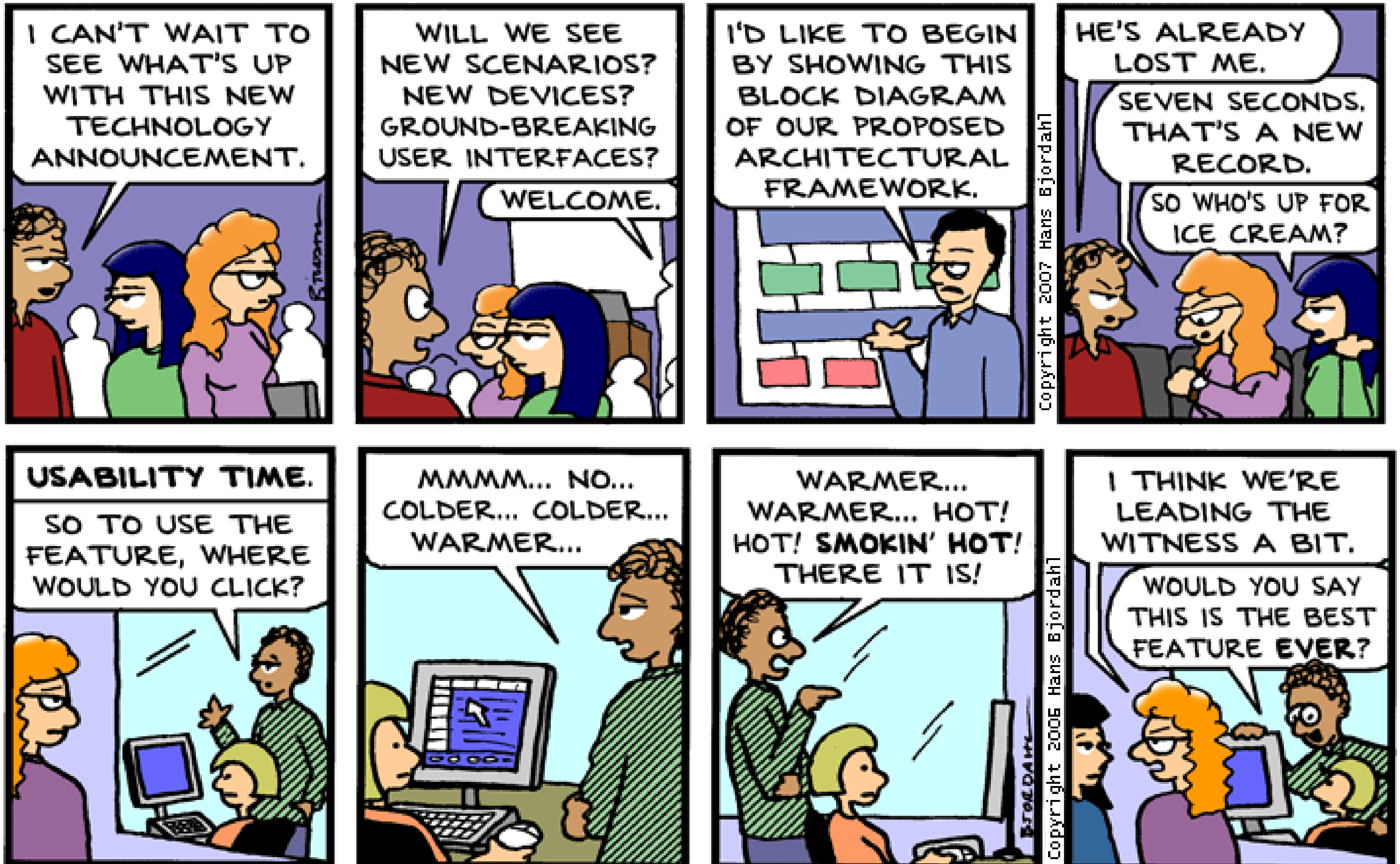
In Our Last Exciting Episode



Bug Bash by Hans Bjordahl

http://www.bugbash.net/

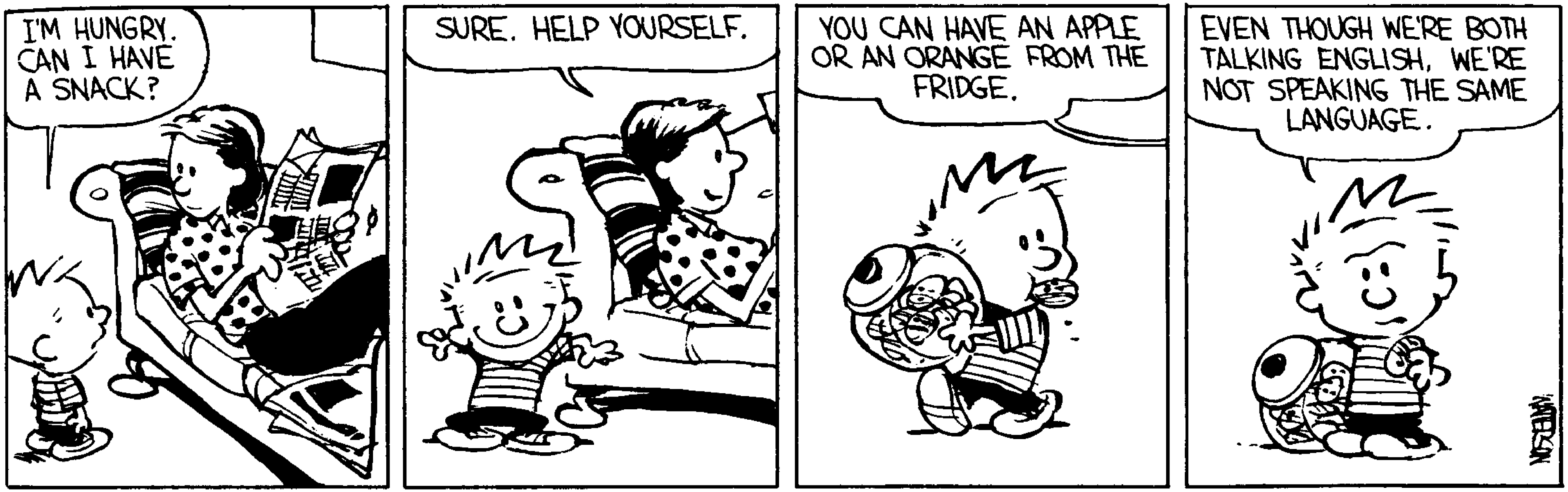
Lessons From Model Checking

- To find **bugs**, we need **specifications**
  - What are some good specifications?
- To **convert** a program into a **model**, we need **predicates**/invariants and a **theorem prover**.
  - Which are the important predicates? Invariants?
  - What should we track when reasoning about a program and what should we abstract?
  - How does a theorem prover work?
- **Simple** algorithms (e.g., depth first search, pushing facts along a CFG) can work well
  - ... under what circumstances?

