Type Checking

One-Slide Summary

- A **type environment** gives types for free variables. You typecheck a let-body with an environment that has been **updated** to contain the new let-variable.
- If an object of type X could be used when one of type Y is acceptable then we say X is a **subtype** of Y, also written $X \leq Y$.
- A type system is **sound** if $\forall$ E. dynamic_type(E) $\leq$ static_type(E)

Lecture Outline

- Typing Rules
- Typing Environments
- "Let" Rules
- Subtyping
- Wrong Rules

Example: 1 + 2

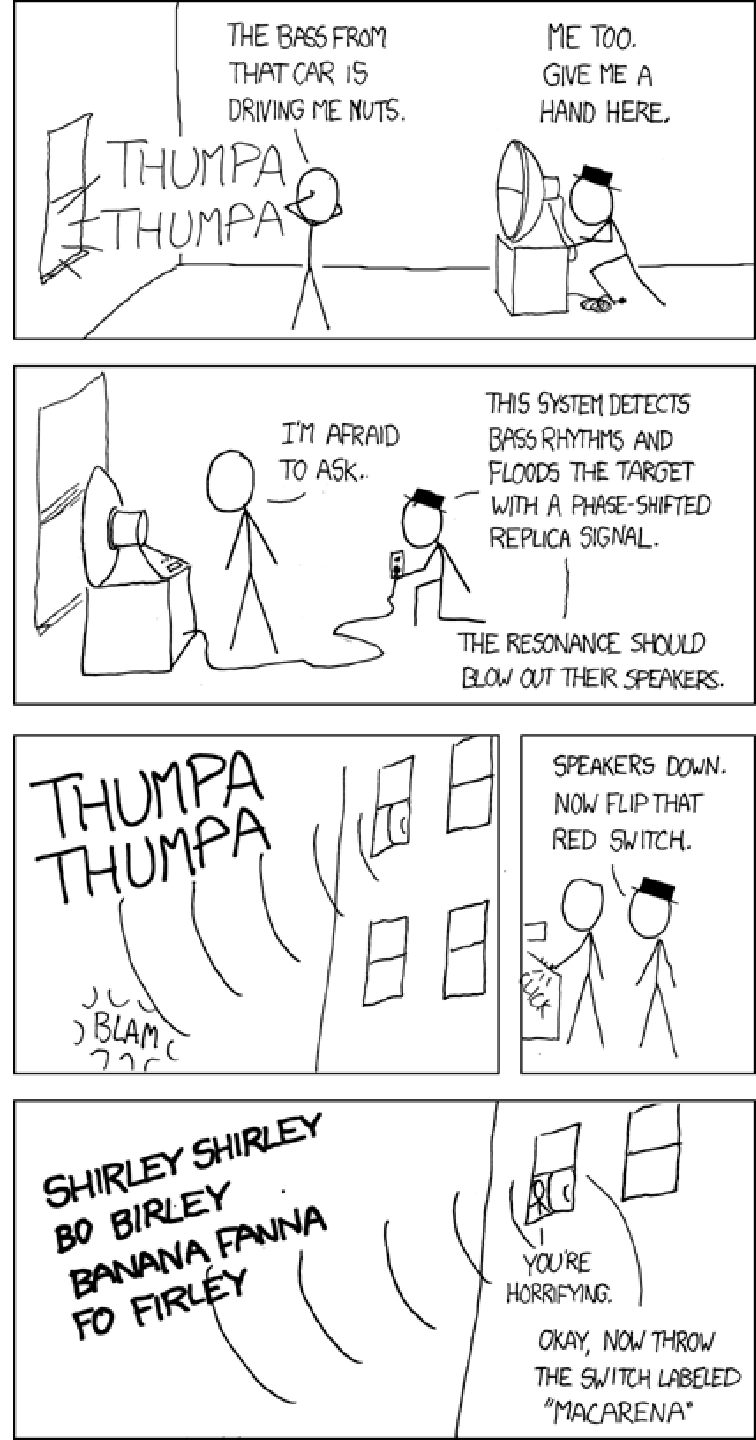
$$\dfrac{\dfrac{}{\vdash 1 : \text{Int}} \qquad \dfrac{}{\vdash 2 : \text{Int}}}{\vdash 1 + 2 : \text{Int}}$$

If we can prove it, then it's true!

Soundness

- A type system is **sound** if
  - Whenever $\vdash e : T$
  - Then e evaluates to a value of type T
- We only want sound rules
  - But some sound rules are better than others:

$$\frac{(i \text{ is an integer})}{\vdash i : \text{Object}}$$



Type Checking Proofs

- Type checking proves facts e : T
  - One type rule is used for each kind of expression

- In the type rule used for a node e
  - The **hypotheses** are the proofs of types of e's subexpressions
  - The **conclusion** is the proof of type of e itself

Rules for Constants

$$\frac{}{\vdash \text{false} : \text{Bool}} \text{[Bool]}$$

$$\frac{}{\vdash s : \text{String}} \text{[String]}$$

(s is a string constant)

Rule for New

new T produces an object of type T

- Ignore SELF_TYPE for now . . .

$$\frac{}{\vdash \text{new } T : T} \quad \text{[New]}$$

Two More Rules

$$\frac{\vdash e : \text{Bool}}{\vdash \text{not } e : \text{Bool}} \quad [\text{Not}]$$

$$\frac{\vdash e_1 : \text{Bool} \qquad \vdash e_2 : T}{\vdash \text{while } e_1 \text{ loop } e_2 \text{ pool} : \text{Object}} \quad [\text{Loop}]$$

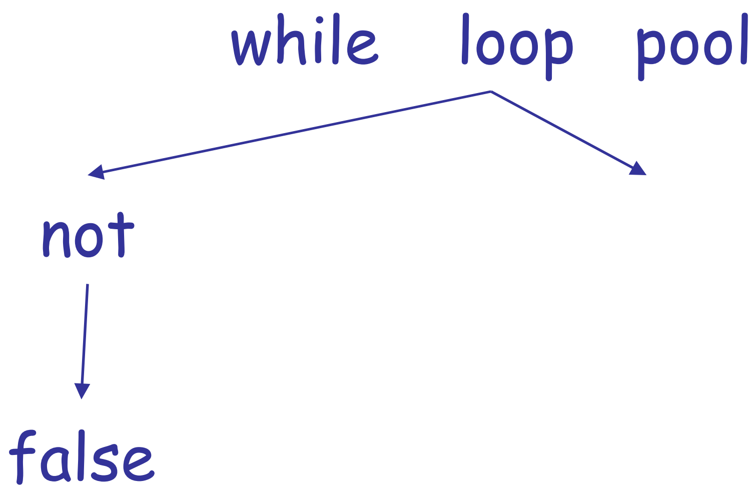
Typing: Example

- Typing for while not false loop 1 + 2 * 3 pool

while loop pool

Typing: Example

- Typing for while not false loop 1 + 2 * 3 pool



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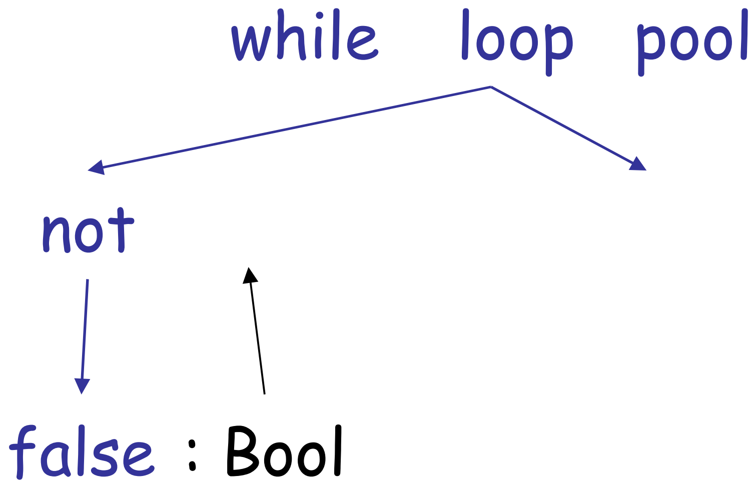

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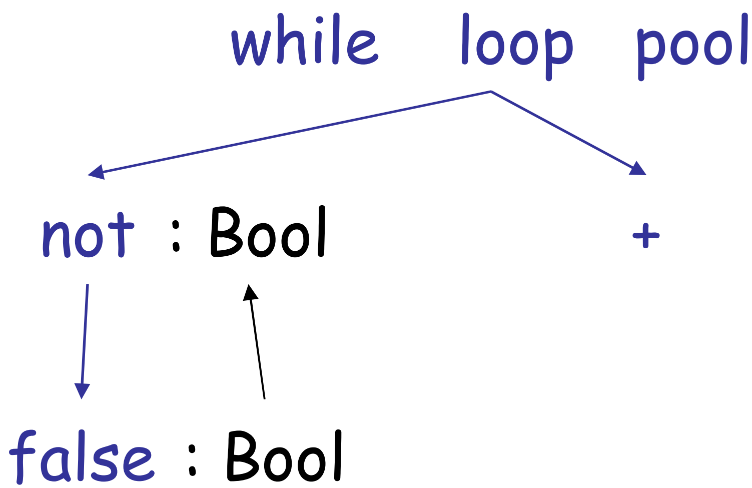

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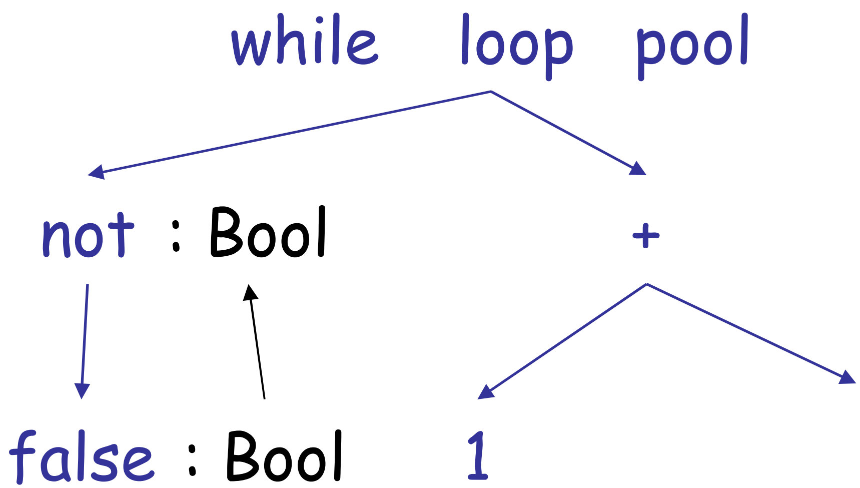

