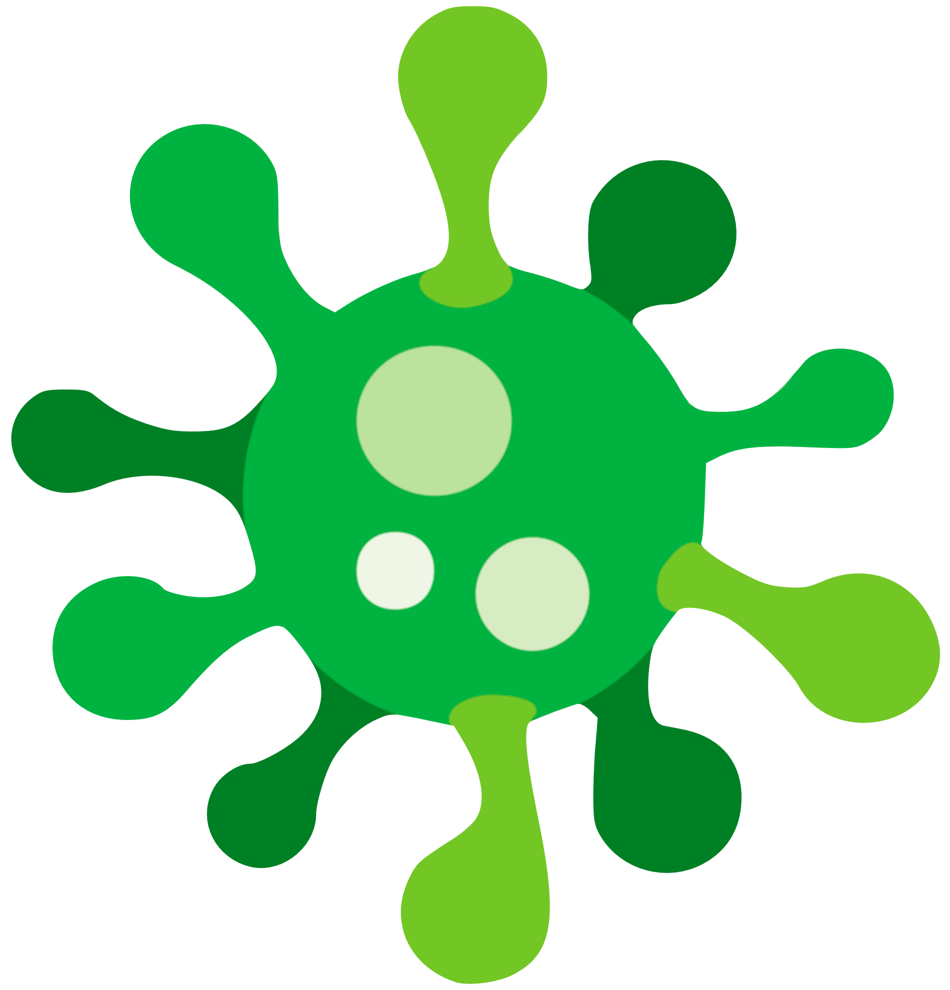
Lecture 16: Image formation

Announcements: coronavirus edition

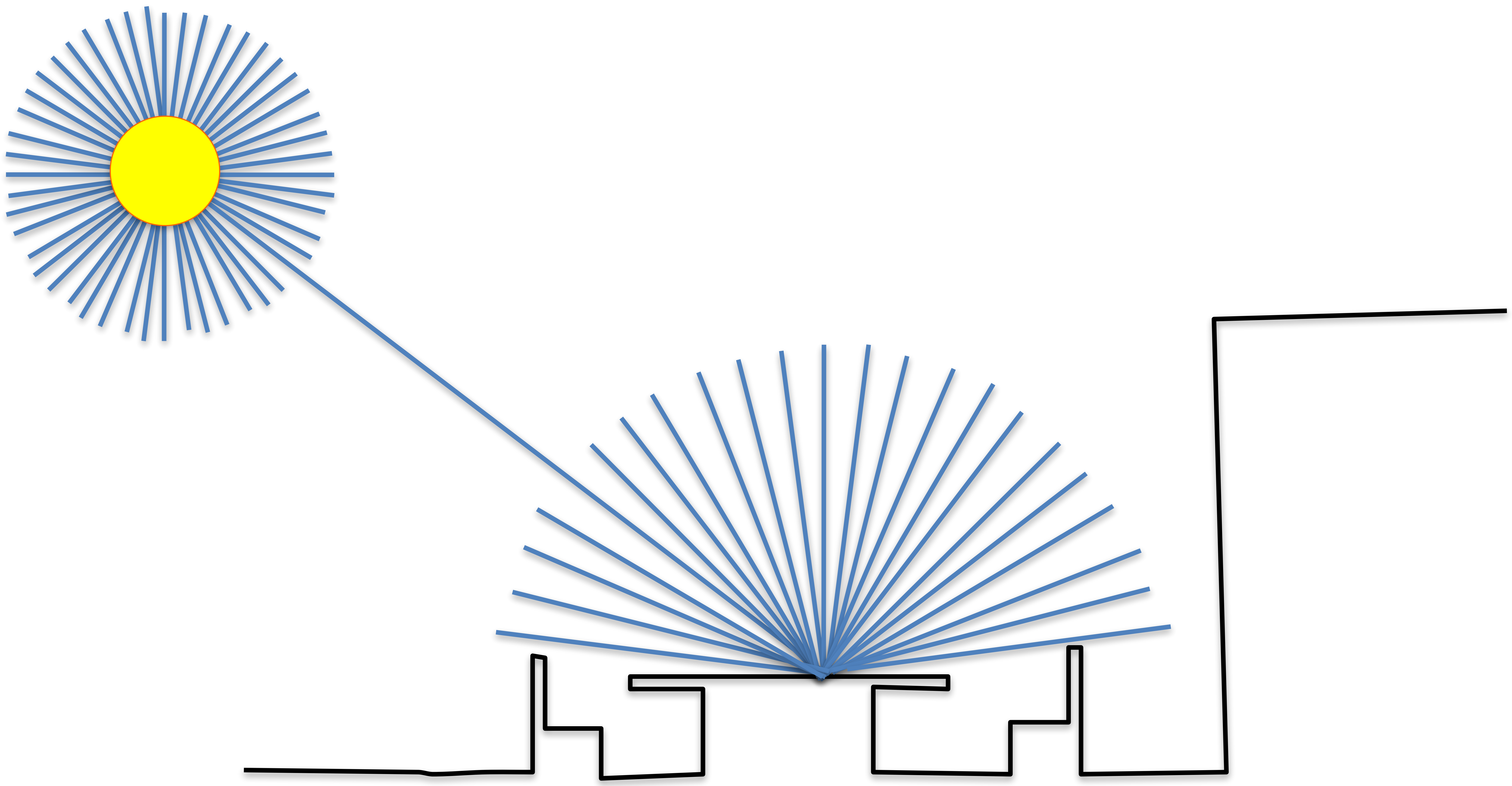
- Coming to class is optional now!
- Will post lecture recordings after class
  – May stream in future
- Project proposal due
- PS7 out tonight
- PS4 grades out



Today

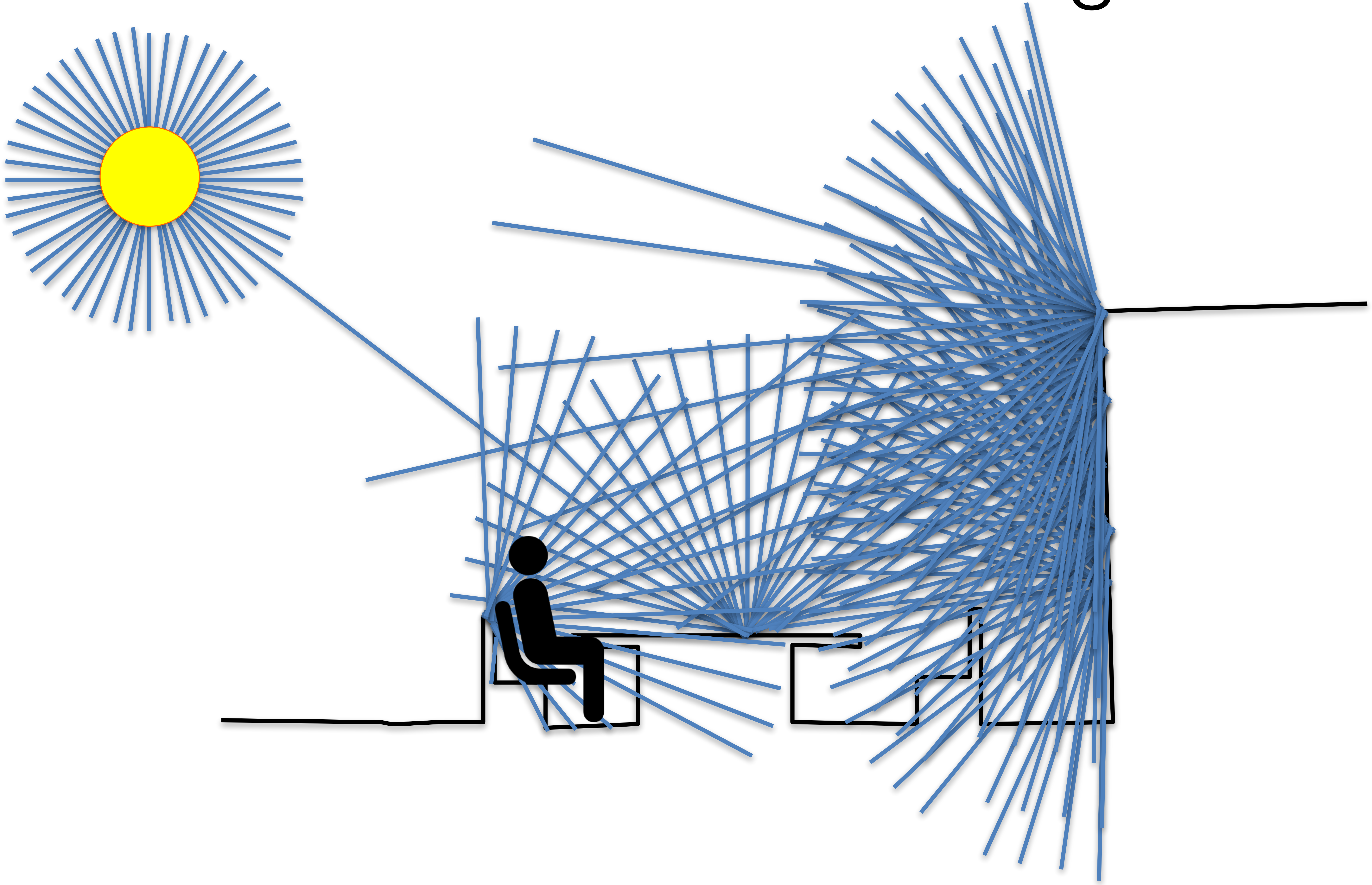
- This section of the course: physical models
- Camera models
- Projection equations

The structure of ambient light



Source: Freeman, Torralba, Isola

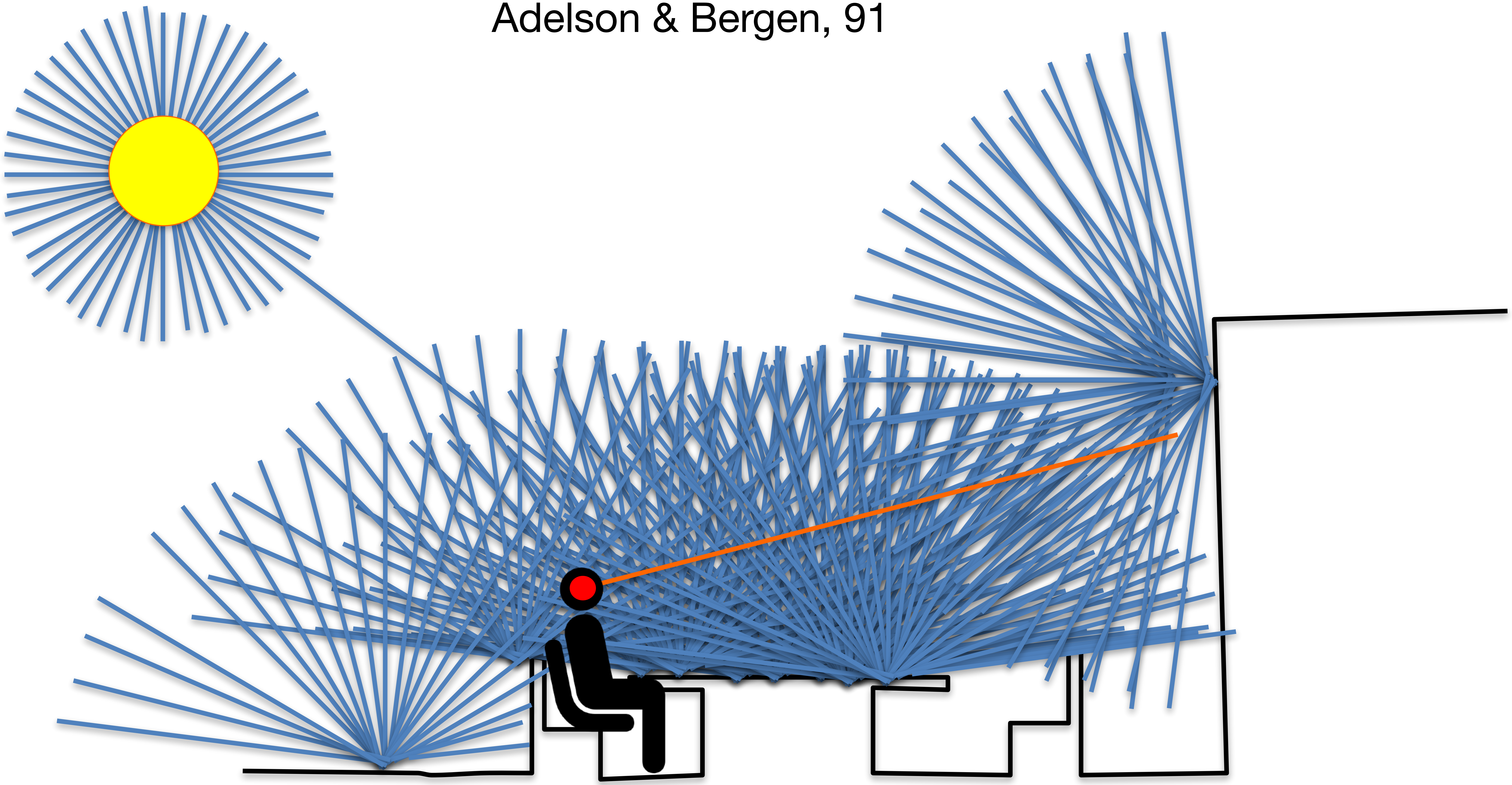
The structure of ambient light



Source: Freeman, Torralba, Isola

The Plenoptic Function

Adelson & Bergen, 91



The intensity P can be parameterized as:

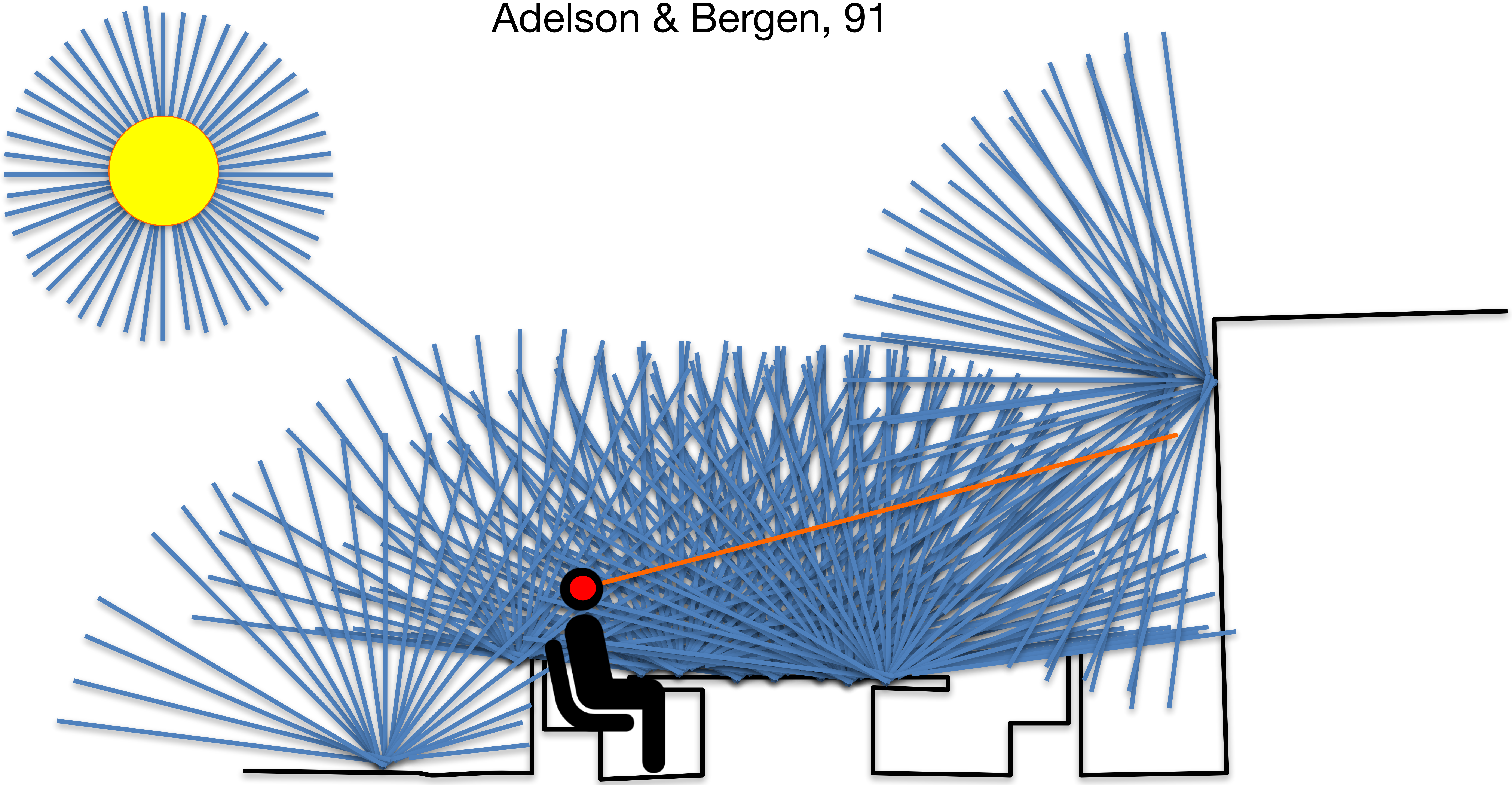
P (X, Y, Z)

Eye position

Source: Freeman, Torralba, Isola

The Plenoptic Function

Adelson & Bergen, 91



The intensity P can be parameterized as:

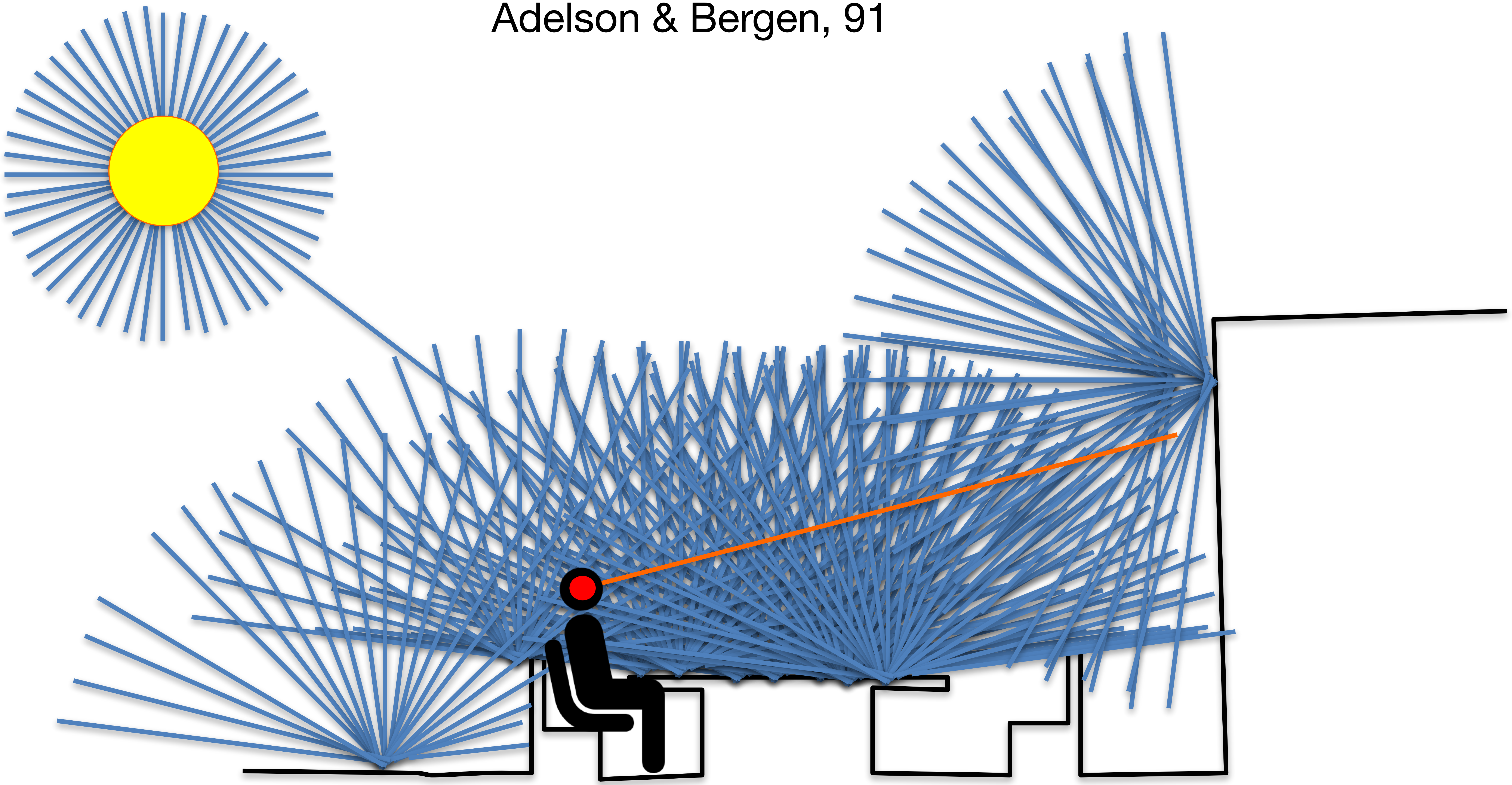
$$P\ (\theta, \phi, \qquad X, Y, Z)$$

Angle

Source: Freeman, Torralba, Isola

The Plenoptic Function

Adelson & Bergen, 91



The intensity P can be parameterized as:

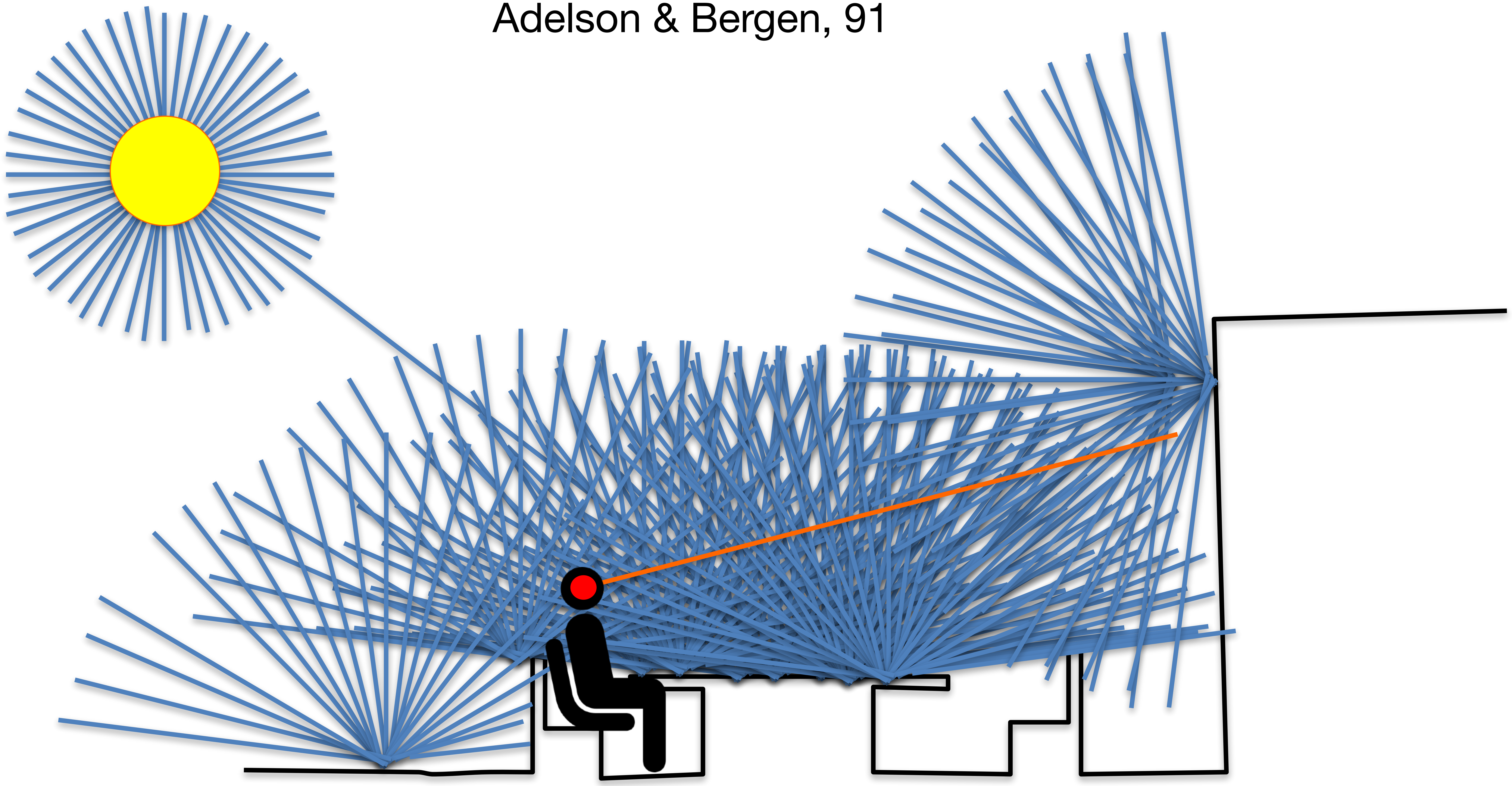
$$P(\theta, \phi, \lambda, t, X, Y, Z)$$

