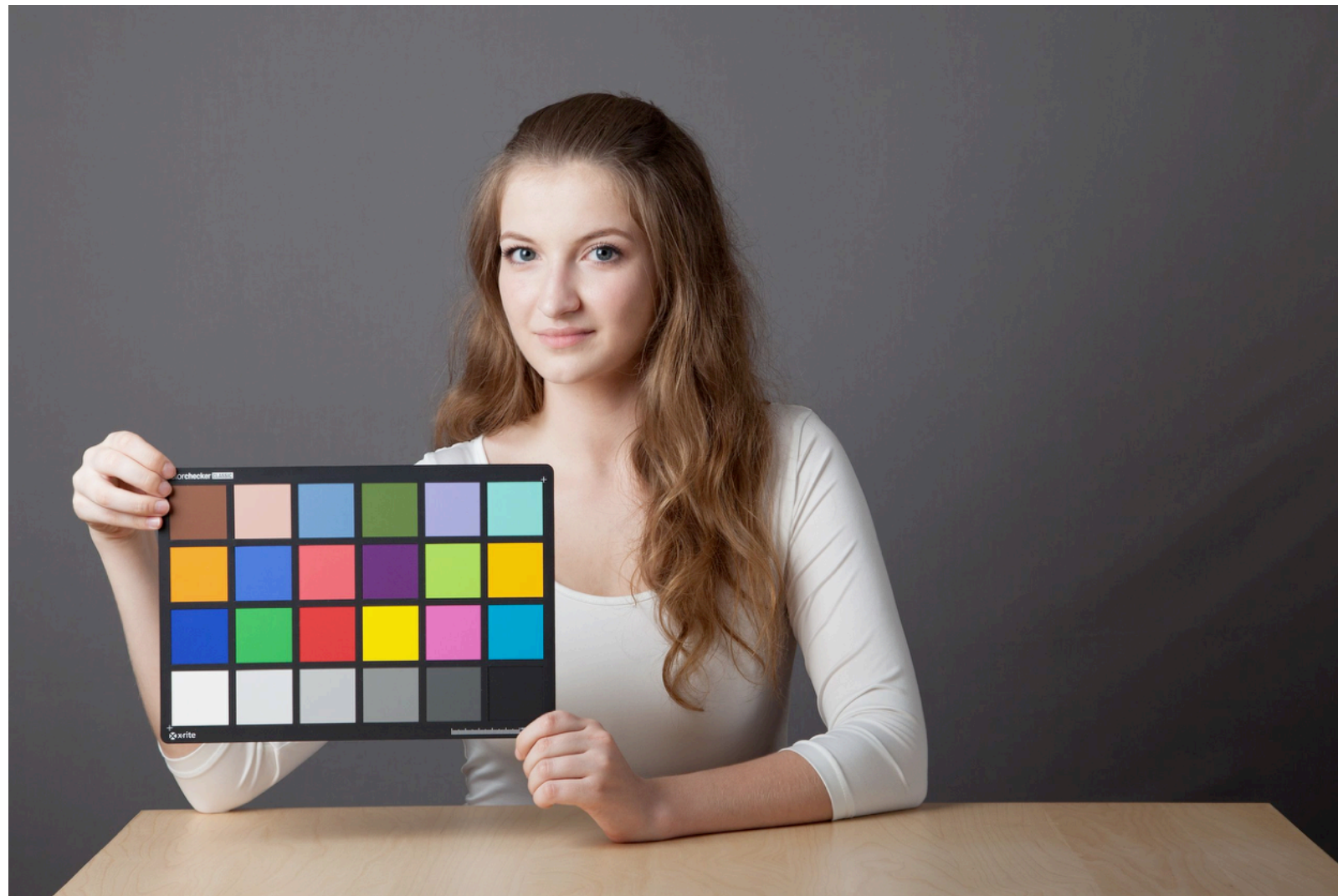
# Light sources



[https://en.wikipedia.org/wiki/Color\\_temperature](https://en.wikipedia.org/wiki/Color_temperature)