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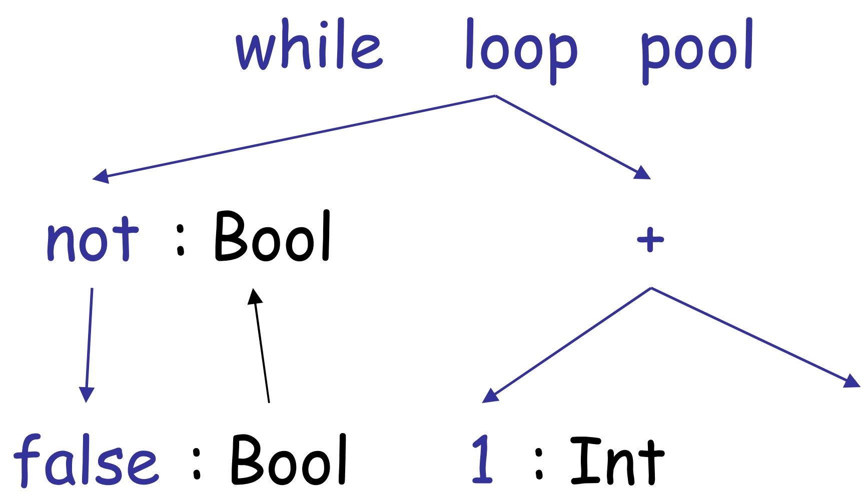

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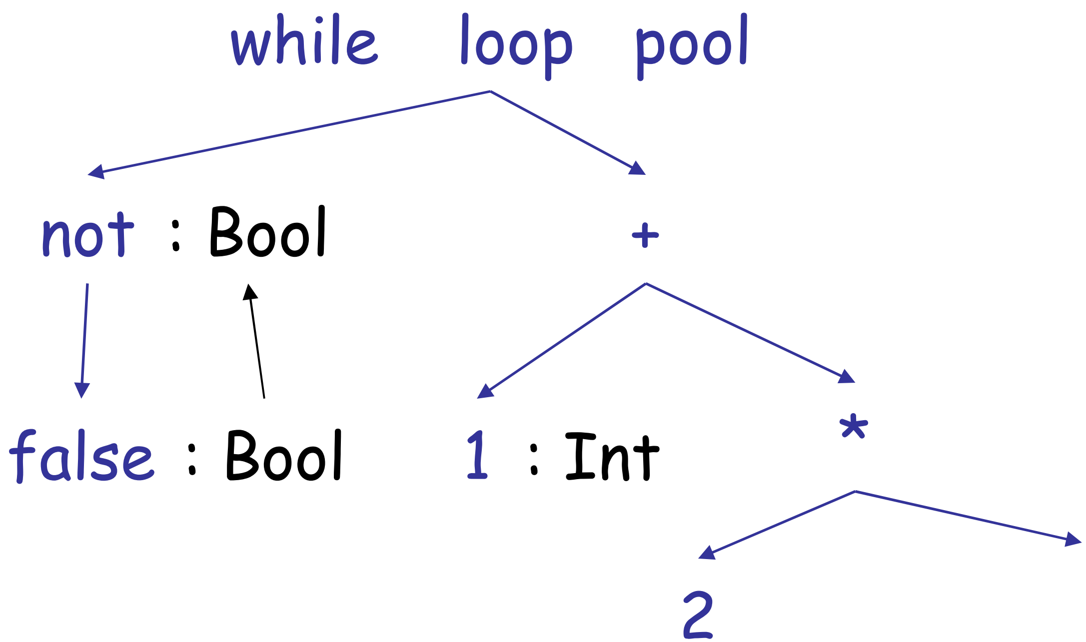

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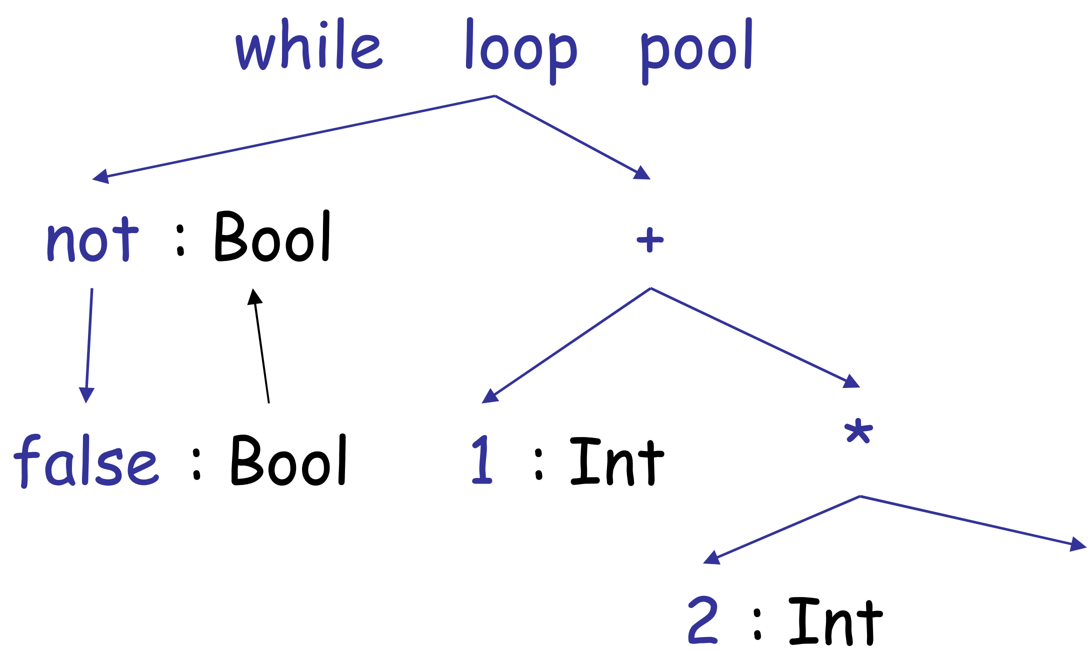

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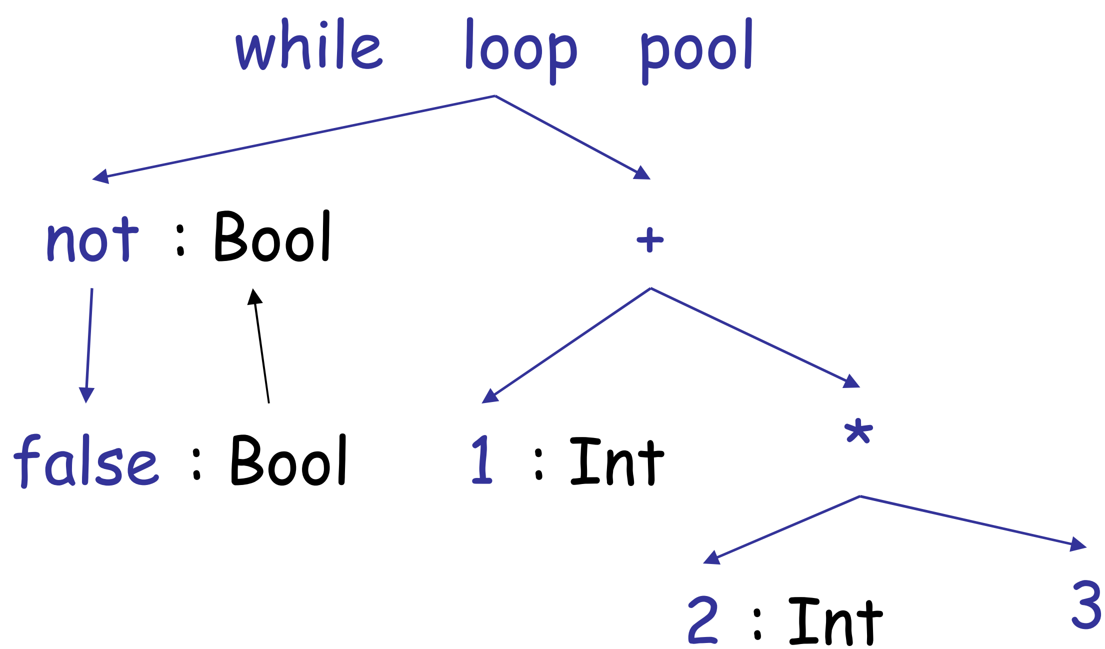

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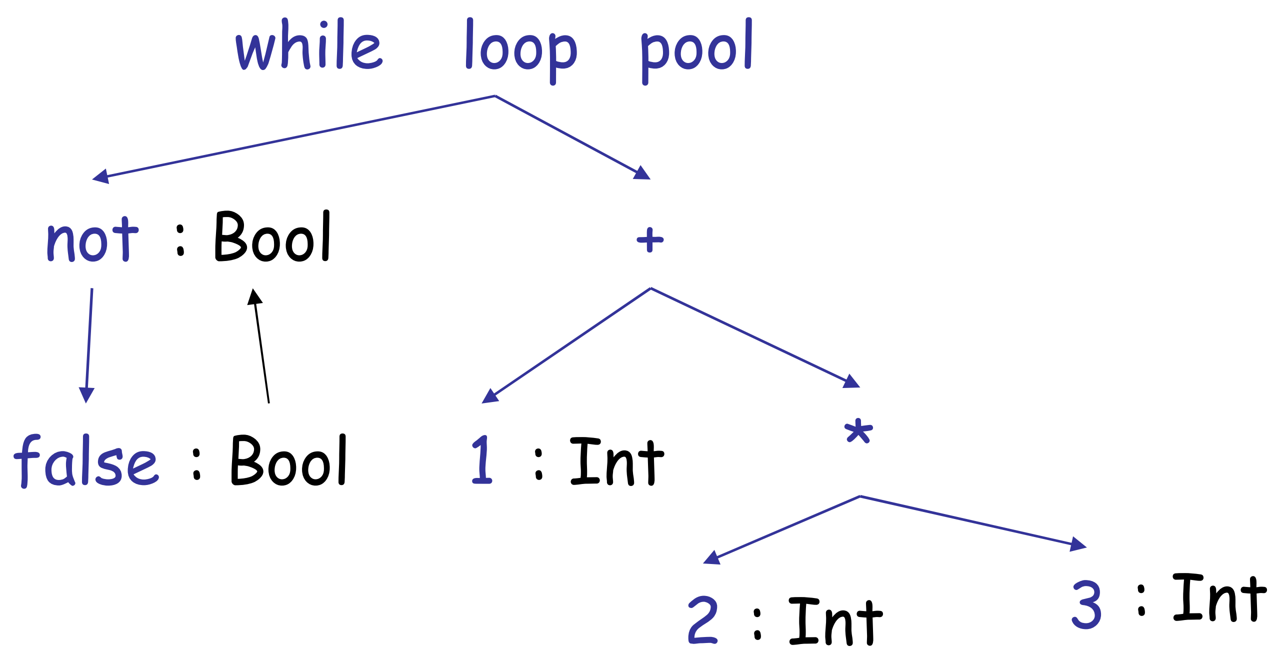

