Generation of a Metamodel for a Micromachined Accelerometer using T-SPICE™ and the I_Z-Optimality Option of I-OPT™

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ABSTRACT

We demonstrate a new method for dynamical parametric simulation of MEMS using I_Z-optimal designs for FEA simulations, fitting using the best linear unbiased predictor, and extraction of relevant coefficients for solution of the appropriate ODE using T-SPICE. A five-factor microaccelerometer example demonstrates that highly accurate dynamical simulations are possible, in some cases, using a very small number of static FEA simulations.

Keywords: Design of experiments, computer simulation, I-optimality, microaccelerometer, metamodel

1 INTRODUCTION

Accurate simulation of microelectromechanical systems (MEMS) usually requires modeling at the structural level using a discretized partial-differential-equation solver using the finite-element or boundary-element method (FEM or BEM). However, application of such analysis often entails a formidable computational burden and designers would benefit from having an accurate, parsimonious, surrogate function that could replace the computationally expensive discretized solver. Such a *metamodel* could be used in place of the full FEA or BEA analysis, for a variety of purposes, such as design optimization.

In this paper we present an intelligent path to generating metamodels and demonstrate the method with a micromachined-accelerometer example. We use the new design-of-computer-experiments capability of I-OPT™ (Version 4, created and available via anonymous FTP in 1999, [1,2]) for determination of the optimal set of *N* inputs at which to exercise the commercial FEA tool ANSYS for automated layout, 3D-model generation, and FEM analysis. The objective used in the optimization of the inputs is called I_Z-optimality. I_Z-optimality rests on an a distinct mathematical foundation from its relative, I_ε-optimality (previously known as simply I-optimality), which has been used in previous, related research [3]. In particular, I_Z-optimality makes the generally correct assumption that the deterministic simulator gives the same result for runs with identical inputs, this being distinct from non-deterministic physical experiments with noise in the responses. Crary et al. presented a review of the procedure applied to a micromachined flow-sensor example [1]. After the *N* FEA simulations are run, the metamodel-generation capability of I-OPT is used to find the best linear unbiased metamodel for use in the system-level simulator T-SPICE.

Our approach has the advantage over previous design-of-experiments approaches that the model function may contain both modeled and unmodeled parts, a minimum of points may be specified, and the choice of points is based on a sound mathematical optimization principle. In addition, we demonstrate how it is possible in some cases, such as our example, to obtain dynamical simulations based on a small number of static FEA simulations.

2 EXAMPLE

The accelerometer used for the example is the same as the one used by Gianchandani and Crary [3] in their comparison of metamodels generated based on simulations performed at I_ε-optimal and D-optimal design points.

2.1 Physical Model

The accelerometer is composed of a rigid proof mass supported by four L-shaped suspension beams. With reference to Figure 1, the lengths of various segments, in microns, are the following:

L1=L2=W1=1000, H1=500, L3=60, W2=50, and H2=10. The response of interest, the z-direction component of the deflection of the proof mass, is modeled as a function of two structural and three environmental parameters, viz., the point of attachment of one support beam to the proof mass ($100 \le a \le 900$ µm in the figure), the width of the short segment of the support beam ($20 \le b \le 100$ µm in the figure), the input- (z-) axis acceleration ($0 \le a_z \le 100$ m/s^2), the cross-axis acceleration ($0 \le a_x \le 100$ m/s^2), and the temperature ($-100 \le T \le 100$ °C). The choice of these parameters was imposed by qualitative engineering knowledge of the relevant parameters for this type of accelerometer.

