DEGREES OF INTERPRETATION*

JOHN N. PHILLIPS

St. Cloud State College

What has been learned about logic by means of "uninterpreted" logistic systems can be supplemented by comparing the latter with systems which are more uninterpreted, as well as with others which are less uninterpreted than the well-known logistic systems. By somewhat extending the meaning of 'uninterpreted', I hope to establish certain claims about the nature of logistic systems and also to cast some light on the nature of "logic itself."

My procedure involves looking at three major "degrees" of interpretation: first, systems uninterpreted *both* semantically and syntactically, second, systems uninterpreted semantically but not syntactically, and third, systems uninterpreted *neither* semantically nor syntactically. We shall be forced to limit ourselves to the truth-functional part of logic in this brief study. What are usually called uninterpreted systems can be seen on a continuum of "degrees" of interpretation, from ordinary reasoning at one extreme to a "thoroughly uninterpreted system" at the other.

"Logic itself" apparently lies nearer to the interpreted, deformalized end of the spectrum than to the uninterpreted, formalized end. Logic is not identical with any particular logistic system, but is that which the particular logistic systems aim to formalize or model or capture. I propose that it is that minimum set of logical, rather than syntactical, primitive terms, definitions, and rules which is needed to generate logically, rather than syntactically, the principles of ordinary reasoning as it is *used* by logicians in their metalinguistic discussions and informal proofs of metatheorems. This minimum set is somewhat larger than the primitive bases of most logistic systems; its truth-functional part is presented in the "Deformalized Logic" at degree twelve.

What has been learned about logic by means of "uninterpreted" logistic systems can be supplemented by comparing the latter with systems which are more uninterpreted, as well as with others which are less uninterpreted than the well-known logistic systems. By somewhat extending the meaning of 'uninterpreted', I hope to establish certain claims about the nature of logistic systems and also to cast some light on the nature of "logic itself."

My procedure will involve looking at three major "degrees" of interpretation: first, systems uninterpreted *both* semantically and syntactically, second, systems uninterpreted semantically but *not* syntactically, and third, systems uninterpreted *neither* semantically nor syntactically. We shall be forced to limit ourselves to the truth-functional part of logic in this brief study. What are usually called uninterpreted systems can be seen on a continuum of "degrees" of interpretation, from ordinary reasoning at one extreme to a "thoroughly uninterpreted system" at the other. Over a certain range of this continuum, it will appear that to interpret is to deformalize and to formalize is to make uninterpreted.

I think this continuum or spectrum will be valuable in itself—as a fresh perspective on logic. In addition, the "more uninterpreted" sector of the continuum will reveal how much syntactical interpretation has been overlooked or de-emphasized, and will also support my claim that, contrary to statements by logicians,

* Received April, 1972.

the rules of concatenation, formation, and transformation, plus other rules, cannot be regarded as "not belonging to the system." The advantages gained by the "less uninterpreted" sector of the continuum include greater economy in the senses that no axioms are needed and no underived rules are needed, proofs and decision procedures are briefer than in other systems, there is little or no nonlogical baggage, such as talk of transformations, introductions, or elimination of signs or of boxes, trees, paths, or lines of truth tables. Further, economies achieved at a certain degree of interpretation or deformalization along with a gain in rigor in the metalanguage can only be pointed out after the continuum has been fully presented.

1. Systems Uninterpreted Both Semantically and Syntactically. Most logistic systems are quasi-uninterpreted rather than actually uninterpreted, since the intended interpretation is given away by calling the constants 'logical operators' and by calling the variables 'propositional variables'. In addition to such familiar terminology, most authors provide a running commentary, without which their systems would not be comprehensible to human understanding—nor manipulable by a machine unless some equivalent to the commentary in design or programming were given. Moreover, conventions of structure and layout make the author's marks on paper meaningful in ways of which neither the author nor the reader maintains awareness. It is my purpose now to make explicit as much as possible of this tacit meaning and structure.

The thoroughly uninterpreted system we are now trying to reach would seem to lack not only the given or hinted-at values of the variables, but also any overt distinction between the variables and the constants. Nor would this distinction be given away by the use of conventional symbols. The system would not be embedded in commentary. The concatention might be unconventional, since conventional concatenation conveys information.

One way of arriving at such a thoroughly uninterpreted system would be by beginning with a specific well-known logistic system and replacing each symbol by an unfamiliar mark, such as '⅄' or '#' or '▮', etc., and then changing the concatenation to a vertical or a nonlinear arrangement. The result might well be incomprehensible to man or beast and, perhaps, unmanipulable by man or machine. My point is that such a system would nevertheless lack nothing more than the syntactical meaning or interpretation that is normally provided by the author through his commentary, his use of conventional symbols, and conventional concatenation.