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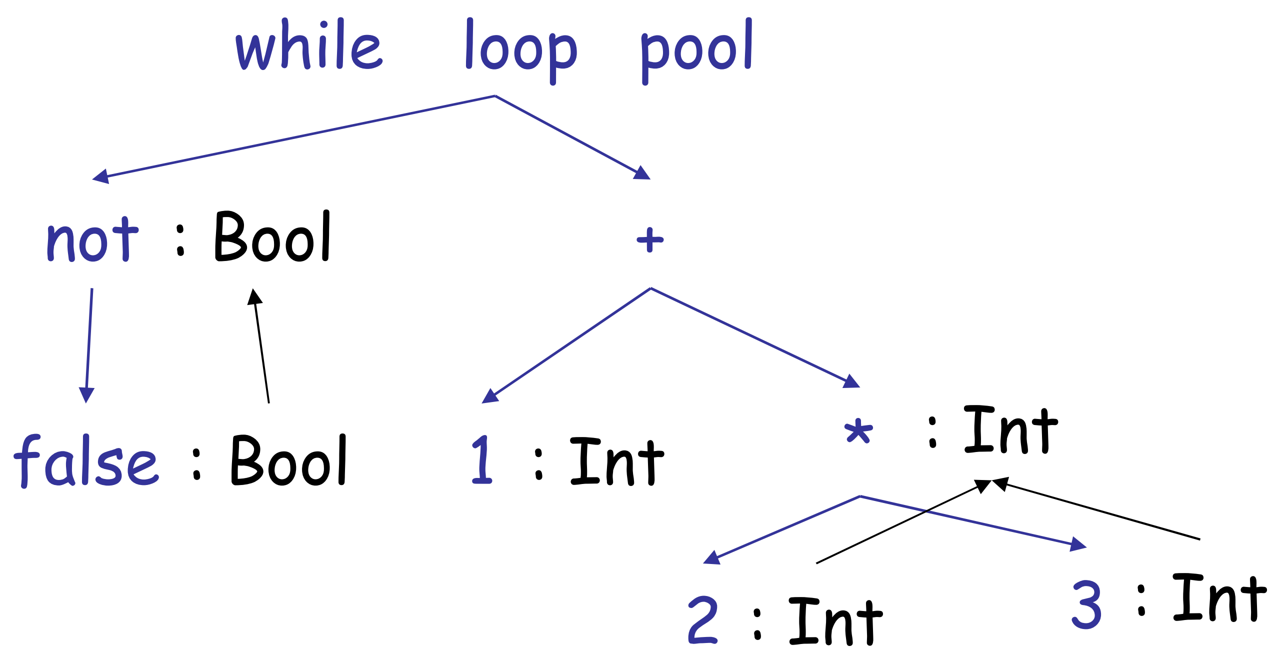

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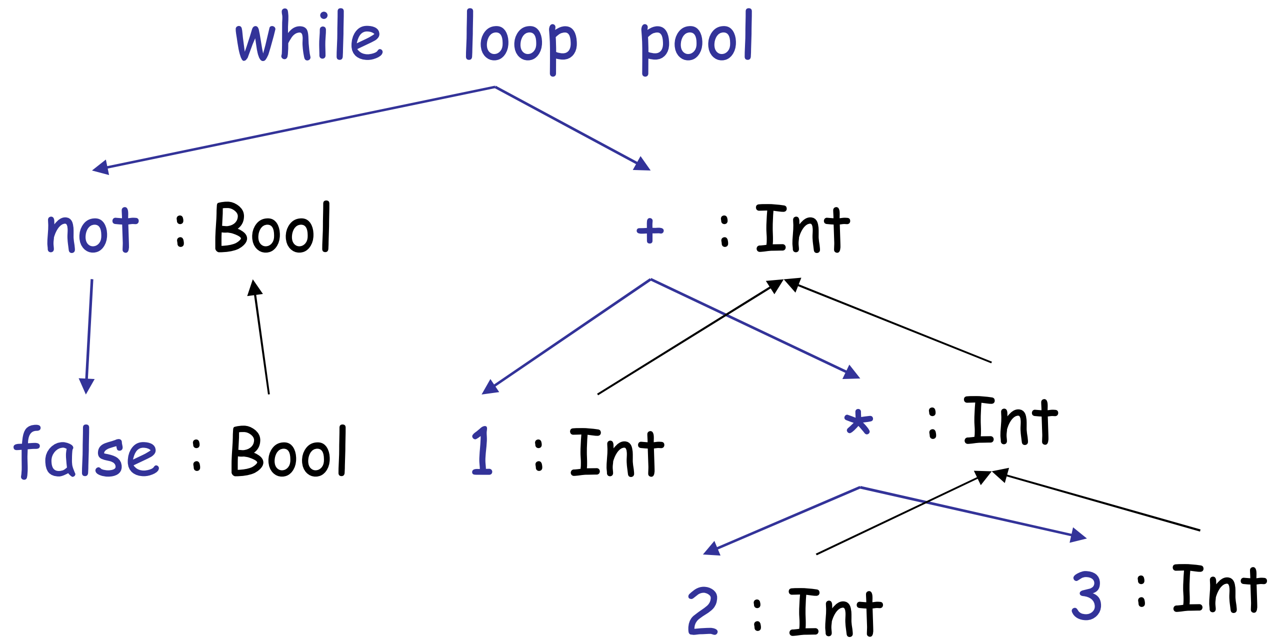

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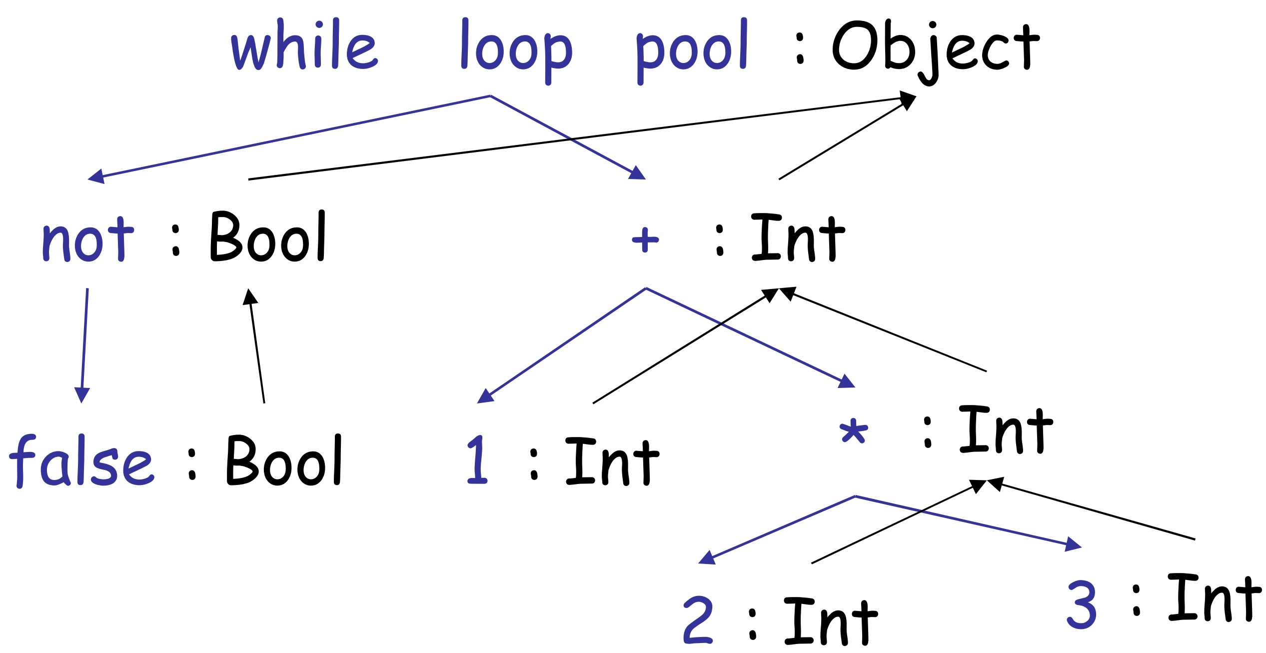


Typing Derivations

- The typing reasoning can be expressed as a tree:

$$\dfrac{\dfrac{\vdash \textbf{false} : \textbf{Bool}}{\vdash \textbf{not false} : \textbf{Bool}} \qquad \dfrac{\vdash \textbf{1} : \textbf{Int} \qquad \dfrac{\vdash \textbf{2} : \textbf{Int} \qquad \vdash \textbf{3} : \textbf{Int}}{\vdash \textbf{2 * 3} : \textbf{Int}}}{\vdash \textbf{1 + 2 * 3} : \textbf{Int}}}{\vdash \textbf{while not false loop 1 + 2 * 3} : \textbf{Object}}$$

- The **root** of the tree is the whole expression
- Each node is an **instance** of a typing rule
- **Leaves** are the rules with no hypotheses

A Problem

- What is the type of a variable reference?

$$\frac{}{\vdash x : ?}\,[\text{Var}]$$

(x is an identifier)

- The local structural rule does *not* carry enough information to give x a type. Fail.

A Solution: Put more information in the rules!

- A **type environment** gives types for **free** variables
  - A **type environment** is a mapping from Object_Identifiers to Types
  - A variable is **free** in an expression if:
    - The expression contains an occurrence of the variable that refers to a declaration *outside* the expression
  - in the expression "x", the variable "x" is free
  - in "let x : Int in x + y" only "y" is free
  - in "x + let x : Int in x + y" both "x", "y" are free

Type Environments

Let O be a function from Object_Identifiers to Types

The sentence $O \vdash e : T$

is read: Under the assumption that variables have the types given by O, it is provable that the expression e has the type T

Modified Rules

The type environment is added to the earlier rules:

$$\frac{}{O \vdash i : \text{Int}} \; [\text{Int}]$$

(i is an integer)

$$\frac{O \vdash e_1 : \text{Int} \qquad O \vdash e_2 : \text{Int}}{O \vdash e_1 + e_2 : \text{Int}} \; [\text{Add}]$$

New Rules

And we can write new rules:

$$\frac{}{O \vdash x : T} \quad [\text{Var}] \ (O(x) = T)$$

Equivalently:

$$\frac{O(x) = T}{O \vdash x : T} \quad [\text{Var}]$$

Let

$$\frac{O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \text{ in } e_1 : T_1} \quad \text{[Let-No-Init]}$$

$O[T_0/x]$ means "O modified to map x to T_0 and behaving as O on all other arguments":

$$O[T_0/x]\,(x) = T_0$$

$$O[T_0/x]\,(y) = O(y)$$

(You can write $O[x/T_0]$ on tests/assignments.)