Wavelength, time

Source: Freeman, Torralba, Isola

The Plenoptic Function

Adelson & Bergen, 91



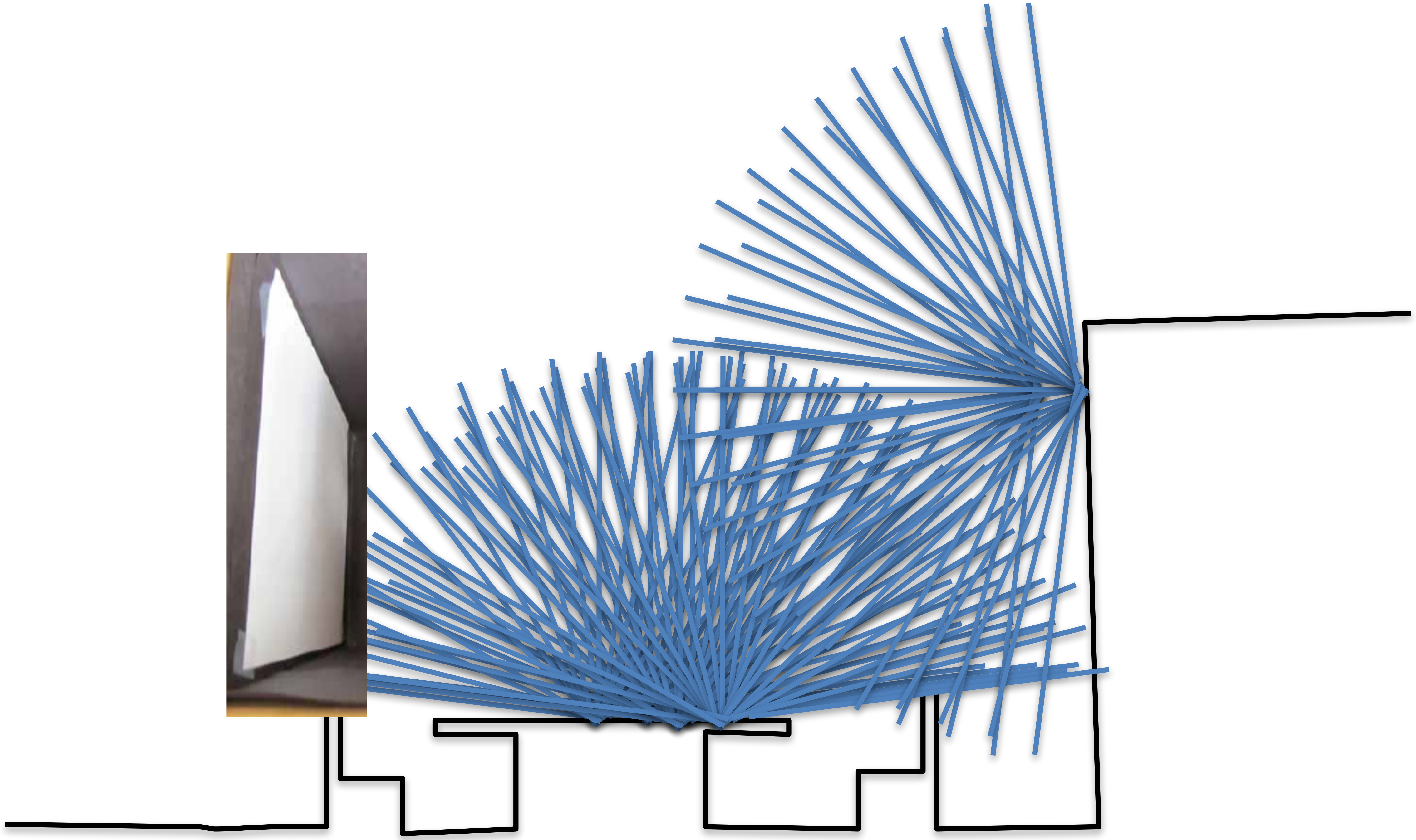
The intensity P can be parameterized as:

$$P\ (\theta, \phi, \lambda, t, X, Y, Z)$$

“The complete set of all convergence points constitutes the permanent possibilities of vision.” Gibson

Source: Freeman, Torralba, Isola

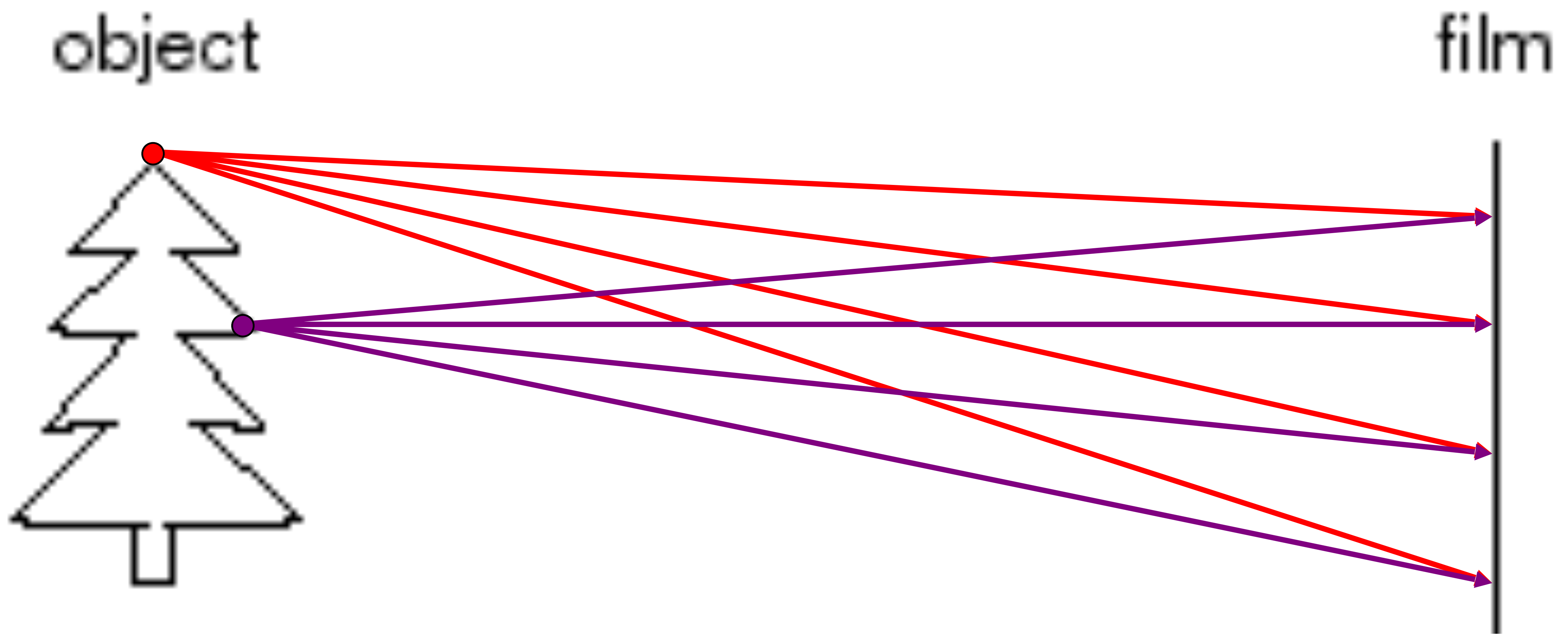
Measuring the Plenoptic function



Why is there no picture appearing on the paper?

Source: Freeman, Torralba, Isola

Image formation

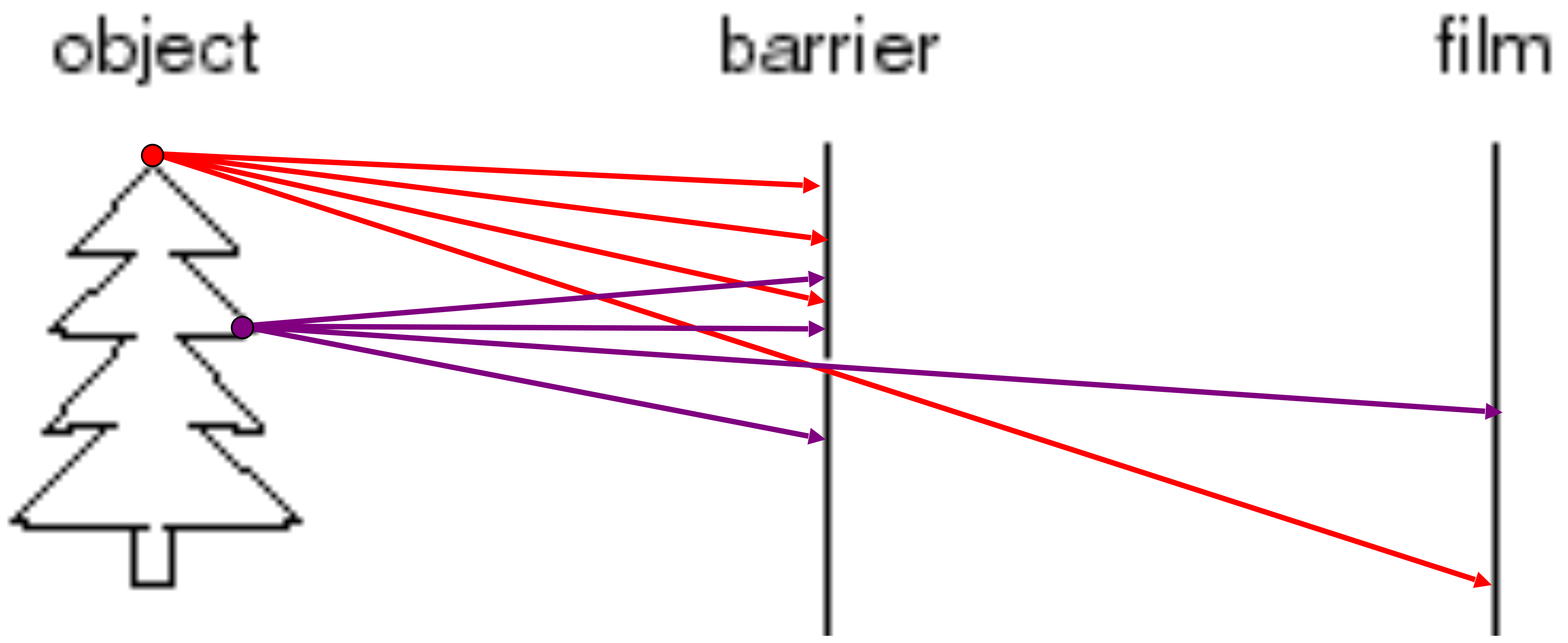


Let's design a camera

- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?
- No. This is a bad camera.

Source: N. Snavely

Pinhole camera

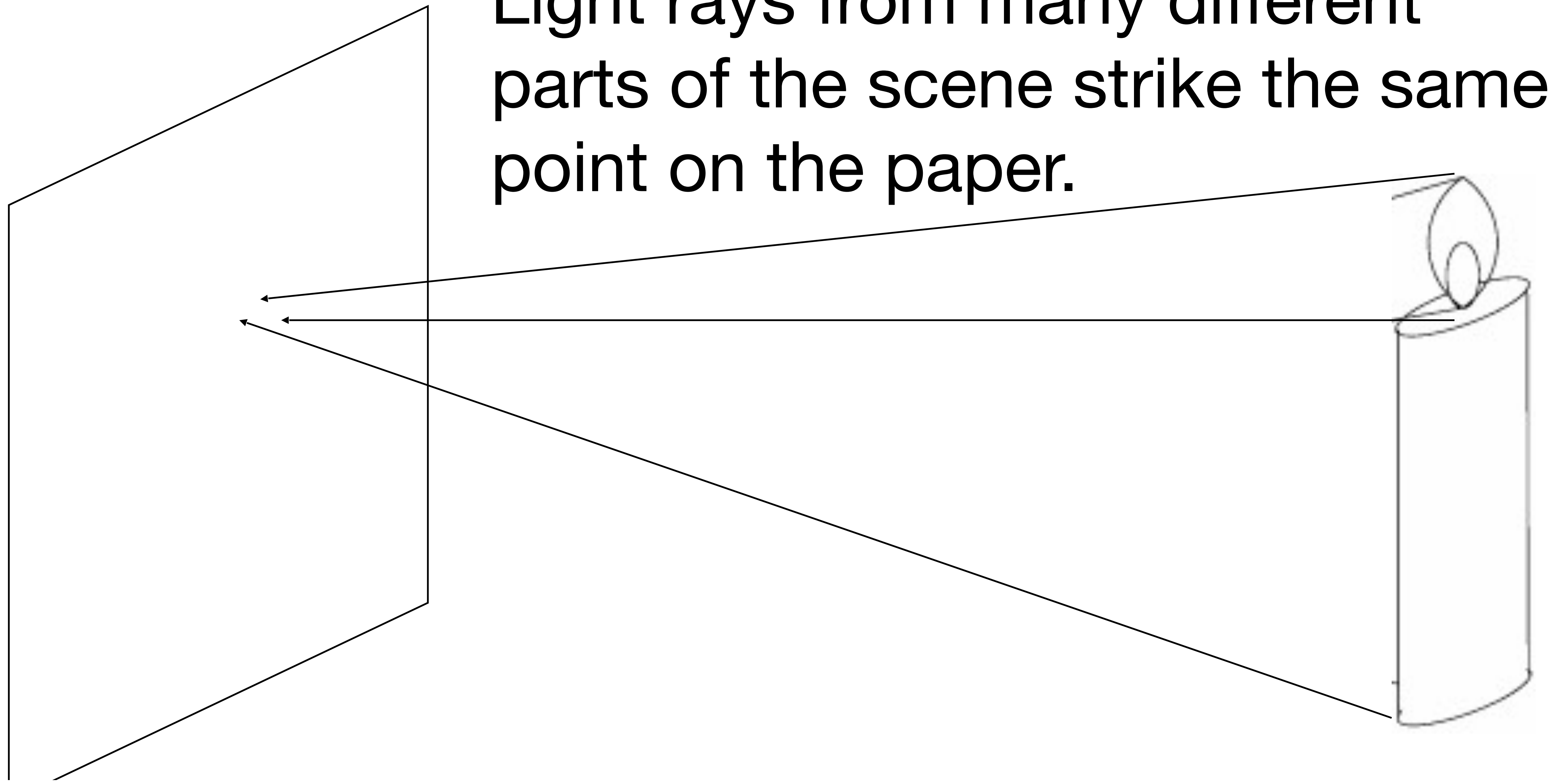


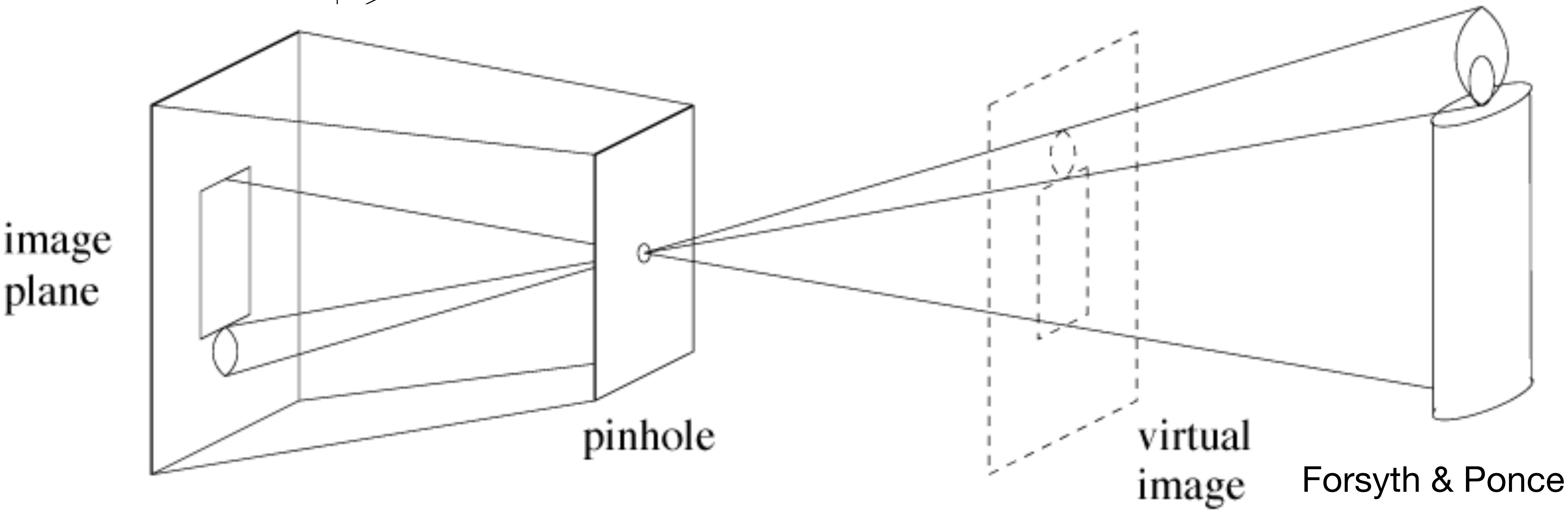
Add a barrier to block off most of the rays

– This reduces blurring
– The opening known as the **aperture**
– How does this transform the image?

Source: N. Snavely

Light rays from many different parts of the scene strike the same point on the paper.





Forsyth & Ponce

The pinhole camera only allows rays from one point in the scene to strike each point of the paper.

