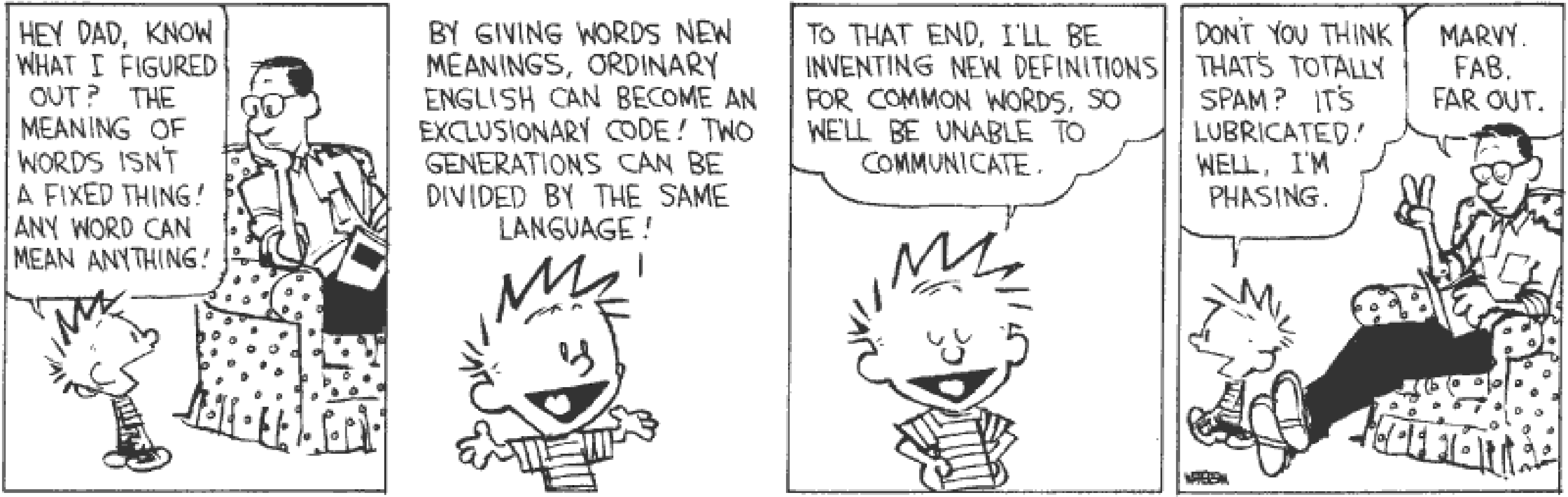
Abstract Interpretation

(Galois, Collections, Widening)


HEY DAD, KNOW WHAT I FIGURED OUT? THE MEANING OF WORDS ISN'T A FIXED THING! ANY WORD CAN MEAN ANYTHING!
BY GIVING WORDS NEW MEANINGS, ORDINARY ENGLISH CAN BECOME AN EXCLUSIONARY CODE! TWO GENERATIONS CAN BE DIVIDED BY THE SAME LANGUAGE!
TO THAT END, I'LL BE INVENTING NEW DEFINITIONS FOR COMMON WORDS, SO WE'LL BE UNABLE TO COMMUNICATE.
DON'T YOU THINK THAT'S TOTALLY SPAM? IT'S LUBRICATED! WELL, I'M PHASING.
MARVY. FAB. FAR OUT.

What Are We Doing?

Sam Block	Variable Renaming for Decompilation and Program Readability
William Burns	Surveying Malware's Relation to Programming Languages
Tim Chaplin	Generalizing the Burrows-Wheeler Transform
Kirti Chawla	Detection of Covert Channels in RFID Supply Chains
Joel Coffman	
Derek Davis	Classification of and Confidence in Repairs to Software Bugs
Daniel Dougherty	Branch Prediction in Dynamic Binary Translation Systems
Zak Fry	A Large-Scale Code Readability Metric
Andrew Jurik	Surveying The Use of Static Analyses to Verify Security Properties
Dan Lepage	A Bayesian Approach to SVBRDF Decomposition
Ming Mao	A Process Execution and Collaboration Policy Assertion Language
Irwin Reyes	The Accuracy of Face-Recognition Systems
Arkaitz Ruiz Alvarez	Verifying Programs that Use Intel's Threading Building Blocks Library
Mona Sergi	Improving Automatic Program Repair with Template-based Mutations
Blake Sheridan	Implementing Support for Hardware-based Profiling in Embedded Systems
Michael Skalak	Implementing and Interpreting Visual Languages
Elizabeth Soechting	Semantic Regression Testing for Tree-Structured Output
Luther Tychonievich	A Unified Compiler Representation to Support Debugging Optimized Code
Kristen Walcott	A Test Case Generation Technique for Multithreaded Programs
Ren Xu	Surveying Refinement Types

Tool Time

- How's Homework 5 going?
- Get started early
- Compilation problems?
  - See FAQ

(trivia: what tool brand is this?)

More Power!

- You can handle it!

Abstract Interpretation

- We have an abstract domain A
  - e.g., A = { positive, negative, zero }
  - An abstraction function $\beta : \mathbb{Z} \to A$
    - $\mathbb{Z}$ is our concrete domain
  - A concretization function $\gamma : A \to \mathcal{P}(\mathbb{Z})$
- Positive + Positive = ???
- Positive + Negative = ???
- Positive / Zero = ???

We don't want security to get suspicious ...

Pibb
+
=
crazy delicious

Review

- We introduced abstract interpretation
- An abstraction mapping from concrete to abstract values
  - Has a concretization mapping which forms a Galois connection

- We'll look a bit more at Galois connections
- We'll lift AI from expressions to programs
- … and we'll discuss the mythic "widening"

Why Galois Connections?

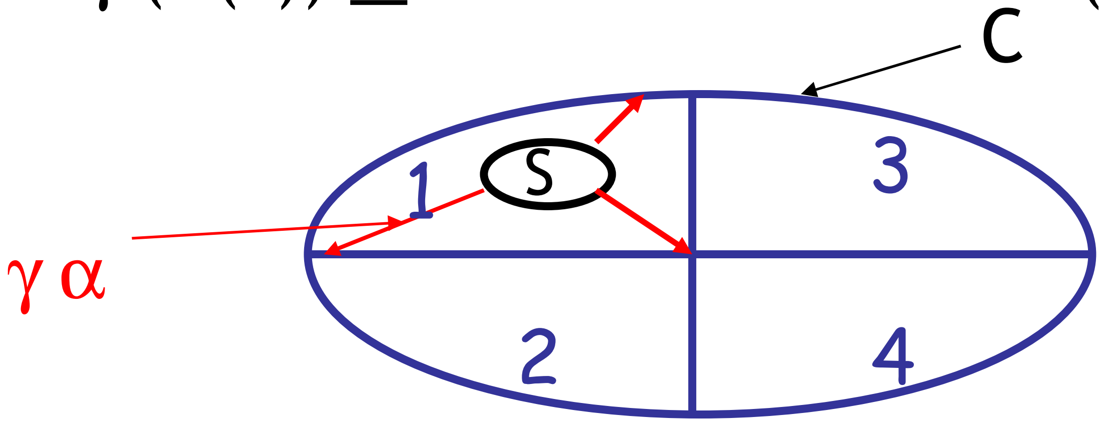
- We have an abstract domain A
  - An abstraction function $\beta : \mathbb{Z} \to A$
  - Induces $\alpha : \mathcal{P}(\mathbb{Z}) \to A$ and $\gamma : A \to \mathcal{P}(\mathbb{Z})$
- We argued that for correctness

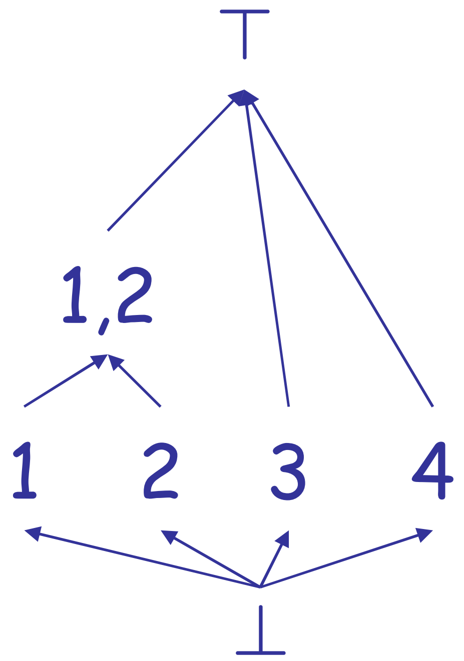
$$\gamma(a_1 \ \underline{op} \ a_2) \supseteq \gamma(a_1) \ op \ \gamma(a_2)$$

