# CS 6120/CS4120: Natural Language Processing

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## Outline

- Vector Semantics
- Sparse representation
  - Pointwise Mutual Information (PMI)
- Dense representation
  - Singular Value Decomposition (SVD)
  - Neural Language Model (Word2Vec)

## Sparse versus dense vectors

- Why dense vectors?
  - Short vectors may be easier to use as features in machine learning (less weights to tune)
  - Dense vectors may generalize better than storing explicit counts (or variations)
  - They may do better at capturing synonymy:
    - *car* and *automobile* are synonyms; but are represented as distinct dimensions; this fails to capture similarity between a word with *car* as a neighbor and a word with *automobile* as a neighbor

## Three methods for getting short dense vectors

- Singular Value Decomposition (SVD) (this lecture)
- "Neural Language Model" – inspired by predictive models
- Brown clustering

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## Singular Value Decomposition (SVD)

## Rank of a Matrix

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$$A = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix}$$

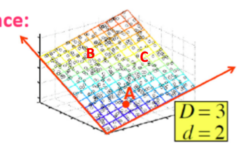
- Rank is 2
- We can rewrite A as two "basis" vectors: [1 2 1] [-2 -3 1]

## Rank as "Dimensionality"

**Cloud of points 3D space:**

- Think of point positions as a matrix: $\begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix}$ A B C

1 row per point:

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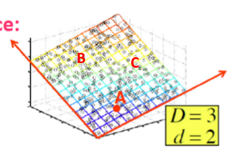


- Rewrite the coordinates in a more efficient way!
  - Old basis vectors: [1 0 0], [0 1 0], [0 0 1]
  - New basis vectors: [1 2 1], [-2 -3 1]

## Intuition of Dimensionality Reduction

- Approximate an N-dimensional dataset using fewer dimensions
- By first rotating the axes into a new space
- In which the highest order dimension captures the most variance in the original dataset
- And the next dimension captures the next most variance, etc.

## Sample Dimensionality Reduction

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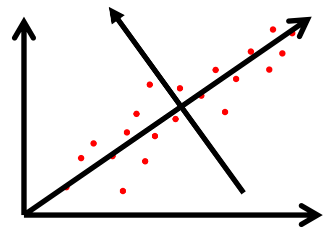
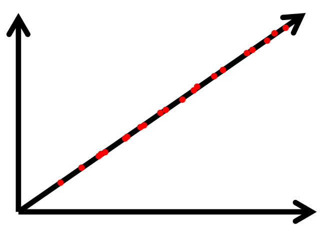
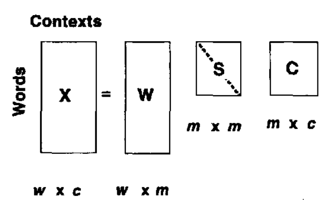


## Sample Dimensionality Reduction



## Singular Value Decomposition




(assuming the matrix has rank m)

Landuaer and Dumais 1997

## Singular Value Decomposition

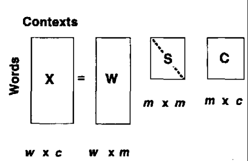
*Any rectangular w x c matrix **X** equals the product of 3 matrices:*

**W**: rows corresponding to original but m columns represents a dimension in a new latent space, such that

- m column vectors are orthogonal to each other
- Columns are ordered by the amount of variance in the dataset each new dimension accounts for

**S**: diagonal *m* x *m* matrix of **singular values** expressing the importance of each dimension.

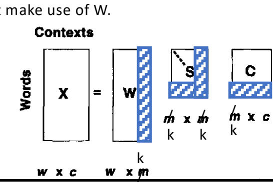
**C**: columns corresponding to original but m rows corresponding to singular values




## SVD applied to term-document matrix: Latent Semantic Analysis

Deerwester et al (1988)

- If instead of keeping all m dimensions, we just keep the top k singular values. Let's say 300.
- The result is a least-squares approximation to the original X
- But instead of multiplying, we'll just make use of W.
- Each row of W:
  - A k-dimensional vector
  - Representing word W



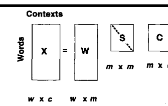
## SVD on Term-Document Matrix: Example

- The matrix X

| | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ | $d_6$ |
|---|---|---|---|---|---|---|
| ship | 1 | 0 | 1 | 0 | 0 | 0 |
| boat | 0 | 1 | 0 | 0 | 0 | 0 |
| ocean | 1 | 1 | 0 | 0 | 0 | 0 |
| wood | 1 | 0 | 0 | 1 | 1 | 0 |
| tree | 0 | 0 | 0 | 1 | 0 | 1 |

