Costs and Sneezewort and Growth

532. Achillea Ptarmica L.

Sneezewort

Breaking News

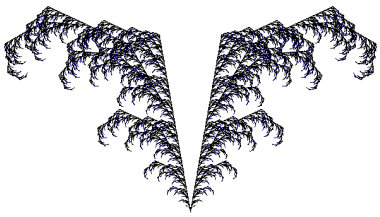
- "Follow-up discussion on the CS education email list has focused on the clothes, the accessories, and whether members of the list or their students might have stuffed the ballot box electronically. (Might have happened!) One person did ask a good question: he asked that anyone with young kids who like Barbie to report back on how they react to CpE Barbie.
  - Tom Horton

One-Slide Summary

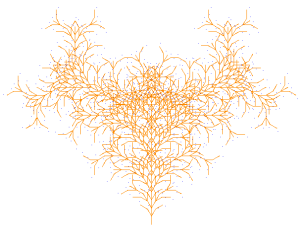
- The basic recursive computation of Fibonacci can take quite a while. **There are faster ways**.
- We can formally measure and evaluate the cost of a computer program. We abstract away details such as processor speed and instead measure how **the solving time increases as the input increases**.
- **g is in O(f)** iff there exist positive constants c and n_0 such that $g(n) \leq cf(n)$ for all $n \geq n_0$.
- If g is in O(f) we say that f is an **upper bound** for g.

Outline

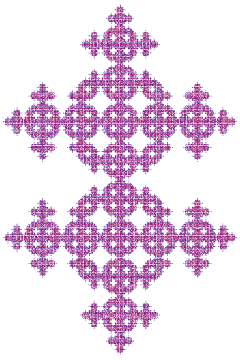
- Sneezewort and Fibonacci
- **Cost** of computing Fibonacci
- **Cost** of sorting
- Intro to Big-Oh Notation



"V" shrubbery
by Andrew Jesien, Becky Elstad



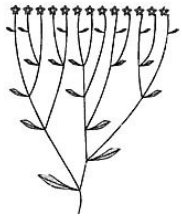
Robot Cav Man
by Jamie Jeon & Walter Borges



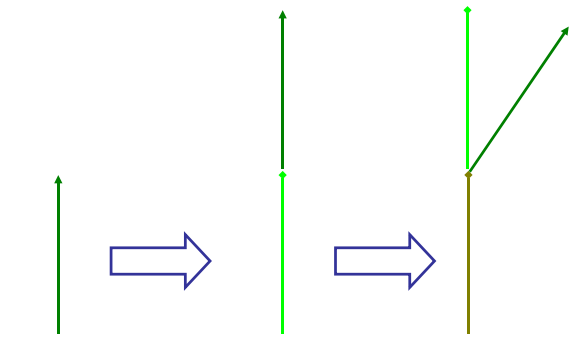
After the Incident
by Ben Morrison and Liz Peterson

Sneezewort

- Achillea ptarmica is real.
- It is "moste efficacious in the inflaming of the braine, and [is] therefore much used in Confusing and Befuddlement Draughts, where the wizard is desirous of producing hot-headedness and recklessness."
  - Order of the Phoenix, p.18
- Sneezewort's pattern of development displays the Fibonacci sequence.



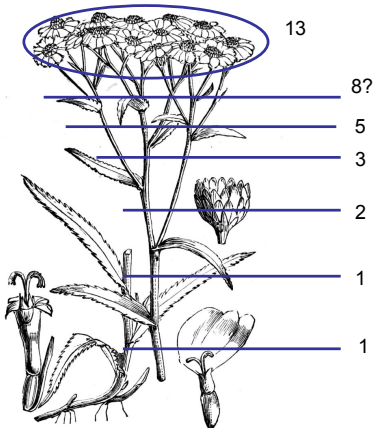
Sneezewort Growth

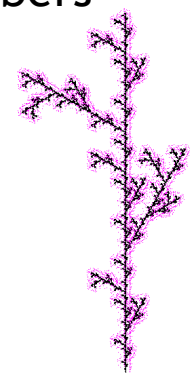


First Time Unit Second Time Unit Offshoot

Could we model Sneezewort with PS3 code?

