Noninteractive Zero Knowledge for NP from Learning With Errors

Chris Peikert   Sina Shiehian

TCS+
1 May 2019

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2. Sound: if $G_0 \not\equiv G_1$, cheating P^* convinces V with prob $\leq 1/2$. Soundness error can be reduced exponentially by (parallel) repetition.
3. Zero Knowledge: can simulate (honest) V's view when $G_0 \equiv G_1$.

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Theorem [GoldreichMicaliWigderson'86,NguyenOngVadhan'06]

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	$b = 0$: open all $h_{i,j}, \rho$ $\longrightarrow$	check $H = \rho(G)$
	$b = 1$: open $h_{i,j}$ for $(i,j) \in \rho(C)$ $\longrightarrow$	check cycle

Noninteractive Zero Knowledge [BlumDeSantisMicaliPersiano'88]

- Interaction is not always possible. What if. . . ?

$\underline{P(x, w)}$ $\quad\quad\quad\quad$ $\underline{V(x)}$

$\xrightarrow{\pi}$ acc/rej

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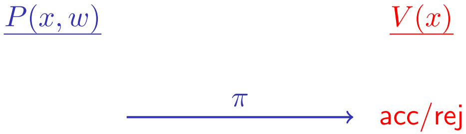

- In 'plain' model, NIZK = BPP (trivial).

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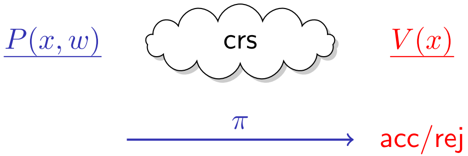

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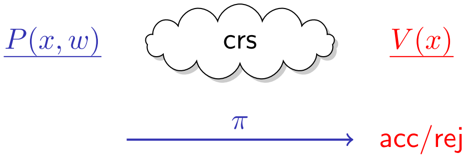

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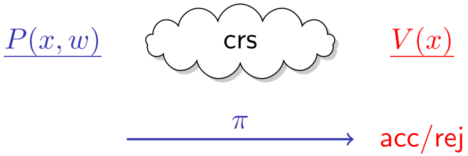

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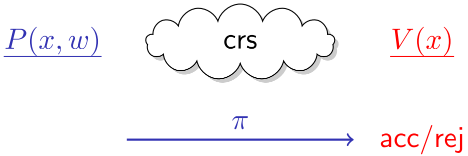

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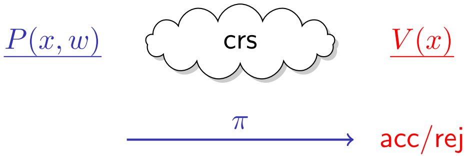

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Our Main Theorem

- NP $\subseteq$ NIZK assuming LWE/worst-case lattice problems are hard.

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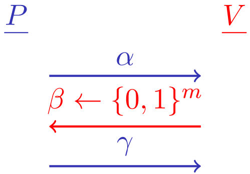

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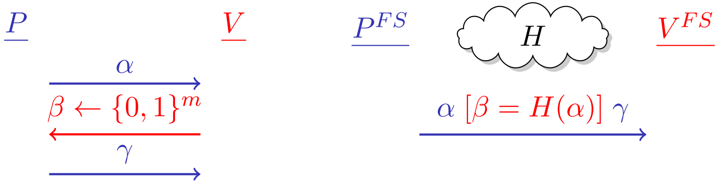

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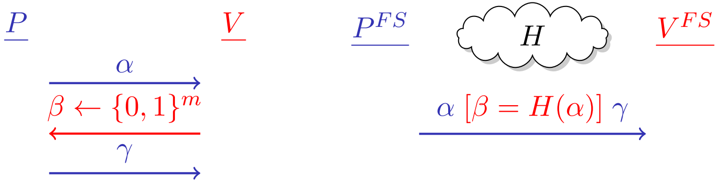

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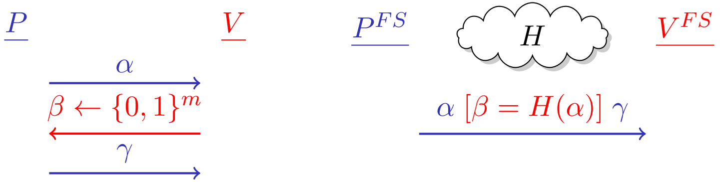