Let Example

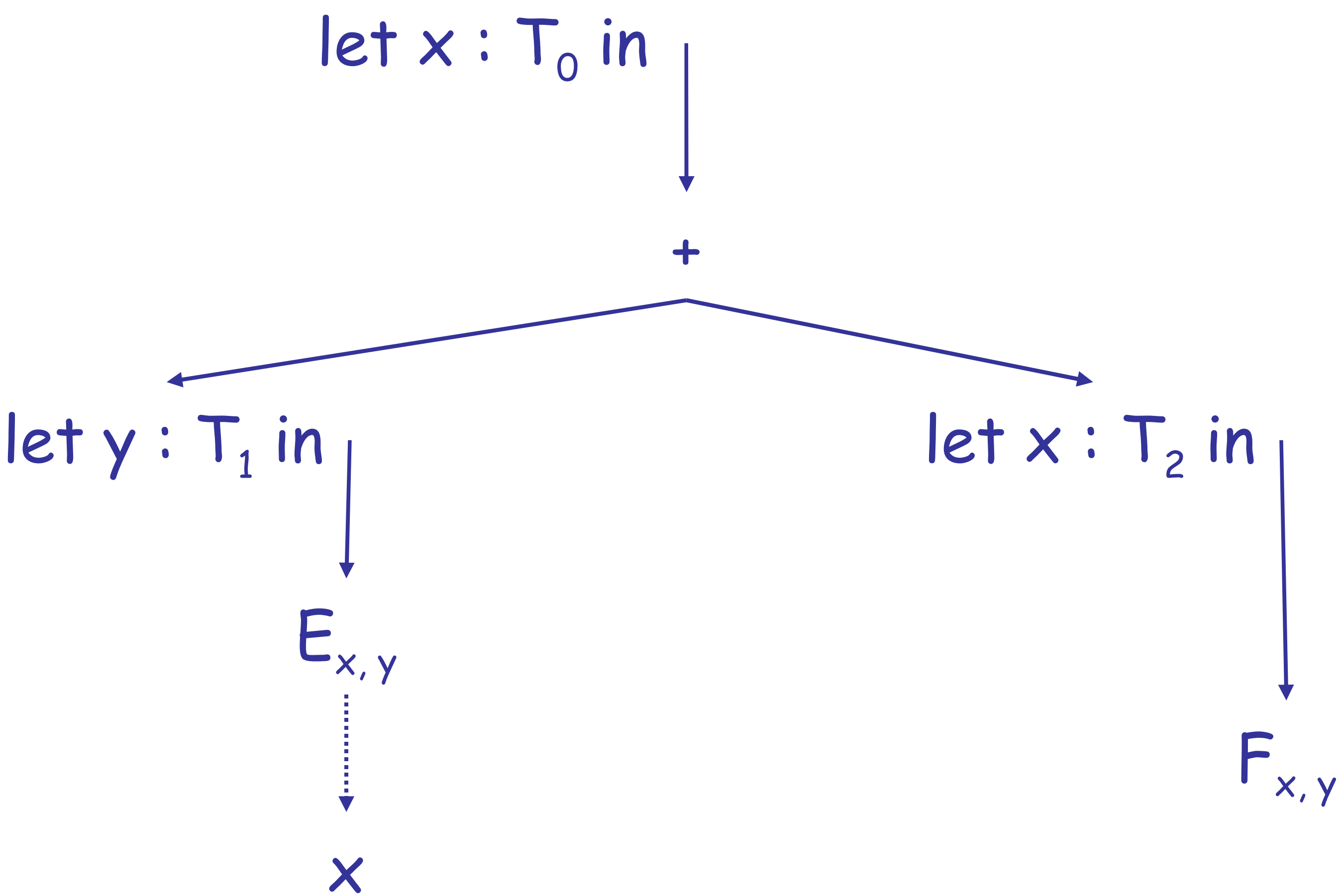
- Consider the Cool expression

  let x : T_0 in (let y : T_1 in $E_{x, y}$) + (let x : T_2 in $F_{x, y}$)

  (where $E_{x, y}$ and $F_{x, y}$ are some Cool expression that contain occurrences of "x" and "y")
- Scope
  - of "y" is $E_{x, y}$
  - of outer "x" is $E_{x, y}$
  - of inner "x" is $F_{x, y}$
- This is captured precisely in the typing rule.

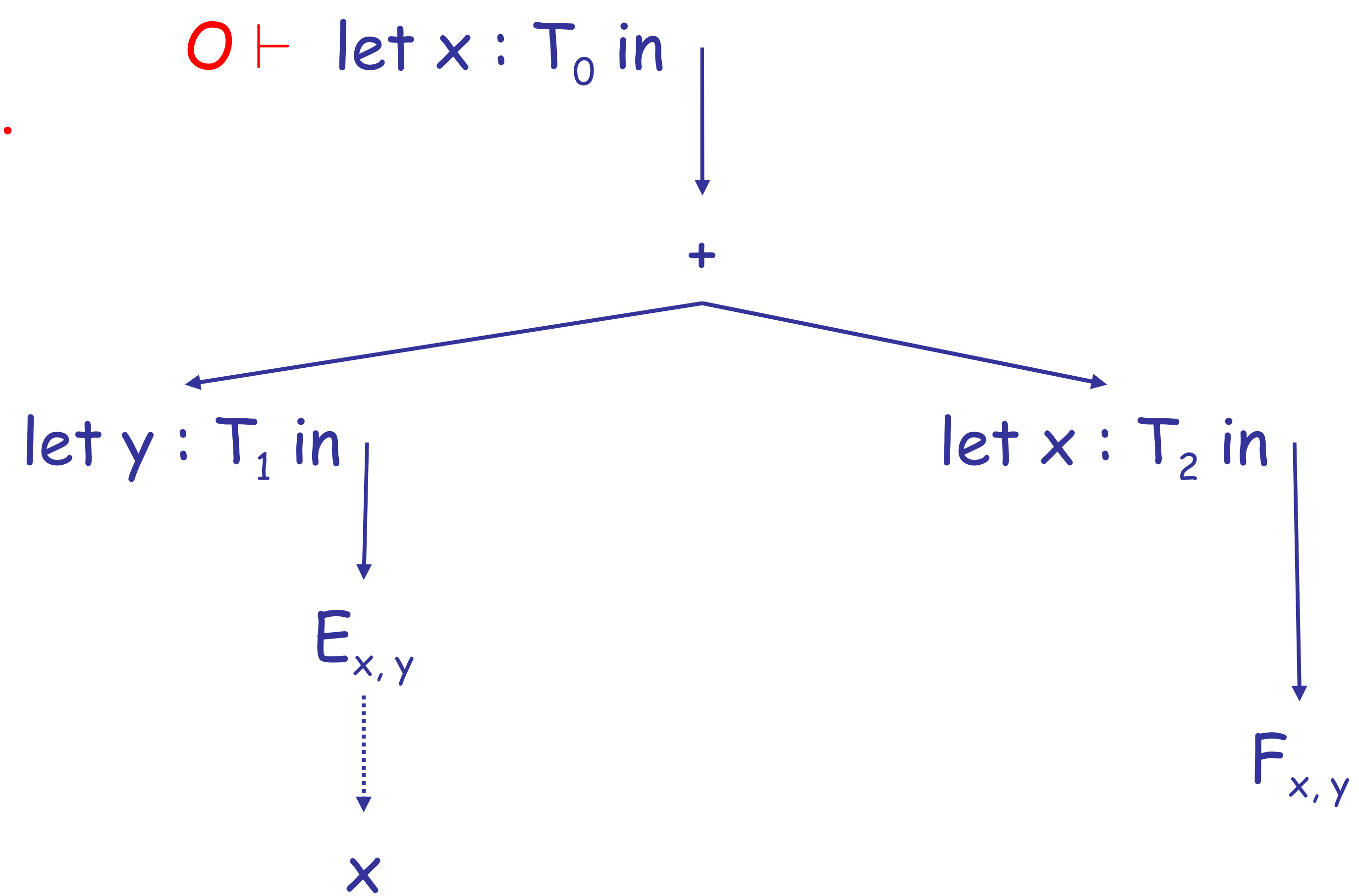
Example of Typing “let”

AST



Example of Typing "let"

AST
Type env.



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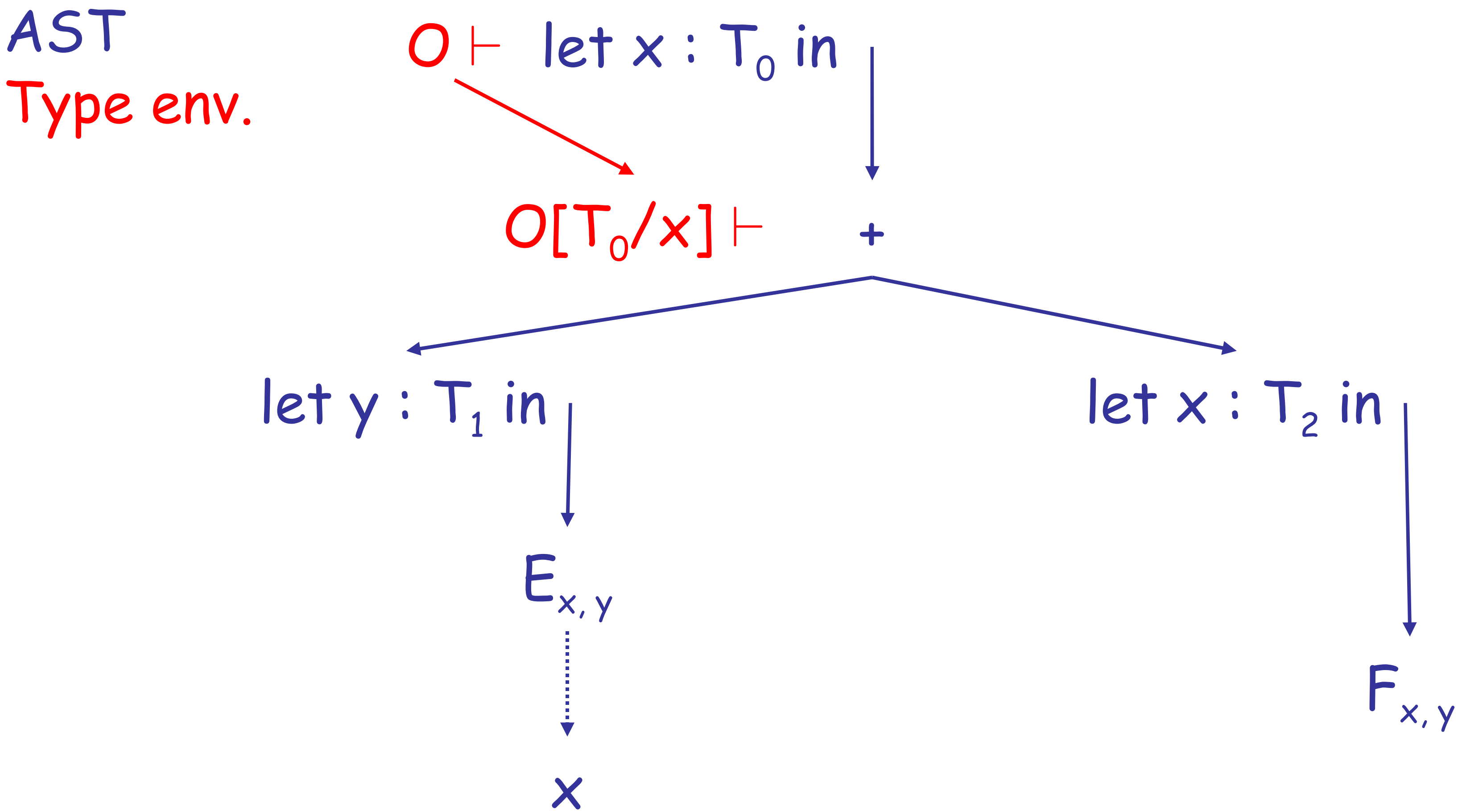