Sneezewort Numbers

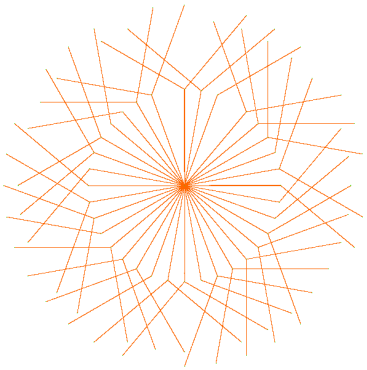




pink
by Jessica Geist, Ellen Clarke

Fibo Results

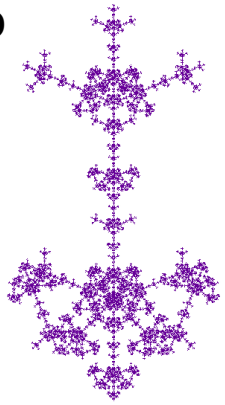
```
> (fibo 2)
1
> (fibo 3)
2
> (fibo 4)
3
> (fibo 10)
55
> (fibo 60)
Still working…
```



At least we finished.
by Dmitriy Semenov and Sara Alspaugh

Tracing Fibo

```
> (require-library "trace.ss")
> (trace fibo)
(fibo)
> (fibo 3)
|(fibo 3)
|       (fibo 2)
|       1
|       (fibo 1)
|       1
|2
2
```

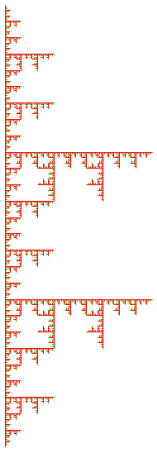


Purple Arrow
by Rachel Lathbury and Andrea Yoon

See end of Class 5 lecture notes for information on "trace.ss"!


```
> (fibo 5)
|(fibo 5)
| (fibo 4)
| |(fibo 3)
| | (fibo 2)
| | 1
| | (fibo 1)
| | 1
| |2
| |(fibo 2)
| |1
| 3
| (fibo 3)
| |(fibo 2)
| |1
| |(fibo 1)
| |1
| 2
|5
5
```

A right-wing Christmas - awwwwww.....
by Andrew Baker & Emily Lam



To calculate (fibo 5) we calculated:

(fibo 4)	1 time	
(fibo 3)	2 times	
(fibo 2)	3 times	5 times total
(fibo 1)	2 times	

= 8 calls to fibo = (fibo 6)

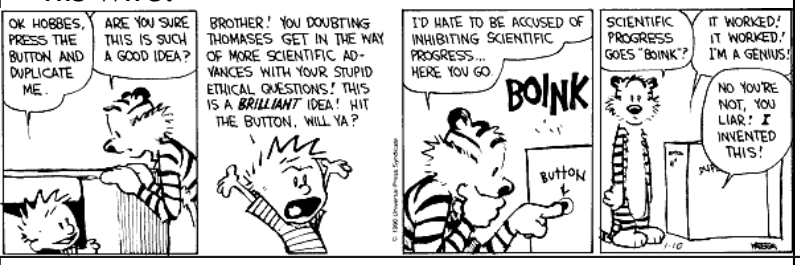
How many calls to calculate (fibo 60)?

Liberal Arts Trivia: History

- This 20th-century American inventor is credited with the phonograph, the carbon telephone transmitter, the practical electric light, and the phrase "Genius is one percent inspiration, ninety-nine percent perspiration." He fought against Nikola Tesla's alternating current in the so-called War of the Currents.

Liberal Arts Trivia: Film Studies

- In this Oscar-nominated 2006 film, David Bowie is almost torched by Thomas Edison's goons but invents a teleportation machine for Wolverine so that he can defeat Batman in a magic trick competition because he thinks Batman killed his wife.



Liberal Arts Trivia: Physics

- Count Alessandro Antonio Anastasio Volta was a 19th-century Italian physicist. Volta studied what we now call capacitance, developing separate means to study both electrical potential *V* and charge *Q*, and discovering that for a given object they are proportional. His experiments in "animal electricity", in which two different metals were connected in series with frog's legs, eventually led to his most famous discovery. What was it?

fast-fibo

```
(define (fast-fibo n)
  (define (fib-helper a b left)
    (if (<= left 0)
        b
        (fib-helper b (+ a b) (- left 1))))
  (fib-helper 1 1 (- n 2)))
```


Fast-Fibo Results

```
> (fast-fibo 10)
55
> (time (fast-fibo 61))
cpu time: 0 real time: 0 gc time: 0
2504730781961
```

