Costs and Sneezewort and Growth

One-Slide Summary

• The basic recursive computation of Fibonacci can take quite a while. There are faster ways.
• We can formally measure and evaluate the cost of a computer program. We abstract away details such as processor speed and instead measure how the solving time increases as the input increases.
• \( g \in O(f) \) iff there exist positive constants \( c \) and \( n_0 \) such that \( g(n) \leq cf(n) \) for all \( n \geq n_0 \).
• If \( g \in O(f) \) we say that \( f \) is an upper bound for \( g \).

Outline

• Sneezewort and Fibonacci
• Cost of computing Fibonacci
• Cost of sorting
• Intro to Big-Oh Notation

Sneezewort

• Achillea ptarmica is real.
• It is “moste efficacious in the inflaming of the braine, and [is] therefore much used in Confusing and Befuddlement Draughts, where the wizard is desirous of producing hot-headedness and recklessness.”
  - Order of the Phoenix, p.18
• Sneezewort’s pattern of development displays the Fibonacci sequence.
Sneezewort Growth

First Time Unit  Second Time Unit  Offshoot

Could we model Sneezewort with PS3 code?

Sneezewort Numbers

1
1
2
3
5
8?
13

pink

by Jessica Geist, Ellen Clarke

Fibo Results

> (fibo 2)
1
> (fibo 3)
2
> (fibo 4)
3
> (fibo 10)
55
> (fibo 60)
Still working…

At least we finished.

by Dmitriy Semenov and Sara Alspaugh

Tracing Fibo

> (require-library "trace.ss")
> (trace fibo)
(fibo)
> (fibo 3)
|(fibo 3)
| (fibo 2)
| 1
| ... Purple Arrow
by Rachel Lathbury and Andrea Yoon

To calculate (fibo 5) we calculated:
(fibo 4) 1 time
(fibo 3) 2 times
(fibo 2) 3 times
(fibo 1) 2 times
5 times total
= 8 calls to fibo = (fibo 6)

How many calls to calculate (fibo 60)?

Liberal Arts Trivia: History

• This 20th-century American inventor is credited with the phonograph, the carbon telephone transmitter, the practical electric light, and the phrase “Genius is one percent inspiration, ninety-nine percent perspiration.” He fought against Nikola Tesla’s alternating current in the so-called War of the Currents.
Liberal Arts Trivia: Film Studies

• In this Oscar-nominated 2006 film, David Bowie is almost torched by Thomas Edison’s goons but invents a teleportation machine for Wolverine so that he can defeat Batman in a magic trick competition because he thinks Batman killed his wife.

Liberal Arts Trivia: Physics

• Count Alessandro Antonio Anastasio Volta was a 19th-century Italian physicist. Volta studied what we now call capacitance, developing separate means to study both electrical potential $V$ and charge $Q$, and discovering that for a given object they are proportional. His experiments in “animal electricity”, in which two different metals were connected in series with frog’s legs, eventually led to his most famous discovery. What was it?

Fast-Fibo Results

> (fast-fibo 10)
55
> (time (fast-fibo 61))
cpu time: 0 real time: 0 gc time: 0
2504730781961

The original fibo would take at least 2.5 Trillion applications. A 2.5 GHz computer does 2.5 Billion simple operations per second, so 2.5 Trillion applications operations take ~1000 seconds. Each application of fibo involves hundreds of simple operations…

Beware the Bunnies!!

According to Bonacci’s model, after less than 10 years, rabbits would out-weigh the Earth!

Beware the Sneezewort!!

---

```scheme
;::: The Earth's mass is 6.0 x 10^24 kg
> (define mass-of-earth (* 6 (expt 10 24)))
::: A typical rabbit's mass is 2.5 kilograms
> (define mass-of-rabbit 2.5)
> (/ (* mass-of-rabbit (fast-fibo 60)) mass-of-earth) 6.450036483e-013
> (/ (* mass-of-rabbit (fast-fibo 120)) mass-of-earth) 2.2326496895795693

According to Bonacci’s model, after less than 10 years, rabbits would out-weigh the Earth!
```
Evaluation Cost

Actual running times vary according to:
- How fast a processor you have
- How much memory you have
- Where data is located in memory
- How hot it is
- What else is running
- etc...

Moore’s “Law” – computing power doubles every 18 months

Measuring Cost

• How does the cost scale with the **size of the input**?
• If the input size increases by one, how much longer will it take?
• If the input size **doubles**, how much longer will it take?

Cost of Fibonacci Procedures

\[
\phi = \frac{\sqrt{5} + 1}{2} = \text{"The Golden Ratio"} \approx 1.618033988749895...
\]

\[
\frac{\text{fast-fibo} (61)}{\text{fast-fibo} (60)} = 1.618033988749895
\]
More Golden Ratios

PS2 Question

```
(define (find-best-hand hands)
  (car (sort hands higher-hand?)))
```

Which is better and by how much?