The Big Lesson

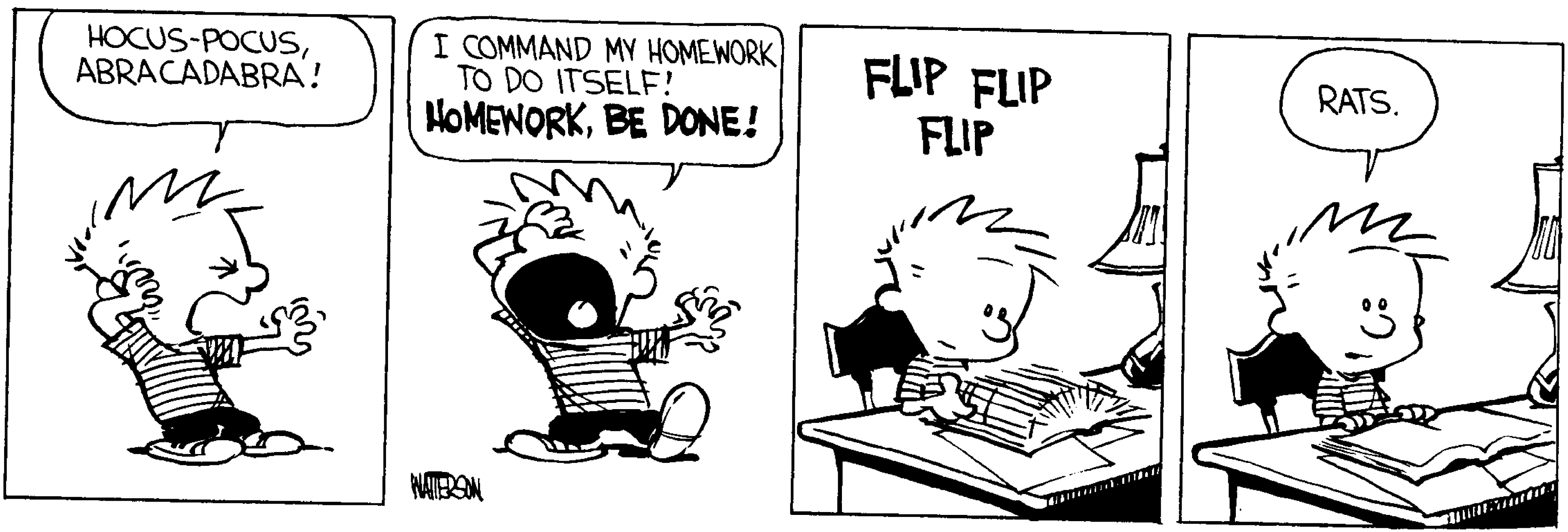
- To reason about a program (= "is it doing the right thing? the wrong thing?") we must *understand what the program means!*

A Simple Imperative Language
Operational Semantics
(= “meaning”)



Homework #0 Due Today

- Can't get BLAST to work?
  - ssh to power1.cs.virginia.edu
  - Plus the BLAST linux binaries
  - cp all of them (e.g., csi*, pblast*, ...) to ~/bin



Medium-Range Plan

- Study a *simp*le *imp*erative language IMP
  - Abstract syntax (today)
  - **Operational semantics** (today)
  - Denotational semantics
  - Axiomatic semantics
  - … and relationships between various semantics (with proofs, peut-être)
  - Today: operational semantics
    - Follow along in Chapter 2 of Winskel

Syntax of IMP

- Concrete syntax: The rules by which programs can be expressed as strings of characters
  - Keywords, identifiers, statement separators vs. terminators (Niklaus!?), comments, indentation (Guido!?)
- Concrete syntax is important in practice
  - For readability (Larry!?), familiarity, parsing speed (Bjarne!?), effectiveness of error recovery, clarity of error messages (Robin!?)
- Well-understood principles
  - Use finite automata and context-free grammars
  - Automatic lexer/parser generators

(Note On Post-LALR Advances)

- If-as-and-when you find yourself making a new language, consider GLR (**elkhound**) instead of LALR(1) (**bison**)
- Scott McPeak, George G. Necula: *Elkhound: A Fast, Practical GLR Parser Generator*. CC 2004: pp. 73-88
- As fast as LALR(1), more natural, handles basically all of C++, etc.

Abstract Syntax

- We ignore parsing issues and study programs given as abstract syntax trees
  - I provide the parser in the homework …
- An abstract syntax tree is (a subset of) the parse tree of the program
  - Ignores issues like comment conventions
  - More convenient for formal and algorithmic manipulation
  - All research papers use ASTs, etc.