The original fibo would take at least 2.5 Trillion applications.
A 2.5 GHz computer does 2.5 *Billion* simple operations per second, so 2.5 Trillion applications operations take ~1000 seconds.
Each application of fibo involves hundreds of simple operations…


```
;;; The Earth's mass is 6.0 x 10^24 kg
> (define mass-of-earth (* 6 (expt 10 24)))
;;; A typical rabbit's mass is 2.5 kilograms
> (define mass-of-rabbit 2.5)
> (/ (* mass-of-rabbit (fast-fibo 60)) mass-of-earth)
6.450036483e-013
> (/ (* mass-of-rabbit (fast-fibo 120)) mass-of-earth)
2.2326496895795693
```

According to Bonacci's model, after less than 10 years, rabbits would out-weigh the Earth!


```
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```
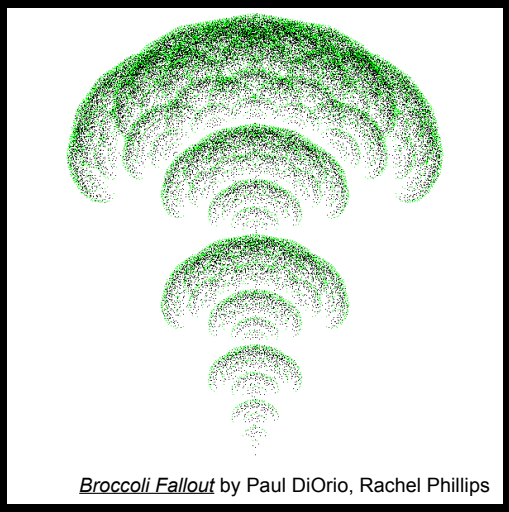

According to Bonacci's model, after less than 10 years, rabbits would out-weigh the Earth!

Beware the Bunnies!! Beware the Sneezewort!!

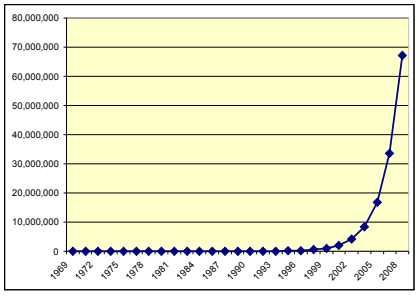


Broccoli Fallout by Paul DiOrio, Rachel Phillips

Evaluation Cost

Actual running times vary according to:

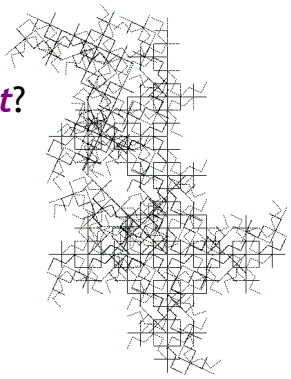
- How fast a processor you have
- How much memory you have
- Where data is located in memory
- How hot it is
- What else is running
- etc...



Moore's "Law" – computing power doubles every 18 months

Measuring Cost

- How does the cost scale with the ***size of the input***?
- If the input size increases by one, how much longer will it take?
- If the input size *doubles*, how much longer will it take?



Untitled
Nokomis McCaskill
Chris Hooe

Cost of Fibonacci Procedures

```
(define (fibo n)
        (if (or (= n 1) (= n 2))
    1 ;;; base case
    (+ (fibo (- n 1))
       (fibo (- n 2)))))
```

```
(define (fast-fibo n)
  (define (fib-helper a b left)
    (if (= left 0)
        b
        (fib-helper b (+ a b) (- left 1))))
  (fib-helper 1 1 (- n 2)))
```

Input	fibo	fast-fibo
m	q	mk
$m+1$		$(m+1)k$
$m+2$	at least q^2	$(m+2)k$

Cost of Fibonacci Procedures

```
(define (fibo n)
        (if (or (= n 1) (= n 2))
    1 ;;; base case
    (+ (fibo (- n 1))
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```

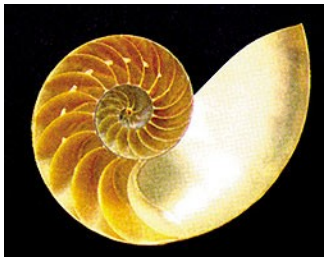
```
(define (fast-fibo n)
  (define (fib-helper a b left)
    (if (= left 0)
        b
        (fib-helper b (+ a b) (- left 1))))
  (fib-helper 1 1 (- n 2)))
```

Input	fibo	fast-fibo
m	q	mk
$m+1$	$q*\Phi$	$(m+1)k$
$m+2$	at least q^2	$(m+2)k$

Φ = (/ (+ 1 (sqrt 5)) 2) = "The Golden Ratio" ~ 1.618033988749895...
~ (/ (fast-fibo 61) (fast-fibo 60)) = 1.618033988749895