  - We wish for the set on the left to be as small as possible
  - To reduce the loss of information through abstraction
- For each set $S \subseteq C$, define $\alpha(S)$ as follows:
  - Pick smallest S' that includes S and is in the image of γ
  - Define $\alpha(S) = \gamma^{-1}(S')$
  - Then we define: $a_1 \ \underline{op} \ a_2 = \alpha(\gamma(a_1) \ op \ \gamma(a_2))$
- Then α and γ form a Galois connection

Galois Connections

- A Galois connection between complete lattices A and $\mathcal{P}(C)$ is a pair of functions α and γ such that:
  - γ and α are monotonic
    - (with the $\subseteq$ ordering on $\mathcal{P}(C)$)
  - $\alpha(\gamma(a)) = a$ for all $a \in A$
  - $\gamma(\alpha(S)) \supseteq S$ for all $S \in \mathcal{P}(C)$





More on Galois Connections

IS FREEDOM OF SPEECH AN ABSOLUTE RIGHT?
"anything which hurts other people's convict-ions, particularly relig-ious convictions, must be avoided. Freedom of expression should be exer-cised in a spirit of responsibility. I must condemn any overt provocation that could dangerously fuel passion
CHIRAC
VOLTAIRE
"I may not agree with what you say but I will defend to the death your right to say it."

- All Galois connections are monotonic
- In a Galois connection one function uniquely and absolutely determines the other

Abstract Interpretation for Imperative Programs

- So far we abstracted the value of expressions
- Now we want to abstract the state at each point in the program
- First we define the concrete semantics that we are abstracting
  - We'll use a collecting semantics

Collecting Semantics

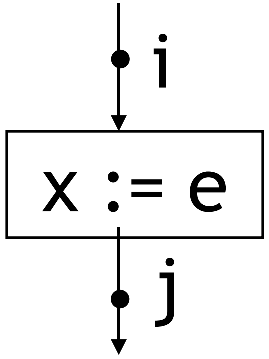
- Recall
  - A state $\sigma \in \Sigma$. Any state σ has type Var $\rightarrow \mathbb{Z}$
  - States vary from program point to program point
- We introduce a set of program points: labels
- We want to answer questions like:
  - Is x always positive at label i?
  - Is x always greater or equal to y at label j?
- To answer these questions we'll construct

  C $\in$ Contexts. C has type Labels $\rightarrow \mathcal{P}(\Sigma)$

  - For each label i, C(i) = all possible states at label i
  - This is called the collecting semantics of the program
  - This is basically what SLAM (and BLAST, ESP, …) approximate (using BDDs to store $\mathcal{P}(\Sigma)$ efficiently)

Defining the Collecting Semantics

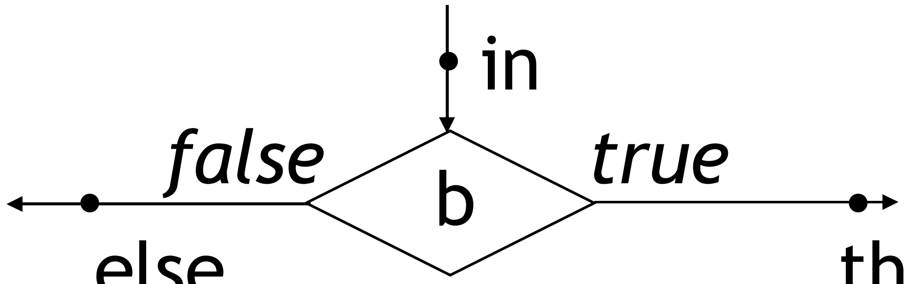
- We first define relations between the collecting semantics at different labels
  - We do it for unstructured CFGs (cf. HW5!)
  - Can do it for IMP with careful notion of program points
- Define a label on each edge in the CFG
- For assignment



$$C_j = \{\sigma[x := n] \mid \sigma \in C_i \wedge [\![e]\!]\sigma = n\}$$

Defining the Collecting Semantics

- For conditionals



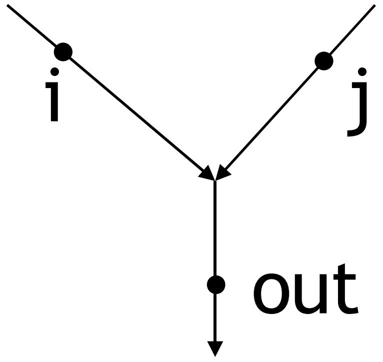
$$C_{else} = \{ \sigma \mid \sigma \in C_{in} \land [\![b]\!]\sigma = \text{false}\}$$

$$C_{then} = \{ \sigma \mid \sigma \in C_{in} \land [\![b]\!]\sigma = \text{true}\}$$

- Assumes b has no side effects (as in IMP or HW5)

Defining the Collecting Semantics

- For a join

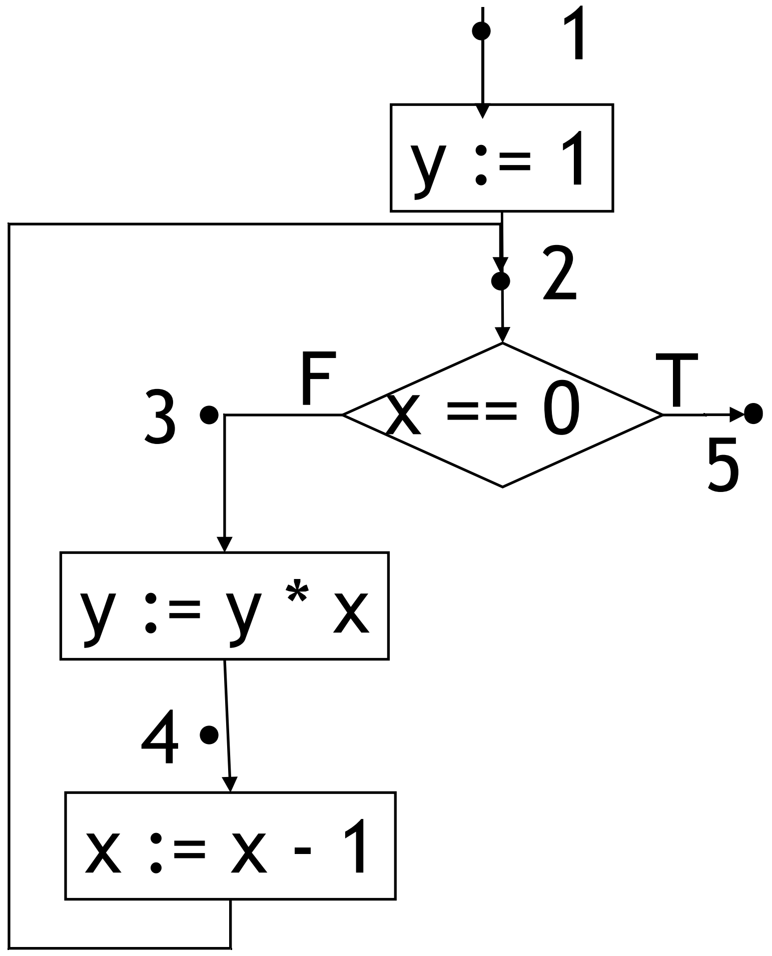


$$C_{out} = C_i \cup C_j$$

- Verify that these relations are monotonic
  - If we increase a C_x all other C_y can only increase

