CS 6120/CS4120: Natural Language Processing

Instructor: Prof. Lu Wang
College of Computer and Information Science
Northeastern University
Webpage: www.ccs.neu.edu/home/luwang

Brown Clusters

Brown Clusters -- Unsupervised

- Goal
  - To learn about regularities in words
  - By clustering words into groups

- Motivation
  - Primarily to deal with word sparsity
  - Also to reduce amount of necessary training data

Brown Clustering Algorithm

- Input: a (large) corpus of words

- Output 1: a partition of words into word clusters

- Output 2 (generalization of 1): a hierarchical word clustering

Example Clusters

- Friday Monday Thursday Wednesday Tuesday Saturday Sunday weekends Sundays Saturdays
- June March July April January December October November September August
- people guys folks fellows CEOs chaps doubters commies unfortunates blokes
- down backwards ashore sideways southward northward overboard aloft downwards adrift
- water gas coal liquid acid sand carbon steam shale iron
- great big vast sudden mere sheer gigantic lifelong scant colossal
- man woman boy girl lawyer doctor guy farmer teacher citizen
- American Indian European Japanese German African Catholic Israeli Italian Arab
- pressure temperature permeability density porosity stress velocity viscosity gravity tension
- mother wife father son husband brother daughter sister boss uncle
- machine device controller processor CPU printer spindle subsystem compiler plotter
- John George James Bob Robert Paul William Jim David Mike
- anyone someone anybody somebody
- feet miles pounds degrees inches barrels tons acres meters bytes
- director chief professor commissioner commander treasurer founder superintendent dean custodian
- liberal conservative parliamentary royal progressive
- Tory provisional separatist federalist PQ

- Assigns each word a binary representation

guava orange onion potato coke pepsi blue

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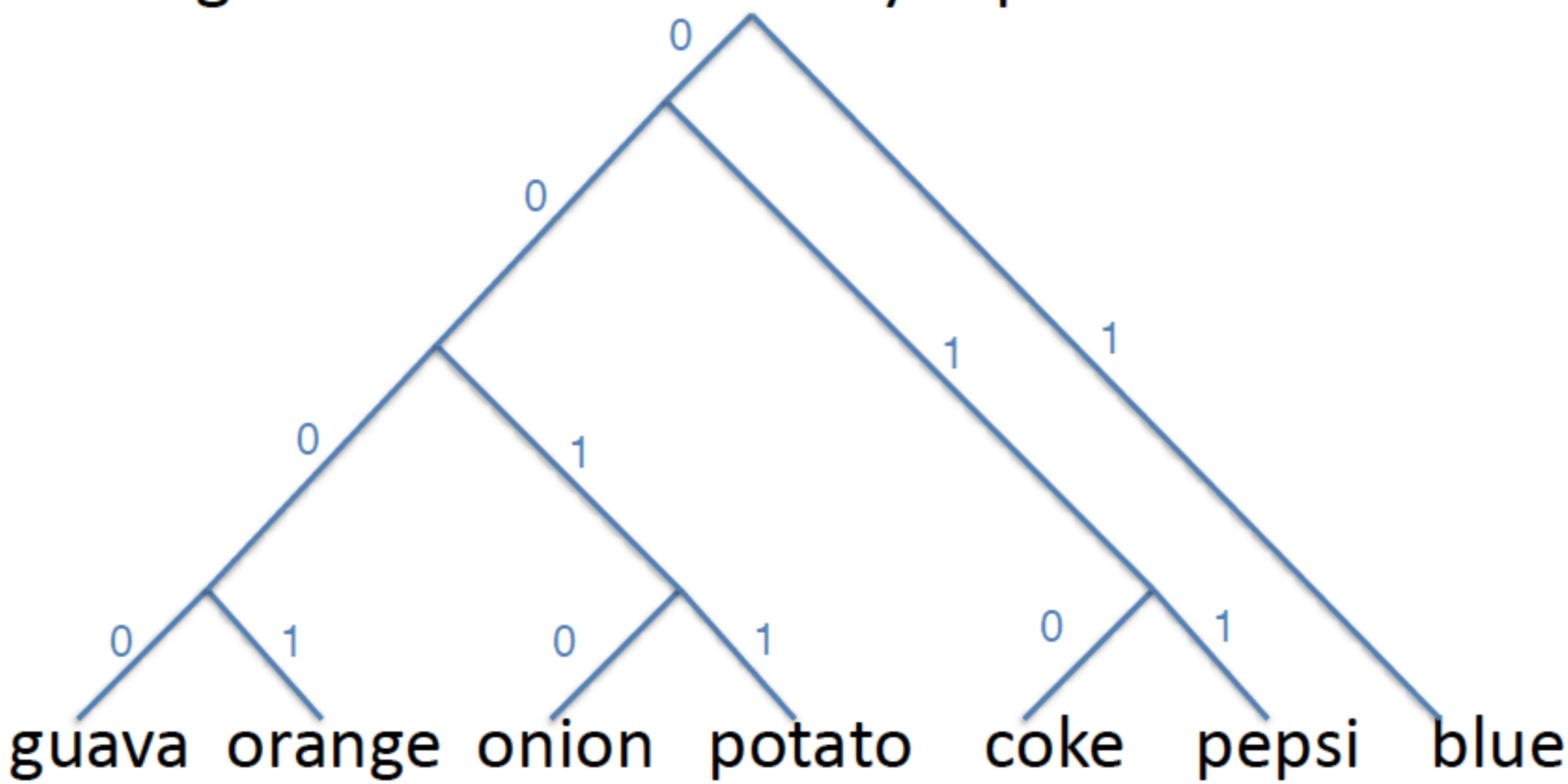