# Same scene under different illuminations



[https://en.wikipedia.org/wiki/Color\\_balance](https://en.wikipedia.org/wiki/Color_balance)



# White balancing

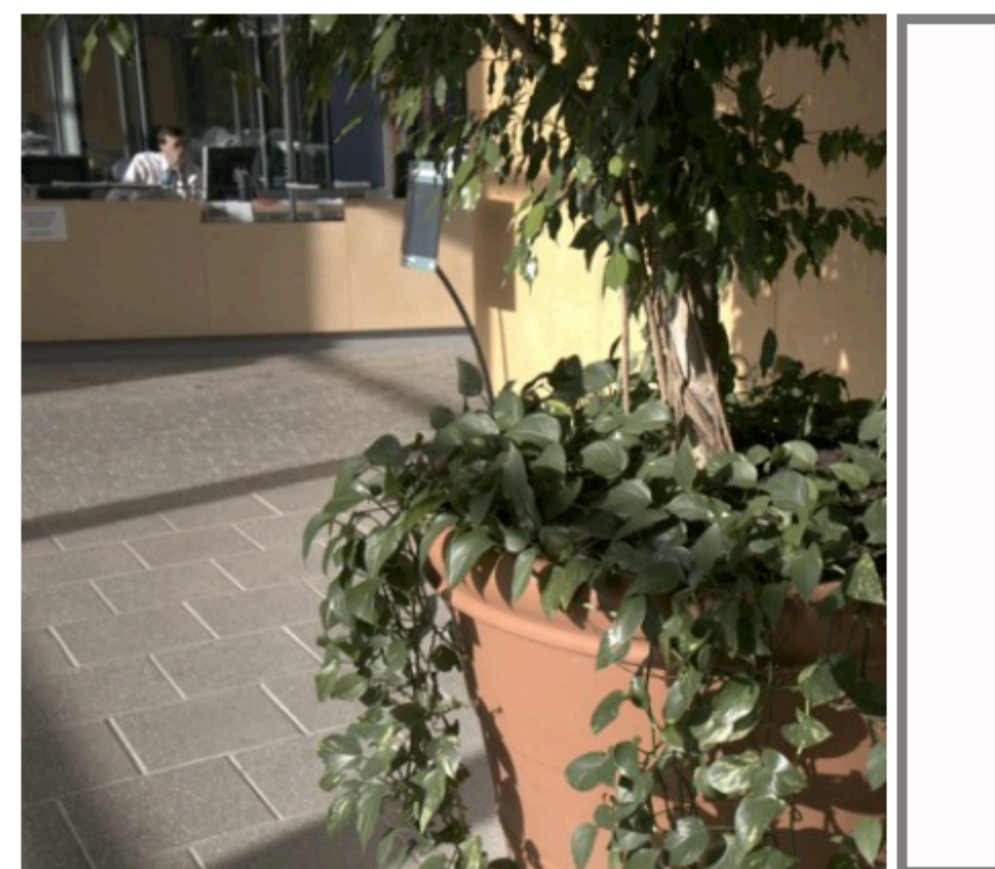


Is this a green light and a white object, or a green object<sup>59</sup> and a white light?