Collecting Semantics: Example

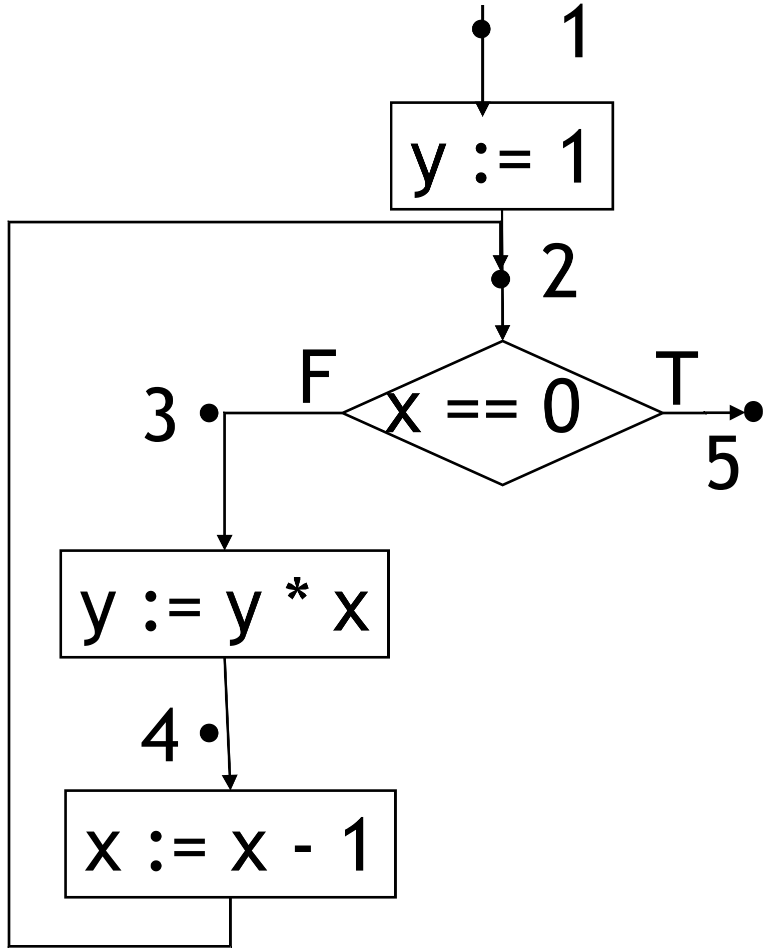
- Assume $x \geq 0$ initially *(explain this?)*



$C_1 = \{\sigma \mid \sigma(x) \geq 0\}$

Collecting Semantics: Example

- Assume $x \geq 0$ initially

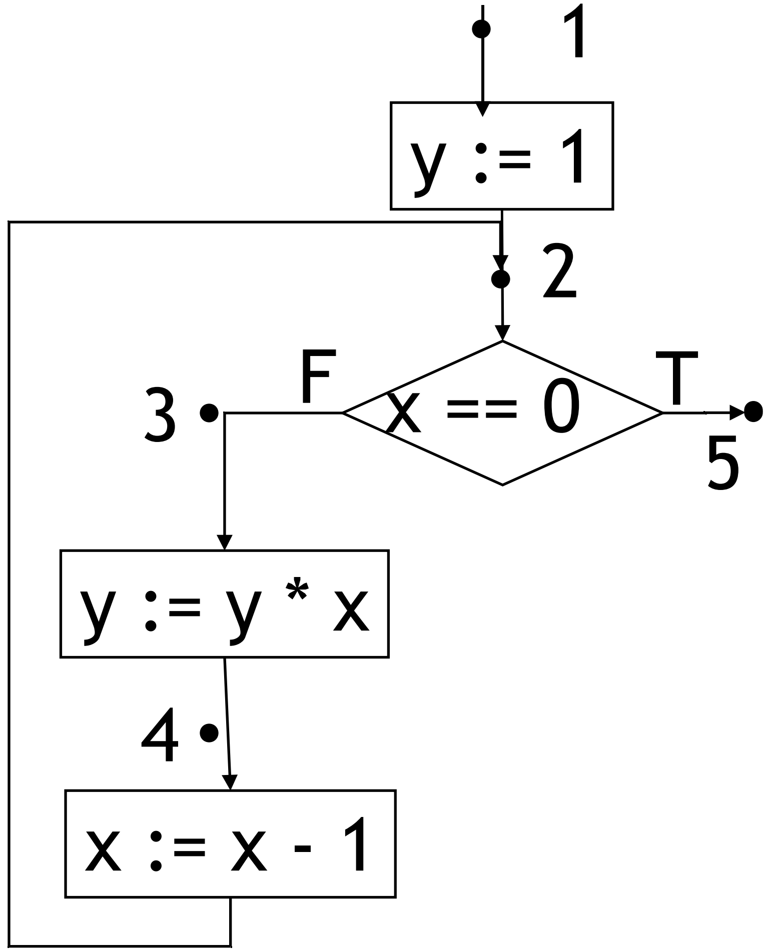


$C_1 = \{\sigma \mid \sigma(x) \geq 0\}$

$C_2 = \{ \sigma[y:=1] \mid \sigma \in C_1\}$

Collecting Semantics: Example

- Assume $x \geq 0$ initially



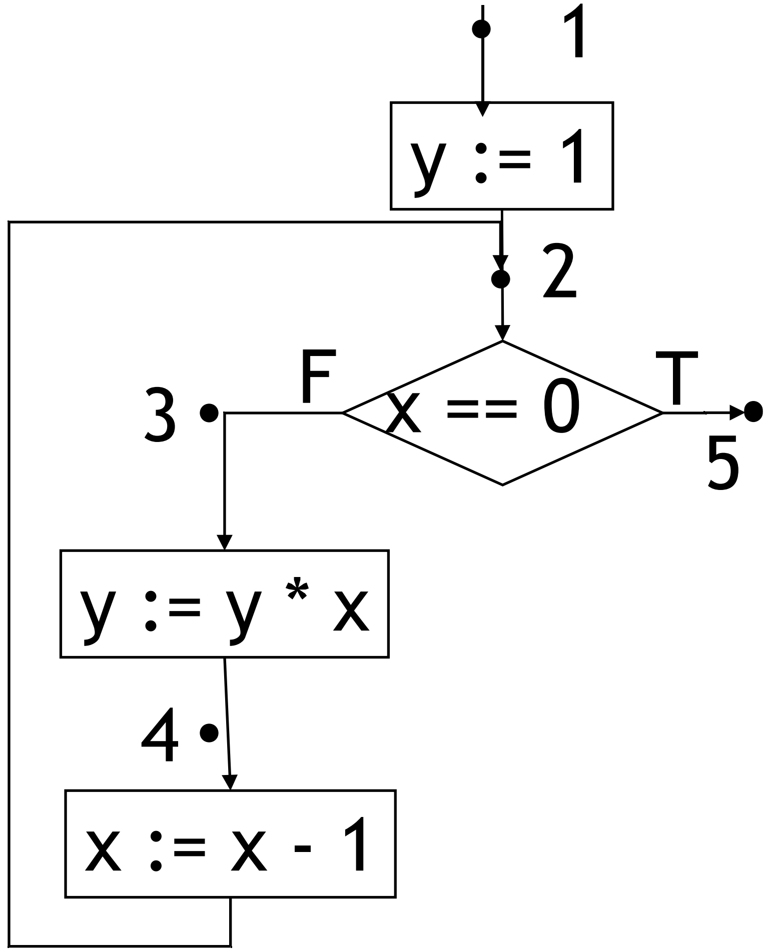
$C_1 = \{\sigma \mid \sigma(x) \geq 0\}$

$C_2 = \{ \sigma[y{:=}1] \mid \sigma \in C_1\}$

$C_3 = C_2 \cap \{\sigma \mid \sigma(x) \neq 0\}$

Collecting Semantics: Example

- Assume x $\geq$ 0 initially



$C_1 = \{\sigma \mid \sigma(x) \geq 0\}$

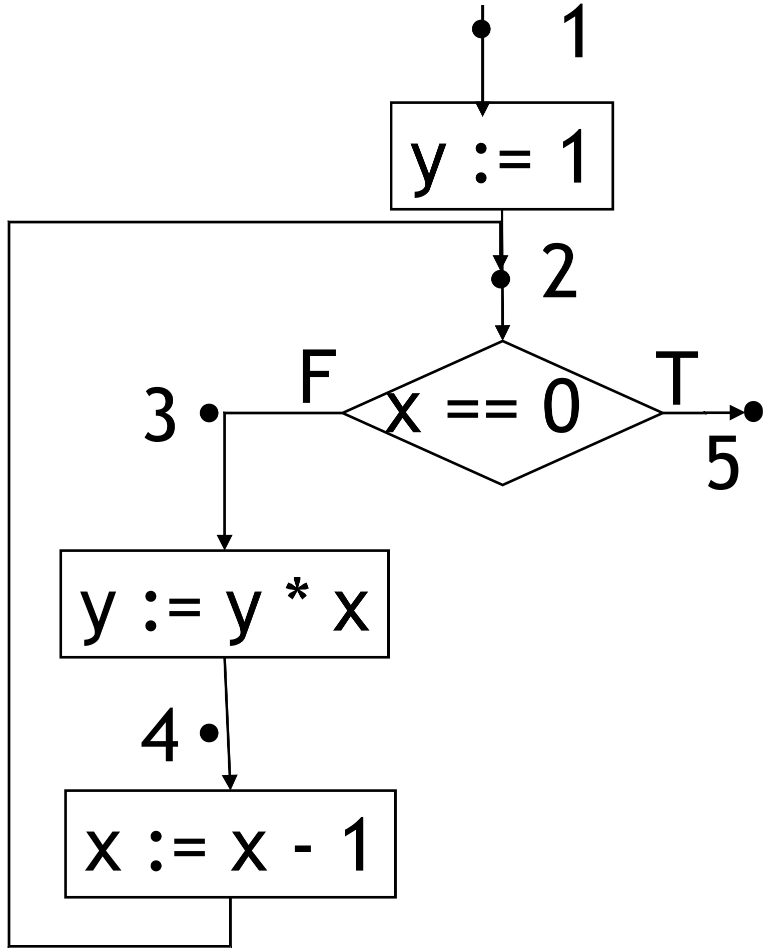
$C_2 = \{ \sigma[y:=1] \mid \sigma \in C_1\}$