- onion: 0010

- Different prefix lengths: different abstractions

- *111111110110000* *slapped*
- 111111110110000 shattered
- 111111110110000 commissioned
- 111111110110000 drafted
- 111111110110000 authorized
- 111111110110000 authorised
- 111111110110000 imposed
- 111111110110000 established
- 111111110110000 developed

- *111111111100110* *officer*
- 111111111100110 acquaintance
- 111111111100110 policymaker
- 111111111100110 instructor
- 111111111100110 investigator
- 111111111100110 advisor
- 111111111100110 aide
- 111111111100110 expert
- 111111111100110 adviser

- *111110100 Clinton*
- 111110100 Aleman
- 111110100 Zeroual
- 111110100 Sampras
- 111110100 Barzani
- 111110100 Cardoso
- 111110100 Kim
- 111110100 King
- 111110100 Saddam
- 111110100 Netanyahu
- 111110100 Dole

- *111111100 Bill*
- 111111100 Boris
- 111111100 Warren
- 111111100 Fidel
- 111111100 Yasser
- 111111100 Kenneth
- 111111100 Viktor
- 111111100 Benjamin
- 111111100 Jacques
- 111111100 Bob
- 111111100 Alexander

Intuition

- Similar words appear in similar contexts

- Similar words have similar distributions of words to their immediate left and right

eat		**juice**
grow		seeds
…		…
	guava	

eat		**juice**
grow		seeds
…		…
	orange	

Formulation

- *V* is the set of all words seen in the corpus

- Say C: V → {1, 2,...k} is a partition of the vocabulary into k classes (k ~ 1000)

- The model: (C(w_0) is a special <s> state)

$$p(\underset{\textbf{\textit{Corpus}}}{w_1, w_2, \ldots, w_N}) = \prod_{t=1}^{N} e(w_t \mid C(w_t))\, q(C(w_t) \mid C(w_{t-1}))$$