Source: Freeman, Torralba, Isola

Pinhole camera

Photograph by Abelardo Morell, 1991

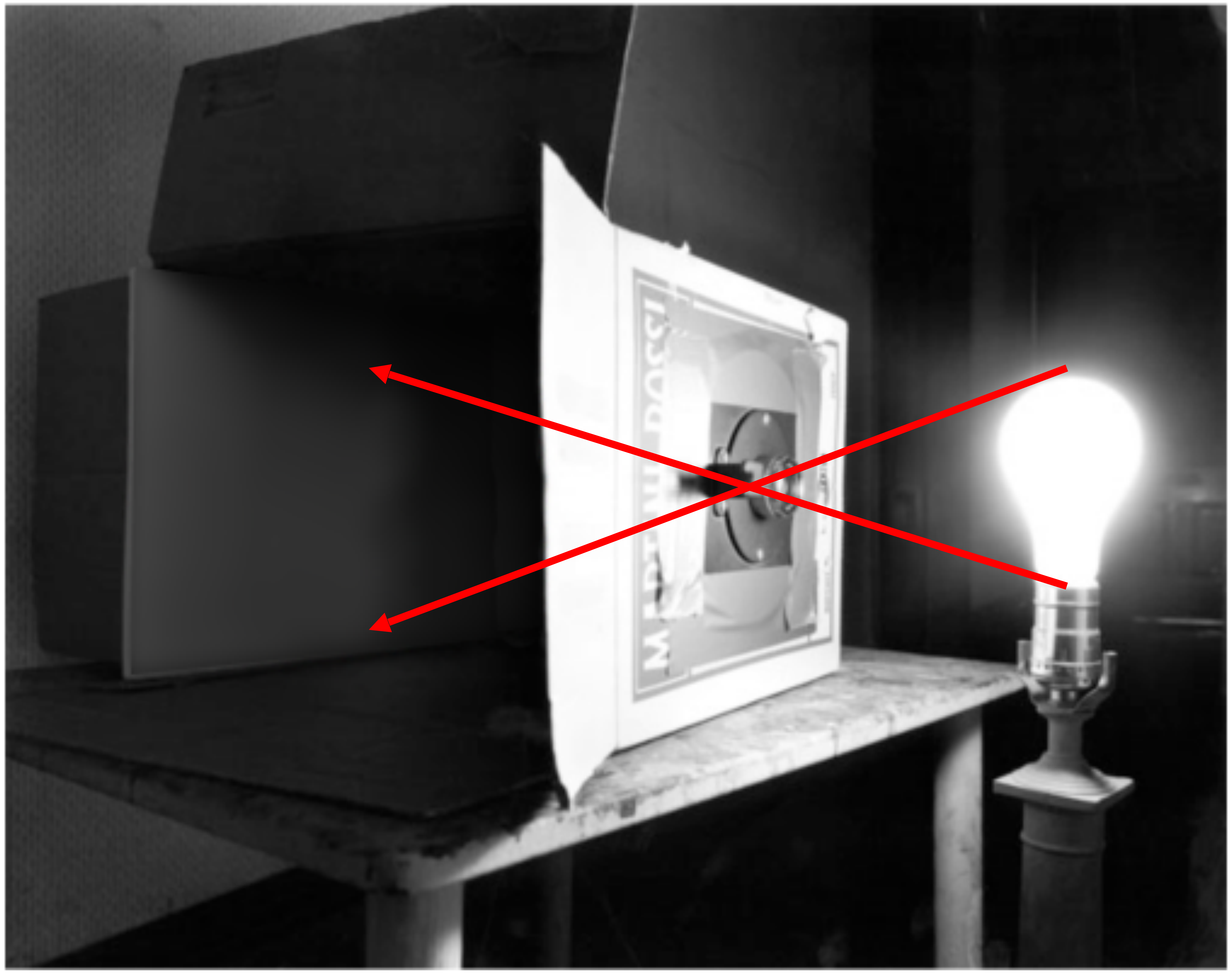
Source: Freeman, Torralba, Isola

Pinhole camera

Photograph by Abelardo Morell, 1991

Source: Freeman, Torralba, Isola

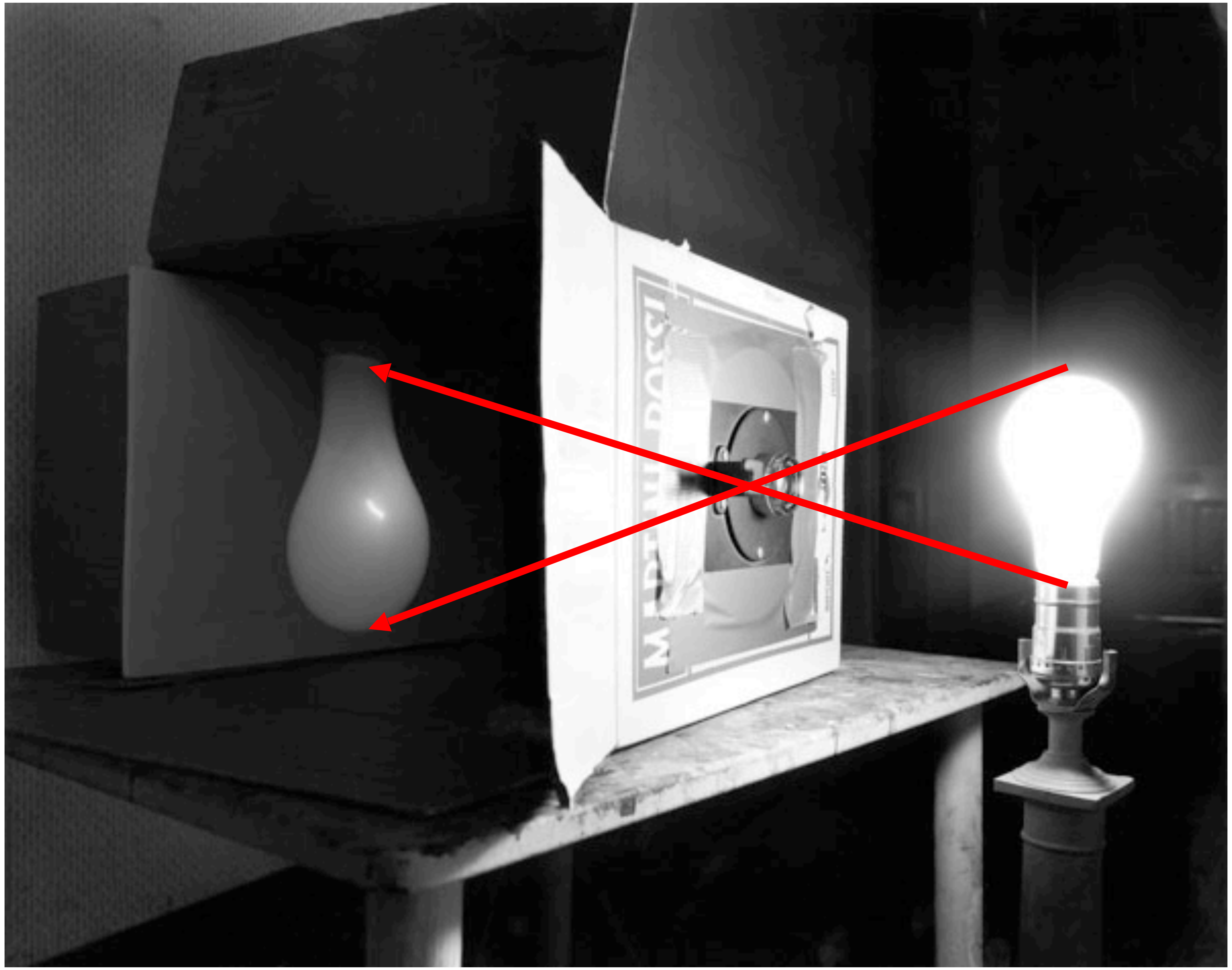
Pinhole camera



Photograph by Abelardo Morell, 1991

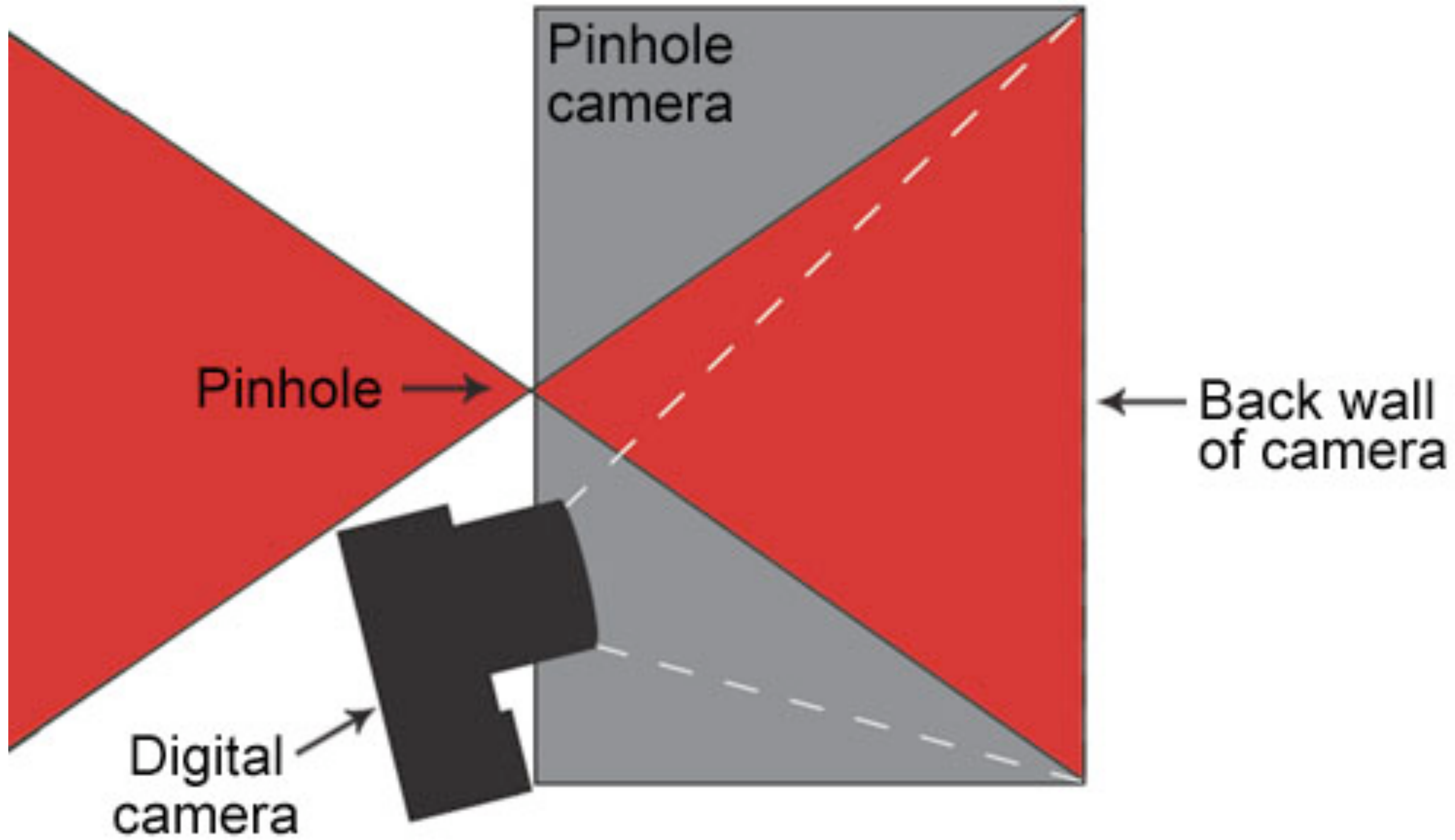
Source: Freeman, Torralba, Isola

Pinhole camera



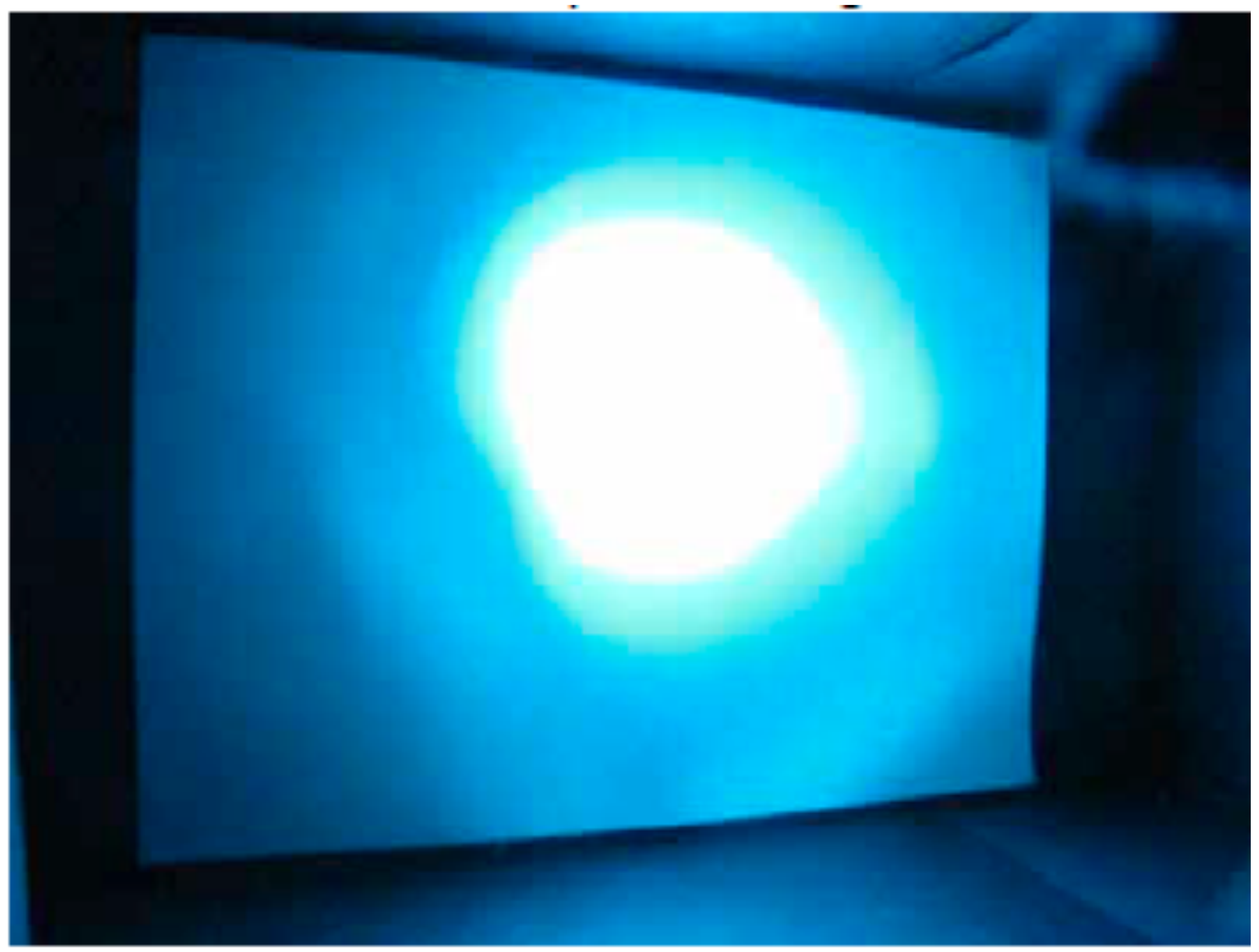
Photograph by Abelardo Morell, 1991

Source: Freeman, Torralba, Isola



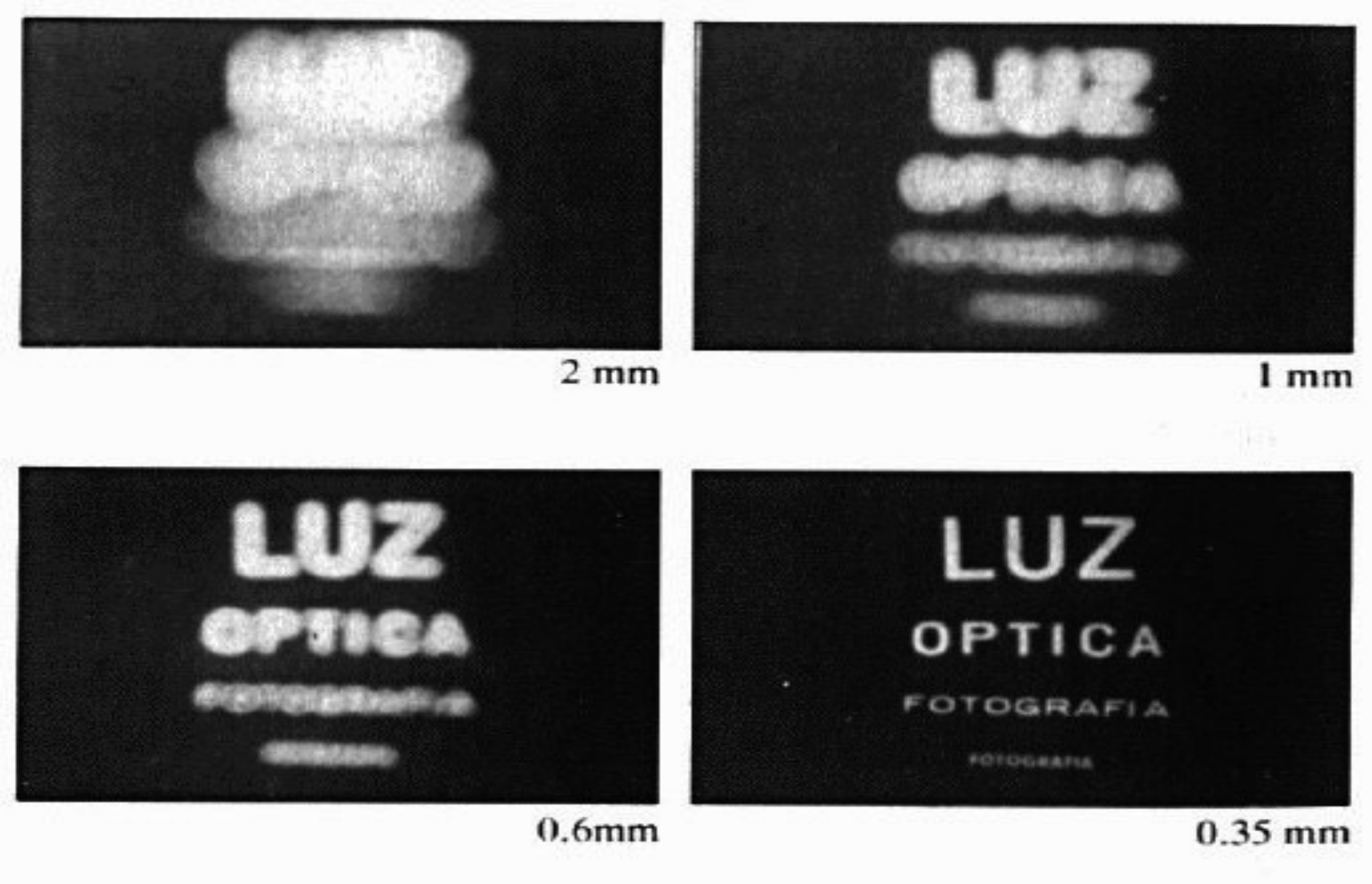
http://www.foundphotography.com/PhotoThoughts/archives/2005/04/pinhole_camera_2.html

Source: Freeman, Torralba, Isola



Source: Freeman, Torralba, Isola

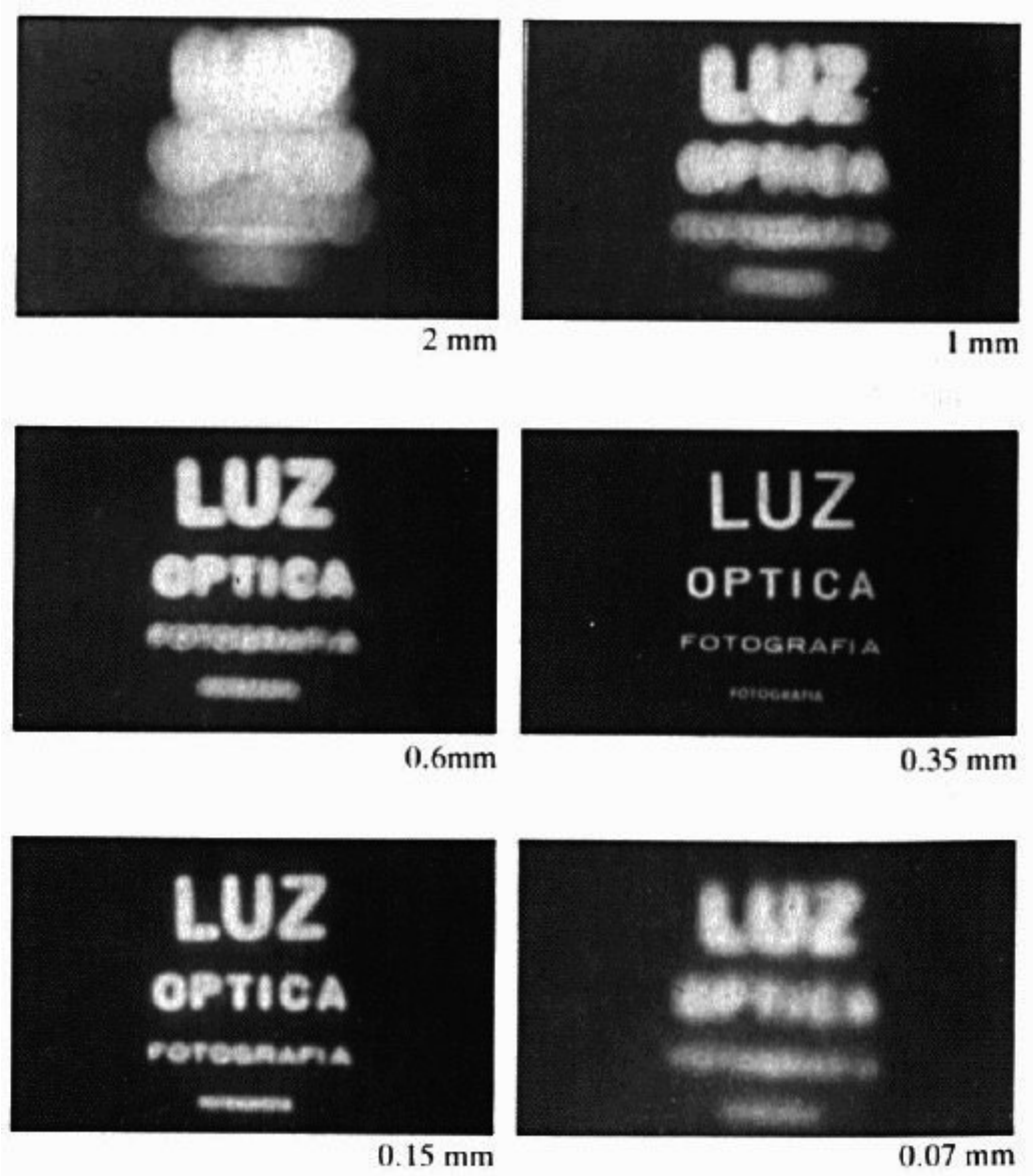
Shrinking the aperture



- Why not make the aperture as small as possible?
  - Less light gets through
  - *Diffraction* effects...

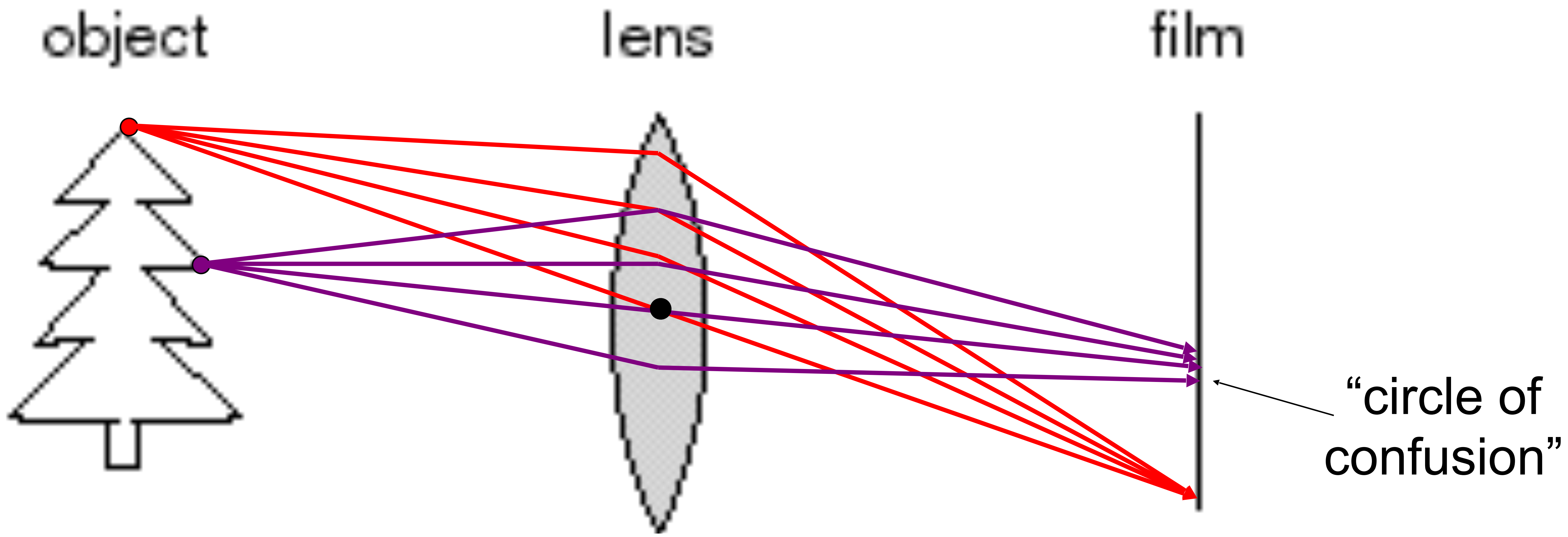
Source: N. Snavely

Shrinking the aperture



Source: N. Snavely

Adding a lens

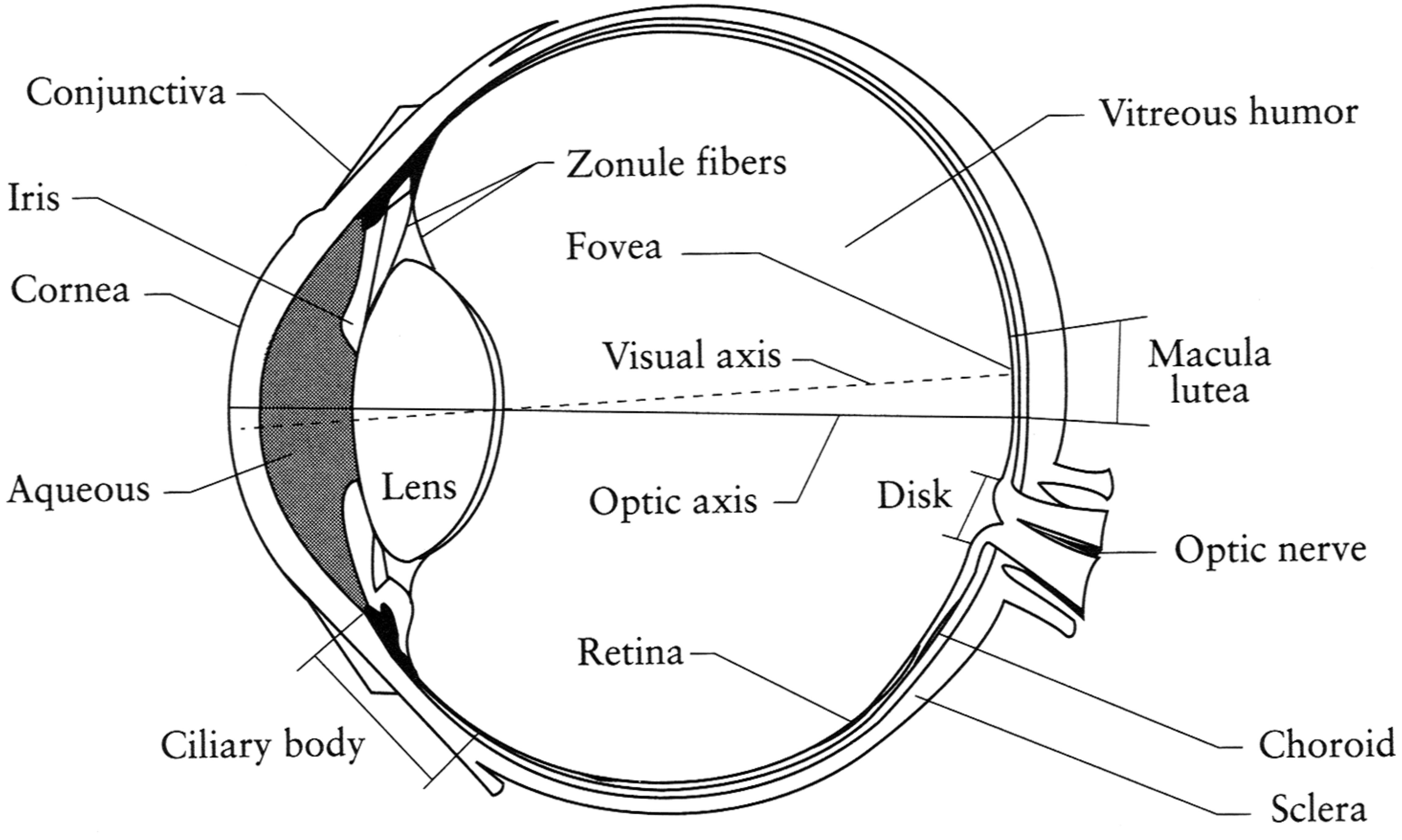


A lens focuses light onto the film

- There is a specific distance at which objects are “in focus”
  - other points project to a “circle of confusion” in the image
- Changing the shape of the lens changes this distance

Source: N. Snavely

The eye



The human eye is a camera

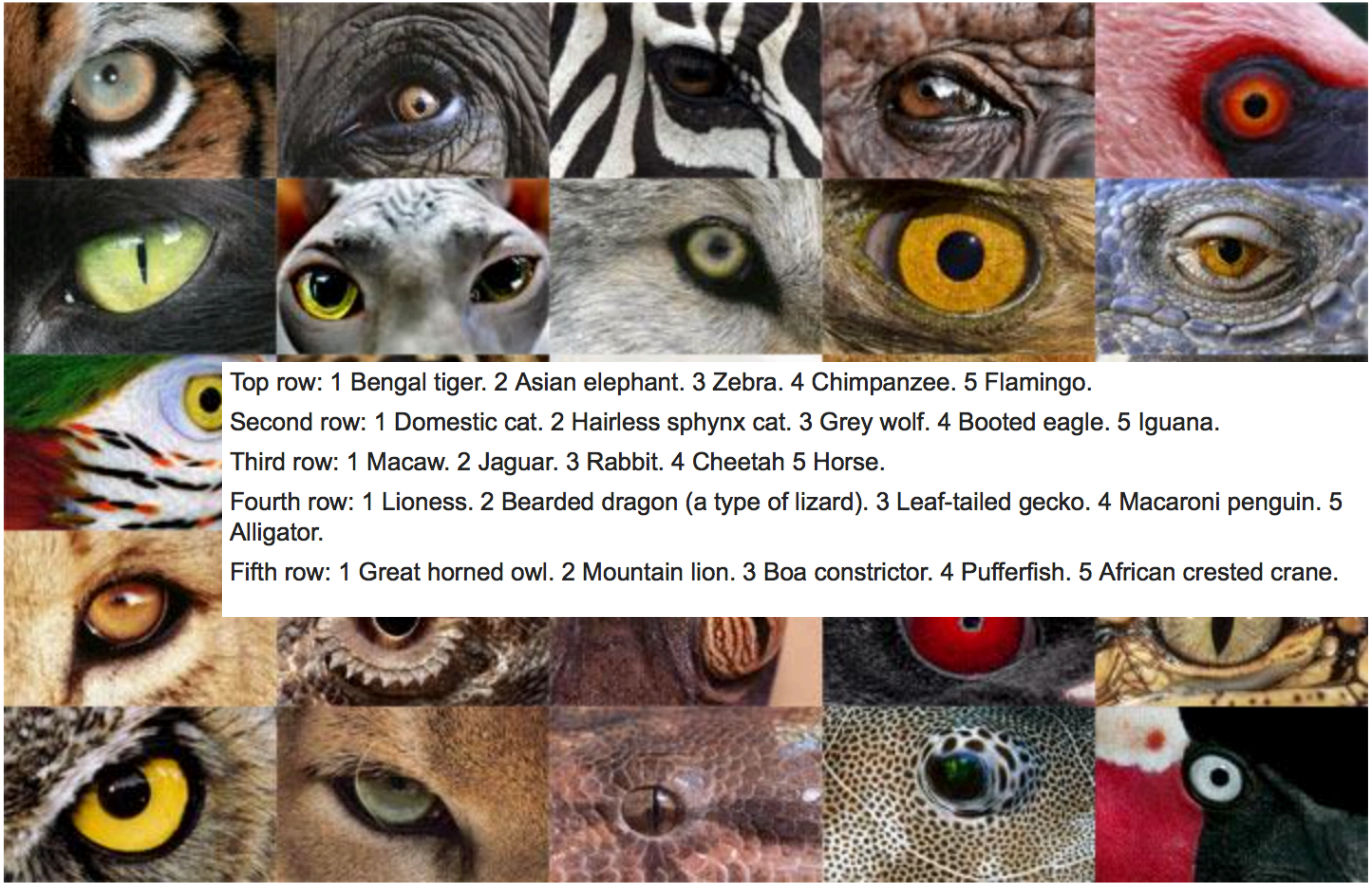
– **Iris -** colored annulus with radial muscles
– **Pupil -** the hole (aperture) whose size is controlled by the iris
– What's the "film"?
  – photoreceptor cells (rods and cones) in the **retina**

Source: N. Snavely

Eyes in nature: eyespots to pinhole camera

http://upload.wikimedia.org/wikipedia/commons/6/6d/Mantis_shrimp.jpg

Source: Freeman, Torralba, Isola



Top row: 1 Bengal tiger. 2 Asian elephant. 3 Zebra. 4 Chimpanzee. 5 Flamingo.