Let us be clear that we are talking about a system, not a chaotic aggregate; one which has as much order as the original from which it was translated. What is lacking is information about the system, which reasonably may be called interpretation. There are other meanings of 'interpretation', of course, one of which is simply the model, and another is the semantical rules which connect the system to the model. What I am now pointing to is all essential information about the system *except* the semantical rules or any recognition of a model of the system.

To illustrate, we might suppose that someone is perfectly taught the correct construction and manipulation of electronic circuit diagrams or the language of chess, without ever being made aware of electronic circuits or of chess games. But this kind of illustration introduces what might be a digressive epistemic perspective, whereas I wish to present only systems and their objective explicit descriptions by syntactical rules.

Instead of beginning with a particular well-known "uninterpreted" system and further disinterpreting it, we might consider the general case. We begin, then, with the set of unselected objects in the world, and restore by degrees the information about any system. At *degree one* we are told that the system consists of a selected set of objects, such as all marks on paper. At *degree two* we are told which marks are to be recognized. The rule which gives us the means for recognizing selected marks will do so by specifying their designs, and so the marks become types rather than tokens. This raises an interesting issue—whether a formal system is really something on paper or rather is a system of platonic forms or intentional objects. Perhaps there are nominalistic ways of accomplishing our purpose at this point, but the issue will recur at other levels. A platonic bias seems inherent in the very concepts of logical forms, formulas, formal systems, formal logic, logical principles, etc., but I shall not try to settle that issue here.

2. Systems Uninterpreted Semantically but not Syntactically. We begin now to treat the relations among signs.

At *degree three* we distinguish two kinds of symbols, which we shall call 'associates' and 'associators'. At *degree four* we distinguish between the replaceable and the unreplaceable associators and are informed how to replace the replaceables with the unreplaceables.

Degree five presents, through rules, the "proper concatenations" of the symbols. Although concatenation rules are not usually included in the "formation rules" of a system, they are presupposed by them, nevertheless. European languages presuppose by convention that the sign designs (letters) will have a two-dimensional, linear, horizontal, left to right succession. By modifying one or more of these conventions, a wide range of "improper" concatenations would be possible, and thus a wide range of alternative formal systems.

Let us remind ourselves that it is beside the point to insist that the system can and does exist without the *statement* of the rules, such as concatenation rules. On the one hand, this is not to say that the system exists without the rules. On the other hand, we are now engaged in the enterprise of making the interpretation of the "uninterpreted" explicit, which calls just for the statement of all the rules of the system.

At *degree six* we are informed of the rules for *acceptable* proper concatenations. This is the level of the customary "rules of formation" for "well-formed formulas." *Degree seven* distinguishes between the preferred and unpreferred acceptable proper concatenations. At *degree eight* the preferred acceptable proper concatenations of symbols are divided into the derived and the underived. Rules for determining the approved derivations are given at *degree nine*.

3. Systems Uninterpreted Neither Semantically Nor Syntactically. If some of the above specifications do not "make sense" to the reader, it is because, of course, we have remained faithful to syntacticalism. Having established the extent of syntactical interpretation to be made explicit, we can now render it more meaningful by providing the semantical interpretation.

When we divulge, at *degree ten*, that the associates are "propositional variables," and the associators are "logical operators," we have become at least quasi-semantical. The conflict between logicians' use of such terminology and their claim to have a semantically uninterpreted system is only partially mitigated by relating such terms to the "intended" interpretation. This practice, in any case, seems to anticipate *degree eleven*, which is the assignment of the terms 'true' and 'false' to indicate the range of the values of the variables (my "associates"). With this, the system can become a version of the calculus of propositions.

Let us refer to the proper concatenations as "associations." Then, when the associates are interpreted as true or false, the accepted association also becomes true or false. More importantly, the "preferred" accepted associations all become true. These things do not happen automatically, of course. Rather, the preferred associations and every degree upon which they are built have been shaped ingeniously by the logician to "correspond" somehow to the logical truths themselves. At each "degree," alternative shapings are possible, leading to other, nonlogistic, formal systems.