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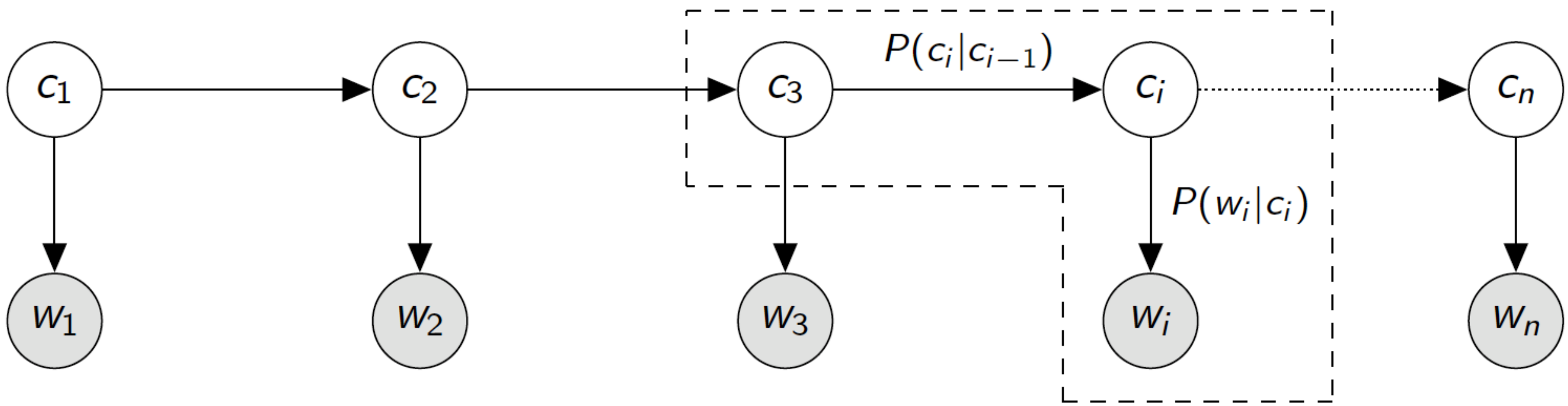
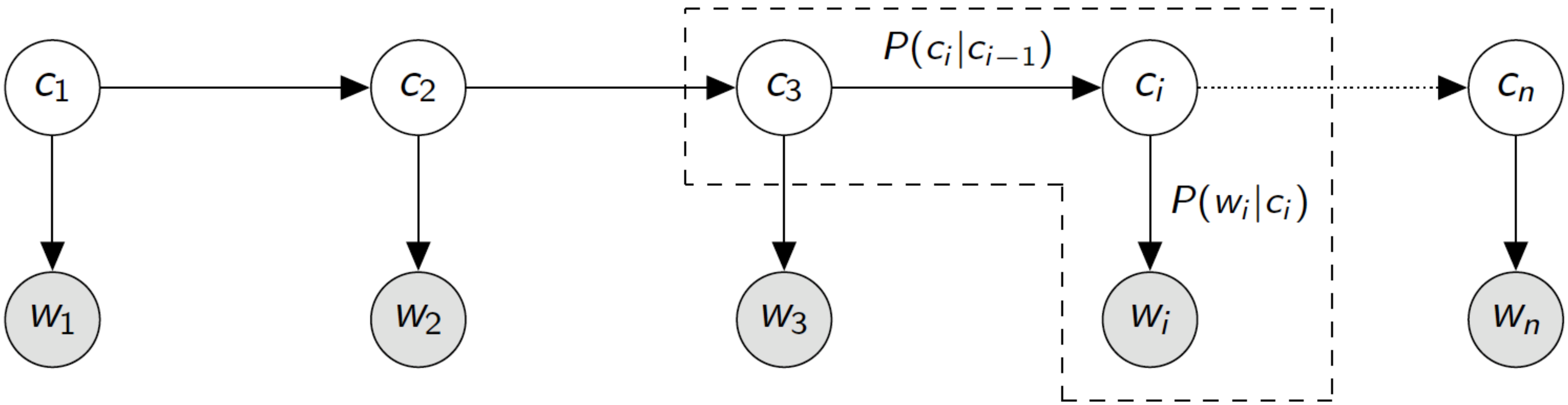
Emission Probability! ***Transition Probability!***

$$\underbrace{p(w_1, w_2, \ldots, w_N)}_{\textit{Corpus}} = \prod_{t=1}^{N} e(w_t \mid C(w_t))\, q(C(w_t) \mid C(w_{t-1}))$$


c_1
c_2
c_3
$P(c_i|c_{i-1})$
c_i
c_n
$P(w_i|c_i)$
w_1
w_2
w_3
w_i
w_n



Difference from HMM: each word can be labeled as one class!

$C(I)=1$, $C(ate)=C(drank)=2$
$C(guava)=C(pepsi)=3$, $C(and)=4$

$e(I|1)=1$, $e(ate|2)= e(drank|2)= 0.3$
$e(guava|3)=e(pepsi|3)=0.1$, $e(and|4)=1$
$q(1|0)=0.2$, $q(2|1)=0.4$, $q(3|2)=0.3$, $q(4|3)=0.1$, $q(2|4)=0.2$

$C(I)=1$, $C(ate)=C(drank)=2$
$C(guava)=C(pepsi)=3$, $C(and)=4$

$e(I|1)=1$, $e(ate|2)= e(drank|2)= 0.3$
$e(guava|3)=e(pepsi|3)=0.1$, $e(and|4)=1$
$q(1|0)=0.2$, $q(2|1)=0.4$, $q(3|2)=0.3$, $q(4|3)=0.1$, $q(2|4)=0.2$

$$P(\text{I ate guava and drank pepsi}) =$$
$$0.2*1*0.4*0.3*0.3*0.1*0.1*1*0.2*0.3*0.3*0.1$$