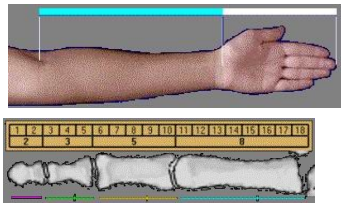
The Golden Ratio

Parthenon



Nautilus Shell

More Golden Ratios



http://www.fenkefeng.org/essaysm18004.html
by Oleksiy Stakhov

PS2 Question

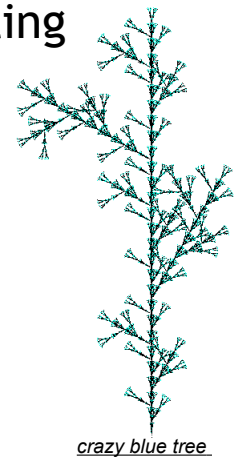
```
(define (find-best-hand hands)
  (car (sort hands higher-hand?)))
```

```
(define (find-best lst cf)
  (if (= 1 (length lst)) (car lst)
      (pick-better cf (car lst) (find-best (cdr lst) cf))))
(define (pick-better cf num1 num2)
  (if (cf num1 num2) num1 num2))
(define (find-best-hand hands)
   (find-best hands higher-hand?))
```

Which is better and by how much?

Simple Sorting

- Can we use find-best to implement sort?
  - Yes!
- Use (find-best lst) to find the best
- Remove it from the list
  - Adding it to the answer
- Repeat until the list is empty



crazy blue tree
by Victor Malaret, Folami Williams

Simple Sort

```
;; cf = comparison function
(define (sort lst cf)   ;; simple sort
  (if (null? lst) lst
      (let ((best (find-best lst cf)))
        (cons
           best
           (sort (delete lst best) cf)))))
;; delete lst x = (filter ... (not (eq? x ...
```


Sorting Hands

```
(define (sort lst cf)
  (if (null? lst) lst
      (let ((best (find-best lst cf)))
        (cons
           best
           (sort (delete lst best) cf)))))

(define (sort-hands lst)
  (sort lst higher-hand?))
```


Sorting

```
(define (sort lst cf)
  (if (null? lst) lst
      (let ((best (find-best lst cf)))
        (cons best (sort (delete lst best) cf)))))

(define (find-best lst cf)
   (if (= 1 (length lst)) (car lst)
       (pick-better cf (car lst) (find-best (cdr lst) cf))))
(define (pick-better cf num1 num2)
  (if (cf num1 num2) num1 num2))
```

How much work is sort?

Sorting Cost

- What grows?
  - n = the number of elements in lst
- How much work are the pieces?
  - find-best:
  - delete:

Sorting Cost

- What grows?
  - n = the number of elements in lst
- How much work are the pieces?
  - find-best: work scales as n (increases by one)
  - delete: work scales as n (increases by one)
- How many times does sort evaluate find-best and delete?

Sorting Cost

- What grows?
  - n = the number of elements in lst
- How much work are the pieces?
  - find-best: work scales as n (increases by one)
  - delete: work scales as n (increases by one)
- How many times does sort evaluate find-best and delete? n
- Total cost: scales as

Sorting Cost

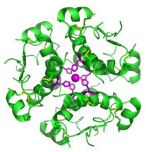
- What grows?
  - n = the number of elements in lst
- How much work are the pieces?
  - find-best: work scales as n (increases by one)
  - delete: work scales as n (increases by one)
- How many times does sort evaluate find-best and delete? n
- Total cost: scales as n^2

Sorting Cost

```
(define (sort lst cf)
  (if (null? lst) lst
      (let ((best (find-best lst cf)))
        (cons best (sort (delete lst best) cf)))))
(define (find-best lst cf)
  (if (= 1 (length lst)) (car lst)
      (pick-better cf (car lst) (find-best (cdr lst) cf))))
```

If we double the length of the list, the amount of work *approximately* quadruples: there are twice as many applications of find-best, and each one takes twice as long

Liberal Arts Trivia: Medicine



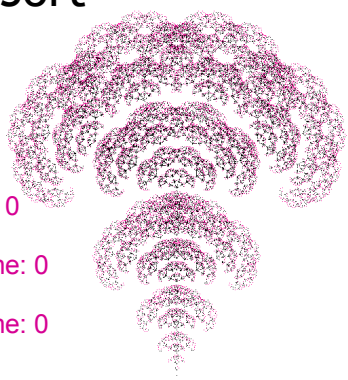
- Nicolae Paulescu was a 20th century physiologist and professor of medicine. He is considered the true discoverer of hormone that causes most of the body's cells to take up glucose from the blood. His first experiments involved an aqueous pancreatic extract which, when injected into a diabetic dog, proved to have a normalizing effect on blood sugar levels. Name the hormone.