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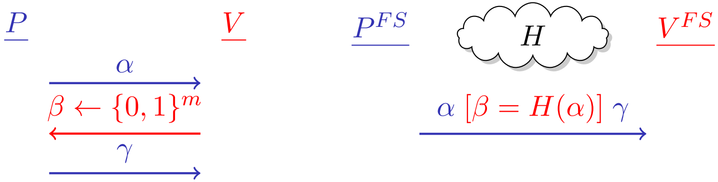

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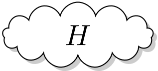
2. Can a cheating P^* find such values, given H? (Proof vs. argument.)

Fiat-Shamir, Soundly [KRR'17,CCRR'18,HL'18,CCHLRRW'19]

P^{FS}  V^{FS}

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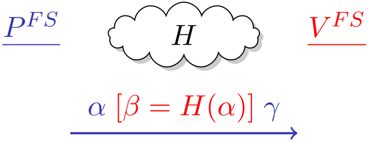

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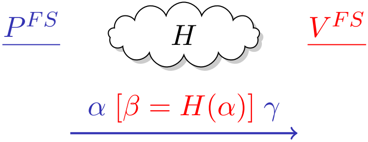

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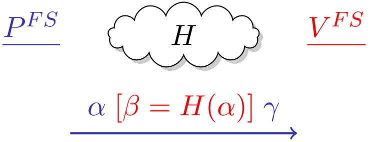

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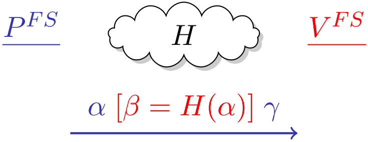

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  Bad β is efficiently computable, using trapdoor for commitments in α.

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  2. Statistical: over $H \leftarrow \mathcal{H}_C \stackrel{c}{\approx} \mathcal{H}$, such α do not exist w/h.p. Yields computationally ZK proof in reference-string model.

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- For NIZK we do not actually need bootstrapping, because the 'bad challenge' functions can be implemented in NC^1 [CCH+'19,Lombardi].

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$$\left(\quad \mathbf{A} \quad \right) \begin{pmatrix} \\ \mathbf{z} \\ \\ \end{pmatrix} = \begin{pmatrix} \mathbf{0} \end{pmatrix} \in \mathbb{Z}_q^n.$$

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Theorems

- Worst-case lattice problems reduce to average-case SIS/LWE.

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Key Point: $c_\alpha \in \mathbb{Z}_q^n$ hides a $\mathbb{Z}_q^n$-value: lets us compare the two directly, not just reason about hidden values (as in [CCH+'19]).

Security Proof from SIS

Hash Key: commitment $\widehat{D}$.

Evaluation: $H(\alpha) := G^{-}(\overline{G(D(\alpha))})$

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- From coins $\mathbf{R}_C$ for $\widehat{C}$ we can compute coins $\mathbf{r}_\alpha$ for c_α, solving SIS.

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- Then

$$\begin{aligned} \widehat{x} \cdot \mathbf{v}_L &= \mathbf{A} \cdot \underbrace{\mathbf{R} \cdot \mathbf{v}_L}_{\mathbf{r}} + \sum_i x_i \cdot \mathbf{G} \cdot \mathbf{G}^{-1}(\mathbf{c}_i) \\ &= \mathbf{A} \cdot \mathbf{r} + L(x) = \overline{L(x)}. \end{aligned}$$

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- Now $H(\alpha) = C(\alpha)$ yields $\mathbf{A}\mathbf{r}_\alpha = \left(\begin{smallmatrix} \mathbf{0} \\ q/2 \end{smallmatrix} \right)$. So $\mathbf{A}'\mathbf{r}_\alpha = \mathbf{0}$ and

$$\tfrac{q}{2} = (\mathbf{s}^t \mathbf{A}' + \mathbf{e}^t) \cdot \mathbf{r}_\alpha = \mathbf{e}^t \cdot \mathbf{r}_\alpha \pmod q,$$

  but $\mathbf{e}, \mathbf{r}_\alpha$ are too small for this: contradiction!

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Thanks!