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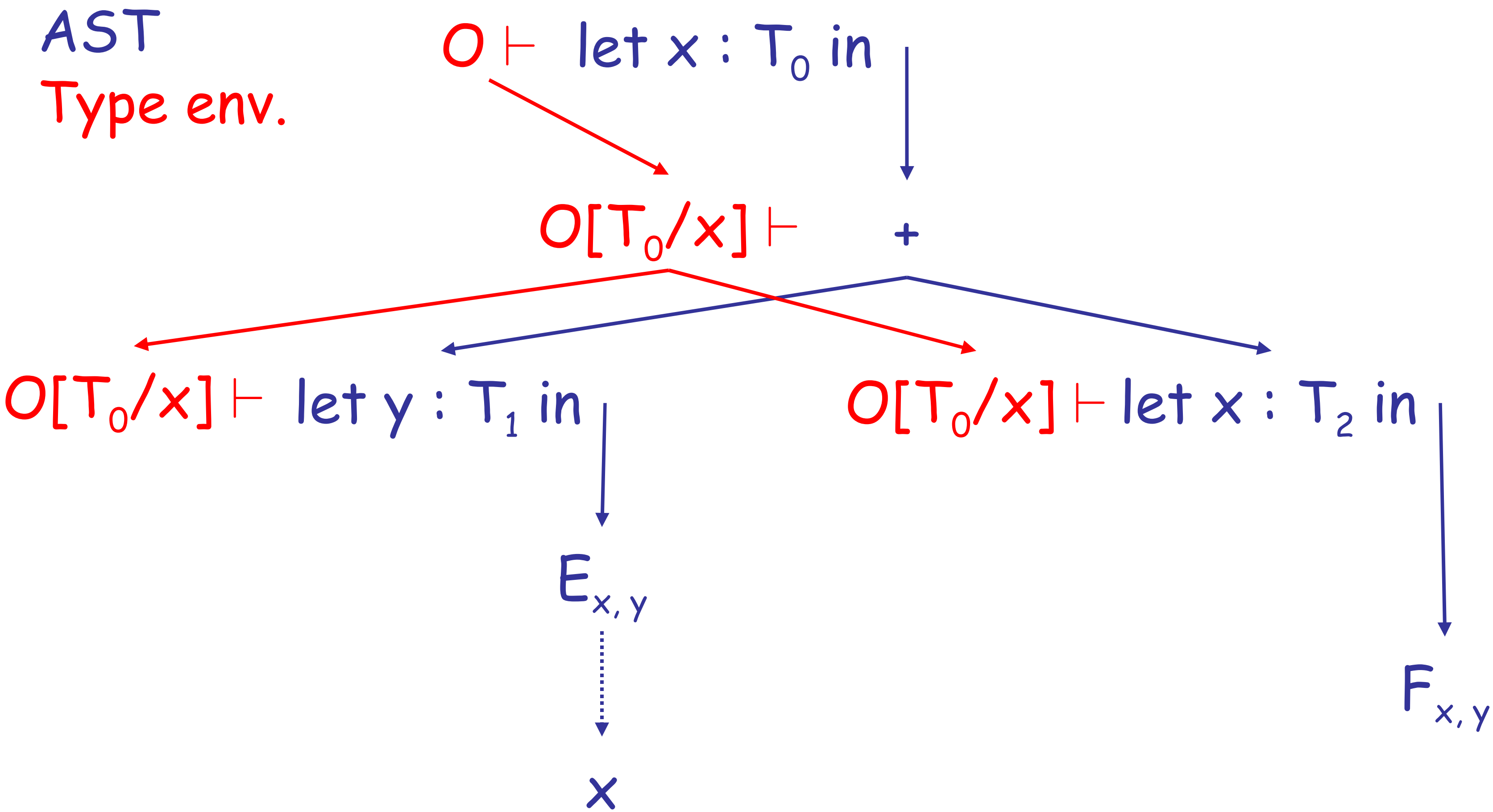

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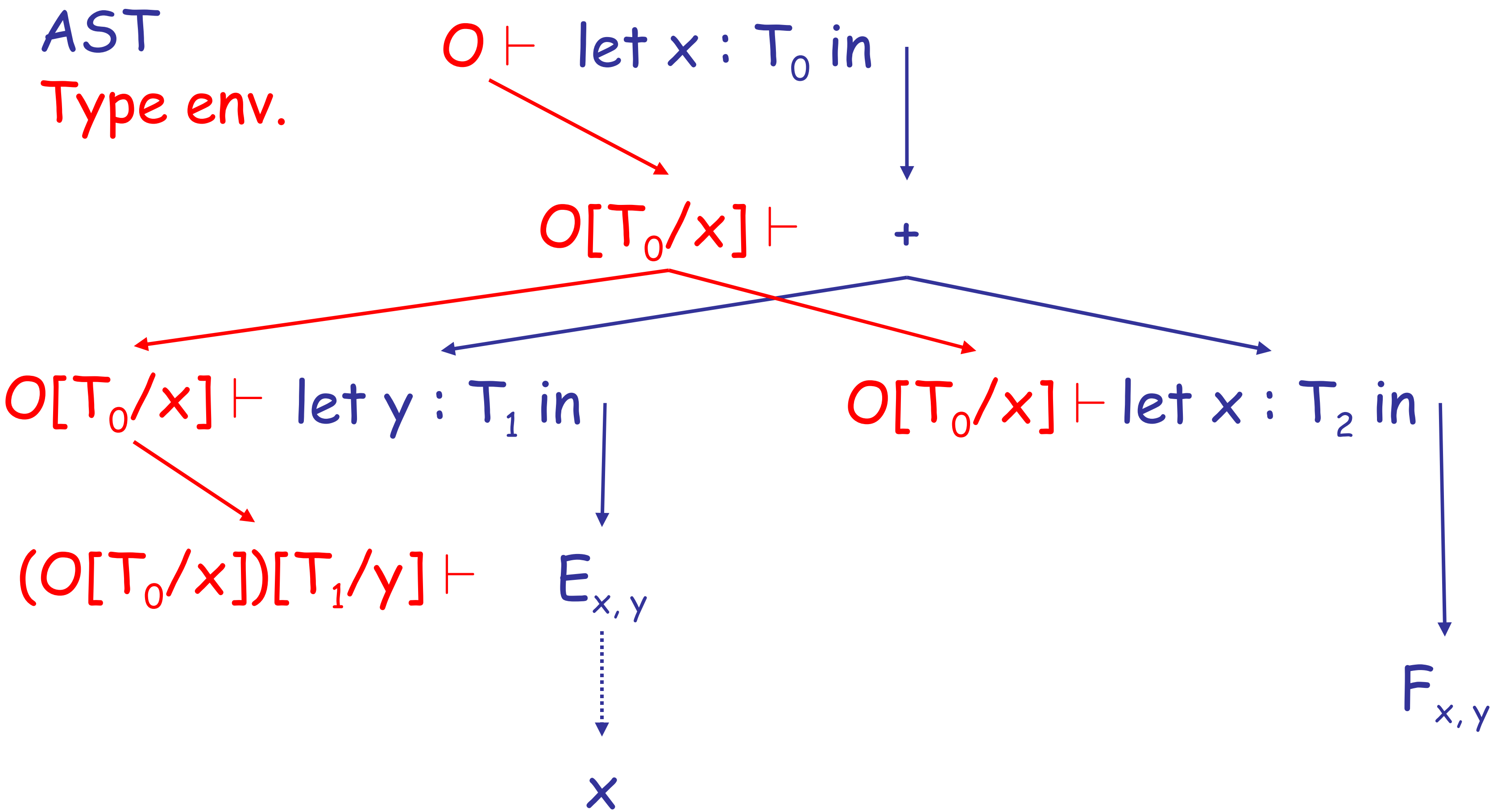

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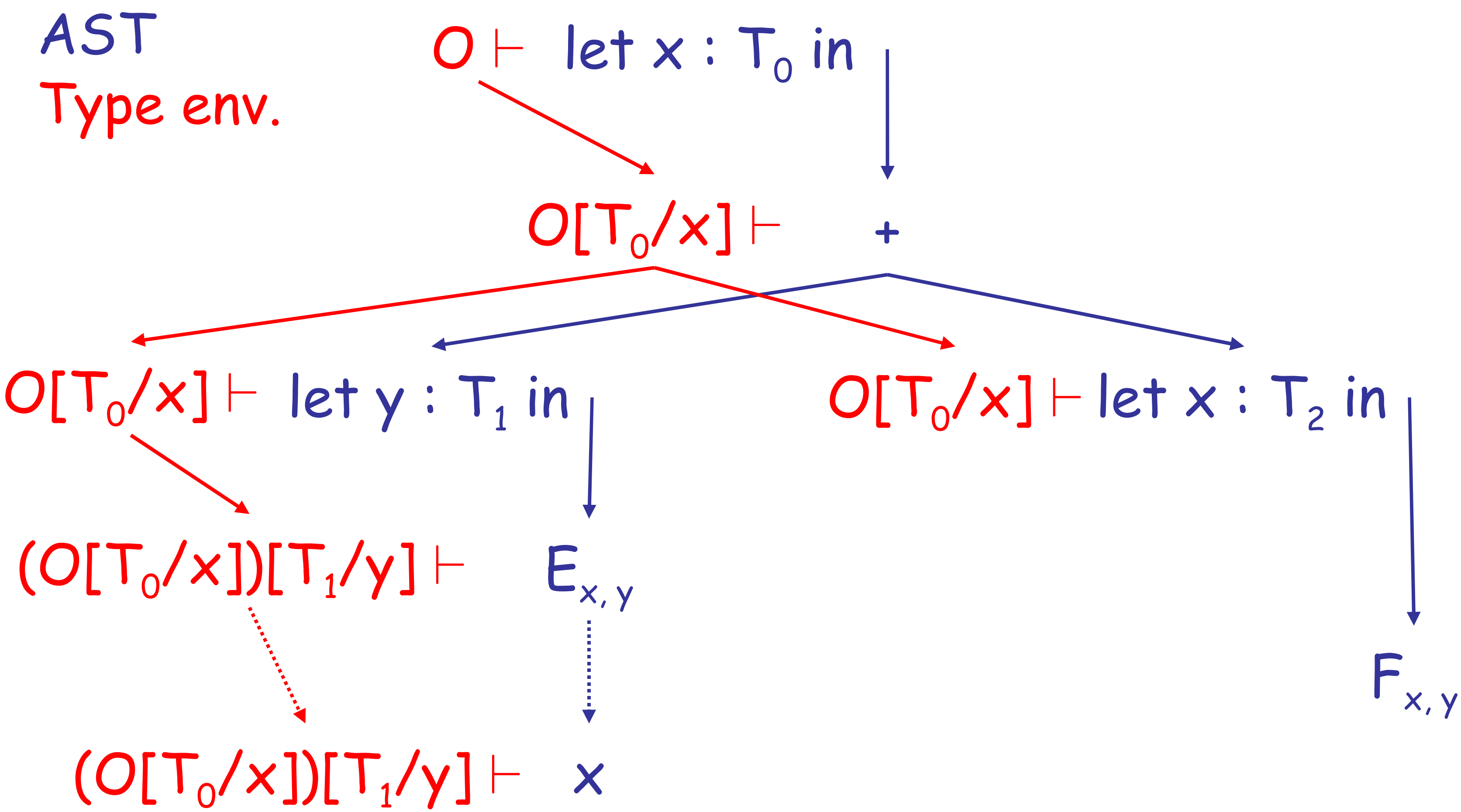