The Model

- Vocabulary *V*
- A function C: *V* → {1..k}
  - partitioning of vocabulary into k classes
- Emission probabilities e(w|C(w))
- Transition probability q(c'|c)

Scoring a Partition: **Quality (C)**

N is the number of words in the corpus

$$\frac{1}{N}\sum_{t=1}^{N}\log\left(e(w_t \mid C(w_t))q(C(w_t) \mid C(w_{t-1}))\right)$$

$$=\sum_{c,c'} p(c,c')\log\left(\frac{p(c,c')}{p(c)p(c')}\right)+\sum_{w} p(w)\log p(w)$$

Mutual information (MI) **constant**

n(c): #occurences of c in corpus under function C
n(c,c'): #occurences of (c,c') in corpus under function C

$$p(c,c')=\frac{n(c,c')}{N} \qquad p(c)=\frac{n(c)}{N}$$

Proof

$$\begin{aligned}\text{Quality}(C) &= \frac{1}{n}\sum_{i=1}^{n} \log P(C(w_i)|C(w_{i-1}))P(w_i|C(w_i)) \\ &= \sum_{w,w'} \frac{n(w,w')}{n} \log P(C(w')|C(w))P(w'|C(w'))\end{aligned}$$

Proof

$$\begin{aligned}
\text{Quality}(C) &= \frac{1}{n}\sum_{i=1}^{n} \log P(C(w_i)|C(w_{i-1}))P(w_i|C(w_i)) \\
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&= \sum_{w,w'} \frac{n(w,w')}{n} \log \frac{n(C(w),C(w'))}{n(C(w))}\frac{n(w')}{n(C(w'))}
\end{aligned}$$

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&= \sum_{w,w'} \frac{n(w,w')}{n} \log \frac{n(C(w),C(w'))n}{n(C(w))n(C(w'))} + \sum_{w,w'} \frac{n(w,w')}{n} \log \frac{n(w')}{n}
\end{aligned}
$$

Proof

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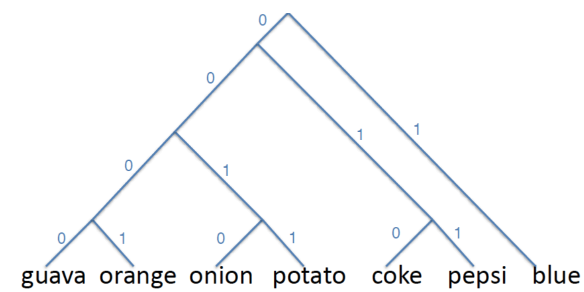
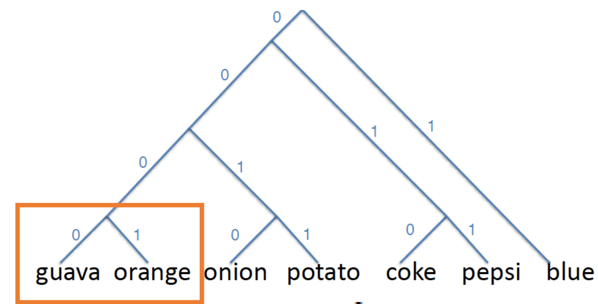
\end{aligned}
$$

A First (Naïve) Algorithm



- Start with |*V*| clusters: each word gets its own cluster
- Our aim is to find k final clusters
- We run |*V*| - k merge steps:
  - At each merge step we pick two clusters c_i and c_j, and merge them into a single cluster
  - We greedily pick merges such that Quality(C) for the clustering C after the merge step is maximized at each stage

A First (Naïve) Algorithm



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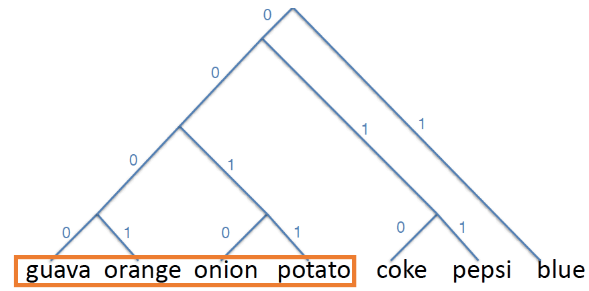
A First (Naïve) Algorithm

- Cost?
  - Naive = $O(|V|^5)$. *Calculate everything on-the-fly!*
  - Improved algorithm gives $O(|V|^3)$ *Store word transitions!*
    - still two slow for realistic values of $|V|$