$C_3 = C_2 \cap \{\sigma \mid \sigma(x) \neq 0\}$

$C_4 = \{\sigma[y:=\sigma(y)*\sigma(x)] \mid$
$\sigma \in C_3\}$

Collecting Semantics: Example

- Assume x $\geq$ 0 initially



$C_1 = \{\sigma \mid \sigma(x) \geq 0\}$

$C_2 = \{ \sigma[y{:=}1] \mid \sigma \in C_1\}$

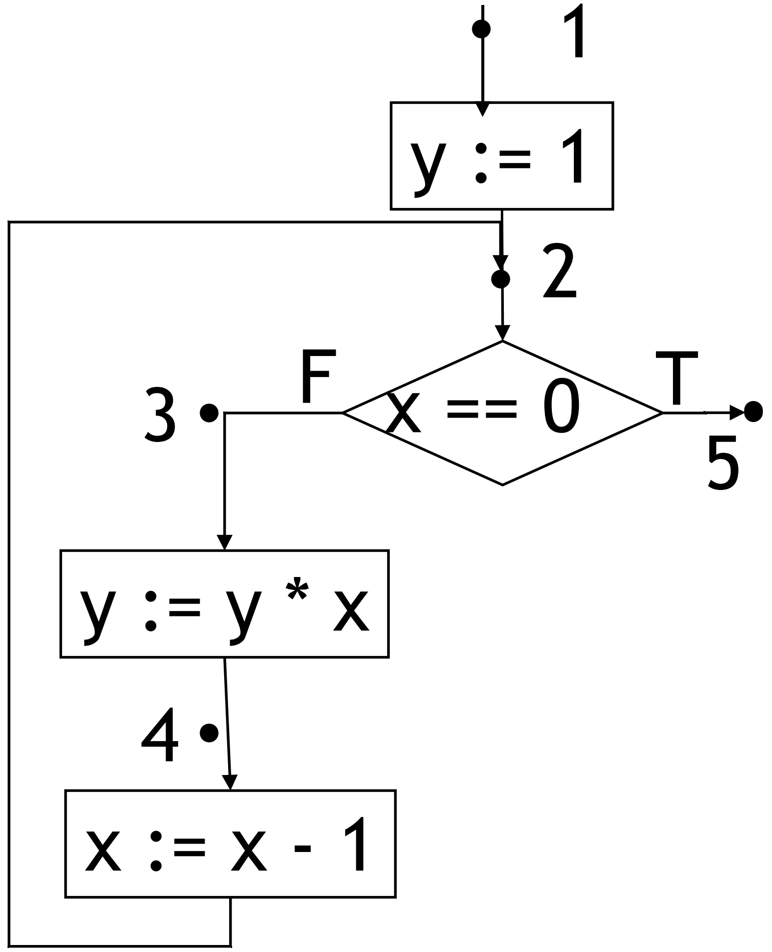
$\cup \{\sigma[x{:=}\sigma(x)-1] \mid \sigma \in C_4\}$

$C_3 = C_2 \cap \{\sigma \mid \sigma(x) \neq 0\}$

$C_4 = \{\sigma[y{:=}\sigma(y)*\sigma(x)] \mid$
$\sigma \in C_3\}$

Collecting Semantics: Example

- Assume x $\geq$ 0 initially



$C_1 = \{\sigma \mid \sigma(x) \geq 0\}$

$C_2 = \{ \sigma[y:=1] \mid \sigma \in C_1\}$
$\cup \{\sigma[x:=\sigma(x)-1] \mid \sigma \in C_4\}$

$C_3 = C_2 \cap \{\sigma \mid \sigma(x) \neq 0\}$

$C_4 = \{\sigma[y:=\sigma(y)*\sigma(x)] \mid \sigma \in C_3\}$

$C_5 = C_2 \cap \{\sigma \mid \sigma(x) = 0\}$

Why Does This Work?

- We just made a system of recursive equations that are defined largely in terms of themselves
  - e.g., $C_2 = F(C_4)$, $C_4 = G(C_3)$, $C_3 = H(C_2)$
- Why do we have any reason to believe that this will get us what we want?

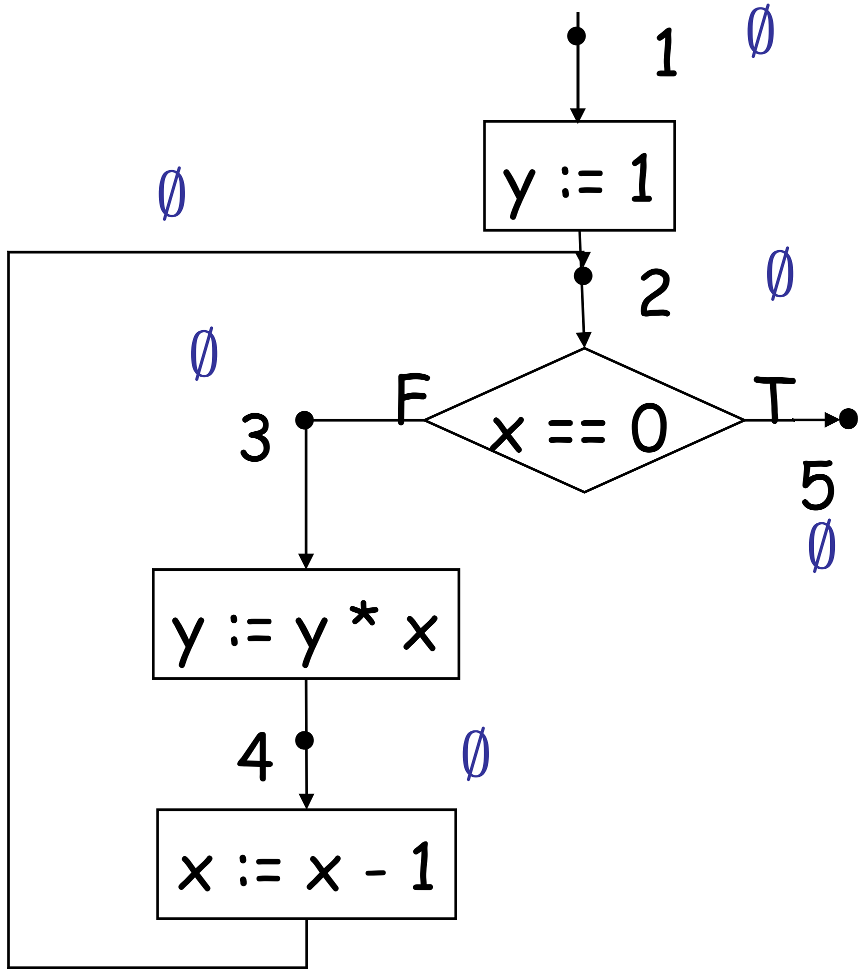
HEY, YOU WANNA PLAY THE QUESTION GAME?
WHAT'S THAT?
YOU'VE NEVER HEARD OF THE QUESTION GAME?
HOW DO YOU PLAY?
DO I REALLY HAVE TO EXPLAIN IT?
HOW CAN I PLAY IF I DON'T KNOW THE RULES?
ARE YOU PUTTING ME ON?
WHAT DO YOU MEAN?