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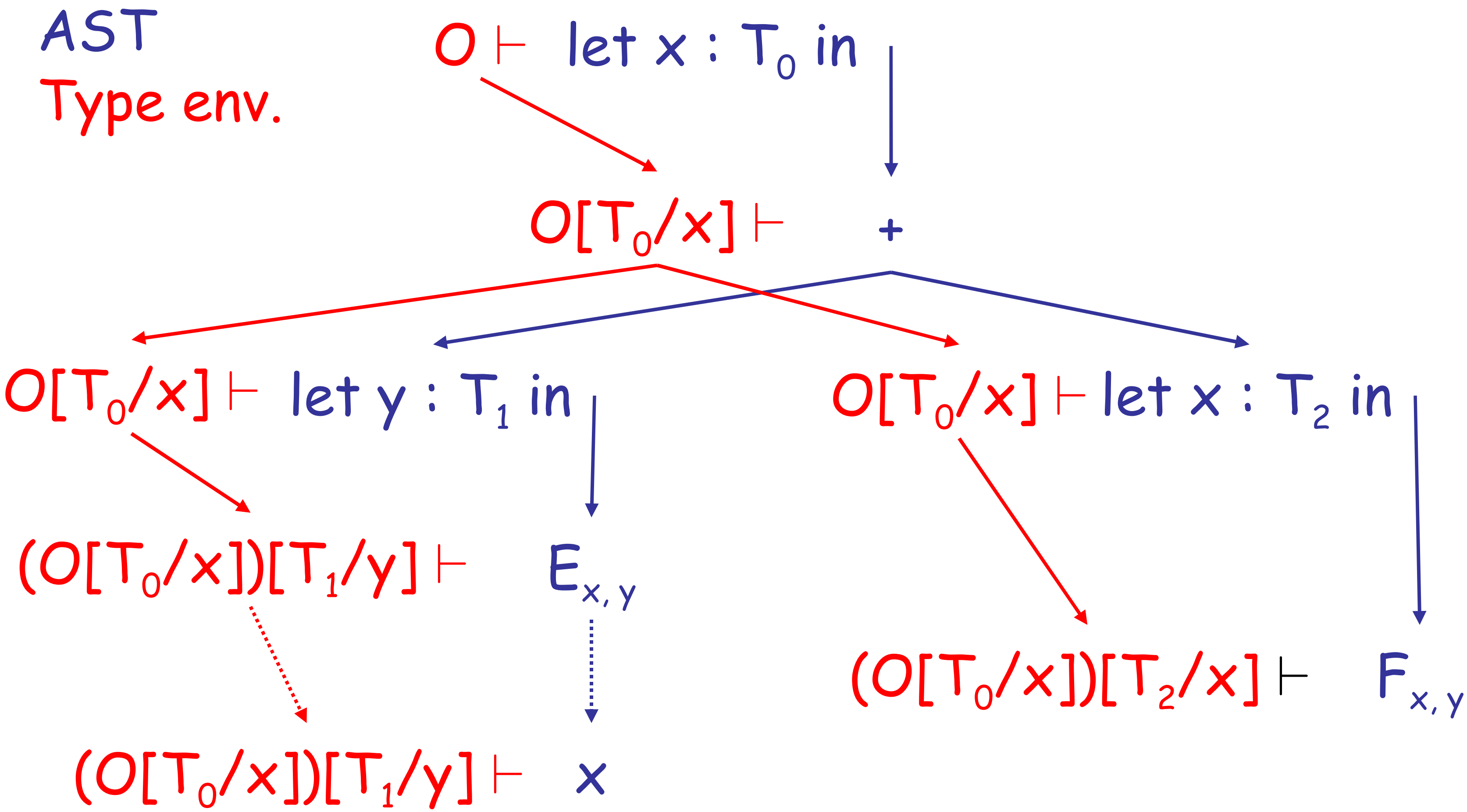


Example of Typing "let"

AST
Type env.
Types

$O \vdash \text{let } x : T_0 \text{ in}$

$O[T_0/x] \vdash \quad +$

$O[T_0/x] \vdash \text{let } y : T_1 \text{ in}$

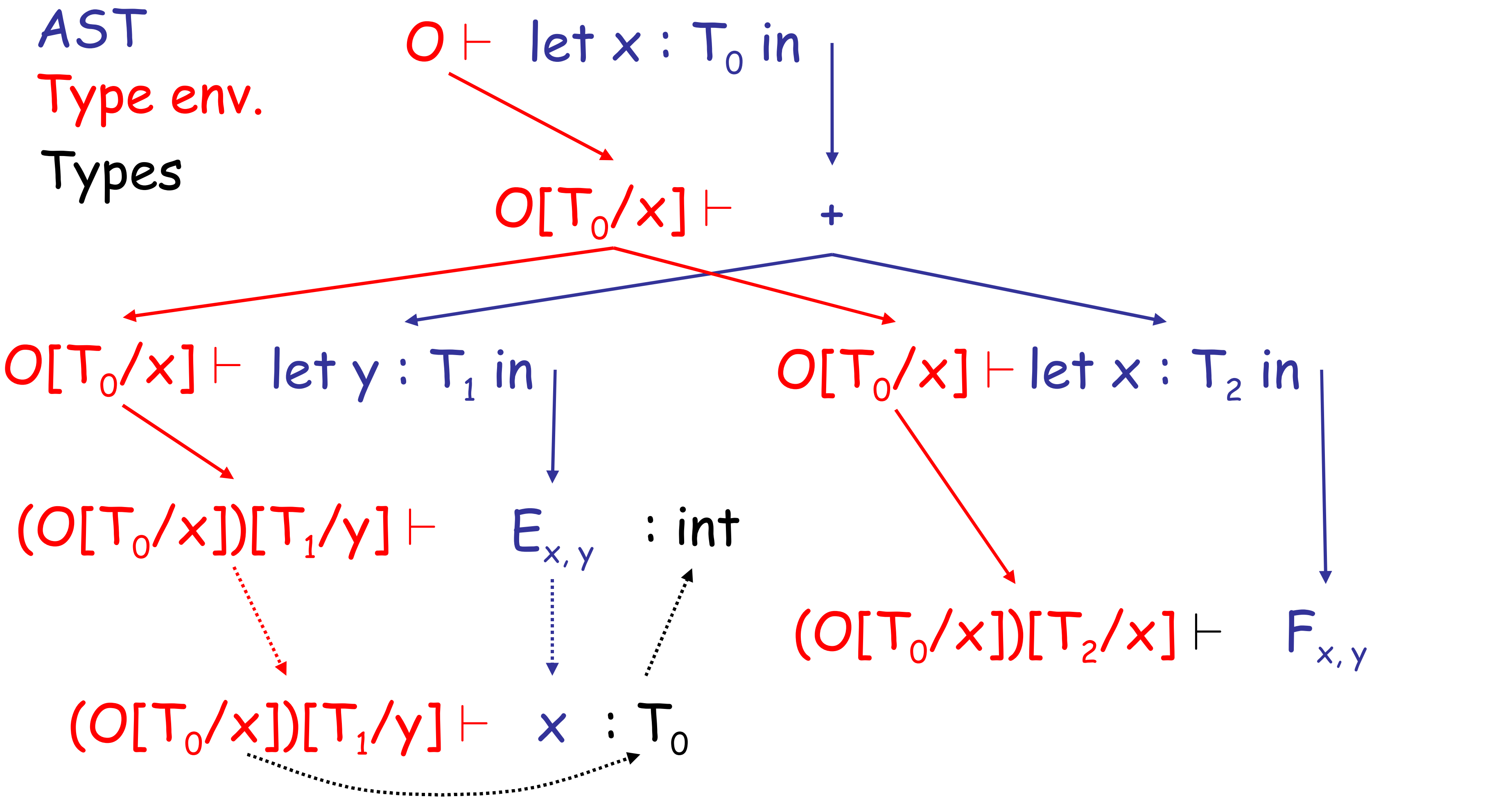
$O[T_0/x] \vdash \text{let } x : T_2 \text{ in}$

$(O[T_0/x])[T_1/y] \vdash \quad E_{x,y}$

$(O[T_0/x])[T_2/x] \vdash \quad F_{x,y}$

$(O[T_0/x])[T_1/y] \vdash \quad x \ : T_0$

Example of Typing "let"



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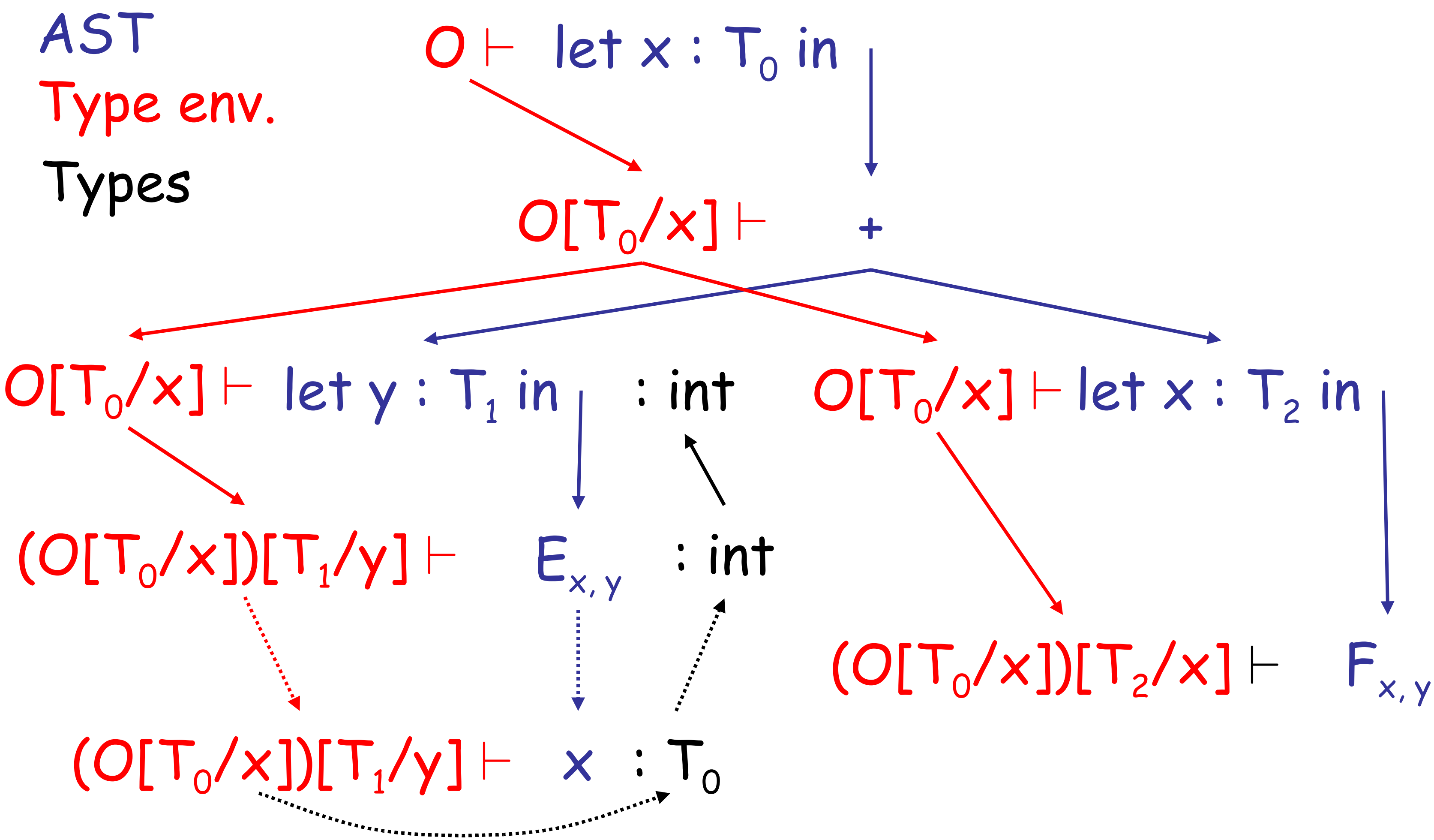