Let us call any system through degree nine a logistic system, provided it is so shaped as to allow the interpretation at degree eleven. The term 'formal systems' will then include logistic systems, along with systems which deviate at one or more of the first nine "degrees." At degree nine, for example, it might be possible to have other than the "approved" type of derivations of theorems for axioms, or at degree seven to have other than the "preferred" accepted associations. Although looking at such alternative formal systems can be instructive, what is more interesting is how strictly the intended goal of formalizing *logic* limits our choices and forces us into certain adaptations, especially when we proceed into quantificational logic.

The semantical interpretation having been provided, it might be asked what further degrees there could be, on our continuum? We now begin to look at systems *less* uninterpreted and less formalized than the well-known logistic systems. The first of these is the most important. At *degree twelve*, then, we decrease the formalization and increase the interpretation by dropping the object language and also the syntax language, leaving only the semantical metalanguage. This drastic step in deformalization makes it possible to assert our logical theses directly, in the semantical metalanguage, rather than model them in an uninterpreted calculus. And this important difference is accompanied by another, that we can now dispense with interpretive or explanatory commentary, since we are talking the language of logic itself, as opposed to talking about an artificial and foreign language.

What will such a system look like? It will consist wholly of semantical rules, or definitions, and their consequences. (If an alert reader detects circularity in speaking of the logical "consequences" of those definitions intended to express the very foundations of the notion of logical consequence, he is quite right. But such circularity is escaped by no system which has rules or definitions. For example, the formation rules of a calculus would be entirely useless without their "logical consequences.") To see the logical consequences of a rule is to interpret that rule; the deformalized system to be presented next differs from other systems in wholly accepting, indeed, founding itself upon, the circular presupposition of logical principles by the basic rules and concepts which attempt to formulate those principles. For the truth-functional part of logic, the only part dealt with in this brief paper, these semantical rules will consist simply of the theses of the well-known truth tables expressed in ordinary language. Here is a version of the system.

Deformalized Logic

(D1) (A), (B), (C), (D), ARE ANYTHING TRUE OR FALSE
(D2) ~(A) IS TRUE IF AND ONLY IF (A) IS FALSE.
(D3) (A & B) IS TRUE IF AND ONLY IF (A) IS TRUE AND (B) IS TRUE.
(D4) (A ∨ B) IS FALSE IFF (A) IS FALSE AND (B) IS FALSE.
(D5) (A ⊃ B) IS FALSE IFF (A) IS TRUE AND (B) IS FALSE.
(D6) (A) IS INCONSISTENT IFF SOMETHING IS BOTH TRUE AND FALSE IF (A) IS TRUE.
(D7) (A) IS TRUE NECESSARILY IFF ~(A) IS INCONSISTENT.
(D8) (A, B, C, & D) IS VALID IFF (A & B & C & ~D) IS INCONSISTENT.

The above set of rules implies all the truth-functional truths, in a sense of 'implies' similar to that in which the rules of a calculus imply the whole calculus. The truth-functional truths will here consist of those things shown to be TRUE NECESSARILY (D7). Somewhat redundantly, all of the valid rules of inference for truth-functional logic are implied by the set, D1–D6, D8. The procedure of the derivations is logical, rather than transformational. It consists of using D6 in connection with the well-known "short-cut" truth-table decision procedure. (That is, we suppose the thesis being tested to be false, then follow through to the consequences of this hypothesis together with D1–D5. If the procedure leads to some component being supposed both true and false, then by D6 and D7 the thesis is TRUE NECESSARILY.) The decision procedure becomes converted to an annotated proof if each step in the procedure is given a justification by citing one of the rules, D1–D8.

The undefined terms in the deformalized system are the following: 'TRUE', 'IF', 'ANYTHING', 'FALSE', 'IS', 'SOMETHING', 'AND', 'OR', and 'ONLY'. None of these undefined terms is uninterpreted. Nor are the variables uninterpreted, since the values for them are specified in the rules D1–D5. These are additional reasons for calling the system "deformalized."

We deformalize still further, at *degree thirteen*, by dropping the metalinguistic variables. Since Aristotle the notion of formal logic, that is, the investigation of logical forms, has depended upon the use of variables of some kind. What might remain of formal logic at degree thirteen is a set of definitions and technical terms. I shall not attempt to present a complete set of these, and I shall not make the claim here which I made at degree twelve, namely, that all of the truth-functional truths and valid forms of truth-functional arguments could be formulated with such a set of technical terms and definitions, without variables. In an informal way, however, perhaps much of logic could be systematized with a few definitions like:

> An *argument* is *valid* if the *negation* of its *conclusion* is *inconsistent* with its *premisses*. (Technical terms emphasized.)