IMP Abstract Syntactic Entities

- int — integer constants ($n \in \mathbb{Z}$)
- bool — bool constants (true, false)
- L — locations of variables (x, y)
- Aexp — arithmetic expressions (e)
- Bexp — boolean expressions (b)
- Com — commands (c)

  – (these also encode the types)

Abstract Syntax (Aexp)

- **Arithmetic expressions (Aexp)**

$$
\begin{array}{lll}
e ::= & n & \text{for } n \in \mathbb{Z} \\
 & \mid x & \text{for } x \in L \\
 & \mid e_1 + e_2 & \text{for } e_1, e_2 \in \text{Aexp} \\
 & \mid e_1 - e_2 & \text{for } e_1, e_2 \in \text{Aexp} \\
 & \mid e_1 * e_2 & \text{for } e_1, e_2 \in \text{Aexp}
\end{array}
$$

- Notes:
  - Variables are not declared
  - All variables have integer type
  - No side-effects (in expressions)

Abstract Syntax (Bexp)

- **Boolean expressions (Bexp)**

$b ::= \text{true}$	
$\mid \text{false}$	
$\mid e_1 = e_2$	for $e_1, e_2 \in$ Aexp
$\mid e_1 \leq e_2$	for $e_1, e_2 \in$ Aexp
$\mid \neg b$	for $b \in$ Bexp
$\mid b_1 \wedge b_2$	for $b_1, b_2 \in$ Bexp
$\mid b_1 \vee b_2$	for $b_1, b_2 \in$ Bexp

“Boolean”

- George Boole
  - 1815-1864
- I’ll assume you know boolean algebra …

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

Abstract Syntax (Com)

- **Commands (Com)**

  c ::= skip

$\mid x := e$	$x \in L \wedge e \in \text{Aexp}$
$\mid c_1 \; ; \; c_2$	$c_1, c_2 \in \text{Com}$
$\mid$ **if** b **then** c_1 **else** c_2	$c_1, c_2 \in \text{Com} \wedge b \in \text{Bexp}$
$\mid$ **while** b **do** c	$c \in \text{Com} \wedge b \in \text{Bexp}$

- Notes:
  - The typing rules are embedded in the syntax definition
  - Other parts are not context-free and need to be checked separately (e.g., all variables are declared)
  - Commands contain all the side-effects in the language
  - Missing: pointers, function calls, what else?

Why Study Formal Semantics?

- Language design (denotational)
- Proofs of correctness (axiomatic)
- Language implementation (operational)
- Reasoning about programs
- Providing a clear behavioral specification
- “All the cool people are doing it.”
  - You need this to understand PL research
- “First one’s free.”

Consider This Legal Java

```
x = 0;
try {
  x = 1;
  break mygoto;
} finally {
  x = 2;
  raise
   NullPointerException;
}
x = 3;
mygoto:
x = 4;
```

- What happens when you execute this code?
- Notably, which assignments are executed?

14.20.2 Execution of try-catch-finally

- A try statement with a finally block is executed by first executing the try block. Then there is a choice:
- If execution of the try block completes normally, then the finally block is executed, and then there is a choice:
  - If the finally block completes normally, then the try statement completes normally.
  - If the finally block completes abruptly for reason *S*, then the try statement completes abruptly for reason *S*.
- If execution of the try block completes abruptly because of a throw of a value *V*, then there is a choice:
  - If the run-time type of *V* is assignable to the parameter of any catch clause of the try statement, then the first (leftmost) such catch clause is selected. The value *V* is assigned to the parameter of the selected catch clause, and the *Block* of that catch clause is executed. Then there is a choice:
    - If the catch block completes normally, then the finally block is executed. Then there is a choice:
      - If the finally block completes normally, then the try statement completes normally.
      - If the finally block completes abruptly for any reason, then the try statement completes abruptly for the same reason.
    - If the catch block completes abruptly for reason *R*, then the finally block is executed. Then there is a choice:
      - If the finally block completes normally, then the try statement completes abruptly for reason *R*.
      - If the finally block completes abruptly for reason *S*, then the try statement completes abruptly for reason *S* (and reason *R* is discarded).
  - If the run-time type of *V* is not assignable to the parameter of any catch clause of the try statement, then the finally block is executed. Then there is a choice:
    - If the finally block completes normally, then the try statement completes abruptly because of a throw of the value *V*.
    - If the finally block completes abruptly for reason *S*, then the try statement completes abruptly for reason *S* (and the throw of value *V* is discarded and forgotten).
- If execution of the try block completes abruptly for any other reason *R*, then the finally block is executed. Then there is a choice:
  - If the finally block completes normally, then the try statement completes abruptly for reason *R*.
  - If the finally block completes abruptly for reason *S*, then the try statement completes abruptly for reason *S* (and reason *R* is discarded).