A Second Algorithm

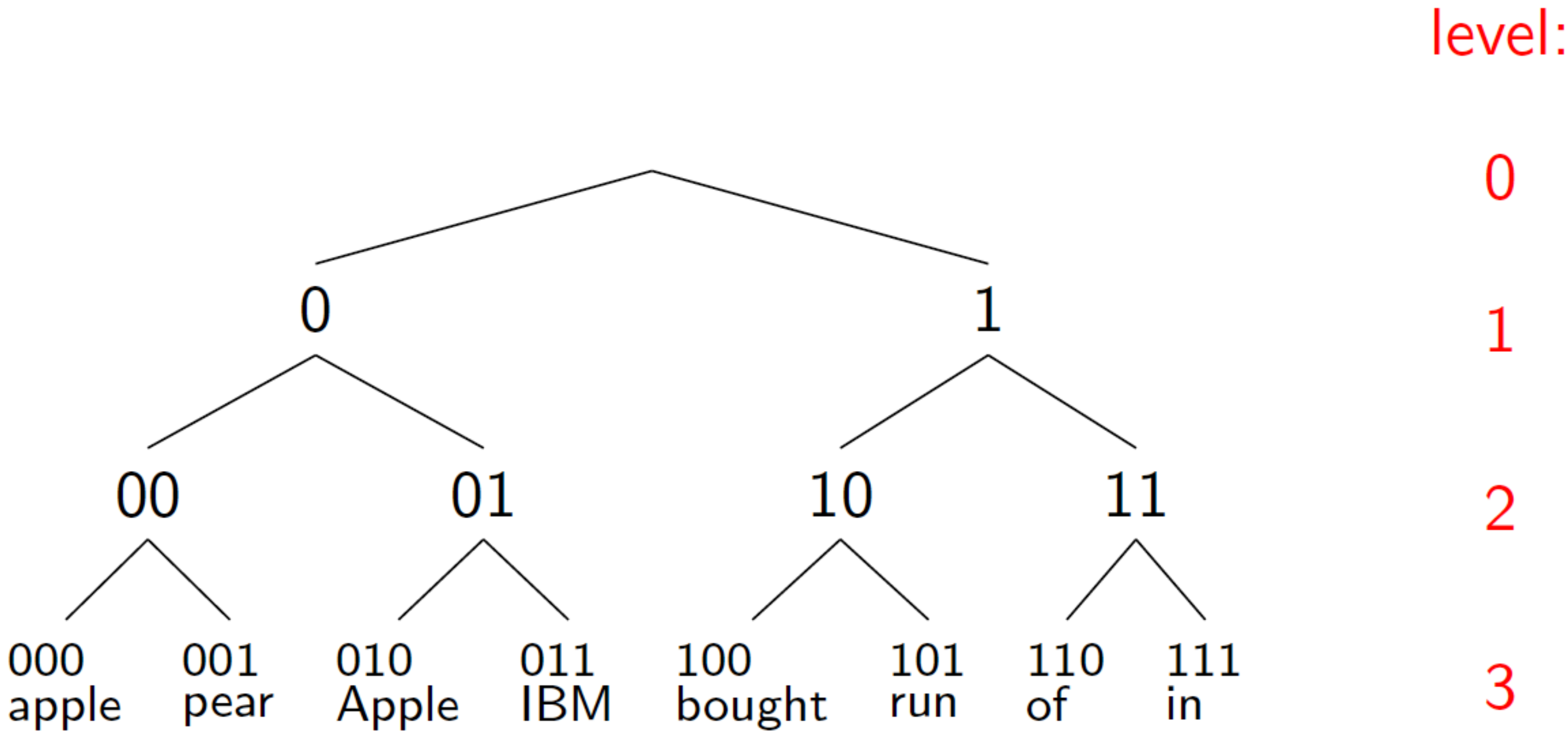


- New parameter: m (e.g., m = 1000)
- Take the top m most frequent words, put each into its own cluster, $c_1, c_2, \ldots c_m$
- For i = (m + 1) … |V|
  - Create a new cluster, c_{m+1}, for the i'th most frequent word. We now have m + 1 clusters
- Choose two clusters from $c_1 \ldots c_{m+1}$ to be merged:
  - pick the merge that gives a maximum value for Quality(C).
  - We're now back to m clusters
- Carry out (m - 1) final merges, to create a full hierarchy