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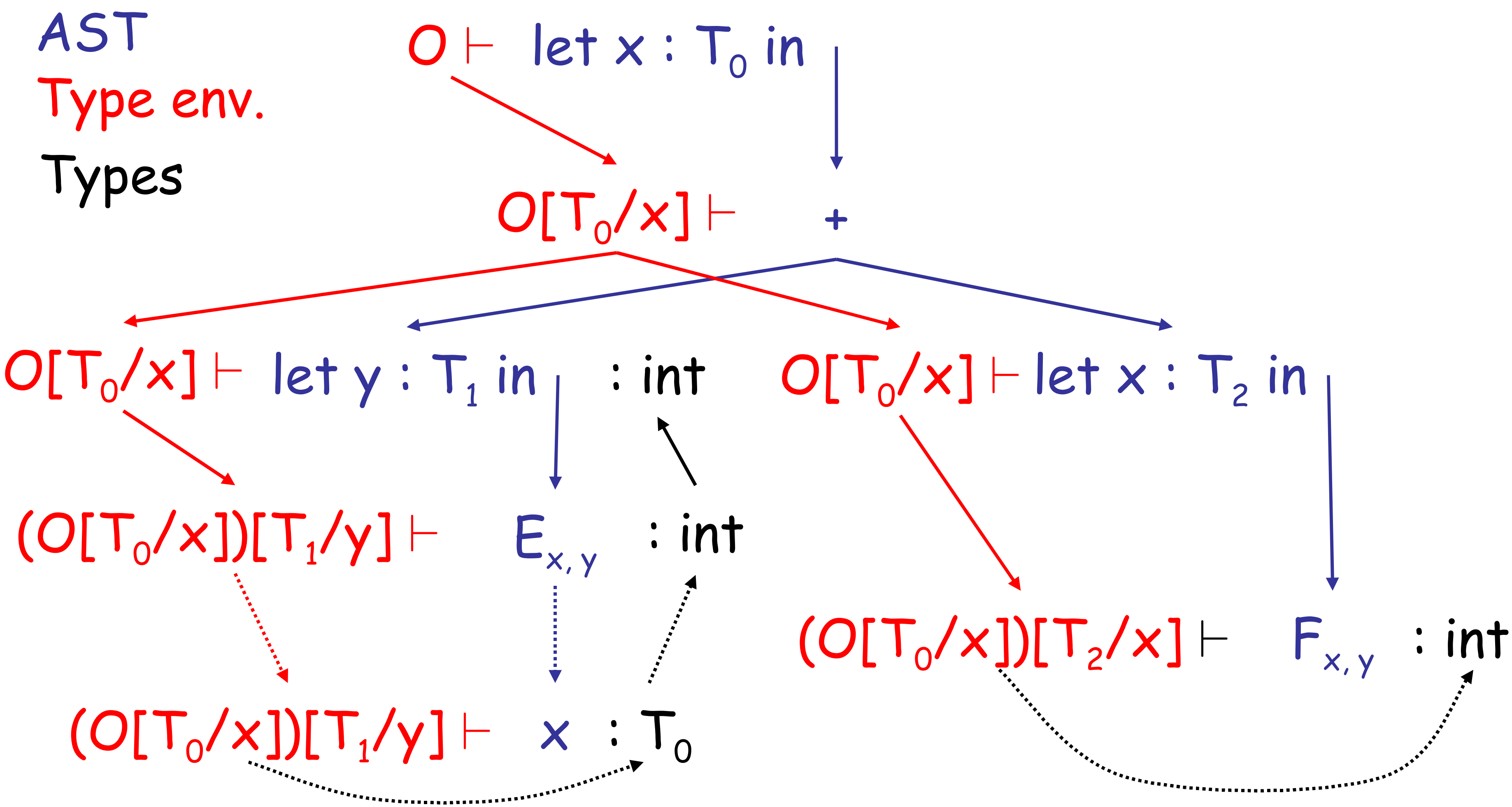

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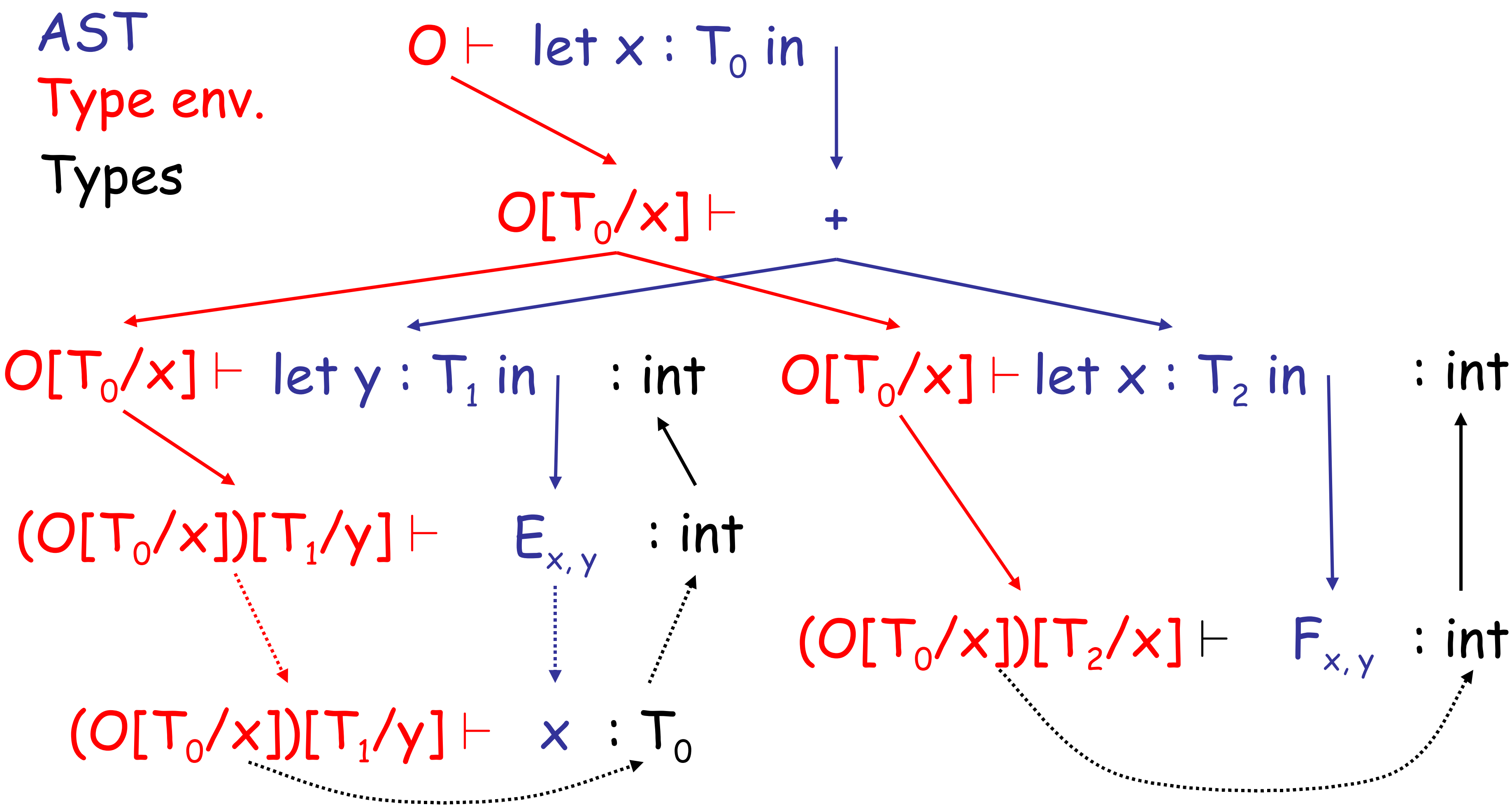

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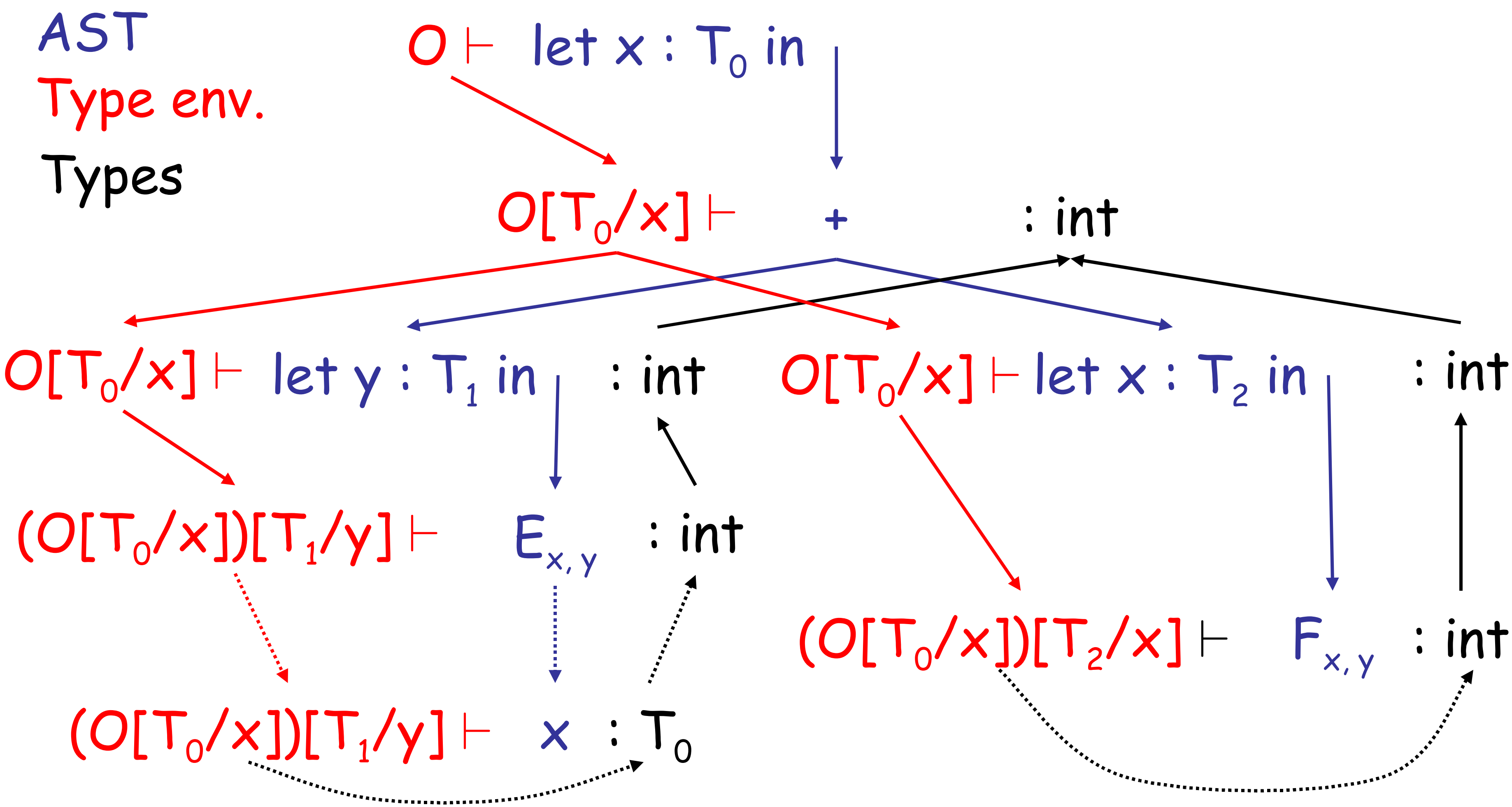

Example of Typing "let"

AST

Type env.