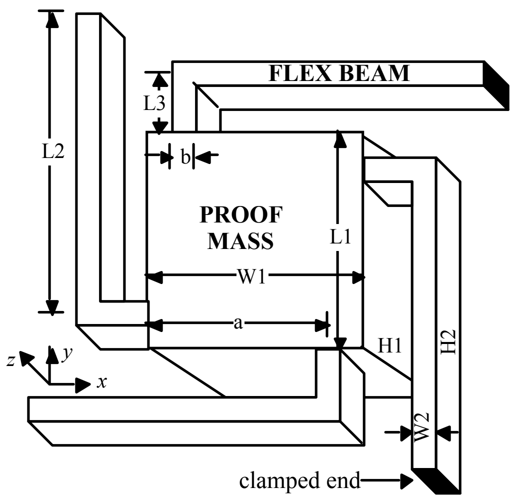

Figure 1. Accelerometer

2.2 Response-Model Function

The model function for the z-component of deflection was assumed to be a second-degree function of a and b and a first-degree function in a_x, a_z, and T, according to the following five-factor form, again following the earlier work [3], but allowing for an additional non-parametric term

$$\Psi(x_1, x_2, x_3, x_4, x_5) = \Psi(a, b, a_z, a_x, T)$$

representing the unknown part of the model function:

$$\begin{aligned}
Z = {} & \beta_0 + \beta_1 a + \beta_2 b + \beta_3 a_z + \beta_4 a_x + \beta_5 T + \beta_6 a^2 \\
& + \beta_7 b^2 + \beta_8 ab + \beta_9 aa_z + \beta_{10} aa_x + \beta_{11} aT + \beta_{12} ba_z \\
& + \beta_{13} ba_x + \beta_{14} bT + \beta_{15} a_z a_x + \beta_{16} a_z T + \beta_{17} a_x T \\
& + \beta_{18} a^2 a_z + \beta_{19} a^2 a_x + \beta_{20} a^2 T + \beta_{21} b^2 a_z + \beta_{22} b^2 a_x \\
& + \beta_{23} b^2 T + \beta_{24} aba_z + \beta_{25} aba_x + \beta_{26} abT + \beta_{27} aa_z a_x \\
& + \beta_{28} aa_z T + \beta_{29} aa_x T + \beta_{30} ba_z a_x + \beta_{31} ba_z T \\
& + \beta_{32} ba_x T + \beta_{33} a_z a_x T + \Psi(a, b, a_x, a_z, T)
\end{aligned} \tag{1}$$

where $\Psi(x_1, \ldots, x_5)$ was modeled as a stochastic process with covariance given by

$$\begin{aligned}
& \operatorname{cov}[\Psi(s_1, \ldots, s_5), \Psi(t_1, \ldots, t_5)] = \\
& \quad \sigma_Z^2 \exp\{-[\theta_1 (s_1-t_1)^2 + \theta_2 (s_2-t_2)^2 + \ldots + \theta_5 (s_5-t_5)^2 \\
& \qquad + \theta_{12} (s_1-t_1)(s_2-t_2) + \theta_{13} (s_1-t_1)(s_3-t_3) + \ldots \\
& \qquad + \theta_{45} (s_4-t_4)(s_5-t_5)]\},
\end{aligned}$$

and $\theta_1, \ldots, \theta_5, \theta_{12}, \ldots, \theta_{45}$, and σ_Z^2 were parameters that were set prior to the search for the optimal design. As in our previous work, we set all the θ_i, and σ_Z^2 to unity and $\theta_{ij}=0$, as initially justified in a two-factor, second-degree-model robustness study of Sacks et al. [4].

2.3 Design of Experiments

Since the response model function had 34 β coefficients, the minimum number of points required to determine the coefficients was 34. In the earlier work [3], allowance was made for four additional points so that the I_ε-optimal and D-optimal designs would include a few points for validation of the metamodel. For the present study, we again chose N=38 points and used I-OPT [2] to generate the design that minimized the integrated-mean-squared error of prediction (IMSE) of the metamodel given in Equation (1), under the assumption of a deterministic simulator. The minimum IMSE is defined as the following:

$$\min_{\omega_N} \frac{1}{\Omega} \int_{\mathbf{x} \in \chi} E\left[\left(\hat{Y}(\mathbf{x}) - Y(\mathbf{x})\right)^2\right] w(\mathbf{x})\, dx_1 \ldots dx_d ,$$

which asks for the N-point design ω_N that minimizes the average over a domain χ of the total expected squared error. $w(\mathbf{x})$ is a weighting function, and

$$\Omega = \int_{\mathbf{x} \in \chi} w(\mathbf{x})\, dx_1 \ldots dx_d .$$

We call a design with this property I_Z-optimal [1].