Liberal Arts Trivia: Sailing

- Name the collection of apparatus through which the force of the wind is transferred to the ship in order to propel it forward – this includes the masts, yardarms, sails, spars and cordage.

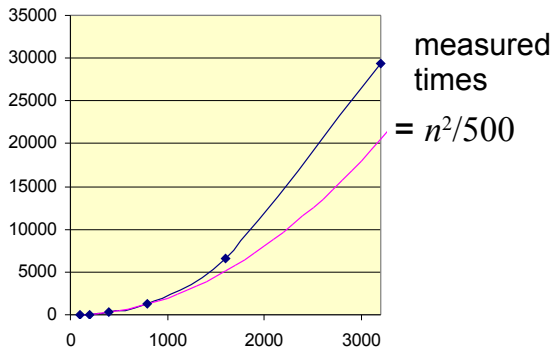
Timing Sort

```
> (time (sort < (revintsto 100)))
cpu time: 20 real time: 20 gc time: 0
> (time (sort < (revintsto 200)))
cpu time: 80 real time: 80 gc time: 0
> (time (sort < (revintsto 400)))
cpu time: 311 real time: 311 gc time: 0
> (time (sort < (revintsto 800)))
cpu time: 1362 real time: 1362 gc time: 0
> (time (sort < (revintsto 1600)))
cpu time: 6650 real time: 6650 gc time: 0
```



Cherry Blossom
by Ji Hyun Lee, Wei Wang

Timing Sort



measured times

= $n^2/500$

Growth Notations

- $g \in O(f)$ ("Big-Oh")
  g grows **no faster than** f (f is upper bound)
- $g \in \Theta(f)$ ("Theta")
  g grows **as fast as** f (f is tight bound)
- $g \in \Omega(f)$ ("Omega")
  g grows **no slower than** f (f is lower bound)

Which one would we most like to know?

Meaning of O ("big Oh")

g is in $O(f)$ iff:

There are positive constants c and n_0 such that

$$g(n) \leq cf(n)$$

for all $n \geq n_0$.

O Examples

g is in $O(f)$ iff there are positive constants c and n_0 such that $g(n) \leq cf(n)$ for all $n \geq n_0$.

Is n in $O(n^2)$?

Is $10n$ in $O(n)$?

Is n^2 in $O(n)$?

O Examples

g is in $O(f)$ iff there are positive constants c and n_0 such that $g(n) \leq cf(n)$ for all $n \geq n_0$.

Is n in $O(n^2)$? Yes, $c = 1$ and n_0=1 works.

Is $10n$ in $O(n)$? Yes, $c = 1/10$ and n_0=1 works.

Is n^2 in $O(n)$? No, no matter what c we pick, $cn^2 > n$ for big enough n ($n > c$)

Revenge of O Examples

g is in $O(f)$ iff there are positive constants c and n_0 such that $g(n) \leq cf(n)$ for all $n \geq n_0$.

Is $n+5$ in $O(n^2)$?

Is n^2-100 in $O(n)$?

Is n^2 in $O(n^3)$?

Revenge of O Examples

g is in $O(f)$ iff there are positive constants c and n_0 such that $g(n) \leq cf(n)$ for all $n \geq n_0$.

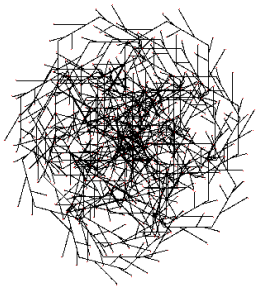
Is $n+5$ in $O(n^2)$? Yes, $c = 1$ and n_0=3 works.

Is n^2-100 in $O(n)$? No, no matter what c we pick, cn^2-100 > n for big enough n.

Is n^2 in $O(n^3)$? Yes, $c = 1$ and n_0=1 works.
Yes, $c = 2$ and n_0=77 works.
Yes, $c = 55$ and n_0=102 works.

Homework

- Read Course Book Chapter 7 before Wednesday
  - Has a formal notation for this kind of analysis!
- Problem Set 3 due!



The Mask
by Zachary Pruckowski, Kristen Henderson