The Collecting Semantics

- We have an equation with the unknown C
  - The equation is defined by a monotonic and continuous function on domain Labels $\to \mathcal{P}(\Sigma)$
- We can use the least fixed-point theorem
  - Start with $C^0(L)=\emptyset$ (aka $C^0 = \lambda L.\emptyset$)
  - Apply the relations between C_i and C_j to get $C^1{}_i$ from $C^0{}_j$
  - Stop when all $C^k = C^{k-1}$
  - Problem: we'll go on forever for most programs
  - But we know the fixed point exists

Collecting Semantics: Example

- (assume x $\geq$ 0 initially)



$C_1 = \{\sigma \mid \sigma(x) \geq 0\}$

$C_2 = \{\sigma[y{:=}1] \mid \sigma \in C_1\}$
$\quad \cup \{\sigma[x{:=}\sigma(x)\text{-}1] \mid \sigma \in C_4\}$

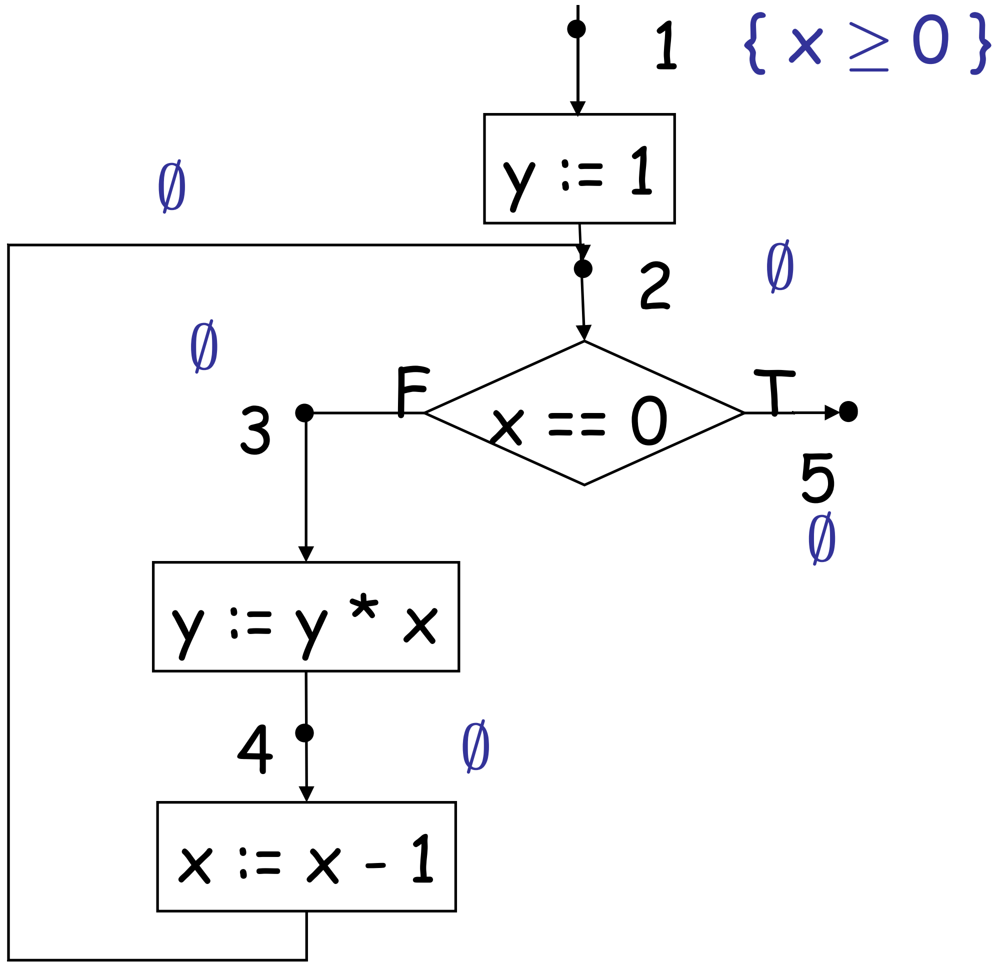
$C_3 = C_2 \cap \{\sigma \mid \sigma(x) \neq 0\}$

$C_5 = C_2 \cap \{\sigma \mid \sigma(x) = 0\}$

$C_4 = \{\sigma[y{:=}\sigma(y)*\sigma(x) \mid \sigma \in C_3\}$

Collecting Semantics: Example

- (assume x $\geq$ 0 initially)



$C_1 = \{\sigma \mid \sigma(x) \geq 0\}$

$C_2 = \quad \{ \sigma[y:=1] \mid \sigma \in C_1\}$

$\qquad \cup \{\sigma[x:=\sigma(x)-1] \mid \sigma \in C_4\}$

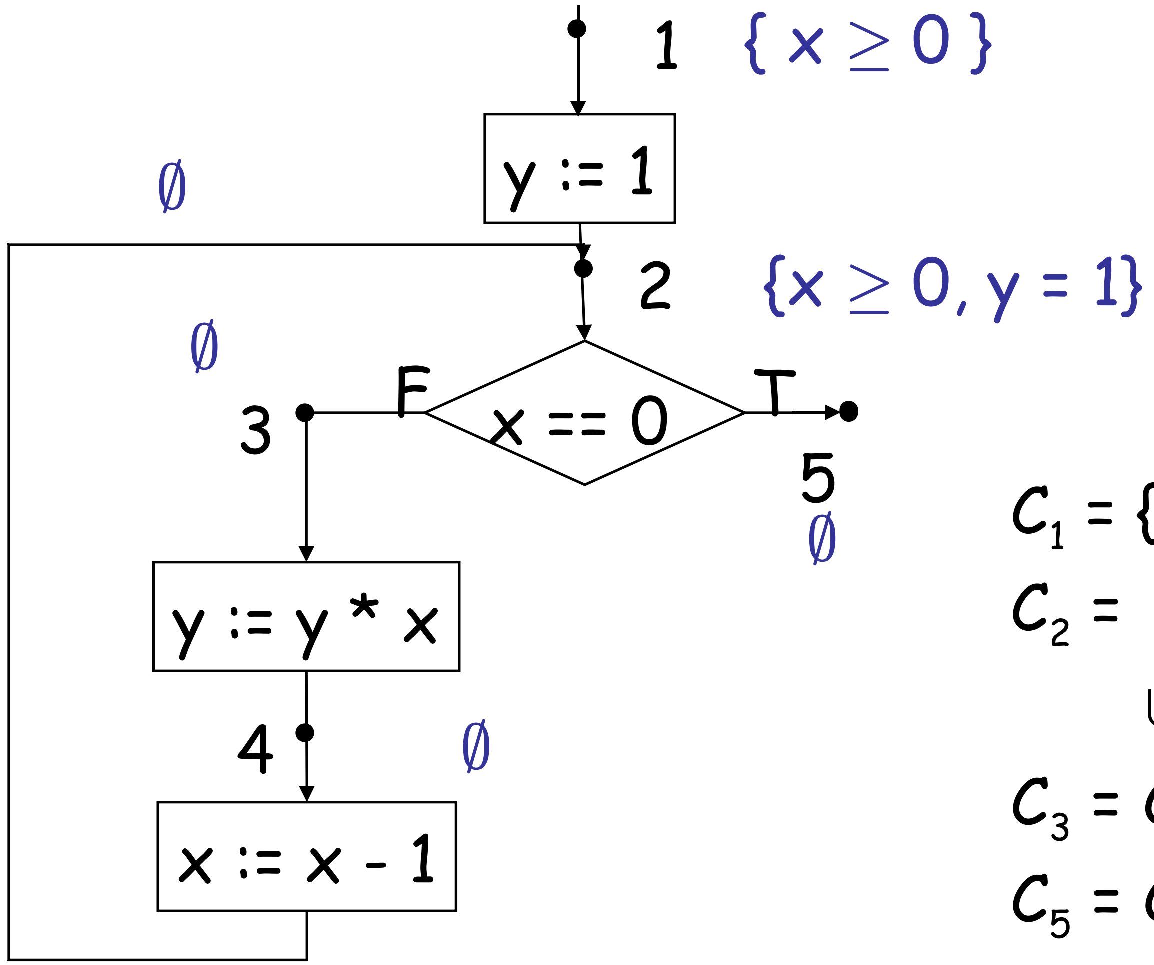
$C_3 = C_2 \cap \{\sigma \mid \sigma(x) \neq 0\}$

$C_5 = C_2 \cap \{\sigma \mid \sigma(x) = 0\}$

$C_4 = \{\sigma[y:=\sigma(y)*\sigma(x) \mid \sigma \in C_3\}$

Collecting Semantics: Example

- (assume x $\geq$ 0 initially)