Second row: 1 Domestic cat. 2 Hairless sphynx cat. 3 Grey wolf. 4 Booted eagle. 5 Iguana.

Third row: 1 Macaw. 2 Jaguar. 3 Rabbit. 4 Cheetah 5 Horse.

Fourth row: 1 Lioness. 2 Bearded dragon (a type of lizard). 3 Leaf-tailed gecko. 4 Macaroni penguin. 5 Alligator.

Fifth row: 1 Great horned owl. 2 Mountain lion. 3 Boa constrictor. 4 Pufferfish. 5 African crested crane.

http://www.telegraph.co.uk/news/earth/earthpicturegalleries/7598120/Animal-eyes-quiz-Can-you-work-out-which-creatures-these-are-from-their-eyes.html?image=25

Source: N. Snavely

Source: Freeman, Torralba, Isola

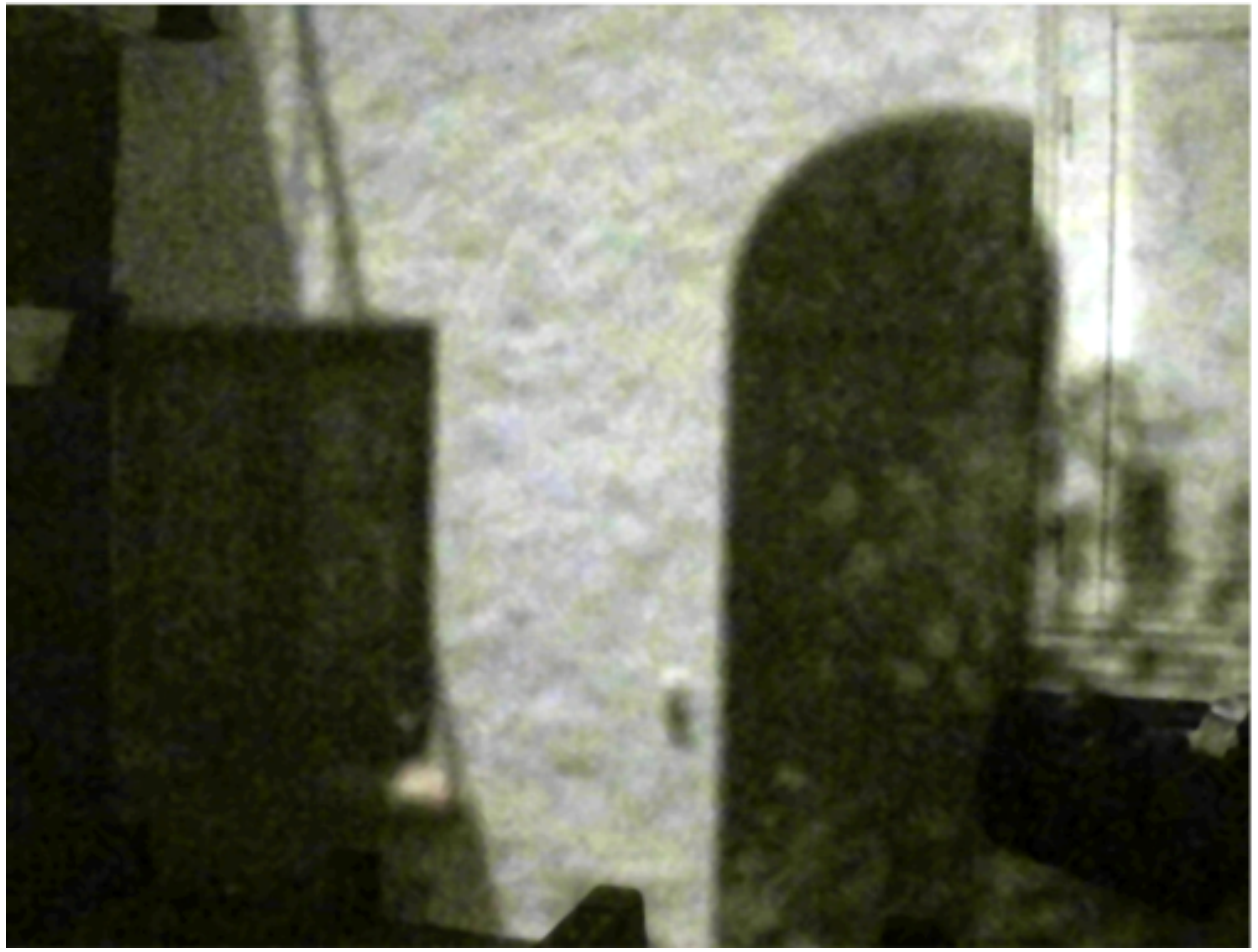
Source: Freeman, Torralba, Isola

Source: Freeman, Torralba, Isola

Accidental pinhole camera

Source: Freeman, Torralba, Isola

Window turned into a pinhole



View outside

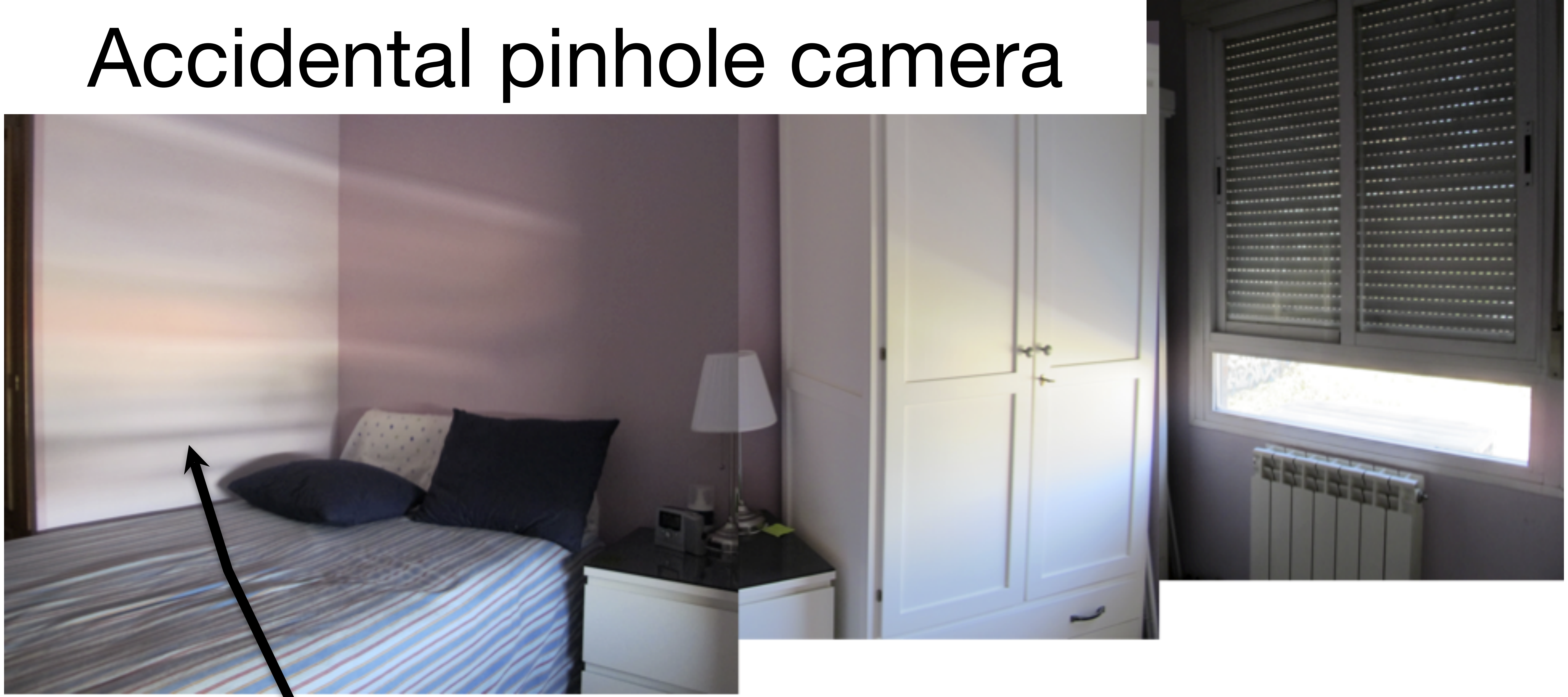
Source: Freeman, Torralba, Isola

Window open

Window turned into a pinhole

Source: Freeman, Torralba, Isola

Accidental pinhole camera



Outside scene

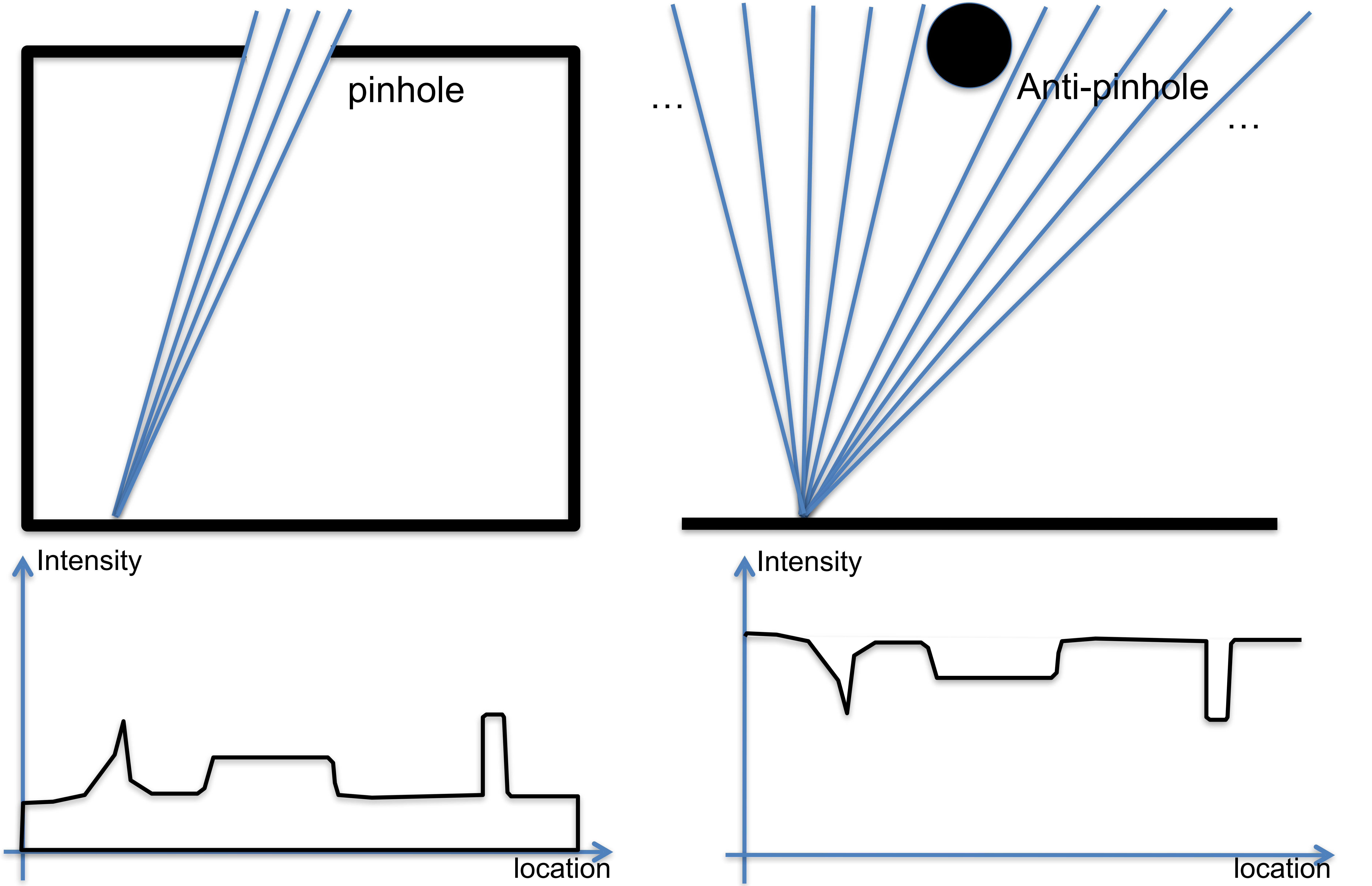
*

Aperture

See Zomet, A.; Nayar, S.K. CVPR 2006 for a detailed analysis.

Source: Freeman, Torralba, Isola

Pinhole and Anti-pinhole cameras



Adam L. Cohen, 1982

Source: Freeman, Torralba, Isola

Mixed accidental pinhole and anti-pinhole cameras

Mixed accidental pinhole and anti-pinhole cameras

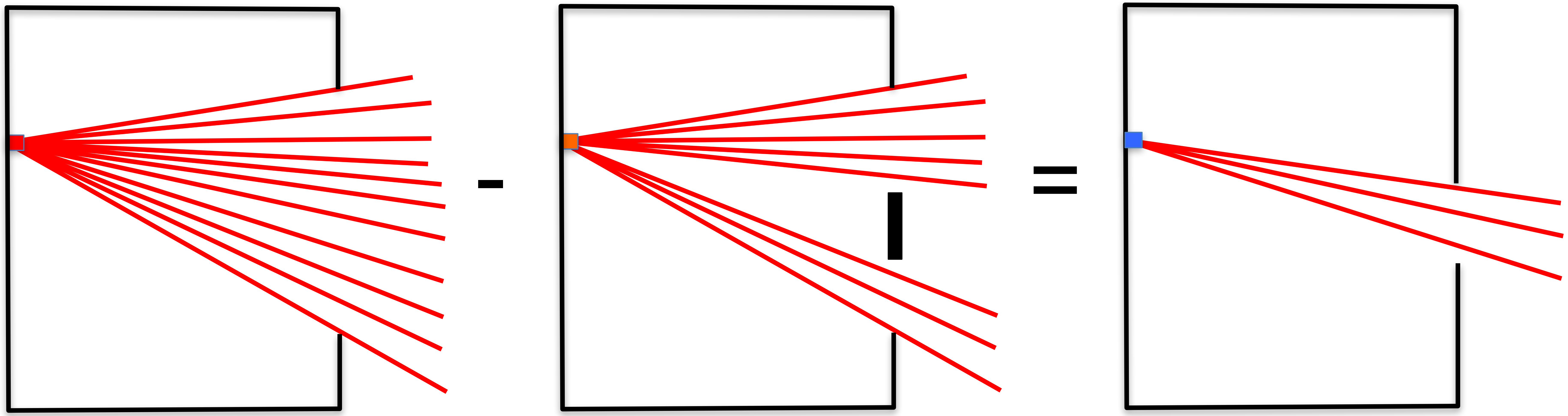
Source: Freeman, Torralba, Isola

Mixed accidental pinhole and anti-pinhole cameras

Room with a window

Person in front of the window

Difference image



Source: Freeman, Torralba, Isola

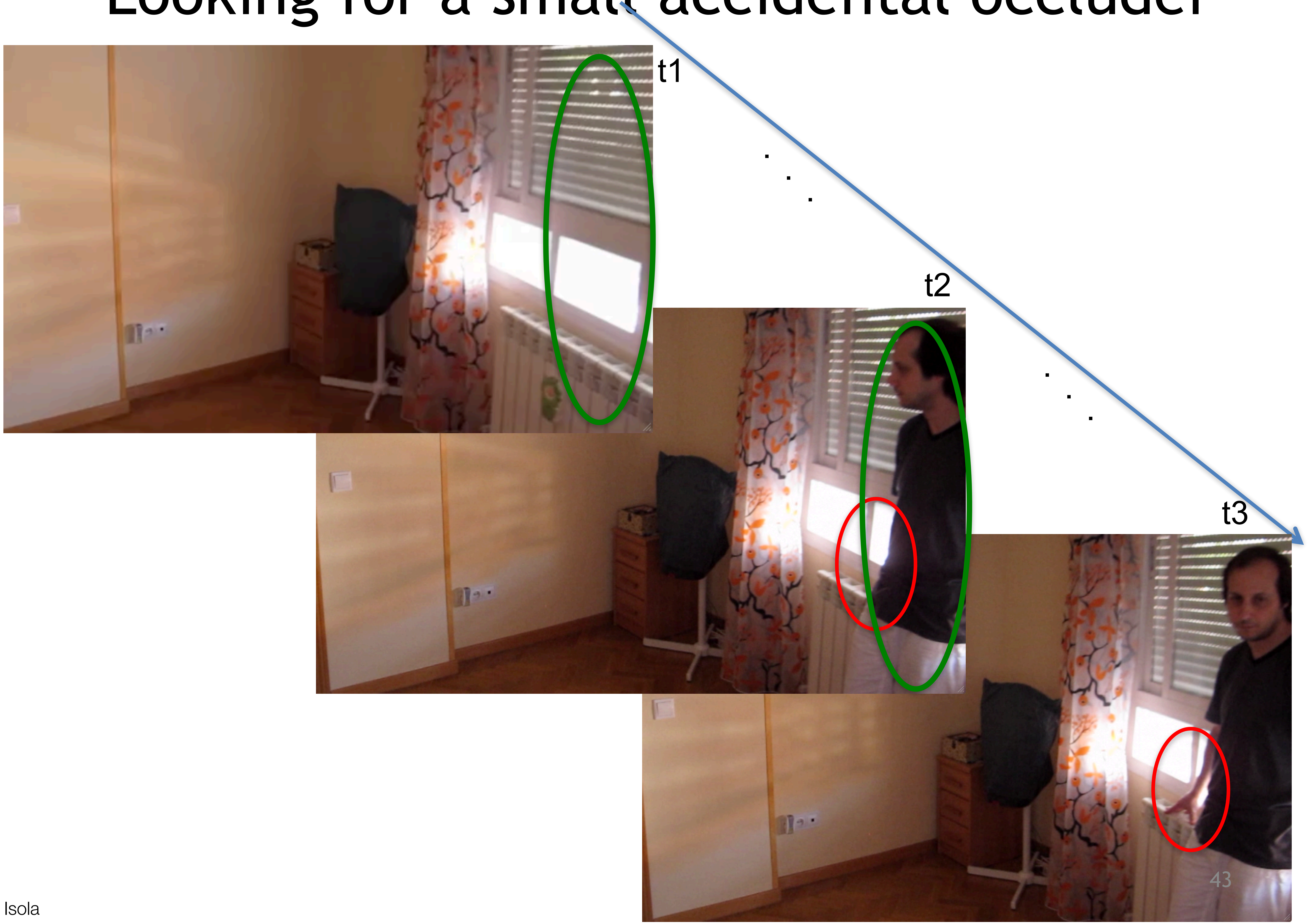
Mixed accidental pinhole and anti-pinhole cameras