2.4 Automated Layout, Finite-Element Model Generation, and Finite-Element Analysis

The geometric parameters (*a* and *b*) required by the layout generator were extracted and saved as a text file in a format that is recognized by the L-Edit/UPI layout generator macro. 38 finite-element models were generated, and for each an ANSYS structural analysis was performed to compute the *z* displacement of the center of the top face of the proof mass.

2.5 Analysis of Experiments

I-OPT was run in its analysis mode to yield the best linear unbiased predictor (BLUP) for the metamodel of Equation 1. The fit function had 34+38=74 terms, as follows:

$$
\begin{aligned}
Y = \beta_0 &+ \beta_1 a + \ldots + \beta_{33} a_z a_x + \\
&\gamma_1 \exp[-(a-a^{(1)})^2 - \ldots -(T- T^{(1)})^2] + \\
&\gamma_2 \exp[-(a-a^{(2)})^2 - \ldots -(T- T^{(2)})^2] + \\
&\ldots \\
&\gamma_{38} \exp[-(a-a^{(38)})^2 - \ldots -(T- T^{(38)})^2],
\end{aligned}
$$

where the $a^{(i)} \ldots T^{(i)}$ are the a and T coordinates of the i'th design point, respectively.

2.6 T-SPICE Model

The approximating metamodel function was implemented in the T-SPICE external functional-modeling language, which enabled the definition of custom-designed behavioral device models. Five sets of input values were chosen arbitrarily and these all had residuals less than 5 nm. This is an improvement of more than two orders of magnitude over the earlier metamodel using D- or I-optimal designs and ordinary-least-squares fitting [3].

2.7 Dynamics

In this example the metamodel, which was constructed based on static analyses only, may be used to construct the ordinary differential equation governing the system dynamics, thus obviating the need for computationally expensive transient FEA analyses.

We considered that the rigid accelerometer package was at rest until t=5 ms, when it was subjected to a constant z-axis acceleration *accel*. The response equation for the displacement of the proof mass in the accelerating reference frame was the following:

$$\frac{\partial^2 z}{\partial t^2} + \frac{b}{m}\frac{\partial z}{\partial t} + \frac{k}{m} z = -\,accel \quad ,$$

where b and k are the damping and stiffness, respectively. To demonstrate the essential concept simply we treated the damping term as a parameter and focused attention on the stiffness, which depended only on the position of the proof mass with respect to the package. In the static analysis the stiffness was simply

$$k = \left(\frac{\partial F}{\partial Z}\right)_{operating\ point} ,$$

where Z was the z displacement in the static analysis performed in an inertial reference frame, as in Equation 1. The static FEA analysis treated an applied acceleration of a_z of the system as a force on the proof mass of $-ma_z$, giving the following:

$$k = -m\left(\frac{\partial Z}{\partial a_z}\right)^{-1} .$$

This procedure led to the following ODE:

$$
\begin{aligned}
&\frac{\partial^2 z}{\partial t^2} + \frac{b}{m}\frac{\partial z}{\partial t} - \{(\beta_3 + \beta_{15}\, a_x)^{-1} \\
&\quad + 2\sum_{i=1}^{38} \gamma_i \left(a_z - a_z^{(i)}\right) \exp\left[-(a - a^{(i)}) - \ldots - (T - T^{(i)})\right]\} z \\
&\quad = -\,accel \quad . \qquad (2)
\end{aligned}
$$

Of particular note is the way that this procedure carries through the dependence of the stiffness on the other factors.

Equation (2) may be subsequently solved at any operating point using an ODE solver such as T-SPICE. An example is shown in Figure 2. The frequency response can be determined, along with a transient analysis and derived quantities such as quality factor.