$C_1 = \{\sigma \mid \sigma(x) \geq 0\}$

$C_2 = \{\sigma[y:=1] \mid \sigma \in C_1\}$

$\cup \{\sigma[x:=\sigma(x)-1] \mid \sigma \in C_4\}$

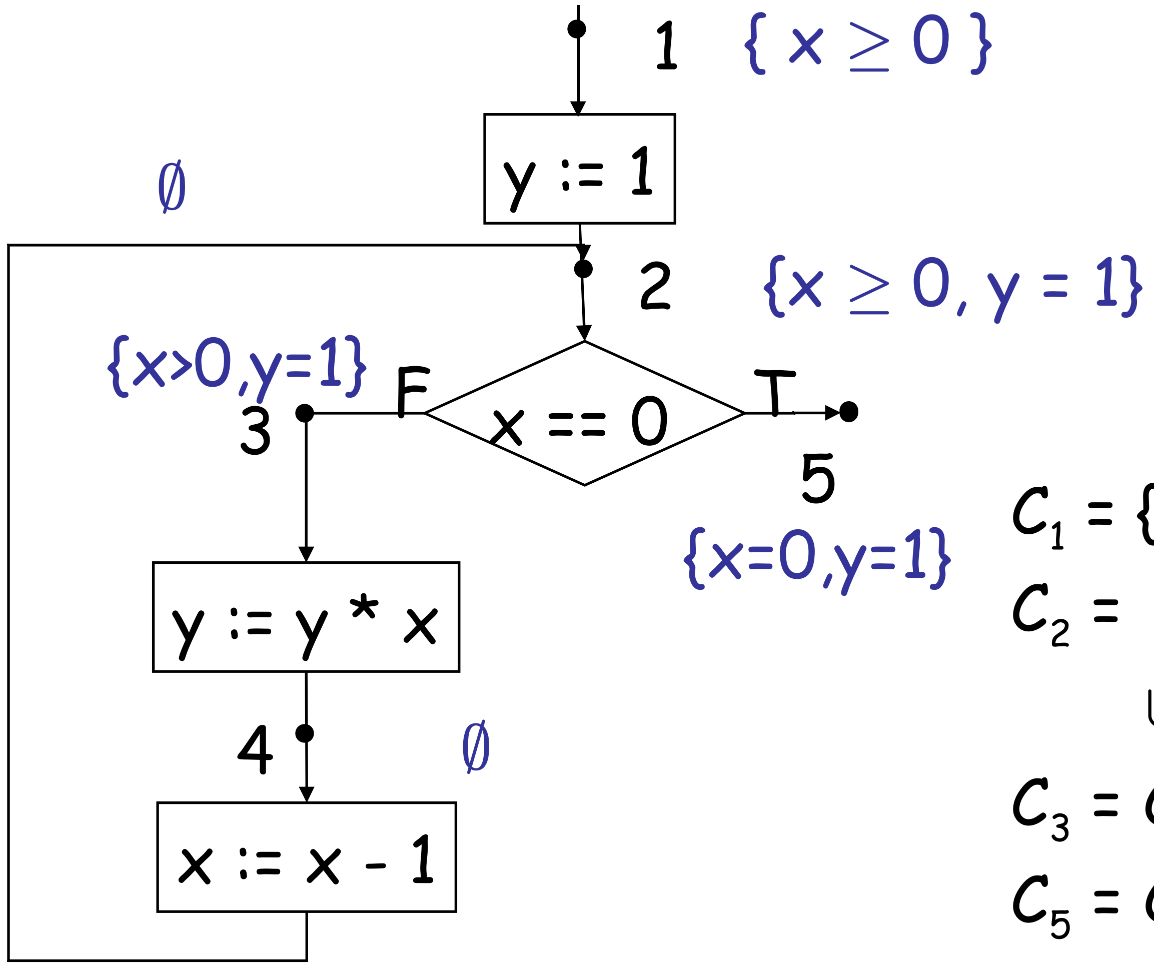
$C_3 = C_2 \cap \{\sigma \mid \sigma(x) \neq 0\}$

$C_5 = C_2 \cap \{\sigma \mid \sigma(x) = 0\}$

$C_4 = \{\sigma[y:=\sigma(y)*\sigma(x) \mid \sigma \in C_3\}$

Collecting Semantics: Example

- (assume $x \geq 0$ initially)



$C_1 = \{\sigma \mid \sigma(x) \geq 0\}$

$C_2 = \{ \sigma[y:=1] \mid \sigma \in C_1\}$
$\quad \cup \{\sigma[x:=\sigma(x)-1] \mid \sigma \in C_4\}$

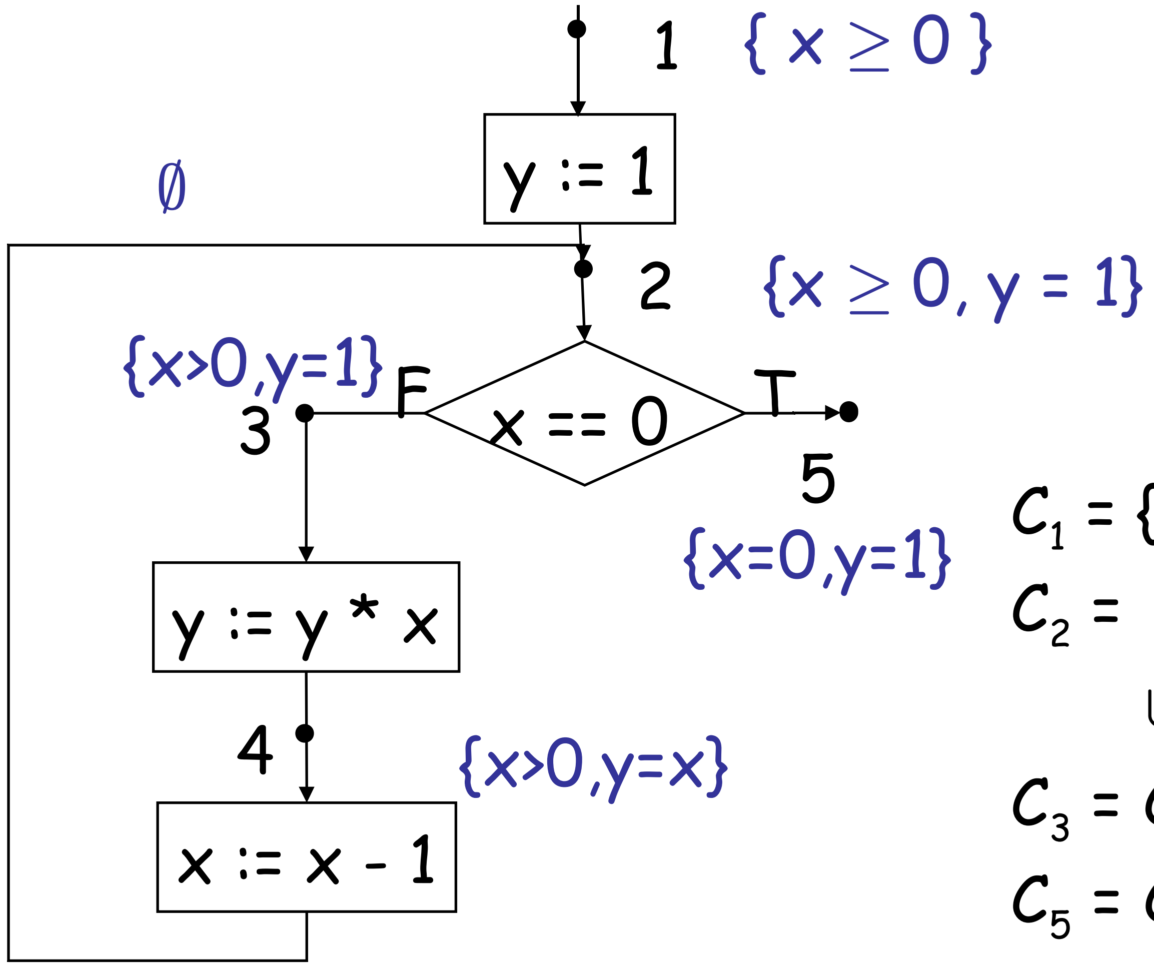
$C_3 = C_2 \cap \{\sigma \mid \sigma(x) \neq 0\}$

$C_5 = C_2 \cap \{\sigma \mid \sigma(x) = 0\}$

$C_4 = \{\sigma[y:=\sigma(y)*\sigma(x) \mid \sigma \in C_3\}$

Collecting Semantics: Example

- (assume $x \geq 0$ initially)



$C_1 = \{\sigma \mid \sigma(x) \geq 0\}$

$C_2 = \{\sigma[y{:=}1] \mid \sigma \in C_1\}$
$\quad \cup \{\sigma[x{:=}\sigma(x)-1] \mid \sigma \in C_4\}$