Can’t we just nail this somehow?

Ouch! Confusing.

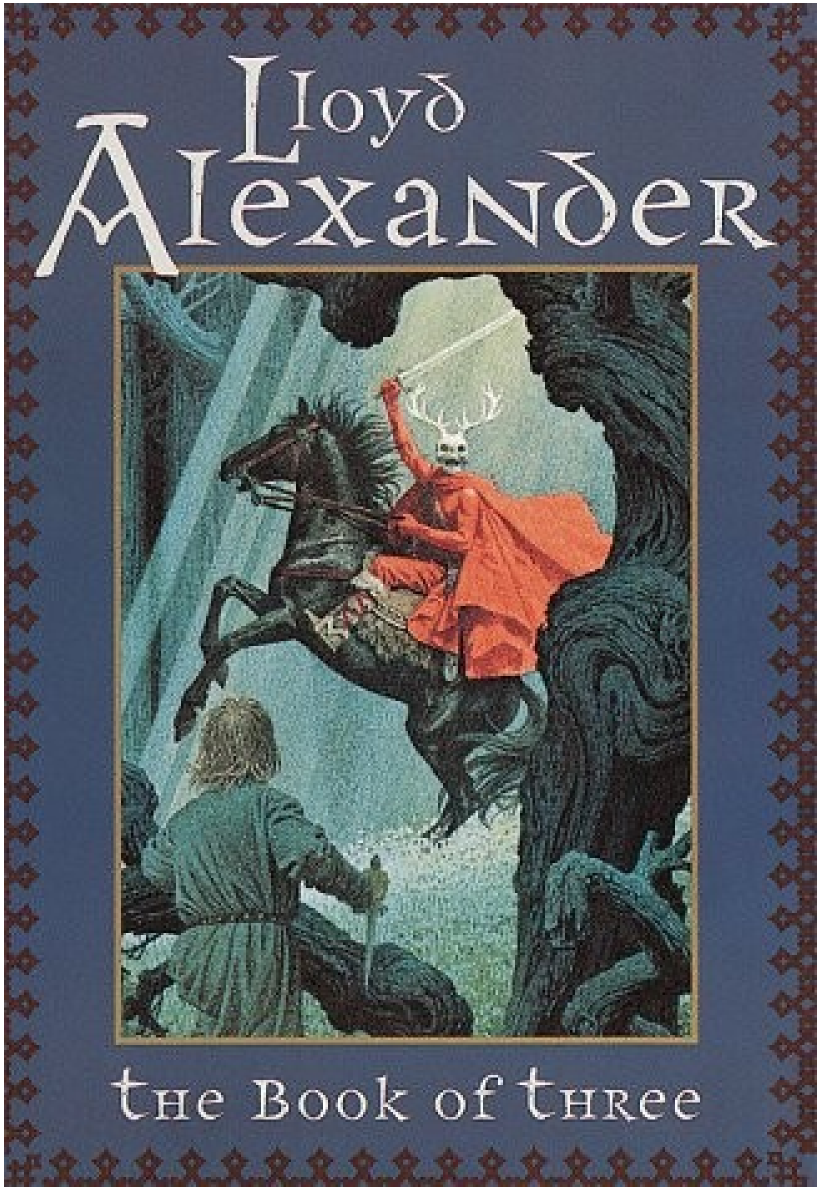
- Wouldn't it be nice if we had some way of describing what a language (feature or program) means …
  - More precisely than English
  - More compactly than English
  - So that you might build a compiler
  - So that you might prove things about programs

Analysis of IMP

- Questions to answer:
  - What is the "meaning" of a given IMP expression/command?
  - How would we go about evaluating IMP expressions and commands?
  - How are the evaluator and the meaning related?