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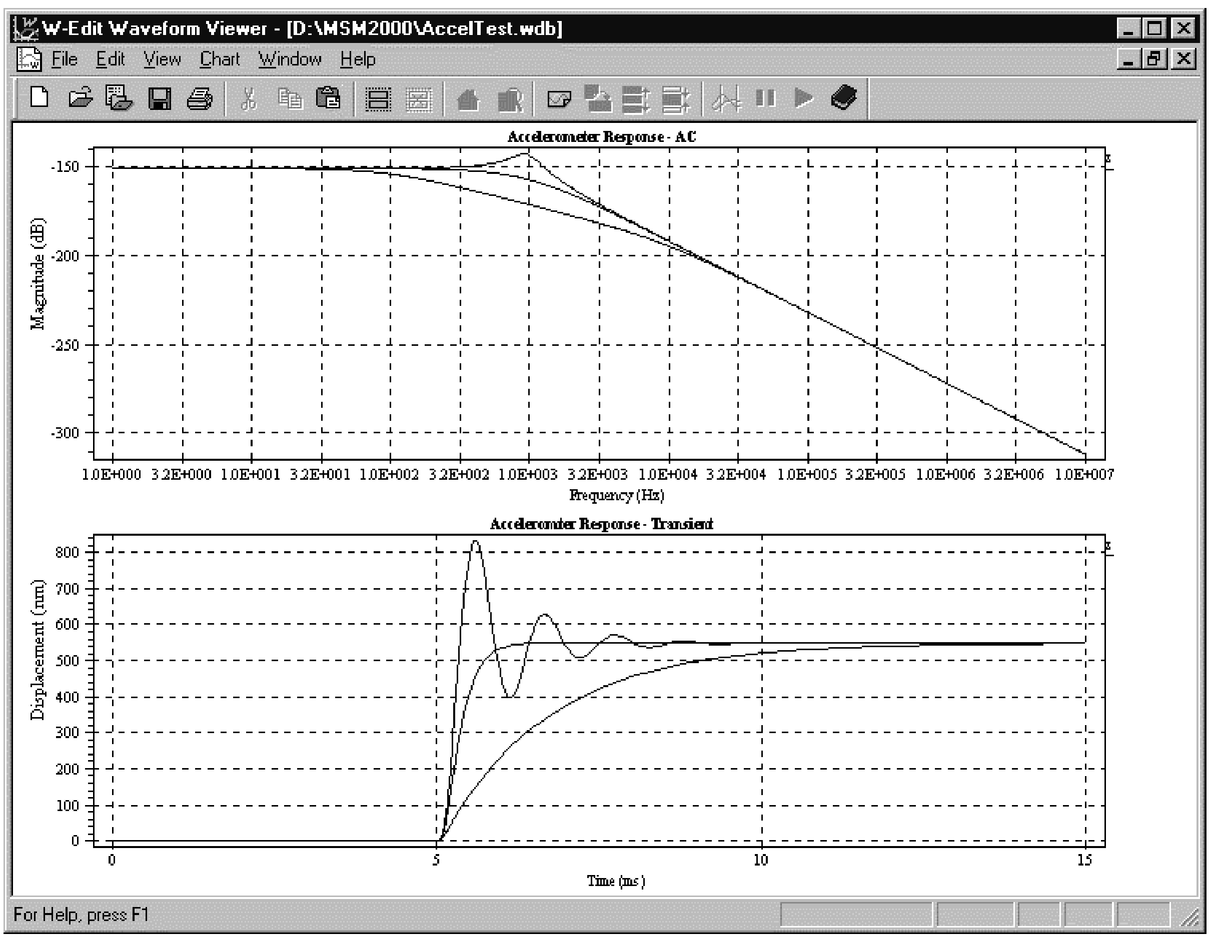

Figure 2. Dynamical analyses of Equation 2 at an operating point [a=500 µm, b=60 µm, a_z=a_x=0, and T=0 °C] as seen from a MEMS Pro W-Edit Waveform Viewer window. A step acceleration of 2g was applied to the micro-accelerometer package at t=5 ms. The values of the damping used in the figure are $b_c/5$ (underdamping), b_c, (critical damping), and $5b_c$, (overdamping).