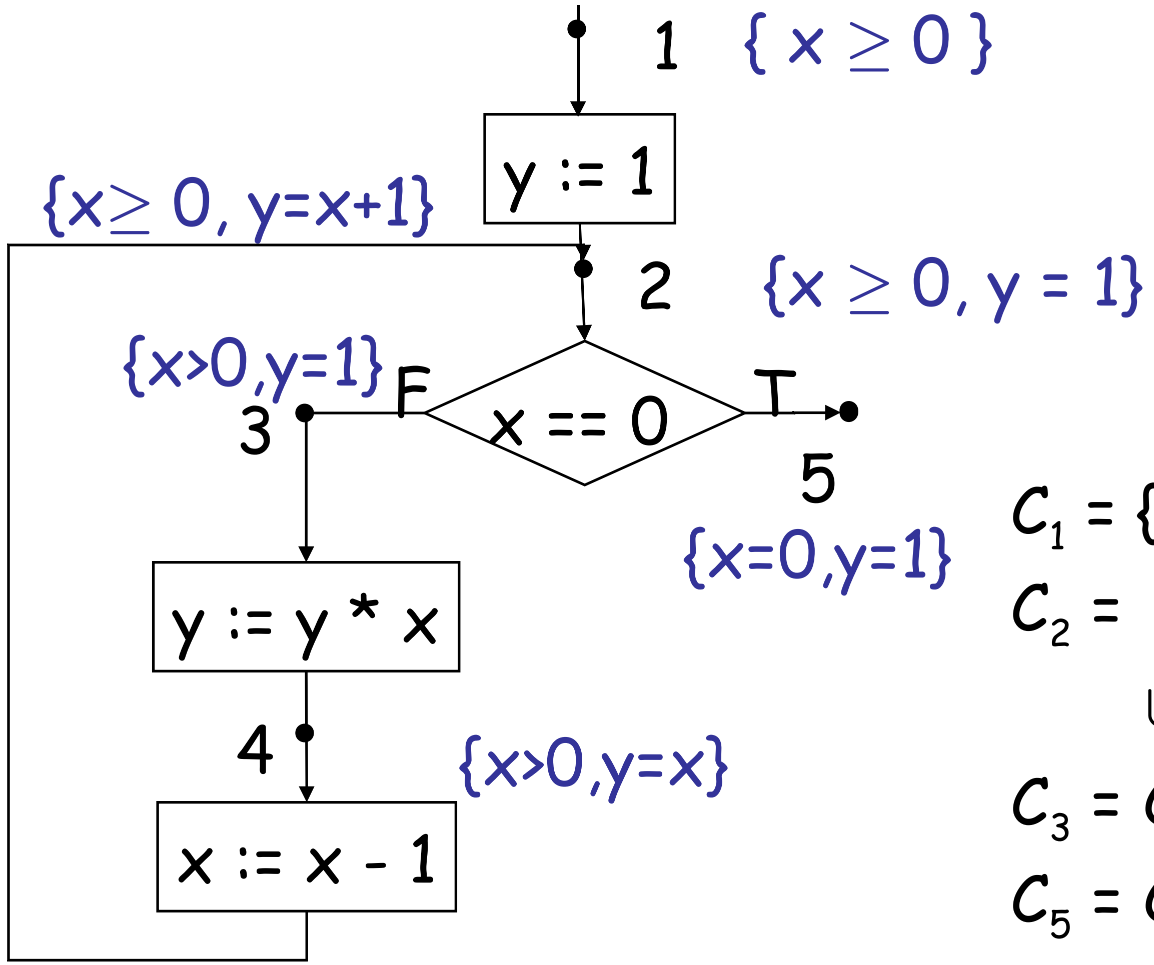
$C_3 = C_2 \cap \{\sigma \mid \sigma(x) \neq 0\}$

$C_5 = C_2 \cap \{\sigma \mid \sigma(x) = 0\}$

$C_4 = \{\sigma[y{:=}\sigma(y)*\sigma(x) \mid \sigma \in C_3\}$

Collecting Semantics: Example

- (assume x $\geq$ 0 initially)



$C_1 = \{\sigma \mid \sigma(x) \geq 0\}$

$C_2 = \{ \sigma[y{:=}1] \mid \sigma \in C_1\}$
$\quad \cup \{\sigma[x{:=}\sigma(x)-1] \mid \sigma \in C_4\}$

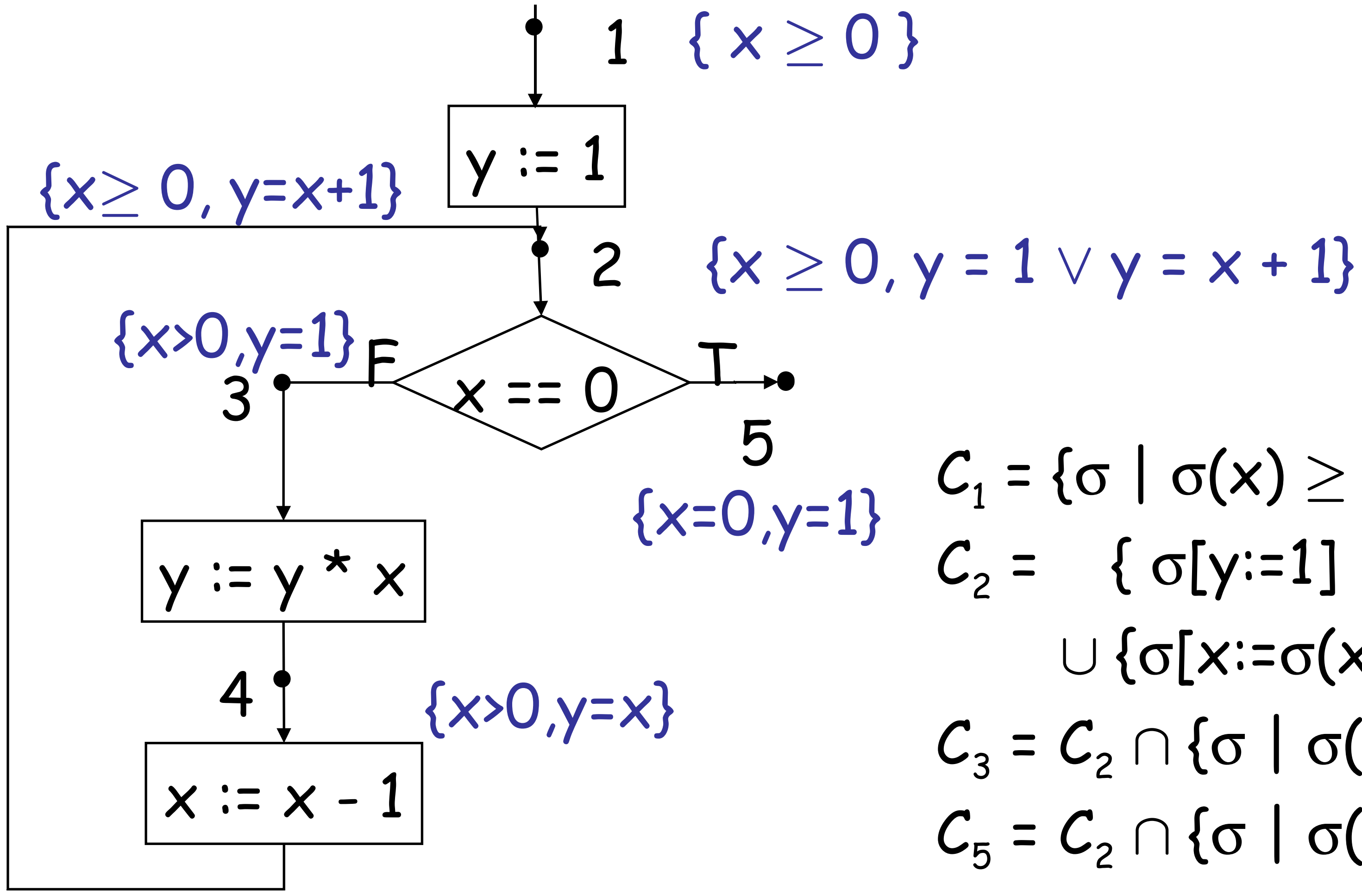
$C_3 = C_2 \cap \{\sigma \mid \sigma(x) \neq 0\}$

$C_5 = C_2 \cap \{\sigma \mid \sigma(x) = 0\}$

$C_4 = \{\sigma[y{:=}\sigma(y)*\sigma(x) \mid \sigma \in C_3\}$

Collecting Semantics: Example

- (assume x $\geq$ 0 initially)



$C_1 = \{\sigma \mid \sigma(x) \geq 0\}$

$C_2 = \{ \sigma[y{:=}1] \mid \sigma \in C_1\}$
$\quad \cup \{\sigma[x{:=}\sigma(x)\text{-}1] \mid \sigma \in C_4\}$

$C_3 = C_2 \cap \{\sigma \mid \sigma(x) \neq 0\}$

$C_5 = C_2 \cap \{\sigma \mid \sigma(x) = 0\}$

$C_4 = \{\sigma[y{:=}\sigma(y)*\sigma(x) \mid \sigma \in C_3\}$

Q: Theatre (006 / 842)