Three Canonical Approaches

- Operational
  - How would I execute this?
- Axiomatic
  - What is true after I execute this?
  - Symbolic Execution
- Denotational
  - What is this trying to compute?



An Operational Semantics

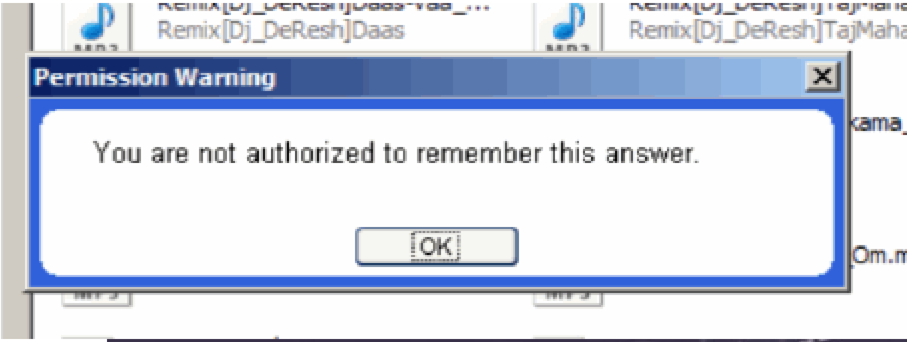
- Specifies how expressions and commands should be evaluated
- Depending on the form of the expression
  - 0, 1, 2, . . . don't evaluate any further.
    - They are normal forms or values.
  - $e_1 + e_2$ is evaluated by first evaluating e_1 to n_1 , then evaluating e_2 to n_2 . (post-order traversal)
    - The result of the evaluation is the literal representing $n_1 + n_2$.
  - Similarly for $e_1 * e_2$
- Operational semantics abstracts the execution of a concrete interpreter
  - Important keywords are colored & underlined in this class.

Semantics of IMP

- The meanings of IMP expressions depend on the values of variables
  - What does "x+5" mean? It depends on "x"!
- The value of variables at a given moment is abstracted as a function from L to $\mathbb{Z}$ (a state)
  - If $x = 8$ in our state, we expect "x+5" to mean 13
- The set of all states is $\Sigma = L \to \mathbb{Z}$
- We shall use σ to range over Σ
  - σ, a state, maps variables to values

Program State

- The **state** σ is somewhat like "**memory**"
  - It holds the current values of all variables
  - Formally, $\sigma : L \to \mathbb{Z}$



Q: Cartoons (682 / 842)

- Why is Gargamel trying to capture the Smurfs?

Q: Computer Science

- This American Turing Award winner is notable for his work in the theory of algorithms, a max-flow solver, a bipartite graph matcher, a string search algorithm, and “Reducibility Among Combinatorial Problems” in which he proved 21 problems to be NP-complete. He introduced the standard methodology for proving problems to be NP-complete.

Notation: Judgment

- We write:

$$\langle e, \sigma \rangle \Downarrow n$$

- To mean that e evaluates to n in state σ.
- This is a judgment. It asserts a relation between e, σ and n.
- In this case we can view $\Downarrow$ as a function with two arguments (e and σ).