Body as the occluder

View outside the window

Source: Freeman, Torralba, Isola

Looking for a small accidental occluder



Source: Freeman, Torralba, Isola

Looking for a small accidental occluder

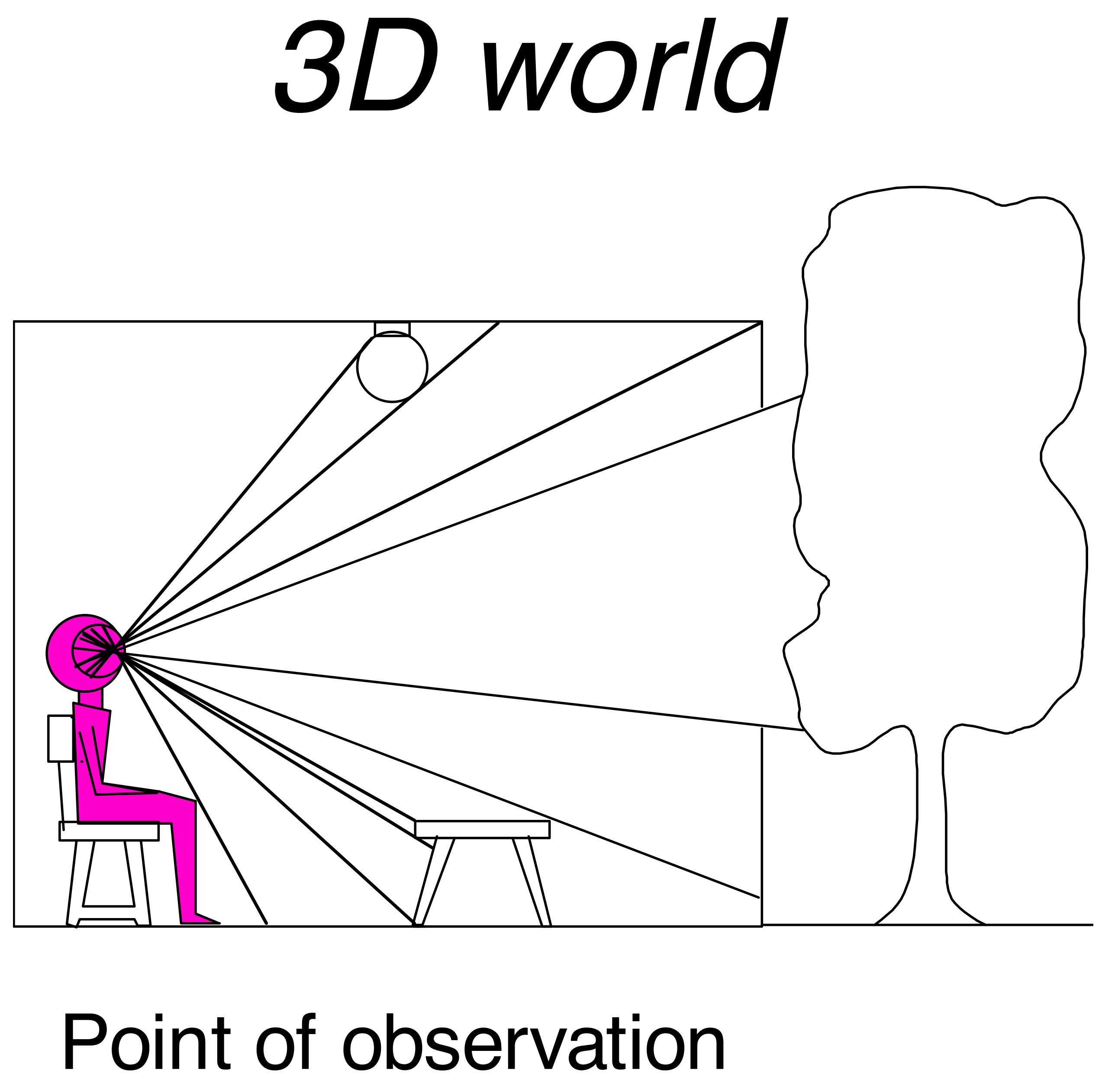
Body as the occluder

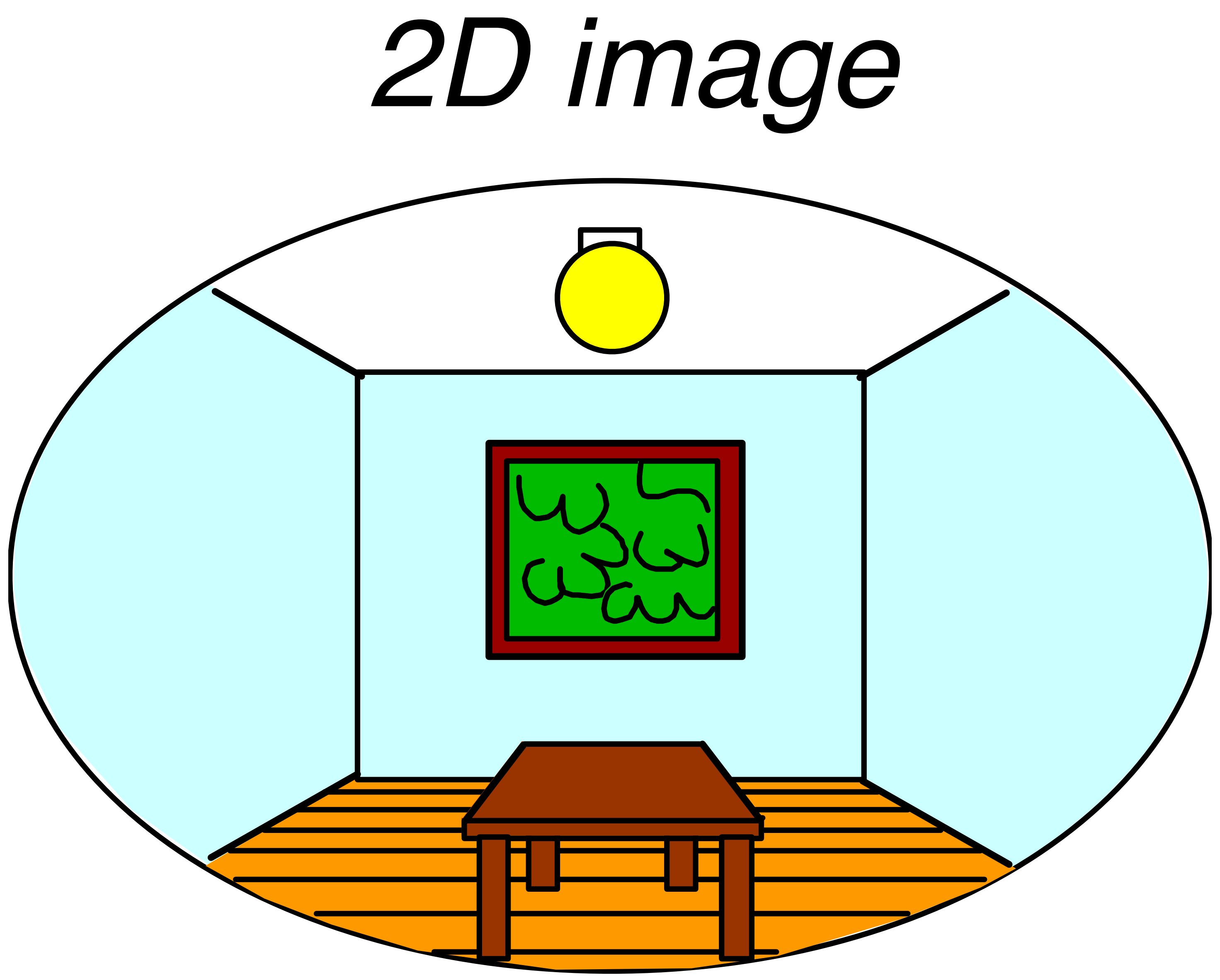
Hand as the occluder

View outside the window

Source: Freeman, Torralba, Isola

Dimensionality Reduction Machine (3D to 2D)





What have we lost?

- Angles
- Distances (lengths)

Slide by A. Efros

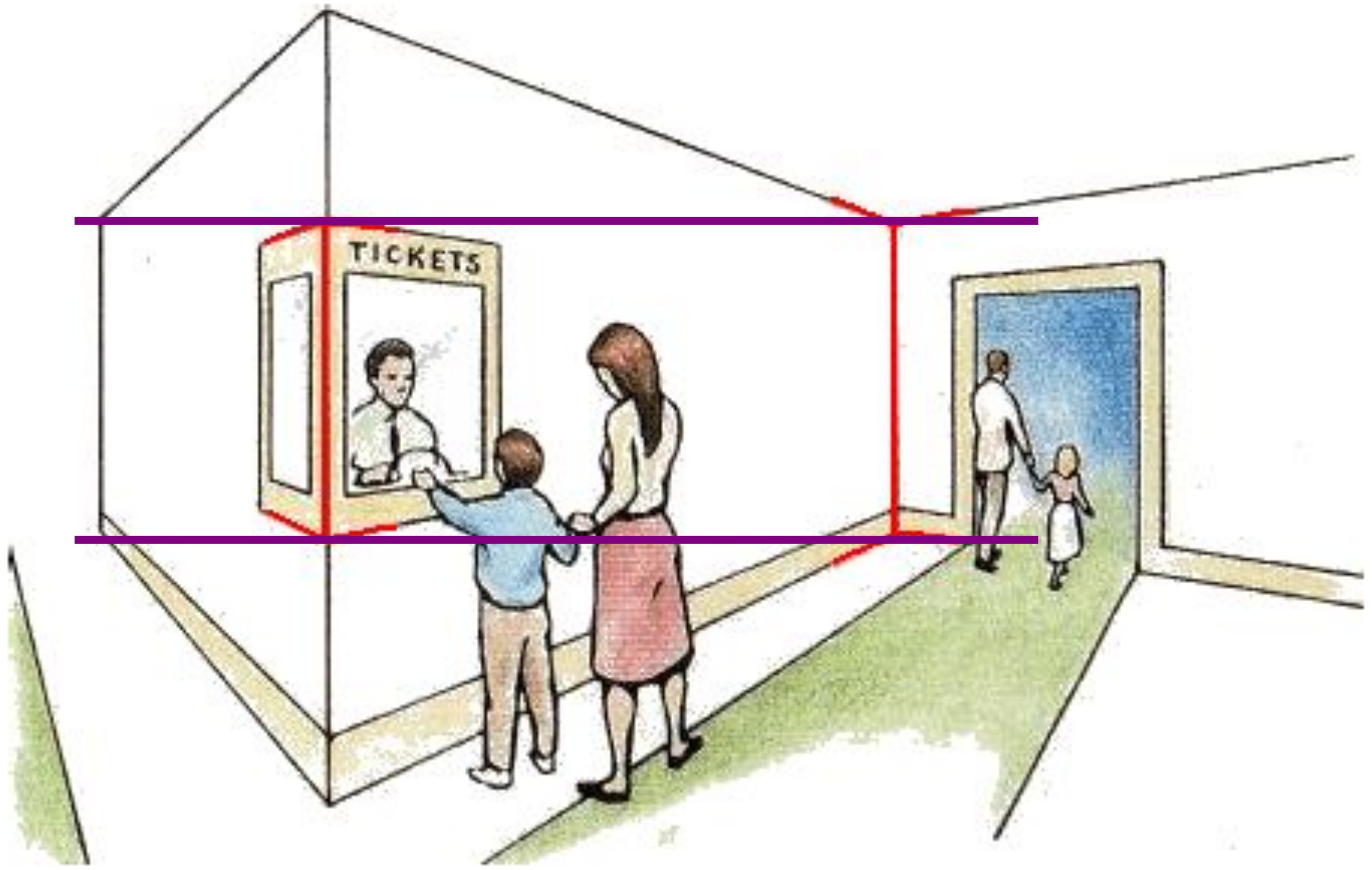
Projection

Source: N. Snavely

Projection

Source: N. Snavely

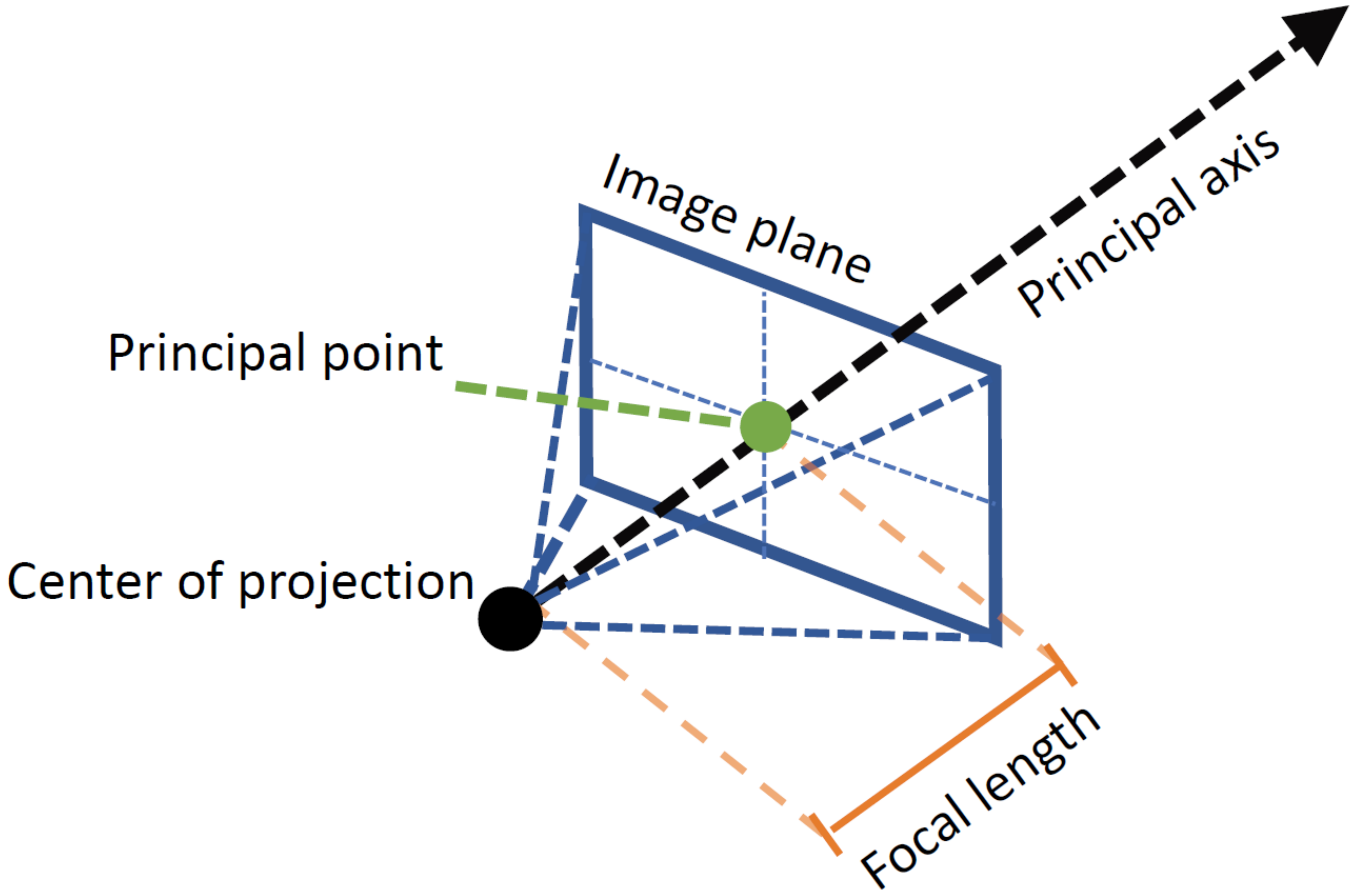
Müller-Lyer Illusion



http://www.michaelbach.de/ot/sze_muelue/index.html

Source: N. Snavely

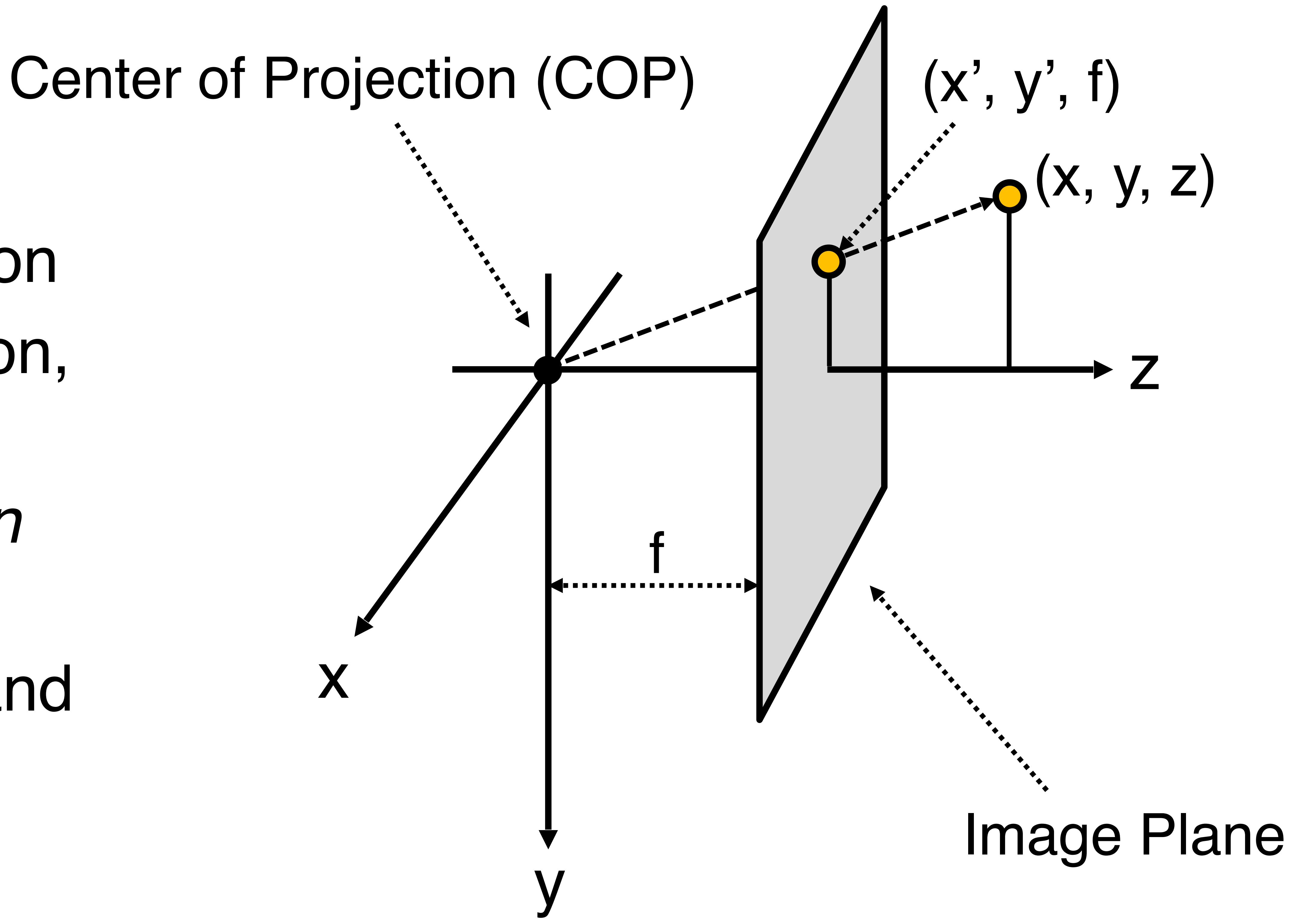
Geometric Model: A Pinhole Camera



Source: N. Snavely Figure credit: Peter Hedman

Modeling projection

- The coordinate system
  - We use the pinhole model as an approximation
  - Put the optical center (aka Center of Projection, or COP) at the origin
  - Put the Image Plane (aka Projection Plane) *in front* of the COP
  - The camera looks down the *positive* z-axis, and the y-axis points down
    - we like this if we want right-handed-coordinates
    - other versions are possible (e.g., OpenGL)



Source: N. Snavely

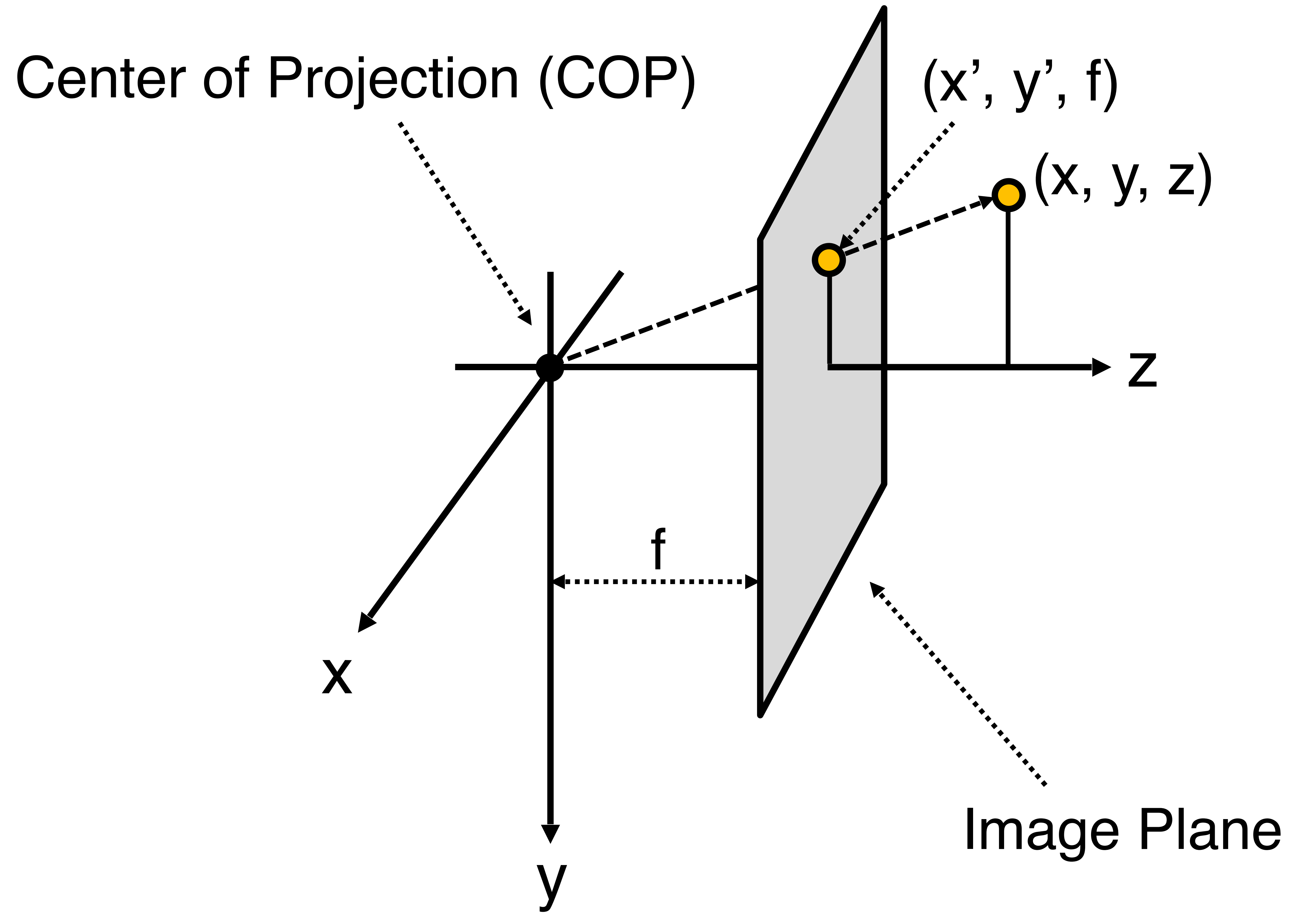
Modeling projection

- Projection equations
  - Compute intersection with PP of ray from (x,y,z) to COP
  - Derived using similar triangles

$$(x,y,z) \rightarrow (f\frac{x}{z}, f\frac{y}{z}, f)$$

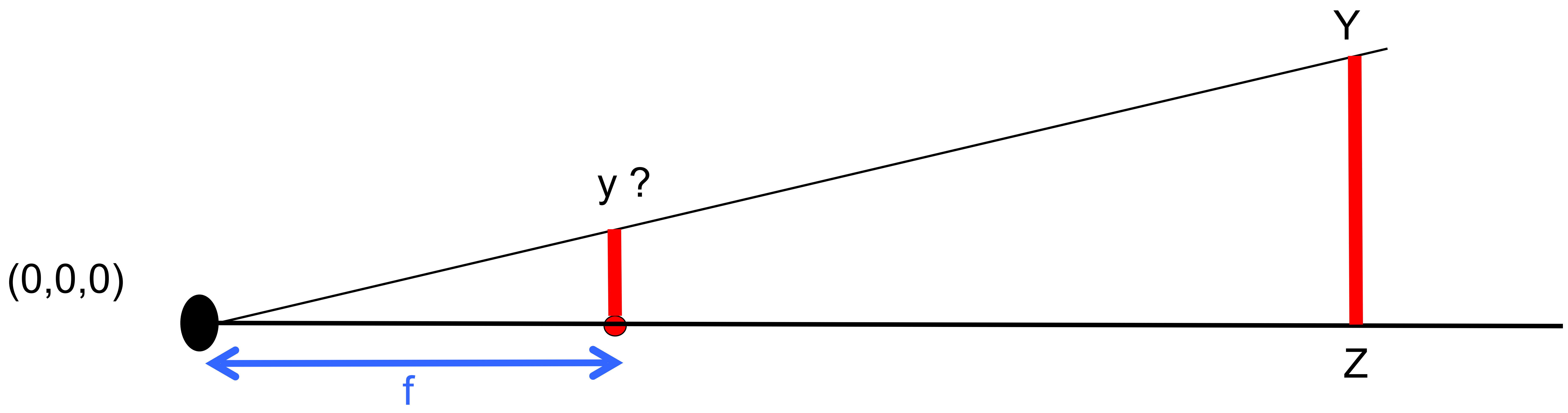
  - We get the projection by throwing out the last coordinate:

$$(x,y,z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$



Source: N. Snavely

Perspective projection



Similar triangles: $y / f = Y / Z$

$y = f\, Y/Z$

How can we represent this more compactly?