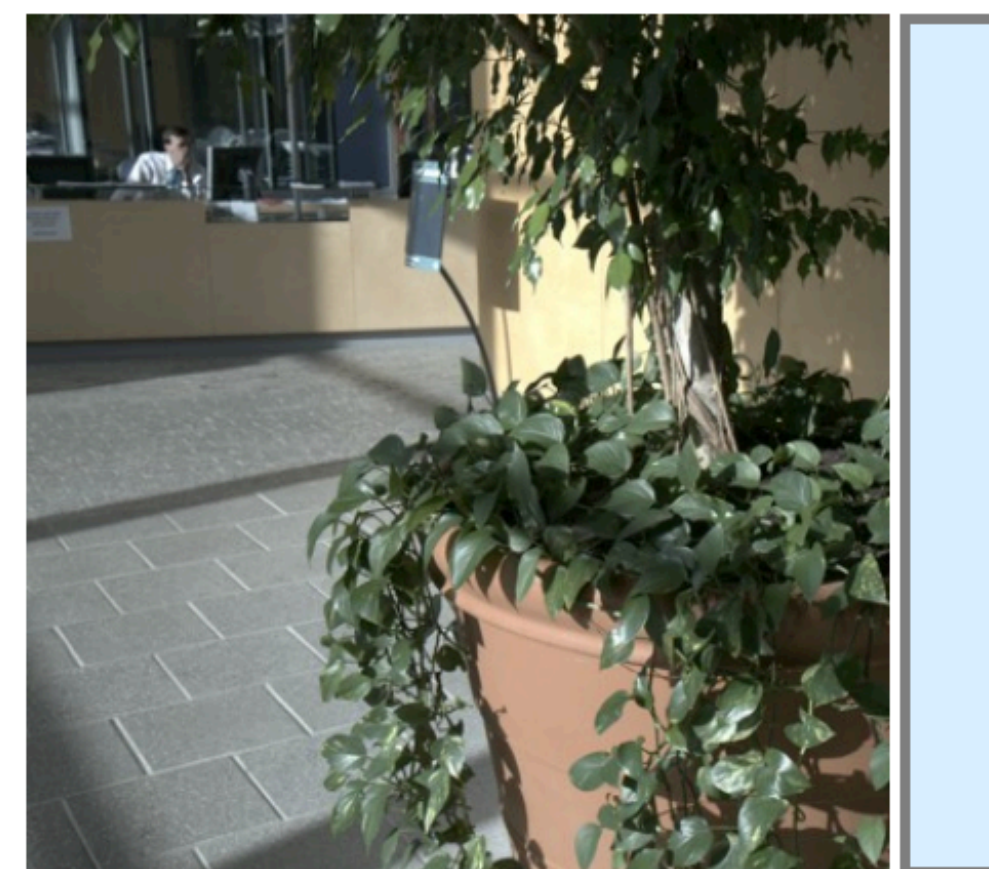


# White balancing


$$I = W \times L$$



our  $\hat{W}$ ,  $\hat{L}$ , err = 0.13°