Matrix **W**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| ship | −0.44 | −0.30 | 0.57 | 0.58 | 0.25 |
| boat | −0.13 | −0.33 | −0.59 | 0.00 | 0.73 |
| ocean | −0.48 | −0.51 | −0.37 | 0.00 | −0.61 |
| wood | −0.70 | 0.35 | 0.15 | −0.58 | 0.16 |
| tree | −0.26 | 0.65 | −0.41 | 0.58 | −0.09 |




Matrix **S**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 2.16 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | 0.00 | 1.59 | 0.00 | 0.00 | 0.00 |
| 3 | 0.00 | 0.00 | 1.28 | 0.00 | 0.00 |
| 4 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 |
| 5 | 0.00 | 0.00 | 0.00 | 0.00 | 0.39 |

Matrix **C**

| | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ | $d_6$ |
|---|---|---|---|---|---|---|
| 1 | −0.75 | −0.28 | −0.20 | −0.45 | −0.33 | −0.12 |
| 2 | −0.29 | −0.53 | −0.19 | 0.63 | 0.22 | 0.41 |
| 3 | 0.28 | −0.75 | 0.45 | −0.20 | 0.12 | −0.33 |
| 4 | 0.00 | 0.00 | 0.58 | 0.00 | −0.58 | 0.58 |
| 5 | −0.53 | 0.29 | 0.63 | 0.19 | 0.41 | −0.22 |

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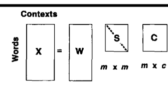


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## Reduce dimension: The Matrix W

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|---|---|---|---|---|---|---|
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| boat | 0 | 1 | 0 | 0 | 0 | 0 |
| ocean | 1 | 1 | 0 | 0 | 0 | 0 |
| wood | 1 | 0 | 0 | 1 | 1 | 0 |
| tree | 0 | 0 | 0 | 1 | 0 | 1 |

⇨

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Similarity between ***ship*** and ***boat*** ***vs*** ***ship*** and ***wood*** ?

## Reduce dimension: The Matrix W

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## More details

- 300 dimensions are commonly used
- The cells are commonly weighted by a product of two weights (TF-IDF)
  - Local weight: Log term frequency
  - Global weight: either idf or an entropy measure

## Let's return to PPMI word-word matrices

- Can we apply to SVD to them?

## SVD applied to term-term matrix

$$\underset{|V|\times|V|}{\begin{bmatrix} \\ X \\ \\ \end{bmatrix}} = \underset{|V|\times|V|}{\begin{bmatrix} \\ W \\ \\ \end{bmatrix}} \underset{|V|\times|V|}{\begin{bmatrix} \sigma_1 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sigma_V \end{bmatrix}} \underset{|V|\times|V|}{\begin{bmatrix} \\ C \\ \\ \end{bmatrix}}$$

(assuming the matrix has rank |V|, may not be true)

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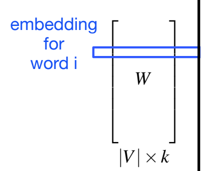
(assuming the matrix has rank |V|, may not be true)

## Truncated SVD on term-term matrix

$$\underset{|V|\times|V|}{\begin{bmatrix} \\ X \\ \\ \end{bmatrix}} = \underset{|V|\times k}{\begin{bmatrix} \\ W \\ \\ \end{bmatrix}} \underset{k\times k}{\begin{bmatrix} \sigma_1 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sigma_k \end{bmatrix}} \begin{bmatrix} C \\ k\times|V| \end{bmatrix}$$

## Truncated SVD produces embeddings

- Each row of W matrix is a k-dimensional representation of each word *w*
- K might range from 50 to 1000
- Generally we keep the top k dimensions, but some experiments suggest that getting rid of the top 1 dimension or even the top 50 dimensions is helpful (Lapesa and Evert 2014).




## Embeddings versus sparse vectors

- Dense SVD embeddings sometimes work better than sparse PPMI matrices at tasks like word similarity
  - Denoising: low-order dimensions may represent unimportant information
  - Truncation may help the models generalize better to unseen data.
  - Having a smaller number of dimensions may make it easier for classifiers to properly weight the dimensions for the task.
  - Dense models may do better at capturing higher order co-occurrence.