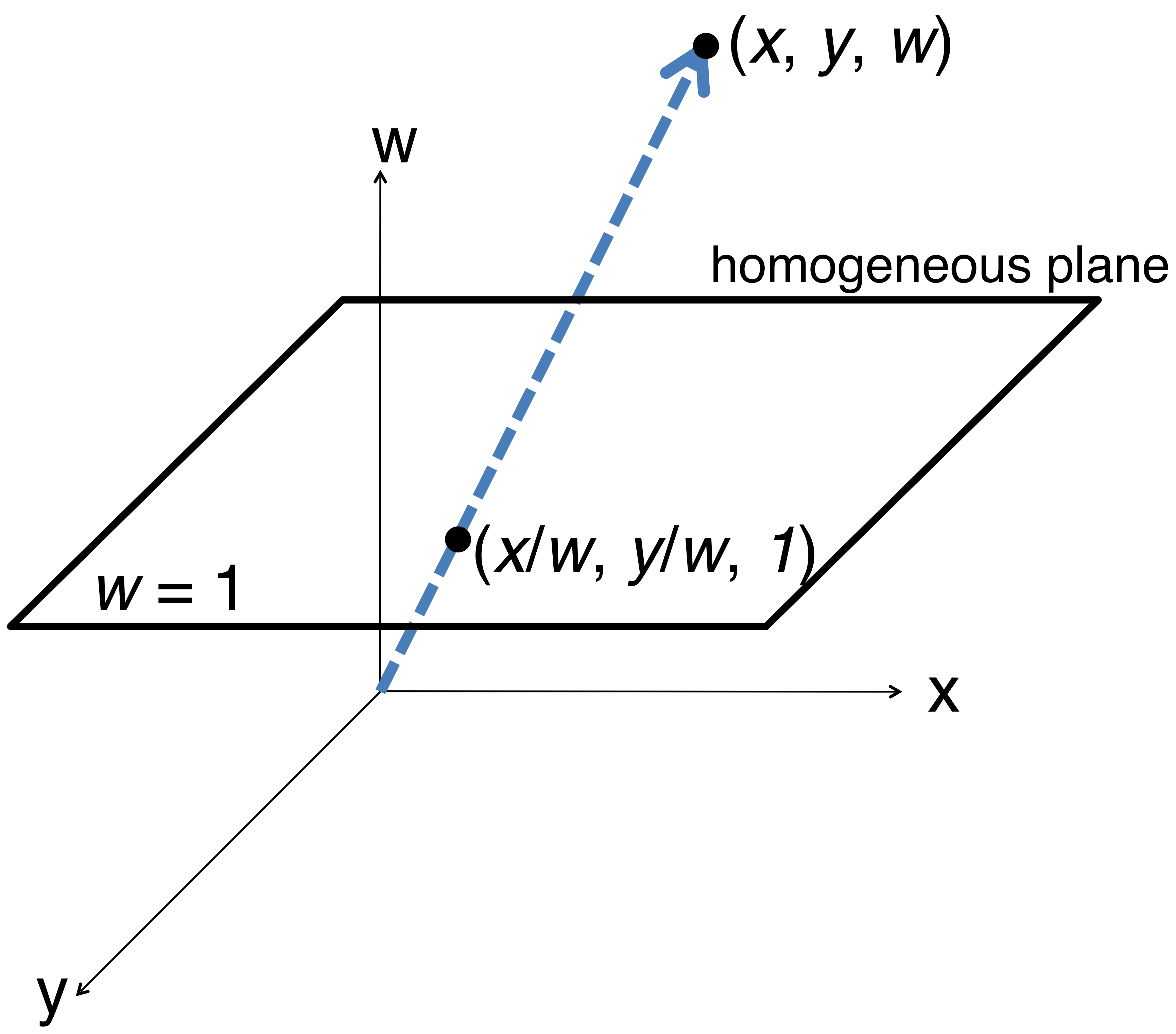
Source: Freeman, Torralba, Isola

Homogeneous coordinates for affine transformations

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates



Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

Source: N. Snavely

Translation

- Solution: homogeneous coordinates to the rescue

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

Source: N. Snavely

Affine transformations

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

any transformation represented by a 3x3 matrix with last row [0 0 1] we call an *affine*

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$

Source: N. Snavely

Basic affine transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{s_x} & 0 & 0 \\ 0 & \boldsymbol{s_y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2D *in-plane* rotation

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \boldsymbol{sh_x} & 0 \\ \boldsymbol{sh_y} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Shear

Source: N. Snavely

Modeling projection

- Is this a linear transformation?
    - no—division by z is nonlinear

Homogeneous coordinates to the rescue!

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Source: N. Snavely

Perspective Projection

Projection is a matrix multiply using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f \end{bmatrix} \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z}\right)$$

divide by third coordinate

This is known as **perspective projection**

- The matrix is the **projection matrix**
- (Can also represent as a 4x4 matrix – OpenGL does something like this)

Source: N. Snavely

Perspective Projection

How does scaling the projection matrix change the transformation?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f \end{bmatrix} \Rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Scale by *f*:

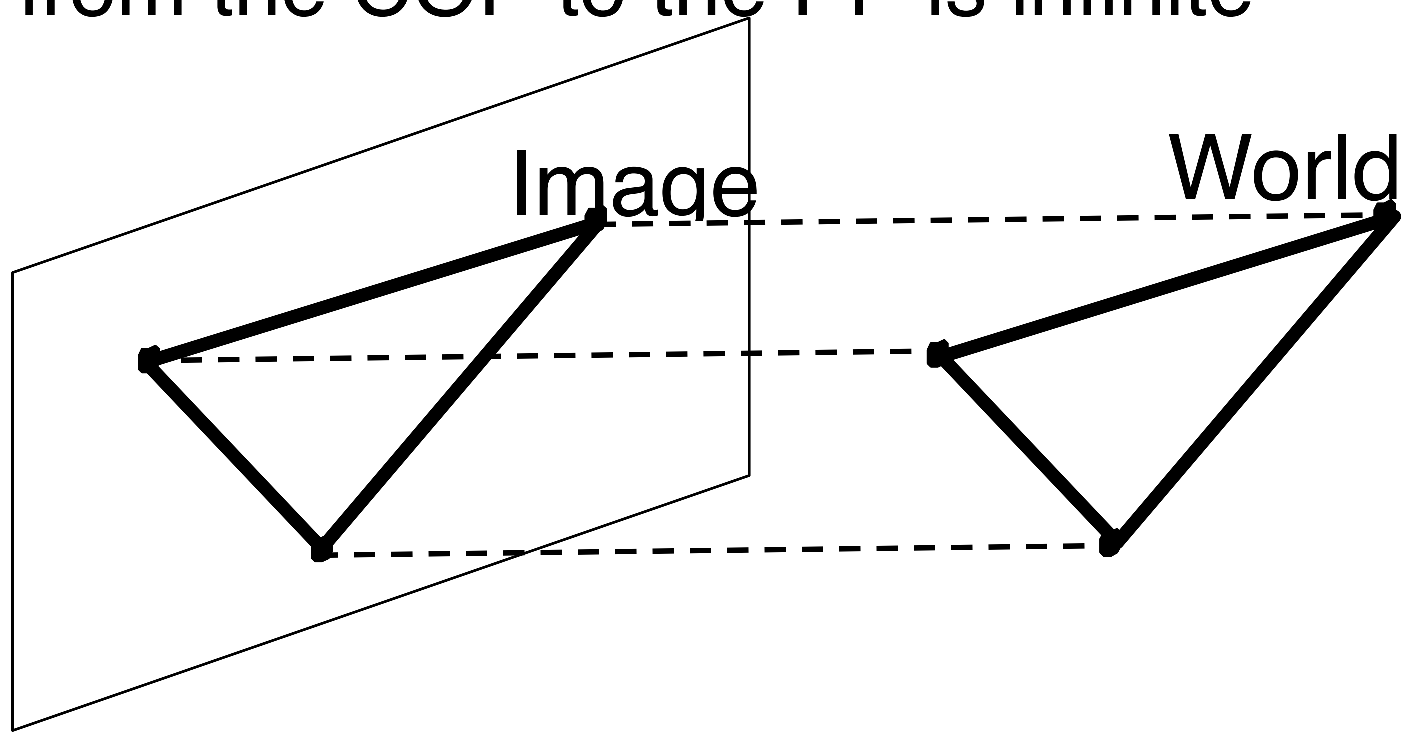
$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \Rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Scaling a projection matrix produces an equivalent projection matrix!

Source: N. Snavely

Orthographic projection

- Special case of perspective projection
  - Distance from the COP to the PP is infinite

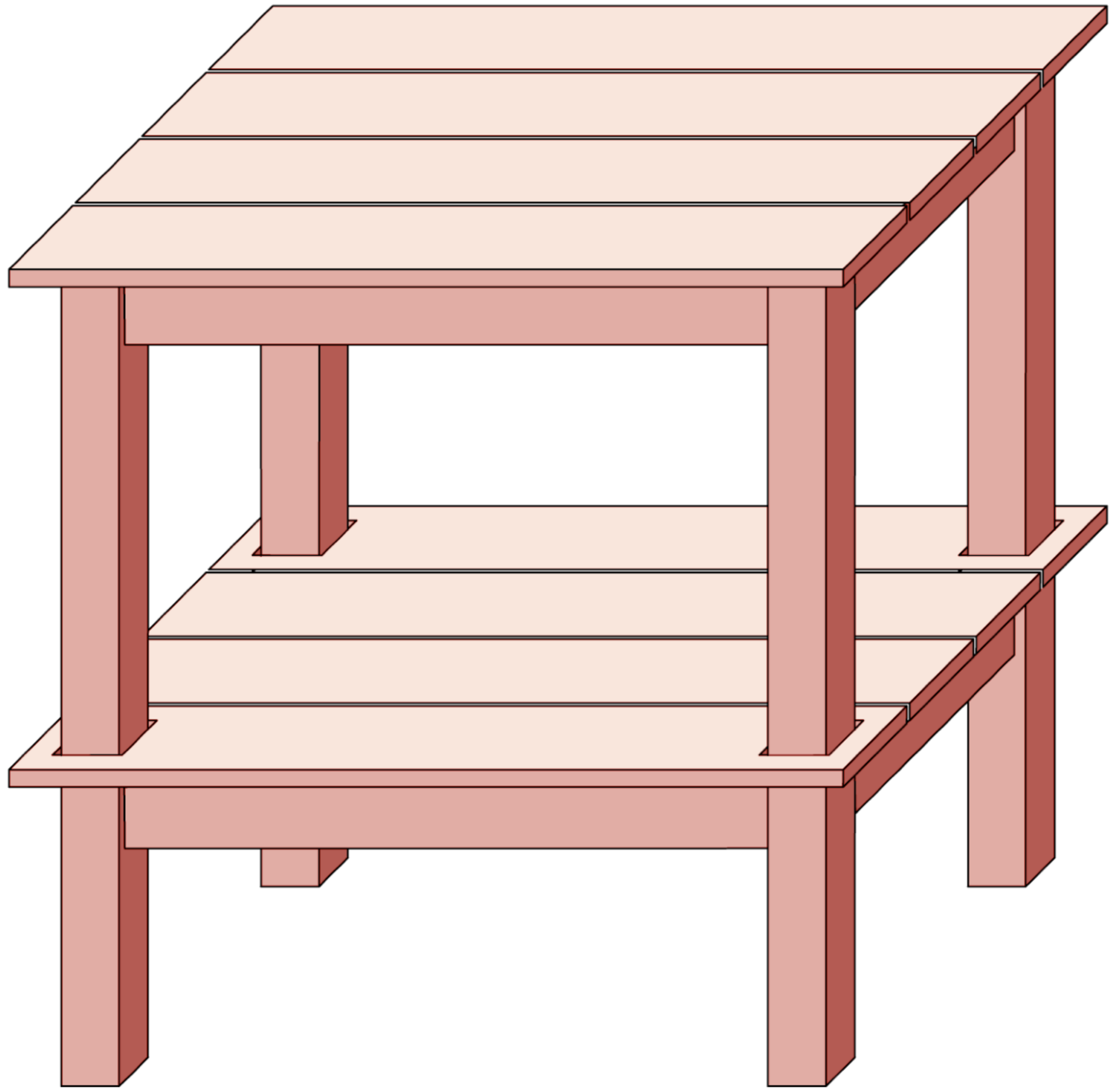


  - Good approximation for telephoto optics
  - Also called “parallel projection”: (x, y, z) → (x, y)
  - What’s the projection matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

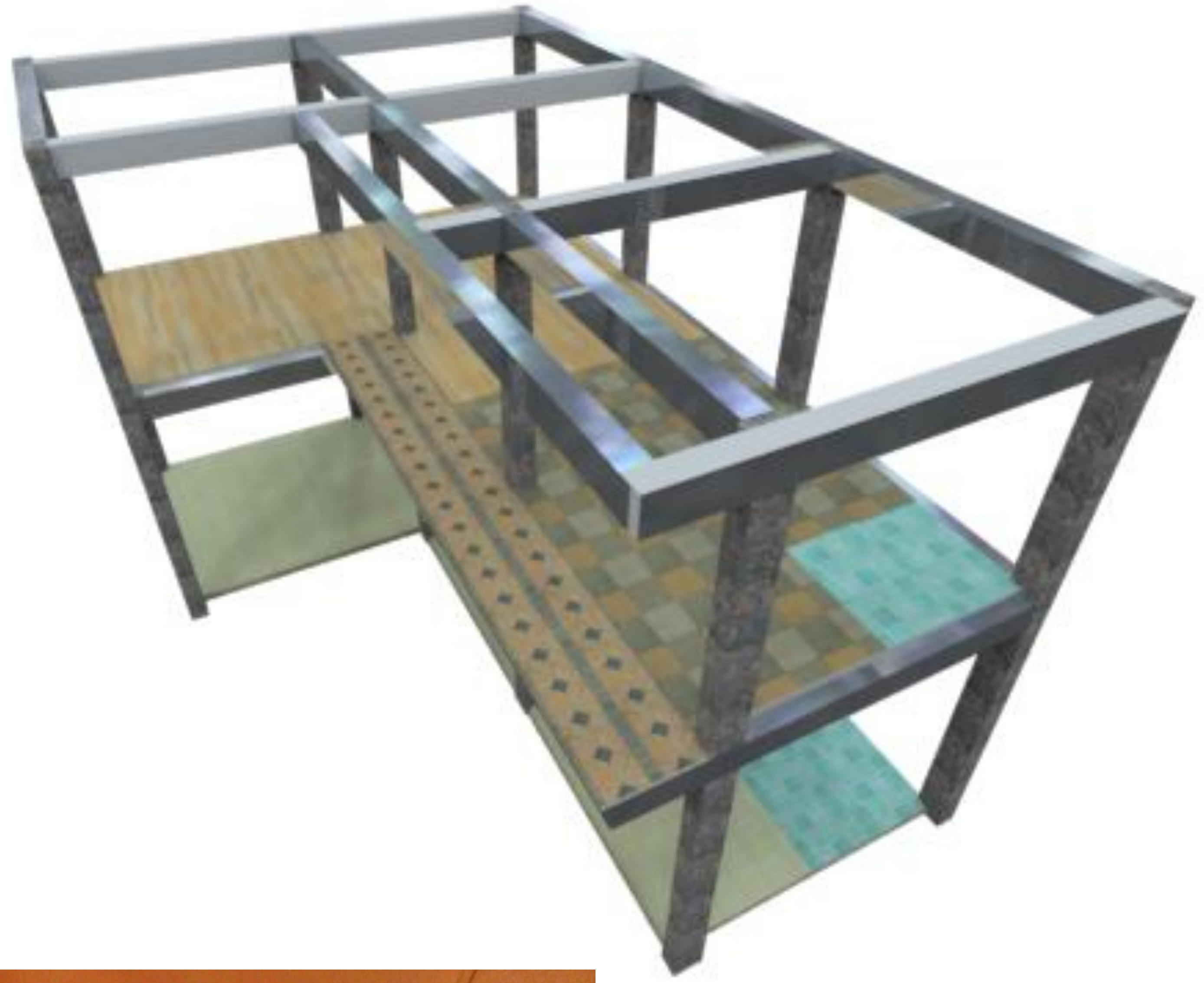
Source: N. Snavely

Orthographic projection



Source: N. Snavely

Perspective projection



Source: N. Snavely

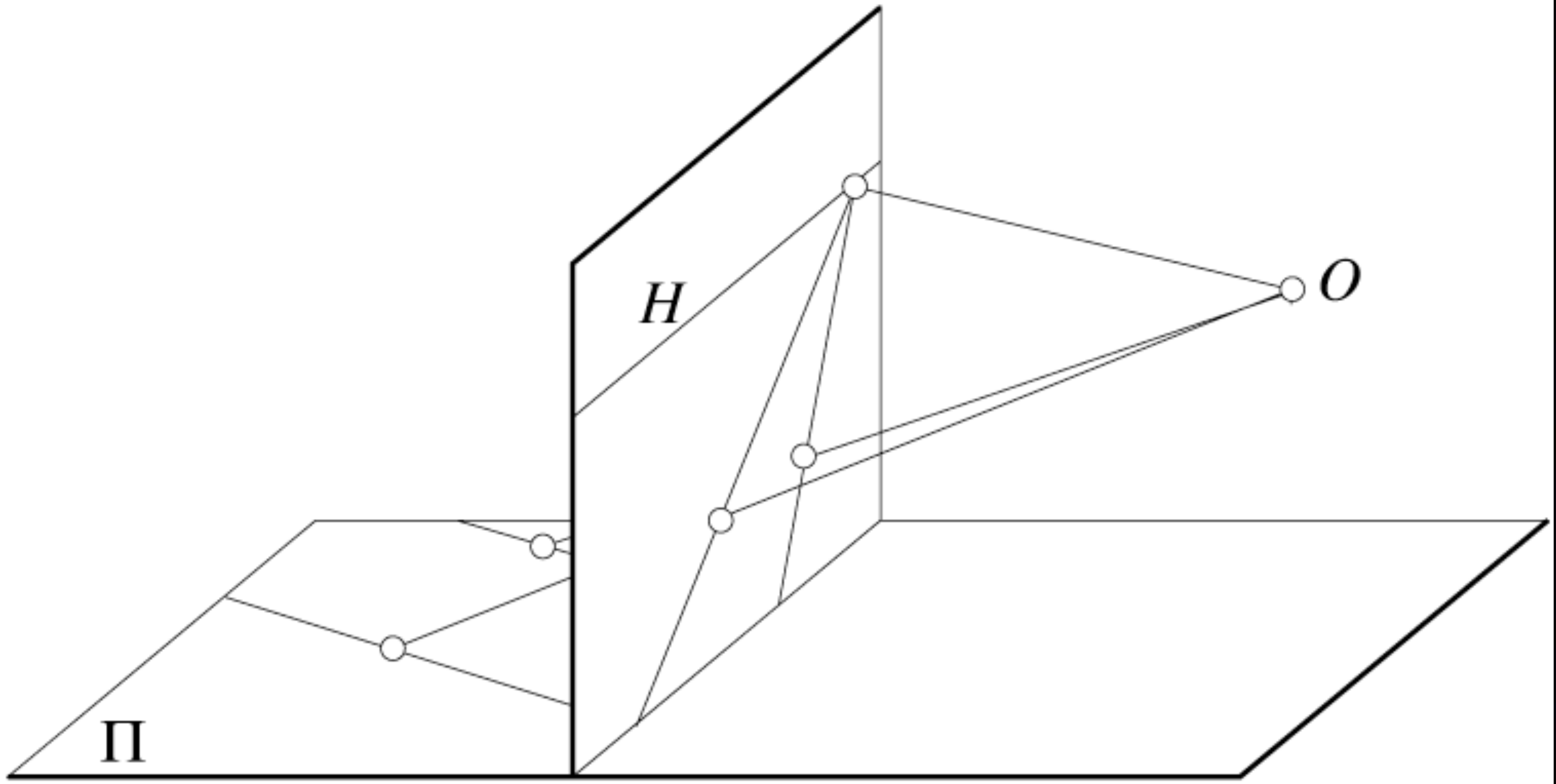
Projection properties

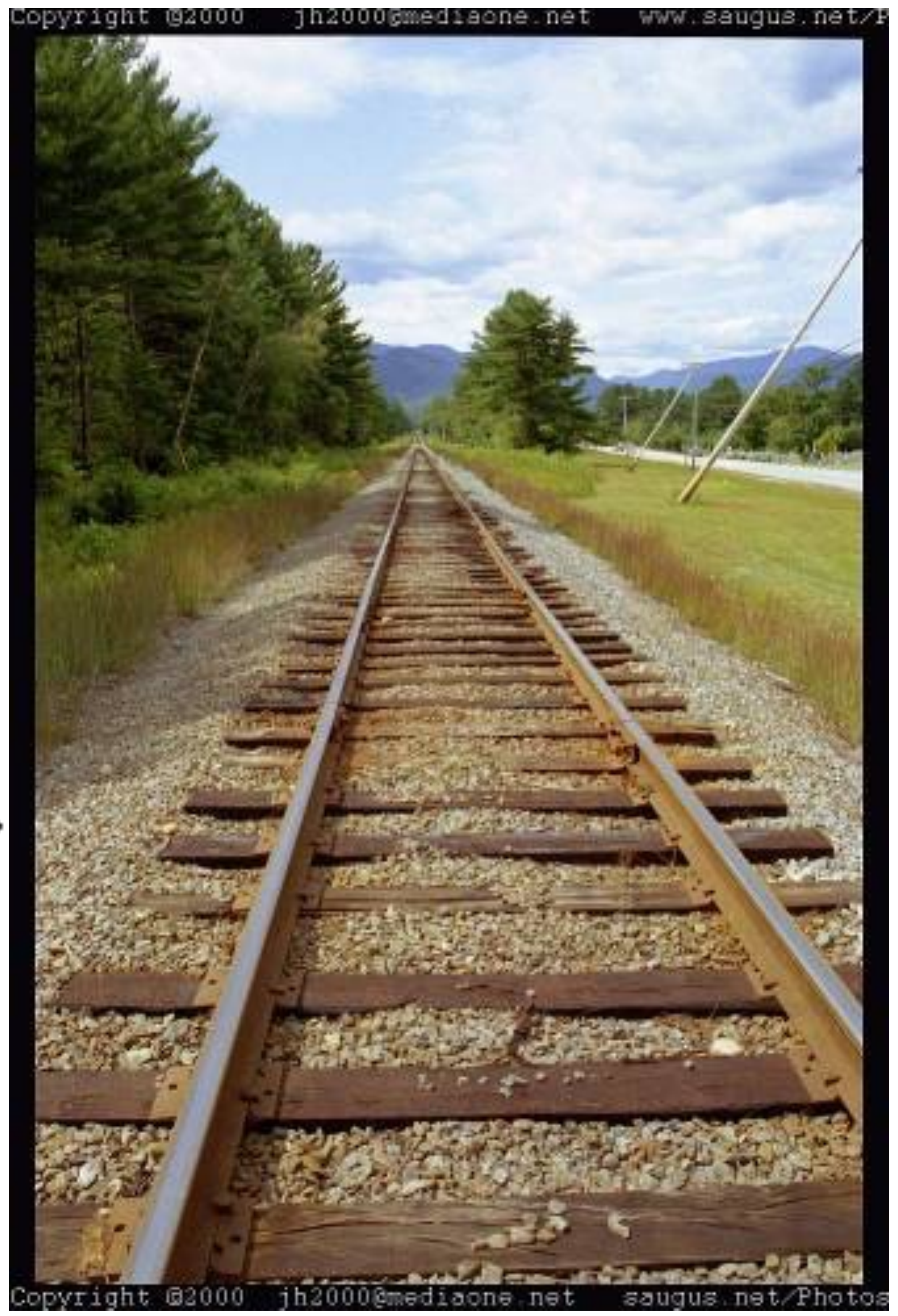
- Many-to-one: any points along same ray map to same point in image
- Points → points
- Lines → lines (collinearity is preserved)
  – But line through focal point projects to a point
- Planes → planes (or half-planes)
  – But plane through focal point projects to line

Source: N. Snavely

Projection properties

- Parallel lines converge at a vanishing point
  - Each direction in space has its own vanishing point
  - But lines parallel to the image plane remain parallel

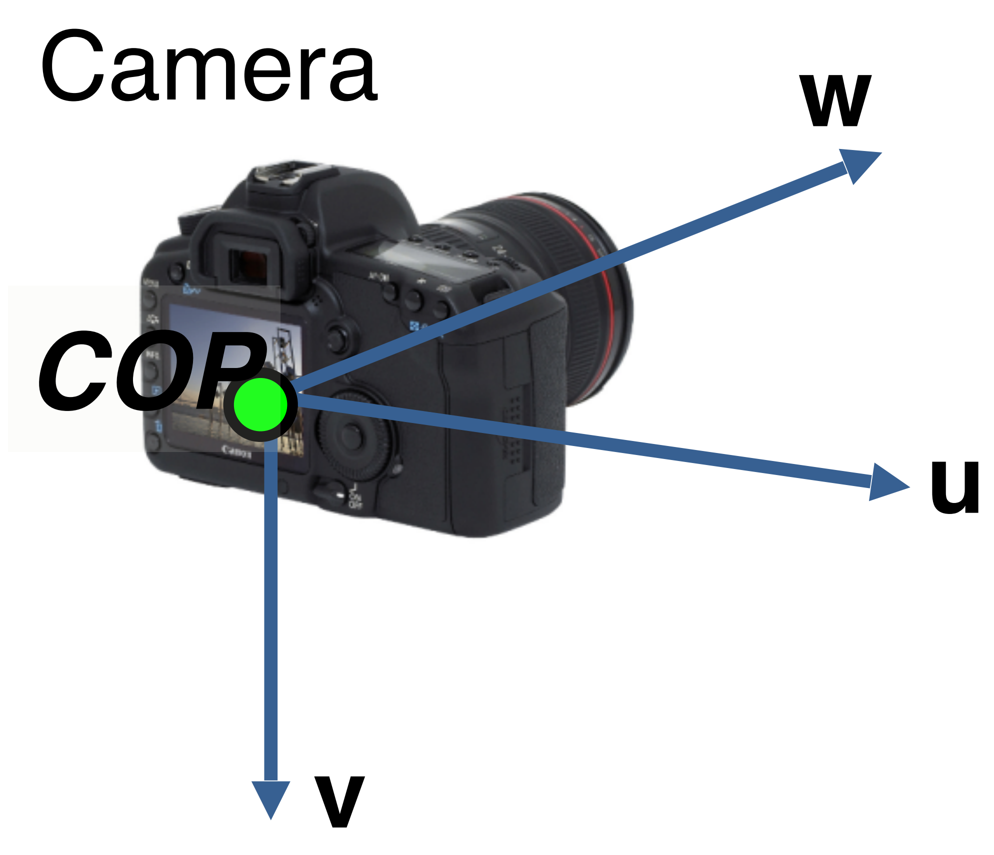


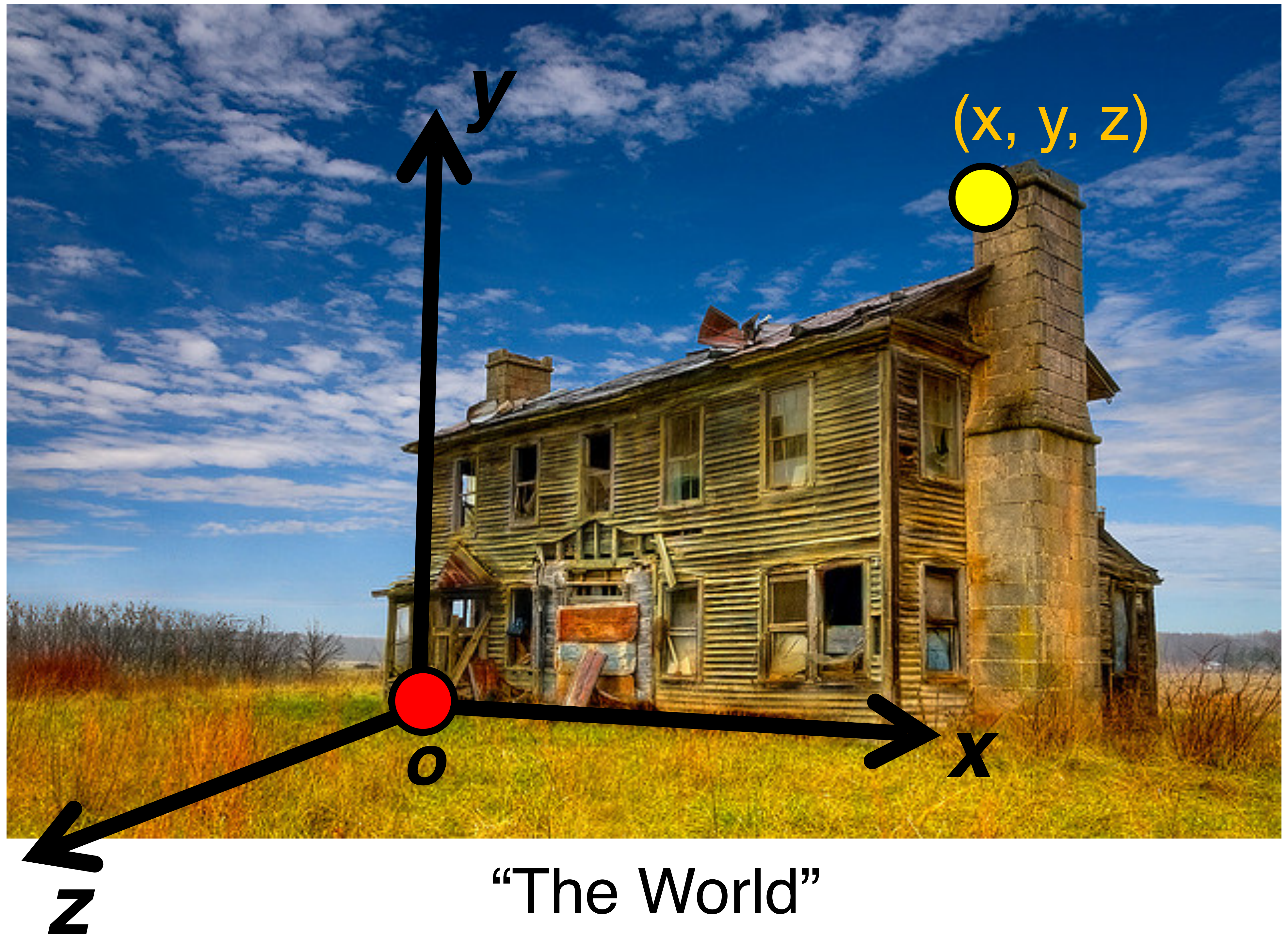


Source: N. Snavely

Camera parameters

- How can we model the geometry of a camera?