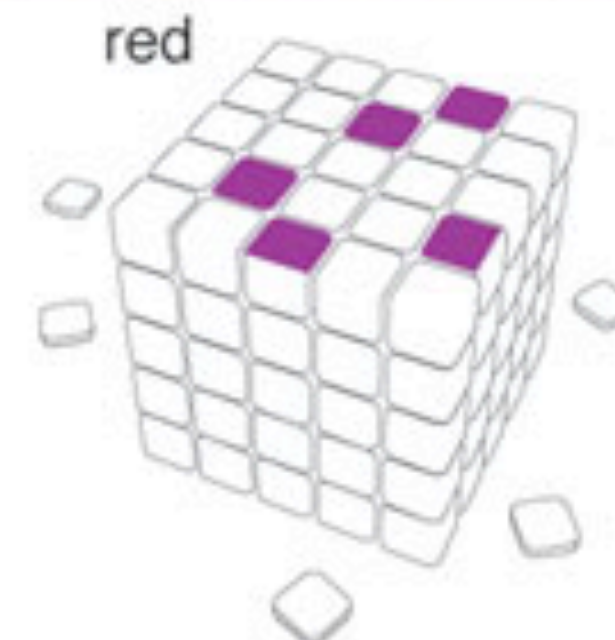
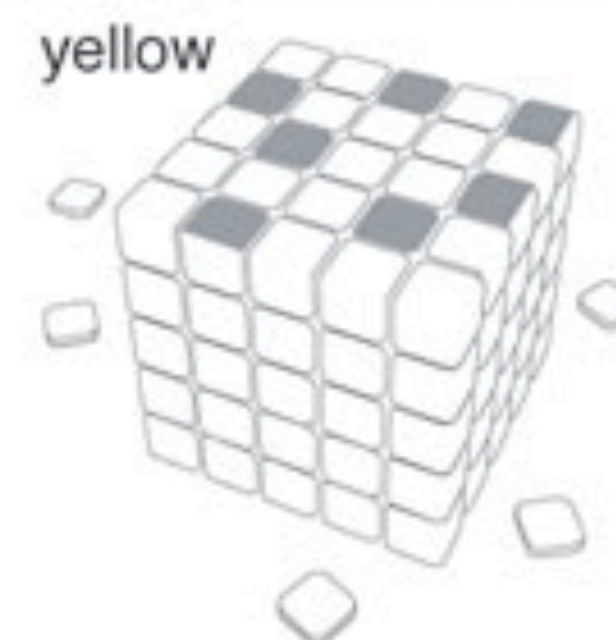