Types

$O \vdash \text{let } x : T_0 \text{ in } \quad : \text{int}$

$O[T_0/x] \vdash \quad + \quad : \text{int}$

$O[T_0/x] \vdash \text{let } y : T_1 \text{ in } \quad : \text{int}$

$O[T_0/x] \vdash \text{let } x : T_2 \text{ in } \quad : \text{int}$

$(O[T_0/x])[T_1/y] \vdash \quad E_{x,y} \quad : \text{int}$

$(O[T_0/x])[T_2/x] \vdash \quad F_{x,y} \quad : \text{int}$

$(O[T_0/x])[T_1/y] \vdash \quad x \quad : T_0$

Notes

- The type environment gives types to the free identifiers in the current scope
- The **type environment** is passed down the AST from the root towards the leaves
- **Types** are computed up the AST from the leaves towards the root

Q: Movies (362 / 842)

- In this 1992 comedy Dana Carvey and Mike Myers reprise a **Saturday Night Live** skit, sing **Bohemian Rhapsody** and say of a guitar: *"Oh yes, it will be mine."*

Q: General (455 / 842)

- This numerical technique for finding solutions to boundary-value problems was initially developed for use in structural analysis in the 1940's. The subject is represented by a model consisting of a number of linked simplified representations of discrete regions. It is often used to determine stress and displacement in mechanical systems.

Q: Movies (377 / 842)

- Identify the subject or the speaker in 2 of the following 3 **Star Wars** quotes.
  - "Aren't you a little short to be a stormtrooper?"
  - "I felt a great disturbance in the Force ... as if millions of voices suddenly cried out in terror and were suddenly silenced."
  - "I recognized your foul stench when I was brought on board."

Let with Initialization

Now consider let with initialization:

$$\frac{O \vdash e_0 : T_0 \qquad O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \text{[Let-Init]}$$

This rule is weak. Why?

Let with Initialization

- Consider the example:

```
class C inherits P { … }
…
let x : P ← new C in …
 …
```

- The previous let rule does not allow this code
  – We say that the rule is **too weak** or **incomplete**

Subtyping

- Define a relation $X \leq Y$ on classes to say that:
  - An object of type X could be used when one of type Y is acceptable, or equivalently
  - X conforms with Y
  - In Cool this means that X is a **subclass** of Y
- Define a relation $\leq$ on classes

  $X \leq X$

  $X \leq Y$ if X inherits from Y

  $X \leq Z$ if $X \leq Y$ and $Y \leq Z$

Let With Initialization (Better)

$$\frac{O \vdash e_0 : T \qquad T \leq T_0 \qquad O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \quad \text{[Let-Init]}$$

- Both rules for let are sound
- But more programs type check with this new rule (it is more complete)

Type System Tug-of-War

- There is a tension between
  - Flexible rules that do not constrain programming
  - Restrictive rules that ensure safety of execution

Expressiveness of Static Type Systems

- A static type system enables a compiler to detect many common programming errors
- The cost is that some correct programs are disallowed
  - Some argue for dynamic type checking instead
  - Others argue for more expressive static type checking
- But more expressive type systems are also more complex

Dynamic And Static Types

- The **dynamic type** of an object is the class C that is used in the "new C" expression that creates the object
  - A **run-time** notion
  - Even languages that are not statically typed have the notion of dynamic type
- The **static type** of an expression is a notation that captures all possible dynamic types the expression could take
  - A **compile-time** notion

Dynamic and Static Types. (Cont.)

- In early type systems the set of static types correspond directly with the dynamic types
- Soundness theorem: for all expressions E

  dynamic_type(E) = static_type(E)

  (in **all** executions, E evaluates to values of the type inferred by the compiler)

- This gets more complicated in advanced type systems (e.g., Java, Cool)

Dynamic and Static Types in COOL

```
class A { … }
class B inherits A {…}
class Main {
   A x ← new A;
   …
   x ← new B;
   …
}
```

x has static type A

Here, x's value has dynamic type A

Here, x's value has dynamic type B

- A variable of static type A can hold values of static type B, if $B \leq A$

Dynamic and Static Types

Soundness theorem for the Cool type system:

$\forall$ E. dynamic_type(E) $\leq$ static_type(E)

Why is this Ok?

- For E, compiler uses static_type(E)
- All operations that can be used on an object of type C can also be used on an object of type C' $\leq$ C
  - Such as fetching the value of an attribute
  - Or invoking a method on the object
- Subclasses can ***only add*** attributes or methods
- Methods can be redefined but with the same types!

Subtyping Example

- Consider the following Cool class definitions

```
Class A { a() : int { 0 }; }
Class B inherits A { b() : int { 1 }; }
```

- An instance of B has methods “a” and “b”
- An instance of A has method “a”
  - A type error occurs if we try to invoke method “b” on an instance of A

Example of Wrong Let Rule (1)

- Now consider a hypothetical wrong let rule:

$$\frac{O \vdash e_0 : T \qquad T \leq T_0 \qquad O \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1}$$

- How is it different from the correct rule?

Example of Wrong Let Rule (1)

- Now consider a hypothetical wrong let rule:

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- How is it different from the correct rule?
- The following good program does *not* typecheck

$$\text{let } x : \text{Int} \leftarrow 0 \text{ in } x + 1$$

- Why?

Example of Wrong Let Rule (2)

- Now consider a hypothetical wrong let rule:

$$\frac{O \vdash e_0 : T \quad T_0 \leq T \quad O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1}$$

- How is it different from the correct rule?

Example of Wrong Let Rule (2)

- Now consider a hypothetical wrong let rule:

$$\frac{O \vdash e_0 : T \quad T_0 \leq T \quad O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1}$$

- How is it different from the correct rule?
- The following *bad* program is well typed

let x : B ← new A in x.b()

- Why is this program bad?

Example of Wrong Let Rule (3)

- Now consider a hypothetical wrong let rule:

$$\frac{O \vdash e_0 : T \quad T \leq T_0 \quad O[T/x] \vdash e_1 : T_1}{O \vdash \textbf{let } x : T_0 \leftarrow e_0 \textbf{ in } e_1 : T_1}$$

- How is it different from the correct rule?

Example of Wrong Let Rule (3)

- Now consider a hypothetical wrong let rule:

$$\frac{O \vdash e_0 : T \quad T \leq T_0 \quad O[T/x] \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1}$$

- How is it different from the correct rule?

- The following good program is *not* well typed

let x : A ← new B in {… x ← new A; x.a(); }

- Why is this program not well typed?

Typing Rule Notation

- The typing rules use **very concise** notation
- They are very carefully constructed
- Virtually any change in a rule either:
  - Makes the type system **unsound**

    (bad programs are accepted as well typed)
  - Or, makes the type system less usable (**incomplete**)

    (good programs are rejected)

- But some good programs will be rejected anyway
  - The notion of a good program is **undecidable**

Assignment

More uses of subtyping:

$$\frac{\begin{array}{c} O(id) = T_0 \\ O \vdash e_1 : T_1 \\ T_1 \leq T_0 \end{array}}{O \vdash id \leftarrow e_1 : T_1} \quad [Assign]$$

Initialized Attributes

- Let $O_C(x) = T$ for all attributes $x{:}T$ in class C
  - O_C represents the class-wide scope
- Attribute initialization is similar to let, except for the scope of names

$$\frac{\begin{array}{c} O_C(id) = T_0 \\ O_C \vdash e_1 : T_1 \\ T_1 \leq T_0 \end{array}}{O_C \vdash id : T_0 \leftarrow e_1 \, ;} \; [\text{Attr-Init}]$$

If-Then-Else

- Consider:

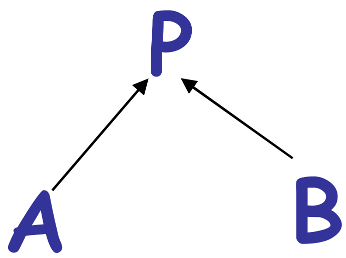
  if e_0 then e_1 else e_2 fi

- The result can be either e_1 or e_2
- The dynamic type is either e_1's or e_2's type

- The best we can do statically is the **smallest supertype** larger than the type of e_1 and e_2

If-Then-Else example

- Consider the class hierarchy



- … and the expression

  if … then new A else new B fi

- Its type should allow for the dynamic type to be both A or B
  - Smallest supertype is P

Least Upper Bounds

- Define: **lub(X,Y)** to be the **least upper bound** of X and Y. lub(X,Y) is Z if
  - $X \leq Z \wedge Y \leq Z$

    Z is an upper bound
  - $X \leq Z' \wedge Y \leq Z' \Rightarrow Z \leq Z'$

    Z is least among upper bounds

- In Cool, the least upper bound of two types is their **least common ancestor** in the **inheritance tree**

If-Then-Else Revisited

$$O \vdash e_0 : \text{Bool}$$

$$O \vdash e_1 : T_1$$

$$O \vdash e_2 : T_2$$

$$\overline{O \vdash \text{if } e_0 \text{ then } e_1 \text{ else } e_2 \text{ fi} : \textbf{lub}(T_1, T_2)}$$

[If-Then-Else]

Case

- The rule for case expressions takes a lub over all branches

$$\frac{\begin{array}{c} O \vdash e_0 : T_0 \\ O[T_1/x_1] \vdash e_1 : T_1' \\ \ldots \\ O[T_n/x_n] \vdash e_n : T_n' \end{array}}{\begin{array}{c} O \vdash \text{case } e_0 \text{ of } x_1{:}T_1 \Rightarrow e_1; \\ \ldots; x_n : T_n \Rightarrow e_n; \text{ esac} : \text{lub}(T_1',\ldots,T_n') \end{array}} \quad [Case]$$

Next Time

- Type checking method dispatch
- Type checking with SELF_TYPE in COOL

Homework

- Today: WA3 due
- Get started on PA4
  - **Checkpoint due Oct 14**
- Before Next Class: Read Chapter 7.2