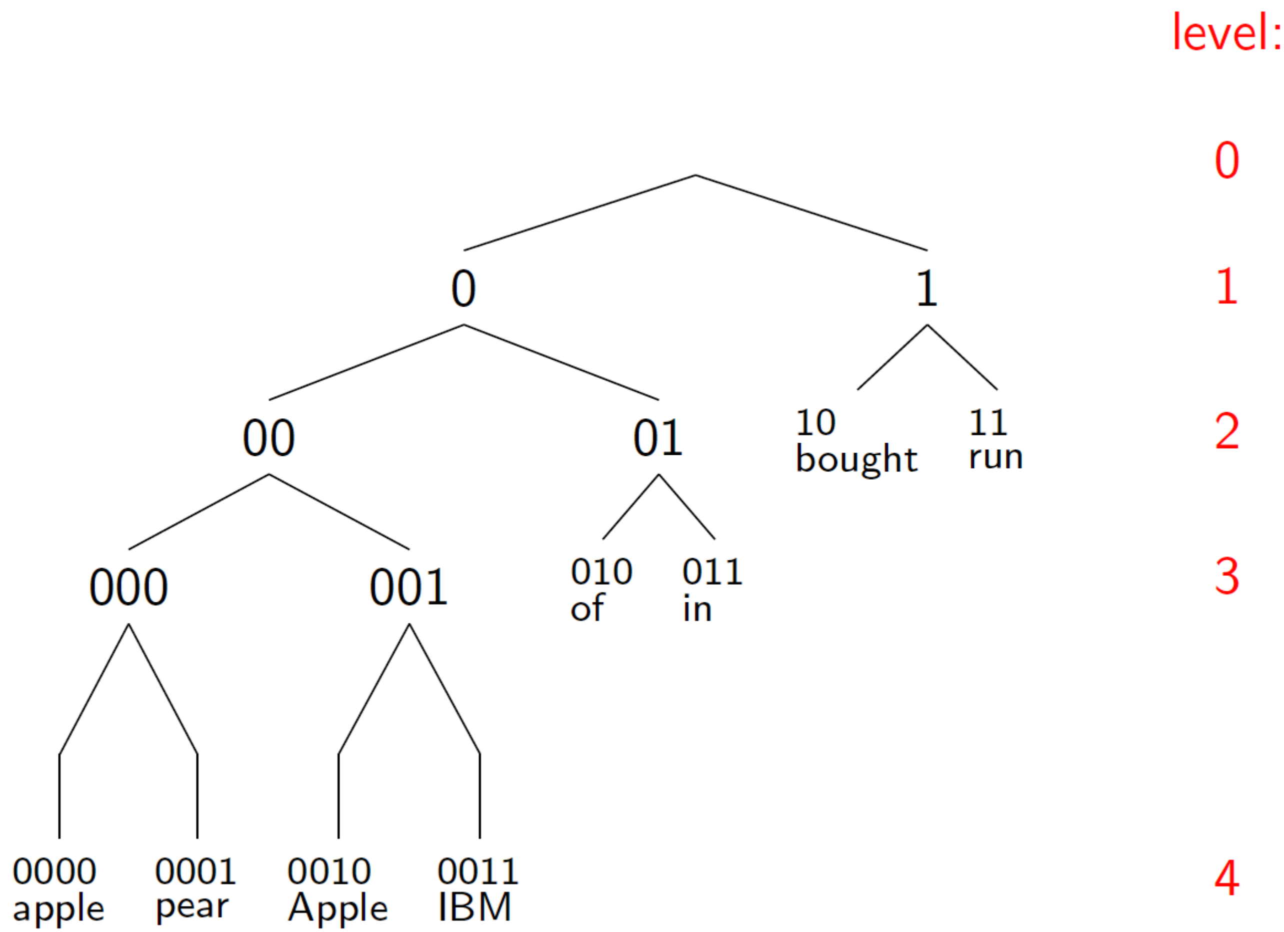
A Second Algorithm

- Running time: $O(|V|m^2 + n)$ where n is corpus length

A perfect balanced binary tree



In reality:


level:
0
1
2
3
4
0
1
00
01
10
bought
11
run
000
001
010
of
011
in
0000
apple
0001
pear
0010
Apple
0011
IBM

More Information on Implementation

- Video: https://www.youtube.com/playlist?list=PLO9y7hOkmmSEAqCc0wrNBrsoJMTmIN98M

- And an application for Named Entity Recognition (NER)