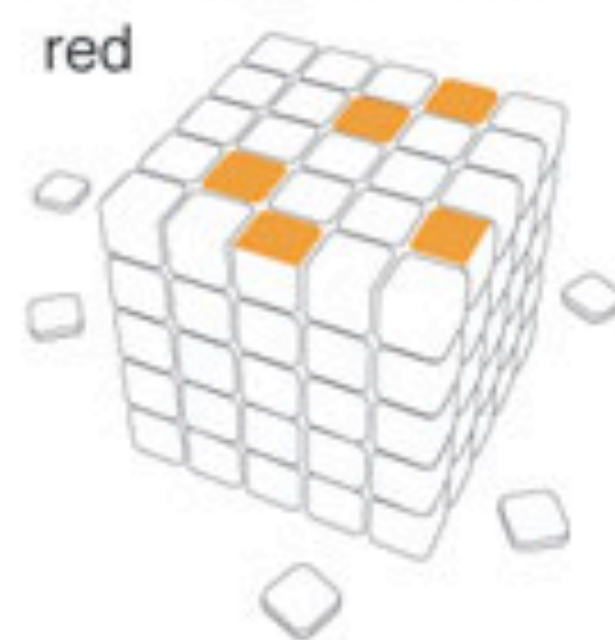
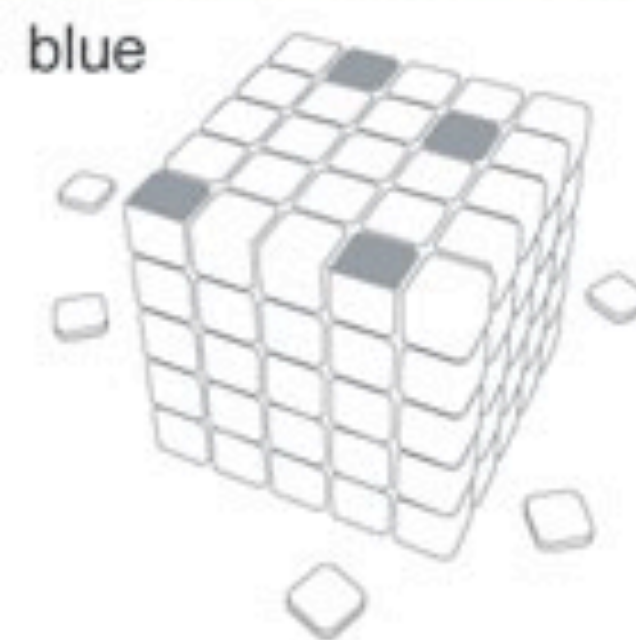
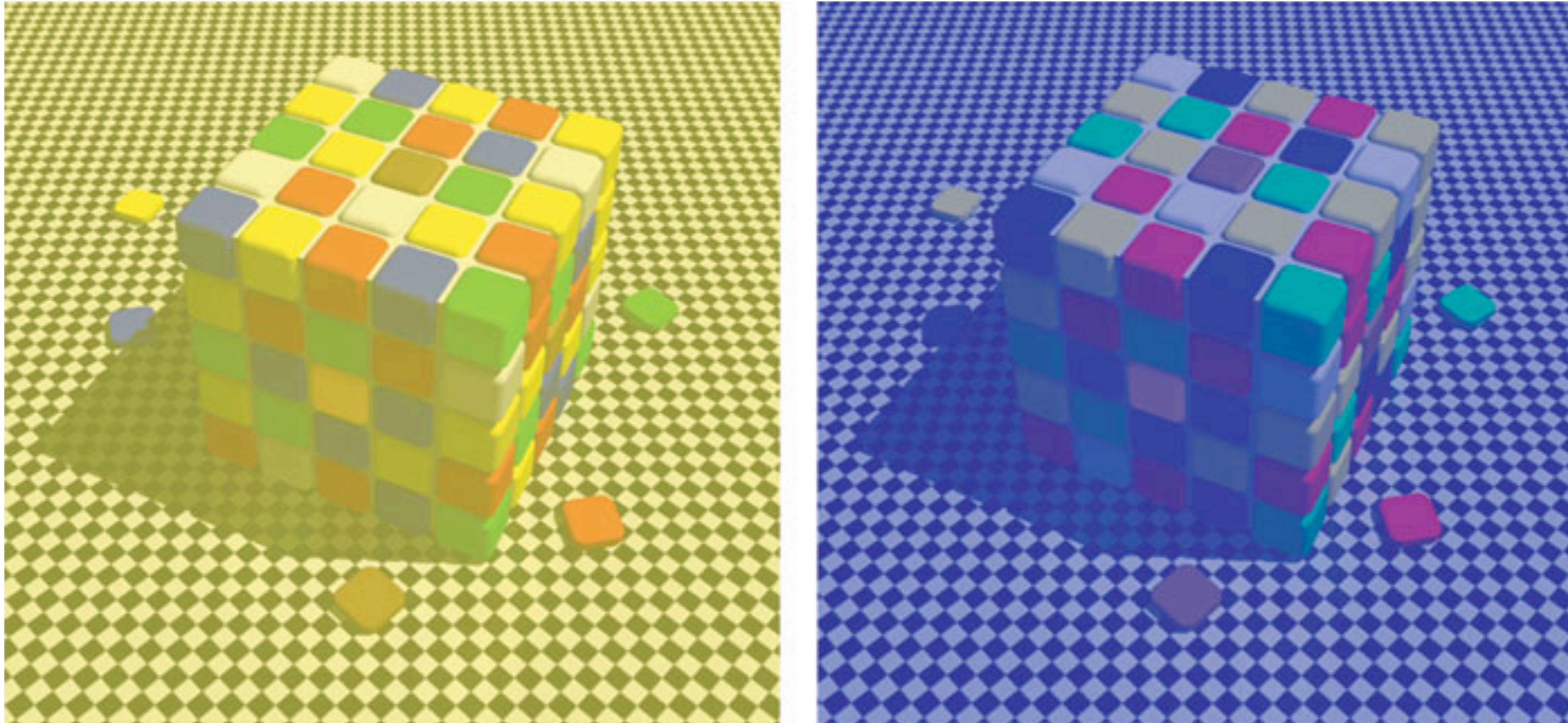