Having thoroughly deformalized logical systems, we now desystematize them, in *degree fourteen*, by dropping all technical terms and stating a few general principles of ordinary reasoning in ordinary language, such as,

> Whatever something true implies is true, and
> Whatever implies something false is false.

A significant amount of scientific and mathematical, as well as commonsense thinking is accomplished in accord with these principles, especially if they are supplemented by a few quantification principles like:

> Whatever is affirmed of all is affirmed of each, and
> Whatever is denied of all is denied of each.

Such principles would constitute our first step in systematizing logic, if we were to proceed in the reverse direction through the "degrees." I would not claim that ordinary reasoners have these principles in mind, but rather that the principles generalize important types of valid ordinary reasoning.

It is significant that the above principles are *true*, rather than arbitrary, like conventions, or neither true nor false, like rules. We are not free to violate them or ignore them without danger of error, as we may confirm by recognizing the *falsity* of the following:

> Whatever implies something true is true.
> Whatever something false implies is false.
> Whatever is affirmed of some is affirmed of all.
> Whatever is denied of some is denied of all.

And so we arrive at *degree fifteen*, the unformalized, unsystematized, fully interpreted phase of logic, that of everyday thinking. We might as well call this "ordinary reasoning," since it is expressed by the open set of ordinary language terms including 'therefore', 'hence', 'since', 'so', 'because', as well as 'if', 'and', 'or', and 'not', plus an indefinite number of other locutions which can be put to similar uses.

4. Summary and Conclusions. Whether my general outline of different degrees of interpretation and formalization of logic is instructive is for each reader to decide for himself. However, I shall itemize some of the specific implications which I believe the views presented above have.

By beginning with the thoroughly uninterpreted system, we were enabled to discover some of the grades or gradations of structural meaning presupposed by a logistic system. Concisely expressed, a logistic system formulates approved ways of deriving all preferred acceptable associations from a small subset of them; and the associations themselves are analyzable into proper concatenations of recognized designs of marks on paper.

The value of the standard uninterpreted logistic systems themselves was not in question in this study. Among their benefits are the effective proofs of consistency and completeness.

The advantages of the system of deformalized logic, at "degree twelve," include the avoidance of nonlogical terms and procedures and avoiding dependence on commentary. Greater parsimony of primitive terms is another advantage of this system. If that seems surprising, it is because, although the standard logistic systems have great parsimony in the object language, they "go out of control" in the metalanguage, where, in their rules, metatheorems, and commentary, they make use of all the undefined metalinguistic terms listed for my deformalized system, as well as a great many more. The deformalized system also dispenses with axioms and rules of inference, since all logical truths (truth-functional) and all valid rules are immediate consequences of the definitions. The circularity in the definitions and in the derivations is not unique to this system: rather, it is only more explicit and acknowledged.

As for the unsystematic ordinary reasoning, in the ranges beyond the deformalized system, it is just this which is used by all logicians in their metalinguistic proofs of metatheorems. This ordinary reasoning is like the scholastic "transcendentals" in its ubiquity, its habit of recurring *within* all attempts to explicate it, through the inevitable use of such terms as 'if', 'and', 'therefore', 'since', or their equivalents. Again, the system of deformalized (degree twelve) logic faces this situation more "candidly."

Concerning the question, "What is a logistic system?", I think the remark often made by logicians that the rules "do not belong to the system itself" is untenable. Is the system, then, the theorems only (including the axioms)? But if these are infinite in number, then the system can never be presented. But if the system is presented as generated (or generateable) from the axioms, this presupposes one or more rules by which the theorems are generated from the axioms. The same argument holds in other "degrees" of the continuum; an infinite set of marks must be given by a finite set of designs of marks and an infinite set of concatenations must be given by a finite set of concatenation rules.

Finally, where in this range of interpreted and uninterpreted systems is "logic itself?" It apparently lies nearer to the interpreted, deformalized end of the spectrum than to the uninterpreted, formalized end. Clearly logic itself is not identical with any particular logistic system. Rather, it is that which the particular logistic systems aim to formalize or model or capture. Is it then ordinary reasoning? Is it the complete infinite set of all logical principles, including rules and logical truths? Is it some subset of these? I propose that it is that minimum set of logical, rather than syntactical, primitive terms, definitions, and rules which is needed to generate logically, rather than syntactically, the principles of ordinary reasoning as ordinary reasoning is *used* by logicians in their metalinguistic discussions and informal proofs of metatheorems. This minimum set is somewhat larger than the primitive bases of most logistic systems; its truth-functional part is presented in the "Deformalized Logic" at degree twelve, above.