Operational Semantics

- This formulation is called natural operational semantics
  - or big-step operational semantics
  - the $\Downarrow$ judgment relates the expression and its "meaning"

- How should we define

$$<e_1 + e_2, \sigma> \Downarrow \ldots ?$$

Notation: Rules of Inference

- We express the evaluation rules as rules of inference for our judgment
  - called the derivation rules for the judgment
  - also called the evaluation rules (for operational semantics)
- In general, we have one rule for each language construct:

$$\frac{<e_1, \sigma> \Downarrow n_1 \qquad <e_2, \sigma> \Downarrow n_2}{<e_1 + e_2, \sigma> \Downarrow n_1 + n_2}$$

This is the *only* rule for $e_1 + e_2$

Rules of Inference

$$\frac{\text{Hypothesis}_1 \ \ldots \ \text{Hypothesis}_N}{\text{Conclusion}}$$

$$\frac{\Gamma \vdash b : \text{bool} \qquad \Gamma \vdash e1 : \tau \qquad \Gamma \vdash e2 : \tau}{\Gamma \vdash \text{if } b \text{ then } e1 \text{ else } e2 : \tau}$$

- For any given proof system, a finite number of rules of inference (or schema) are listed somewhere
- Rule instances should be easily checked
- What is the definition of "NP"?

Derivation

$$
\dfrac{
\dfrac{
\dfrac{\Gamma(x) = int}{\Gamma \vdash x : int}\,\text{var}
\qquad
\dfrac{}{\Gamma \vdash 3 : int}\,\text{int}
}{\Gamma \vdash x > 3 : bool}\,\text{gt}
\qquad
\dfrac{
\dfrac{\Gamma(x) = int}{\Gamma \vdash x : int}\,\text{var}
\qquad
\dfrac{
\dfrac{\Gamma(x) = int}{\Gamma \vdash x : int}\,\text{var}
\qquad
\dfrac{}{\Gamma \vdash 1 : int}\,\text{int}
}{\Gamma \vdash x - 1 : int}\,\text{sub}
}{\Gamma \vdash x := x - 1}\,\text{assign}
}{\Gamma \vdash \texttt{while}\ x > 3\ \texttt{do}\ x := x - 1\ \texttt{done}}\,\text{while}
$$

- Tree-structured (conclusion at bottom)
- May include multiple sorts of rules-of-inference
- Could be constructed, typically are not
- Typically verified in polynomial time

Evaluation Rules (for Aexp)

$$\frac{}{\langle n, \sigma \rangle \Downarrow n} \qquad \frac{}{\langle x, \sigma \rangle \Downarrow \sigma(x)}$$

$$\frac{\langle e_1, \sigma \rangle \Downarrow n_1 \quad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 + e_2, \sigma \rangle \Downarrow n_1 + n_2} \qquad \frac{\langle e_1, \sigma \rangle \Downarrow n_1 \quad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 - e_2, \sigma \rangle \Downarrow n_1 - n_2}$$

$$\frac{\langle e_1, \sigma \rangle \Downarrow n_1 \quad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 * e_2, \sigma \rangle \Downarrow n_1 * n_2}$$

- This is called structural operational semantics
  - rules defined based on the structure of the expression
- These rules do not impose an order of evaluation!

Evaluation Rules (for Bexp)

$$\frac{}{\langle \texttt{true}, \sigma \rangle \Downarrow \text{true}}$$

$$\frac{\langle e_1, \sigma \rangle \Downarrow n_1 \qquad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 \leq e_2, \sigma \rangle \Downarrow n_1 \leq n_2}$$

$$\frac{}{\langle \texttt{false}, \sigma \rangle \Downarrow \text{false}}$$

$$\frac{\langle e_1, \sigma \rangle \Downarrow n_1 \qquad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 = e_2, \sigma \rangle \Downarrow n_1 = n_2}$$

$$\frac{\langle b_1, \sigma \rangle \Downarrow \text{false}}{\langle b_1 \wedge b_2, \sigma \rangle \Downarrow \text{false}}$$

$$\frac{\langle b_2, \sigma \rangle \Downarrow \text{false}}{\langle b_1 \wedge b_2, \sigma \rangle \Downarrow \text{false}}$$

$$\frac{\langle b_1, \sigma \rangle \Downarrow \text{true} \qquad \langle b_2, \sigma \rangle \Downarrow \text{true}}{\langle b_1 \wedge b_2, \sigma \rangle \Downarrow \text{true}}$$

(show: candidate $\vee$ rule)

How to Read the Rules?