"The World"

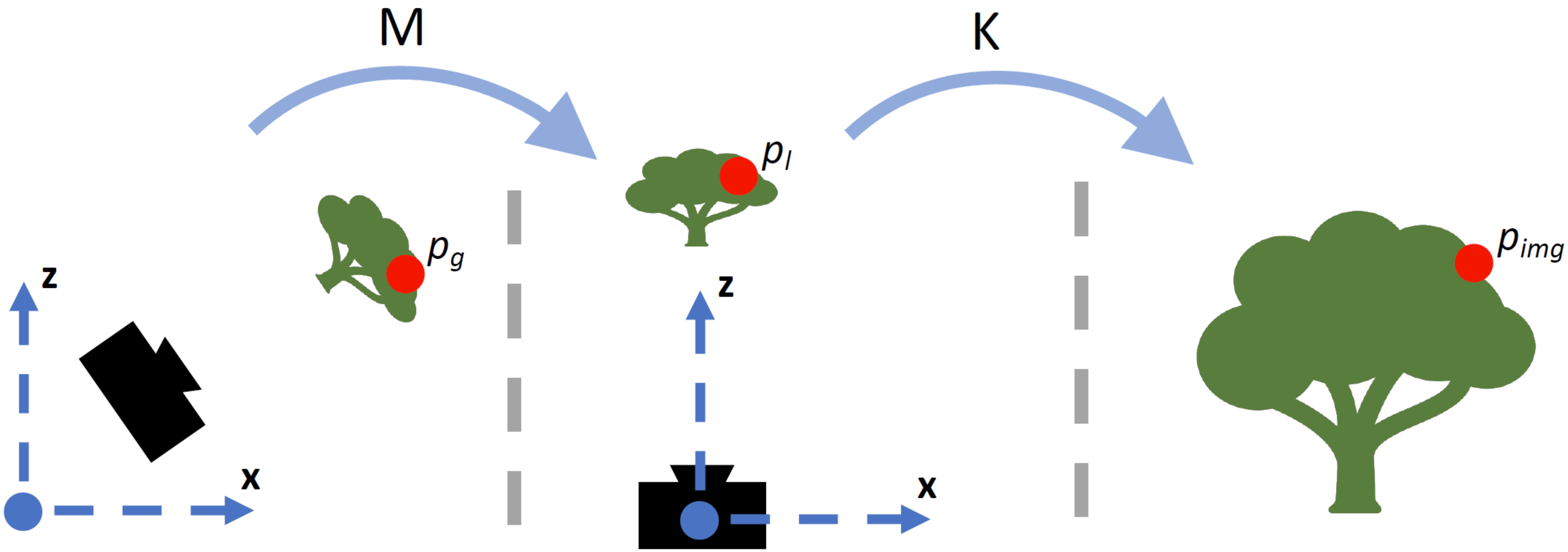
Three important coordinate systems:

1. *World* coordinates
2. *Camera* coordinates
3. *Image* coordinates

How do we project a given world point (x, y, z) to an image point?

Source: N. Snavely

Coordinate frames



World coordinates Camera coordinates Image coordinates

Source: N. Snavely

Figure credit: Peter Hedman

Camera parameters

To project a point (*x*, *y*, *z*) in *world* coordinates into a camera

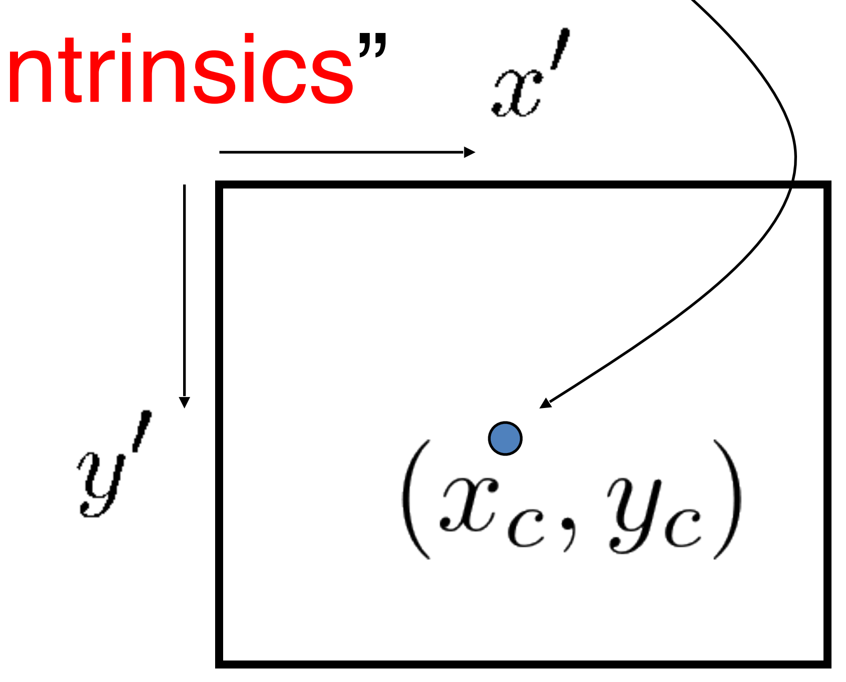
- First transform (*x*, *y*, *z*) into *camera* coordinates
- Need to know
  - Camera position (in world coordinates)
  - Camera orientation (in world coordinates)
- Then project into the image plane to get *image (pixel) coordinates*
  - Need to know camera *intrinsics*

Source: N. Snavely

Camera parameters

A camera is described by several parameters

- Translation T of the optical center from the origin of world coords
- Rotation R of the image plane
- focal length f, principal point (c_x, c_y), pixel aspect size α
- blue parameters are called “extrinsics,” red are “intrinsics”



Projection equation

$$\mathbf{x} = \begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{\Pi X}$$

- The projection matrix models the cumulative effect of all parameters
- Useful to decompose into a series of operations

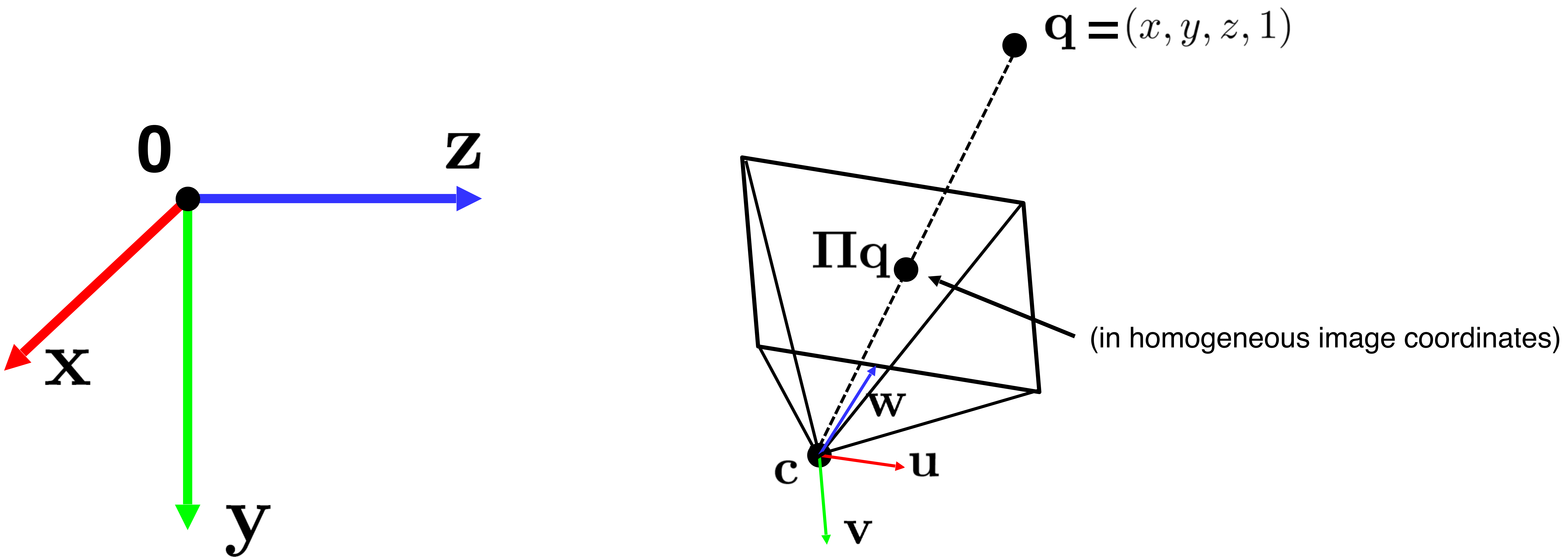
identity matrix

$$\mathbf{\Pi} = \underbrace{\begin{bmatrix} f & s & c_x \\ 0 & \alpha f & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsics}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{R}_{3\times3} & \mathbf{0}_{3\times1} \\ \mathbf{0}_{1\times3} & 0 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3\times3} & \mathbf{T}_{3\times1} \\ \mathbf{0}_{1\times3} & 0 \end{bmatrix}}_{\text{translation}}$$

- The definitions of these parameters are **not** completely standardized
  - especially intrinsics—varies from one book to another

Source: N. Snavely

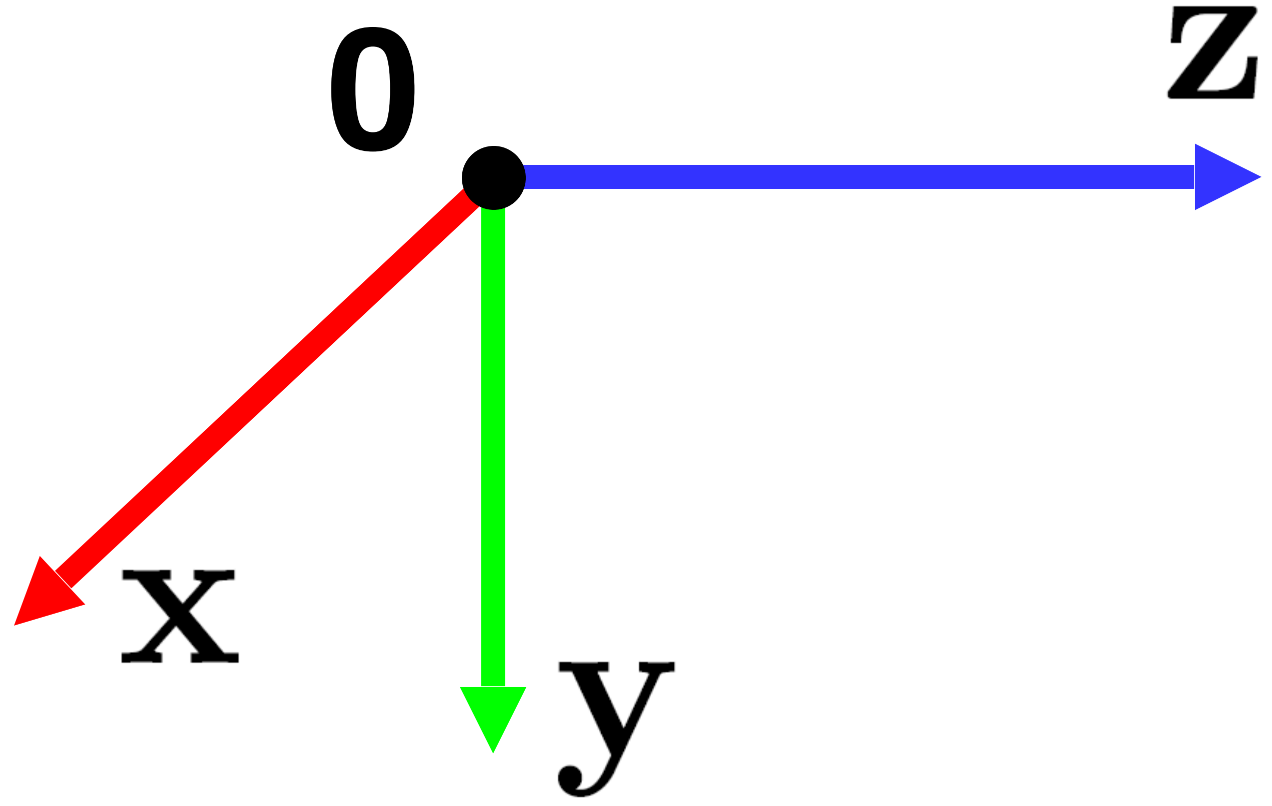
Projection matrix



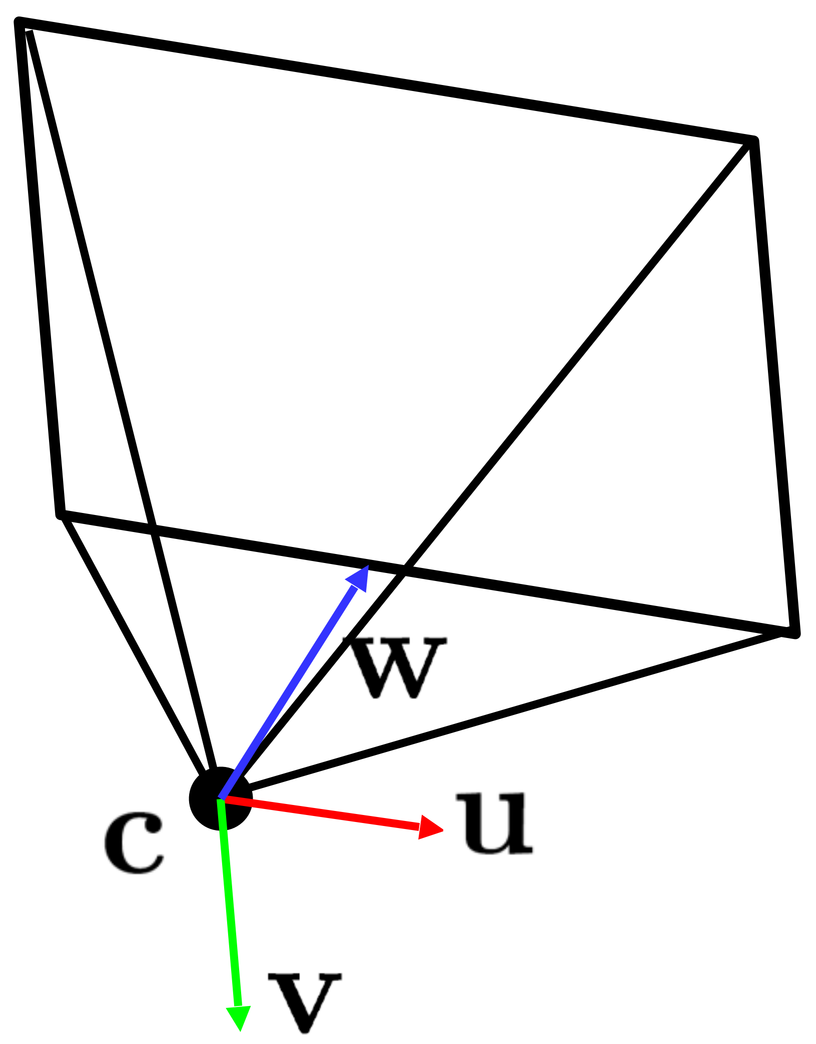
Source: N. Snavely

Extrinsics

- How do we get the camera to "canonical form"?
  - (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)



Step 1: Translate by **-c**



Source: N. Snavely

Extrinsics

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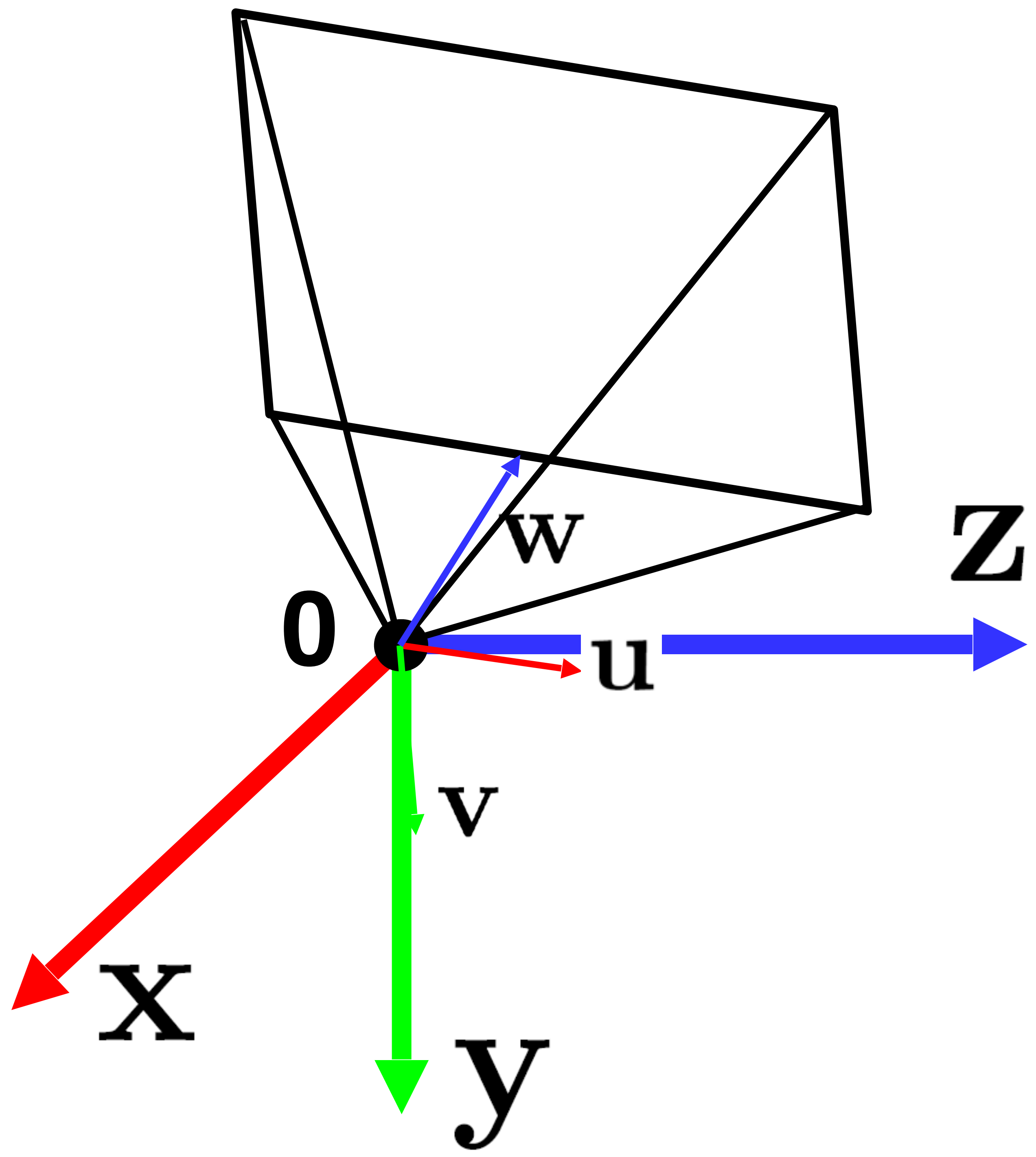


Step 1: Translate by **-c**

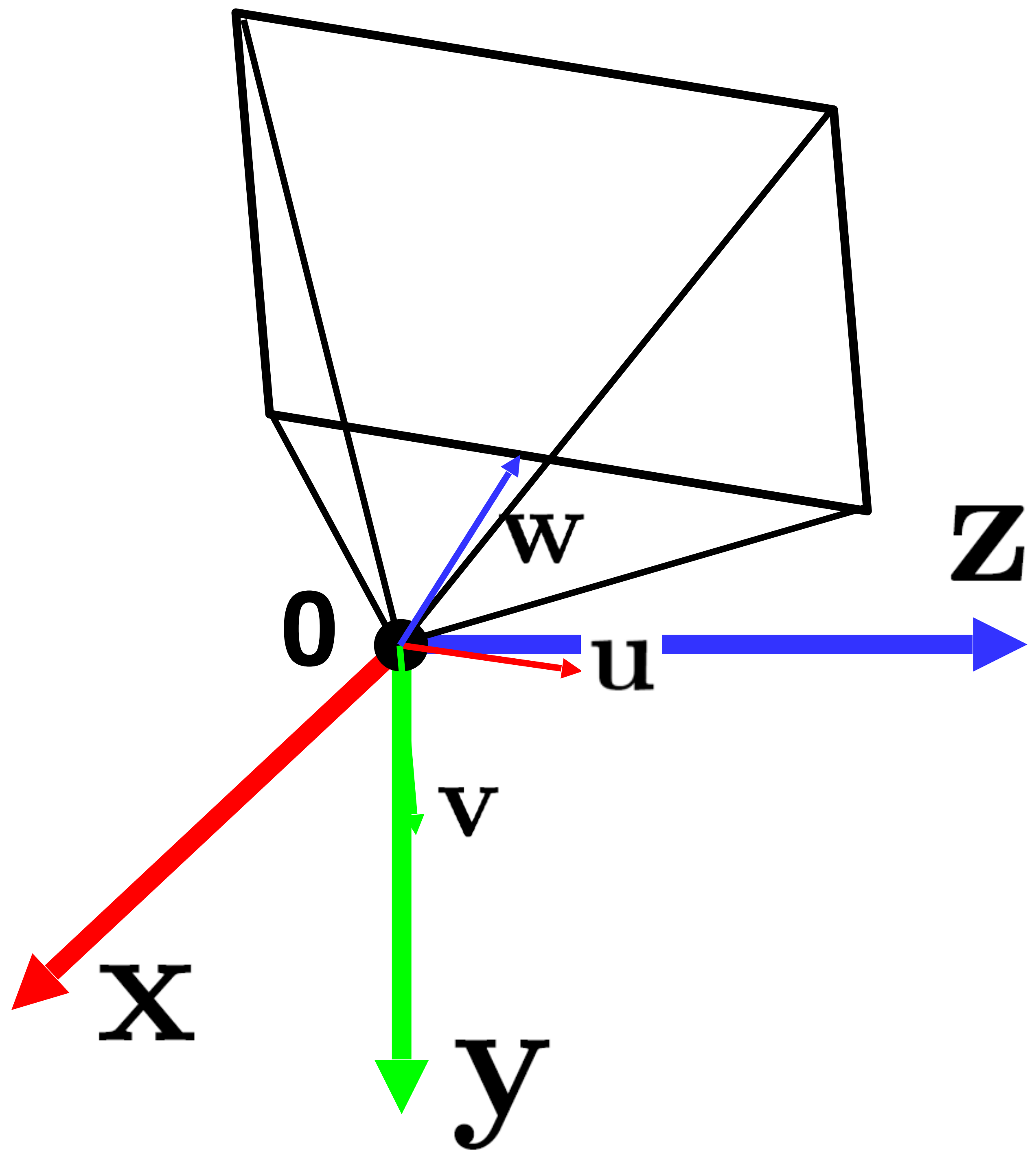
How do we represent translation as a matrix multiplication?