baseline  $\hat{W}$ ,  $\hat{L}$ , err = 5.34°

[Barron, “Convolutional Color Constancy”, 2015]

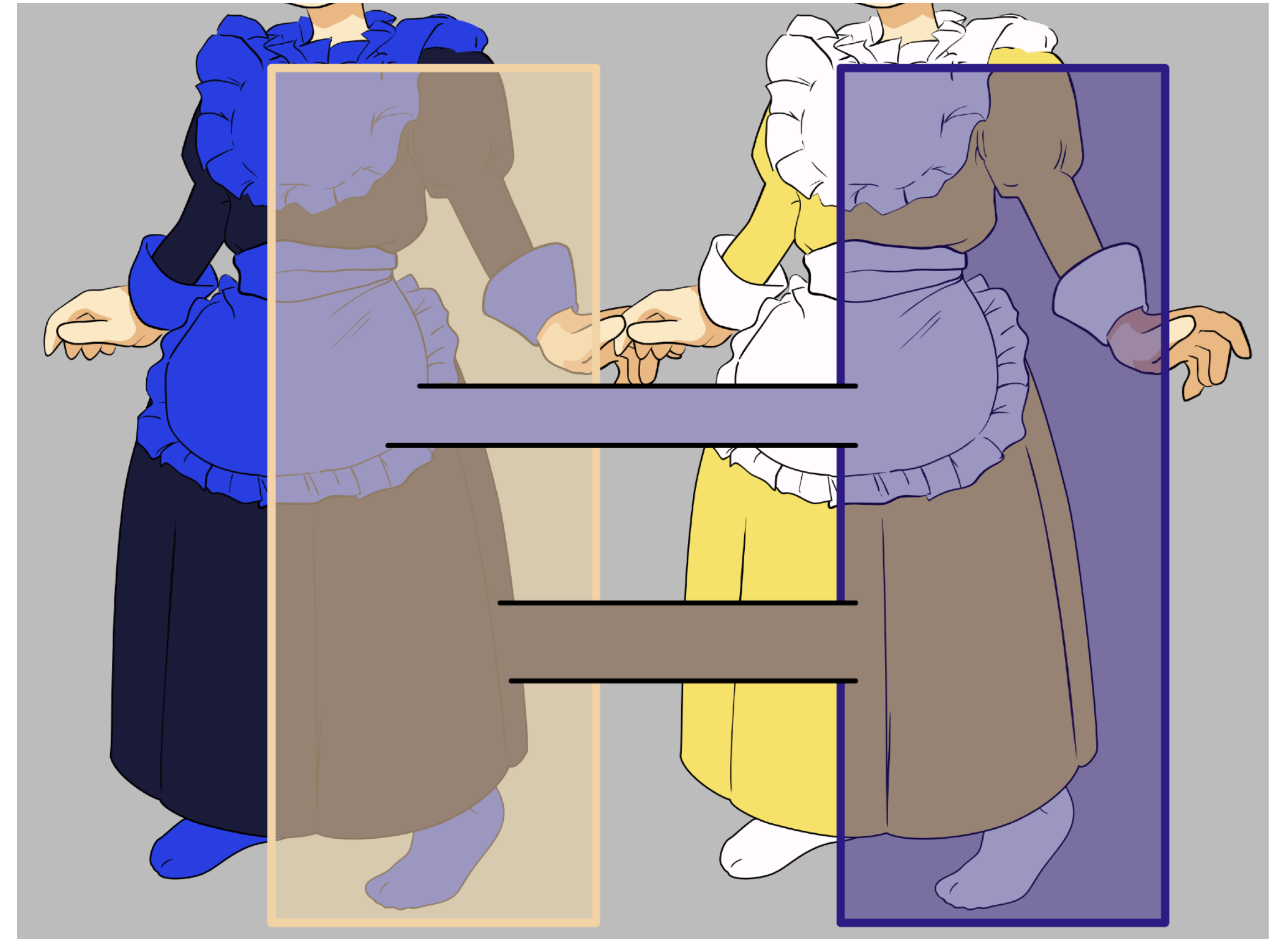


# Color constancy





# The dress



Two interpretations  
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**Next class:** light and shading