- Forward (top-down) = inference rules
  - if we know that the hypothesis judgments hold then we can infer that the conclusion judgment also holds
  - If we know that
    $<e_1, \sigma> \Downarrow 5$ and
    $<e_2, \sigma> \Downarrow 7$, then we can infer that
    $<e_1 + e_2, \sigma> \Downarrow 12$

How to Read the Rules?

- Backward (bottom-up) = evaluation rules
  - Suppose we want to evaluate $e_1 + e_2$, i.e., find n s.t. $e_1 + e_2 \Downarrow n$ is derivable using the previous rules
  - By inspection of the rules we notice that the last step in the derivation of $e_1 + e_2 \Downarrow n$ **must be** the addition rule
    - the other rules have conclusions that would not match $e_1 + e_2 \Downarrow n$
    - this is called reasoning by **inversion** on the derivation rules

Evaluation By Inversion

- Thus we must find n_1 and n_2 such that $e_1 \Downarrow n_1$ and $e_2 \Downarrow n_2$ are derivable
  - This is done recursively
- If there is exactly one rule for each kind of expression we say that the rules are syntax-directed
  - At each step at most one rule applies
  - This allows a simple evaluation procedure as above (recursive tree-walk)
  - True for our Aexp but not Bexp. *Why?*

Evaluation of Commands

- The evaluation of a Com may have side effects but has no direct result
  - What is the result of evaluating a command ?
- The "result" of a Com is a new state:

$$<c, \sigma> \Downarrow \sigma'$$

  - But the evaluation of Com might not terminate! Danger Will Robinson! (huh?)

Com Evaluation Rules 1

$$\frac{}{\langle \text{skip}, \sigma \rangle \Downarrow \sigma} \qquad \frac{\langle c_1, \sigma \rangle \Downarrow \sigma' \quad \langle c_2, \sigma' \rangle \Downarrow \sigma''}{\langle c_1 ; c_2, \sigma \rangle \Downarrow \sigma''}$$

$$\frac{\langle b, \sigma \rangle \Downarrow \text{true} \quad \langle c_1, \sigma \rangle \Downarrow \sigma'}{\langle \text{if } b \text{ then } c_1 \text{ else } c_2, \sigma \rangle \Downarrow \sigma'}$$

$$\frac{\langle b, \sigma \rangle \Downarrow \text{false} \quad \langle c_2, \sigma \rangle \Downarrow \sigma'}{\langle \text{if } b \text{ then } c_1 \text{ else } c_2, \sigma \rangle \Downarrow \sigma'}$$

Com Evaluation Rules 2

$$\frac{\langle e, \sigma\rangle \Downarrow n}{\langle x := e, \sigma\rangle \Downarrow \sigma[x := n]}$$

Def: $\sigma[x{:=}\ n](x) = n$

$\sigma[x{:=}\ n](y) = \sigma(y)$

- Let's do while together

Com Evaluation Rules 3

$$\frac{\langle e, \sigma\rangle \Downarrow n}{\langle x := e, \sigma\rangle \Downarrow \sigma[x := n]}$$

Def: $\sigma[x{:=}\ n](x) = n$

$\sigma[x{:=}\ n](y) = \sigma(y)$

$$\frac{\langle b, \sigma\rangle \Downarrow \text{false}}{\langle \text{while } b \text{ do } c, \sigma\rangle \Downarrow \sigma}$$

$$\frac{\langle b, \sigma\rangle \Downarrow \text{true} \quad \langle c; \text{while } b \text{ do } c, \sigma\rangle \Downarrow \sigma'}{\langle \text{while } b \text{ do } c, \sigma \rangle \Downarrow \sigma'}$$

Homework

- Homework 0 Due Today
- Homework 1 **Due In One Week**
- Reading!
  - If this wasn't intuitive, try some of the optional readings for more context.