$$\mathbf{T} = \begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{c} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

Source: N. Snavely

Extrinsics

- How do we get the camera to “canonical form”?
  – (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)



Step 1: Translate by **-c**
Step 2: Rotate by **R**

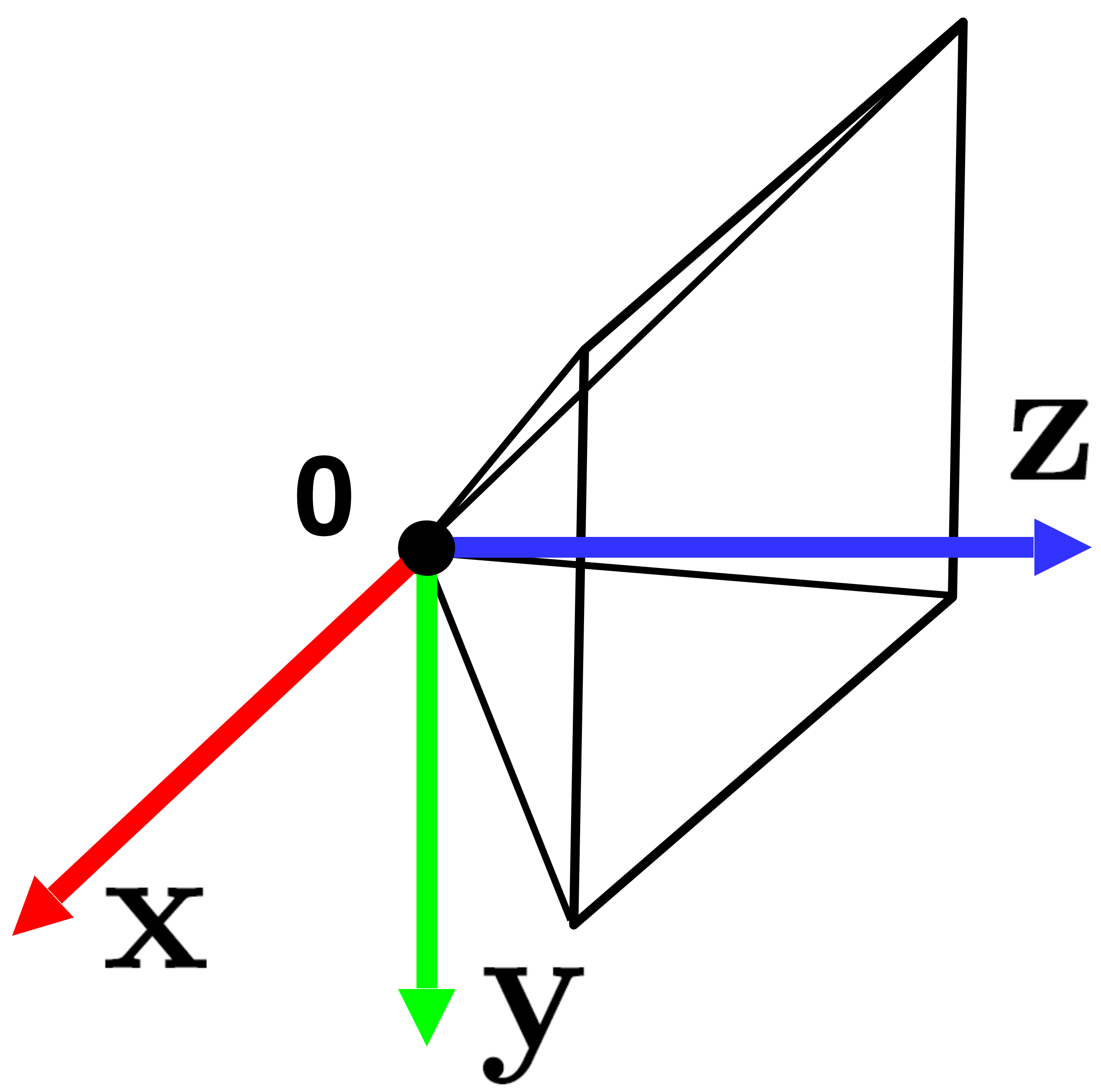
$$\mathbf{R} = \begin{bmatrix} \mathbf{u}^T \\ \mathbf{v}^T \\ \mathbf{w}^T \end{bmatrix}$$

3x3 rotation matrix

Source: N. Snavely

Extrinsics

- How do we get the camera to "canonical form"?
  – (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)



Step 1: Translate by **-c**
Step 2: Rotate by **R**

$$\mathbf{R} = \begin{bmatrix} \mathbf{u}^T \\ \mathbf{v}^T \\ \mathbf{w}^T \end{bmatrix}$$

(with extra row/column of [0 0 0 1])

Source: N. Snavely

Perspective projection

$$\underbrace{\begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{\substack{\mathbf{K} \\ \text{(intrinsics)}}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(converts from 3D rays in camera coordinate system to pixel coordinates)

in general, $\mathbf{K} = \begin{bmatrix} f & s & c_x \\ 0 & \alpha f & c_y \\ 0 & 0 & 1 \end{bmatrix}$ (upper triangular matrix)

α: **aspect ratio** (1 unless pixels are not square)

s : **skew** (0 unless pixels are shaped like rhombi/parallelograms)

(c_x, c_y): **principal point** ((w/2,h/2) unless optical axis doesn't intersect projection plane at image center)

Source: N. Snavely

Typical intrinsics matrix

$$\mathbf{K} = \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

- **2D affine transform** corresponding to a scale by *f* (focal length) and a translation by (c_x, c_y) (principal point)
- Maps 3D rays to 2D pixels

Source: N. Snavely

Focal length

- Can think of as "zoom"


24mm

50mm

200mm

800mm

- Also related to *field of view*

Source: N. Snavely

Wide angle

Standard

Telephoto

Source: N. Snavely

http://petapixel.com/2013/01/11/how-focal-length-affects-your-subjects-apparent-weight-as-seen-with-a-cat/

Source: N. Snavely

Projection matrix

$$\mathbf{\Pi} = \underset{\text{intrinsics}}{\mathbf{K}} \underbrace{\underset{\text{projection}}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}} \underset{\text{rotation}}{\begin{bmatrix} & \mathbf{R} & & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{\text{translation}}{\begin{bmatrix} & \mathbf{I}_{3\times 3} & & -\mathbf{c} \\ 0 & 0 & 0 & 1 \end{bmatrix}}}$$

The **K** matrix converts 3D rays in the camera's coordinate system to 2D image points in image (pixel) coordinates.

This part converts 3D points in world coordinates to 3D rays in the camera's coordinate system. There are 6 parameters represented (3 for position/translation, 3 for rotation).

Source: N. Snavely

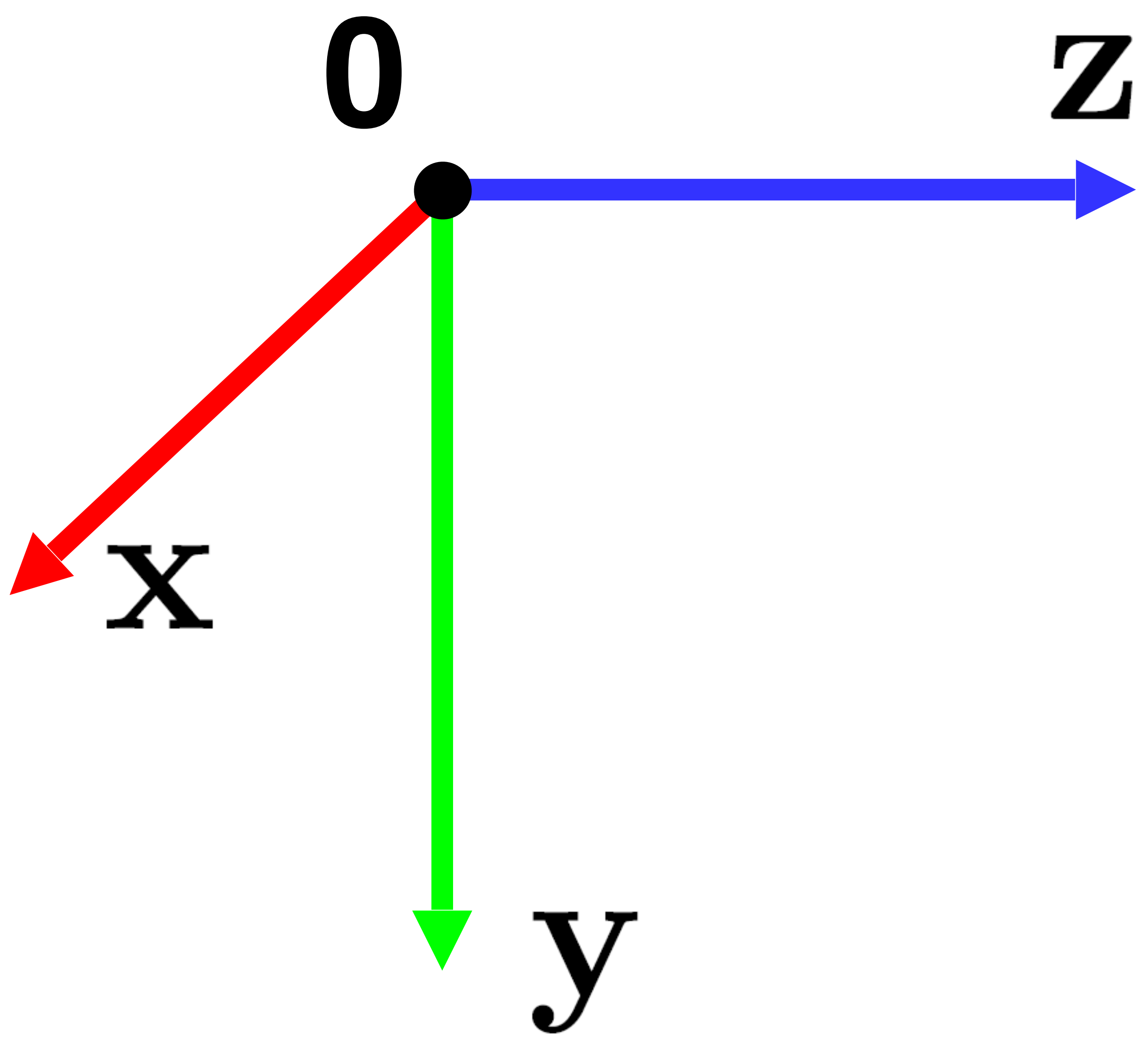
Projection matrix

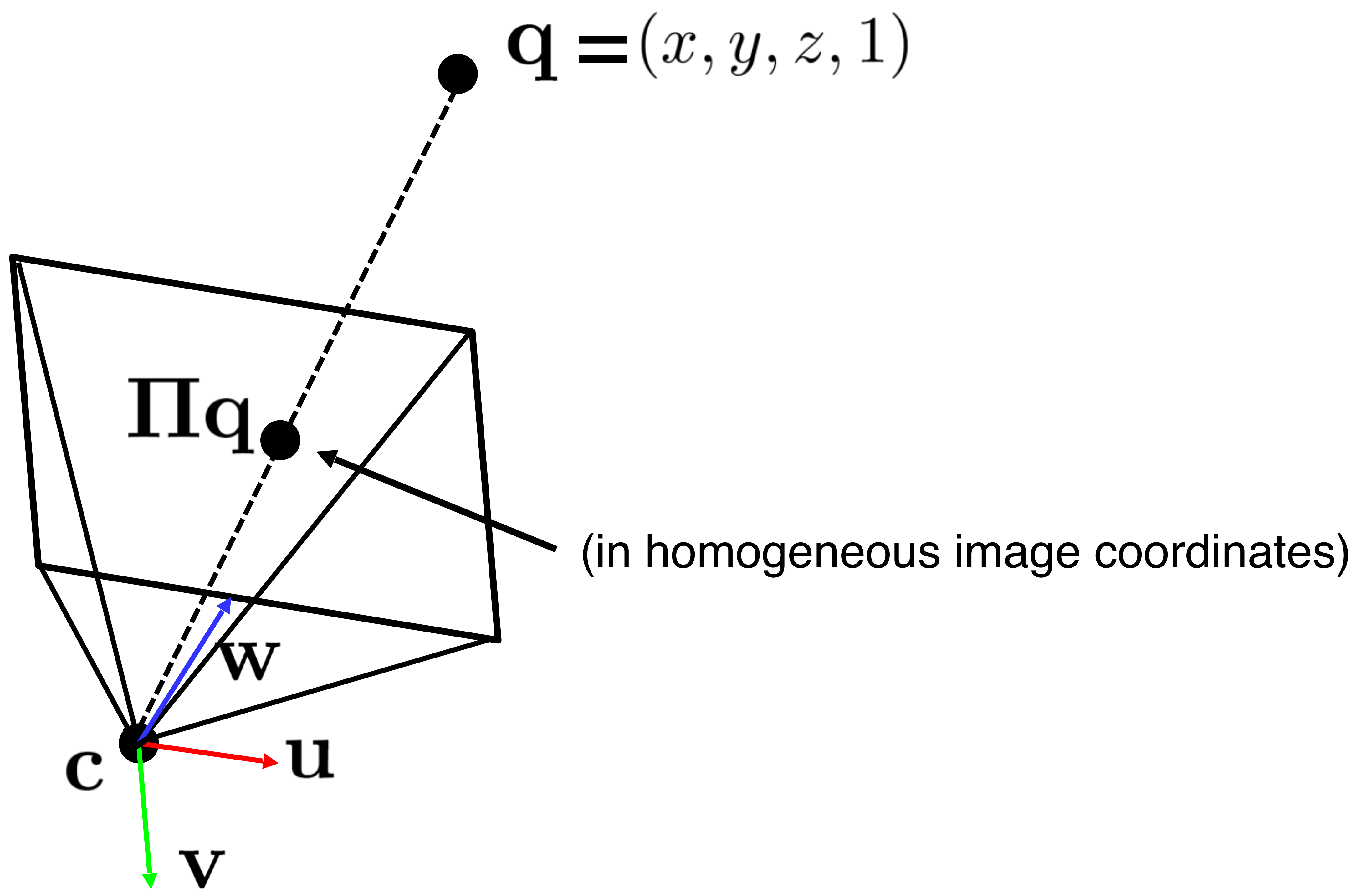
$$\mathbf{\Pi} = \underset{\text{intrinsics}}{\mathbf{K}} \underbrace{\underset{\text{projection}}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}} \underset{\text{rotation}}{\begin{bmatrix} \mathbf{R} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}} \underset{\text{translation}}{\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{c} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}}}_{\left[\ \mathbf{R}\ \middle|\ \underbrace{-\mathbf{Rc}}_{\text{(sometimes called } \mathbf{t}\text{)}}\ \right]}$$

$$\mathbf{\Pi} = \mathbf{K} \left[\ \mathbf{R}\ \middle|\ -\mathbf{Rc}\ \right]$$

Source: N. Snavely

Projection matrix





Source: N. Snavely

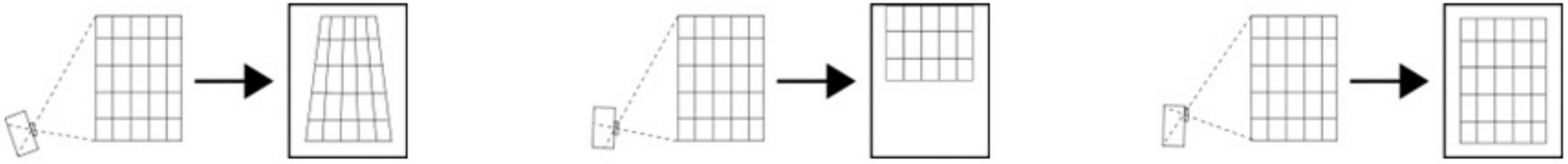
Perspective distortion

- Problem for architectural photography: converging verticals

Source: F. Durand

Perspective distortion

- Problem for architectural photography: converging vertic



Tilting the camera upwards results in converging verticals

Keeping the camera level, with an ordinary lens, captures only the bottom portion of the building

Shifting the lens upwards results in a picture of the entire subject

- Solution: view camera (lens shifted w.r.t. film)

http://en.wikipedia.org/wiki/Perspective_correction_lens

Source: F. Durand

Perspective distortion

- What does a sphere project to?

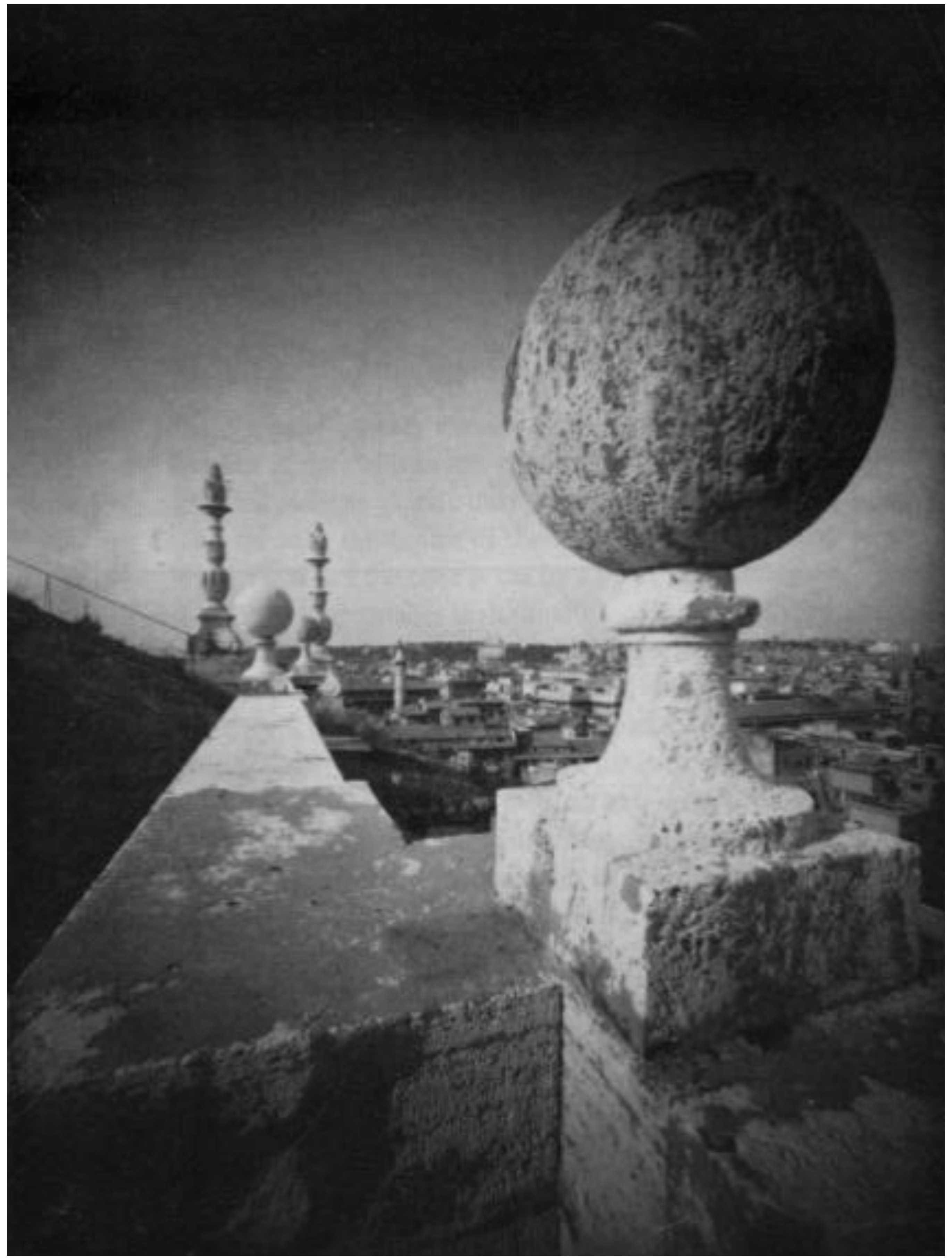
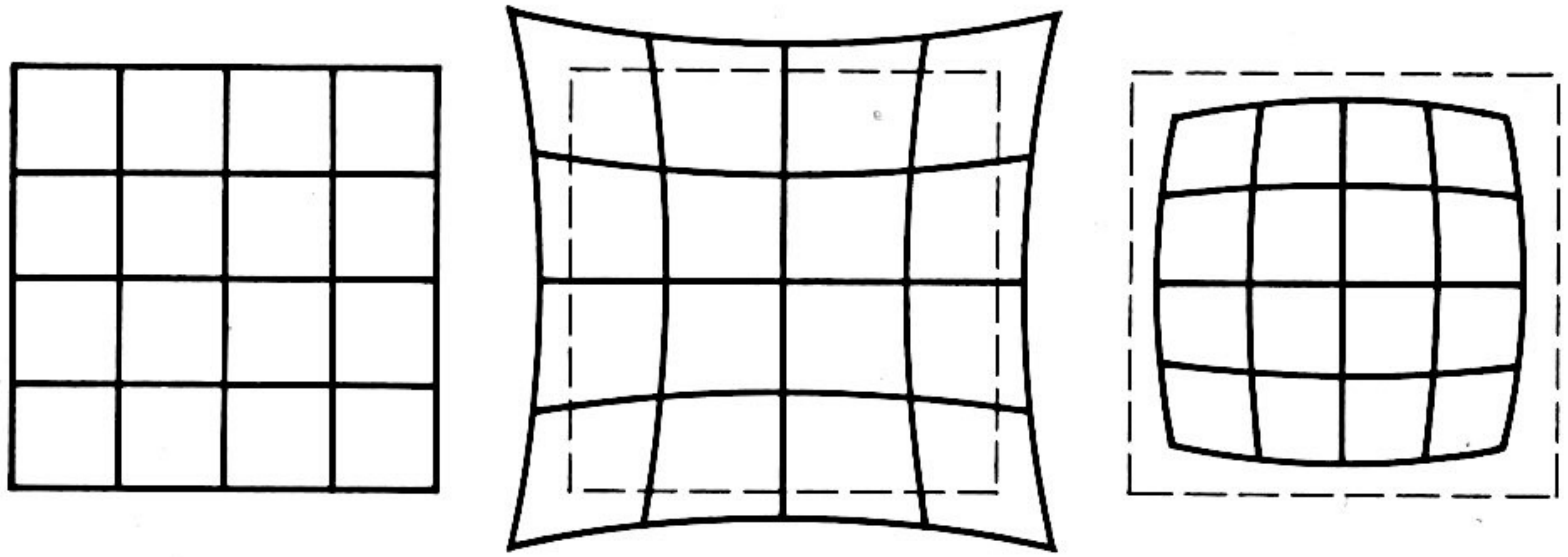


Image source: F. Durand

Source: N. Snavely

Distortion



No distortion Pin cushion Barrel

- Radial distortion of the image
  - Caused by imperfect lenses
  - Deviations are most noticeable for rays that pass through the edge of the lens

Source: N. Snavely

Source: N. Snavely

Modeling distortion

$(\widehat{x}, \widehat{y}, \widehat{z})$

Project to "normalized" image coordinates

$$x'_n = \widehat{x}/\widehat{z}$$
$$y'_n = \widehat{y}/\widehat{z}$$

Apply radial distortion

$$r^2 = {x'_n}^2 + {y'_n}^2$$
$$x'_d = x'_n(1 + \kappa_1 r^2 + \kappa_2 r^4)$$
$$y'_d = y'_n(1 + \kappa_1 r^2 + \kappa_2 r^4)$$

Apply focal length translate image center

$$x' = f x'_d + x_c$$
$$y' = f y'_d + y_c$$

- To model lens distortion
  - Use above projection operation instead of standard projection matrix multiplication

Source: N. Snavely

Correcting radial distortion

from [Helmut Dersch]

Next lecture: More geometry!