Simple Sorting

- Can we use find-best to implement sort?
  - Yes!

- Use (find-best lst) to find the best
- Remove it from the list
  - Adding it to the answer
- Repeat until the list is empty

```
(define (sort lst cf)
  (if (null? lst) lst
      (let ((best (find-best lst cf)))
        (cons best (sort (delete lst best) cf)))))
```

Sorting Hands

```
(define (sort lst cf)
  (if (null? lst) lst
      (let ((best (find-best lst cf)))
        (cons best (sort (delete lst best) cf)))))
```

Sorting

```
(define (sort lst cf)
  (if (null? lst) lst
      (let ((best (find-best lst cf)))
        (cons best (sort (delete lst best) cf)))))
```

How much work is sort?
### Sorting Cost

- **What grows?**
  - \( n \) = the number of elements in lst
- **How much work are the pieces?**
  - find-best:
  - delete:

- **How many times does sort evaluate find-best and delete?** \( n \)
- **Total cost:** scales as \( n^2 \)

### Liberal Arts Trivia: Medicine

- Nicolae Paulescu was a 20th century physiologist and professor of medicine. He is considered the true discoverer of hormone that causes most of the body’s cells to take up glucose from the blood. His first experiments involved an aqueous pancreatic extract which, when injected into a diabetic dog, proved to have a normalizing effect on blood sugar levels. Name the hormone.
Liberal Arts Trivia: Sailing

- Name the collection of apparatus through which the force of the wind is transferred to the ship in order to propel it forward - this includes the masts, yardarms, sails, spars and cordage.

Timing Sort

> (time (sort < (revintsto 100)))
cpu time: 20 real time: 20 gc time: 0
> (time (sort < (revintsto 200)))
cpu time: 80 real time: 80 gc time: 0
> (time (sort < (revintsto 400)))
cpu time: 311 real time: 311 gc time: 0
> (time (sort < (revintsto 800)))
cpu time: 1362 real time: 1362 gc time: 0
> (time (sort < (revintsto 1600)))
cpu time: 6650 real time: 6650 gc time: 0

Timing Sort

\[
\text{Measured times } = \frac{n^2}{500}
\]

Growth Notations

- \( g \in O(f) \) ("Big-Oh")
  \( g \) grows no faster than \( f \) (\( f \) is upper bound)
- \( g \in \Theta(f) \) ("Theta")
  \( g \) grows as fast as \( f \) (\( f \) is tight bound)
- \( g \in \Omega(f) \) ("Omega")
  \( g \) grows no slower than \( f \) (\( f \) is lower bound)

Which one would we most like to know?

Meaning of \( O \) ("big Oh")

\[ g \text{ is in } O(f) \text{ iff:} \]
There are positive constants \( c \) and \( n_0 \) such that
\[ g(n) \leq cf(n) \]
for all \( n \geq n_0 \).

\[ g \text{ is in } O(f) \text{ iff there are positive constants } c \text{ and } n_0 \text{ such that } g(n) \leq cf(n) \text{ for all } n \geq n_0. \]

\[ g \text{ is in } O(n^2) \text{?} \]
\[ g \text{ is } 10n \text{ in } O(n) \text{?} \]
\[ g \text{ is } n^2 \text{ in } O(n) \text{?} \]
### O Examples

$g$ is in $O(f)$ iff there are positive constants $c$ and $n_0$ such that $g(n) \leq cf(n)$ for all $n \geq n_0$.

- **Is $n$ in $O(n^2)$?**
  - Yes, $c = 1$ and $n_0 = 1$ works.

- **Is $10n$ in $O(n)$?**
  - Yes, $c = 1/10$ and $n_0 = 1$ works.

- **Is $n^2$ in $O(n)$?**
  - No, no matter what $c$ we pick, $cn^2 > n$ for big enough $n$ ($n > c$).

### Revenge of $O$ Examples

$g$ is in $O(f)$ iff there are positive constants $c$ and $n_0$ such that $g(n) \leq cf(n)$ for all $n \geq n_0$.

- **Is $n+5$ in $O(n^2)$?**
  - Yes, $c = 1$ and $n_0 = 3$ works.

- **Is $n^2 - 100$ in $O(n)$?**
  - Yes, $c = 2$ and $n_0 = 77$ works.

- **Is $n^2$ in $O(n^3)$?**
  - Yes, $c = 55$ and $n_0 = 102$ works.

### Homework

- Read Course Book Chapter 7 before Wednesday
  - Has a formal notation for this kind of analysis!
- Problem Set 3 due!