- Name the 1879 Gilbert & Sullivan operetta parodied by the following quote:
  - *I am the very model of a Newsgroup personality.*
  - *I intersperse obscenity with tedious banality.*
  - *Addresses I have plenty of, both genuine and ghosted too,*
  - *On all the countless newsgroups that my drivel is cross-posted to.*

Q: Movies (387 / 842)

- Name the movie quoted below and also name either character or either character's actor. In this 1987 Mel Brooks spoof, one character is revealed to be another character's *"father's brother's nephew's cousin's former roommate."*

Q: TV Music (040 / 842)

- Fill in the three blanks in this **Flintstones** theme song snippet:
  - *Let's ride with the family down the street*
  - *Through the courtesy of <u>blank</u> <u>blank</u> <u>blank</u>*
  - *When you're with the Flintstones*
  - *Have a yabba dabba doo time*

Abstract Interpretation

- Pick a complete lattice A (abstractions for $\mathcal{P}(\Sigma)$)
  - Along with a monotonic abstraction $\alpha : \mathcal{P}(\Sigma) \to A$
  - Alternatively, pick $\beta : \Sigma \to A$
  - This uniquely defines its Galois connection γ
- Take the relations between C_i and move them to the abstract domain:

$$a : \text{Label} \to A$$

- Assignment

Concrete: $C_j = \{\sigma[x := n] \mid \sigma \in C_i \wedge [\![e]\!]\sigma = n\}$

Abstract: $a_j = \alpha \{\sigma[x := n] \mid \sigma \in \gamma(a_i) \wedge [\![e]\!]\sigma = n\}$

Abstract Interpretation

- Conditional

  Concrete: $C_j = \{ \sigma \mid \sigma \in C_i \wedge [\![b]\!]\sigma = \text{false}\}$ and

  $C_k = \{ \sigma \mid \sigma \in C_i \wedge [\![b]\!]\sigma = \text{true}\}$

  Abstract: $a_j = \alpha \{ \sigma \mid \sigma \in \gamma(a_i) \wedge [\![b]\!]\sigma = \text{false}\}$ and

  $a_k = \alpha \{ \sigma \mid \sigma \in \gamma(a_i) \wedge [\![b]\!]\sigma = \text{true}\}$

- Join

  Concrete: $C_k = C_i \cup C_j$

  Abstract: $a_k = \alpha (\gamma(a_i) \cup \gamma(a_j)) = \text{lub } \{a_i, a_j\}$

Least Fixed Points In The Abstract Domain

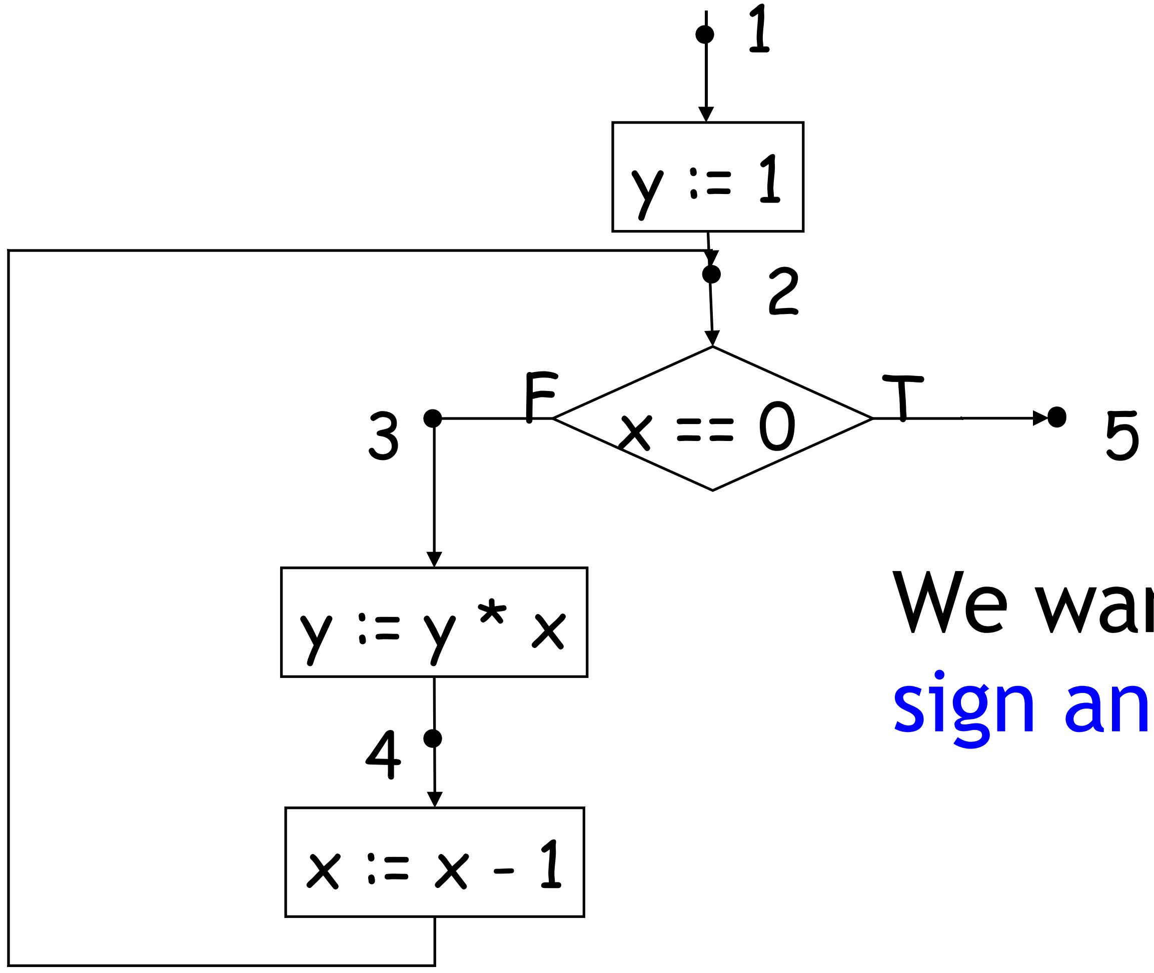
- We have a recursive equation with unknown "a"
  - Defined by a monotonic and continuous function on the domain Labels $\to$ A
- We can use the least fixed-point theorem:
  - Start with $a^0 = \lambda L.\bot$ (aka: $a^0(L) = \bot$)
  - Apply the monotonic function to compute a^{k+1} from a^k
  - Stop when $a^{k+1} = a^k$
- Exactly the same computation as for the collecting semantics
  - What is new?
  - *"There is nothing new under the sun but there are lots of old things we don't know."* – Ambrose Bierce

Least Fixed Points In The Abstract Domain

- We have a hope of termination!
- Classic setup: A has only uninteresting chains (finite number of elements in each chain)
  - A has finite height h (= "finite-height lattice")
- The computation takes $O(h \times |\text{Labels}|^2)$ steps
  - At each step "a" makes progress on at least one label
  - We can only make progress h times
  - And each time we must compute |Labels| elements
- This is a quadratic analysis: good news
  - This is exactly the same as Kildall's 1973 analysis of dataflow's polynomial termination given a finite-height lattice and monotonic transfer functions.

Abstract Interpretation: Example

- Consider the following program, x>0



We want to do the sign analysis on it.

Abstract Domain for Sign Analysis

- Invent the complete sign lattice

$$S = \{ \bot, -, 0, +, \top \}$$

- Construct the complete lattice

$$A = \{x, y\} \to S$$

  - With the usual point-wise ordering
  - Abstract state gives the sign for x and y
- We start with $a^0 = \lambda L.\lambda v \in \{x,y\}.\bot$
  - aka: $a^0(L,v) = \bot$

Let’s Do It!

Label		Iterations →										
1	x	+									+	
	y	⊤									⊤	
2	x	⊥	+			⊤					⊤	
	y	⊥	+						⊤		⊤	
3	x	⊥		+			⊤				⊤	
	y	⊥		+						⊤	⊤	
4	x	⊥			+			⊤			⊤	
	y	⊥			+			⊤			⊤	
5	x	⊥					0				0	
	y	⊥					+			⊤	⊤	

Notes, Weaknesses, Solutions

- We abstracted the state of each variable independently

$$A = \{x, y\} \to \{\bot, -, 0, +, \top\}$$

- We lost relationships between variables
  - e.g., at a point x and y may always have the same sign
  - In the previous abstraction we get $\{x := \top, y := \top\}$ at label 2 (when in fact y is always positive!)
- We can also abstract the state as a whole

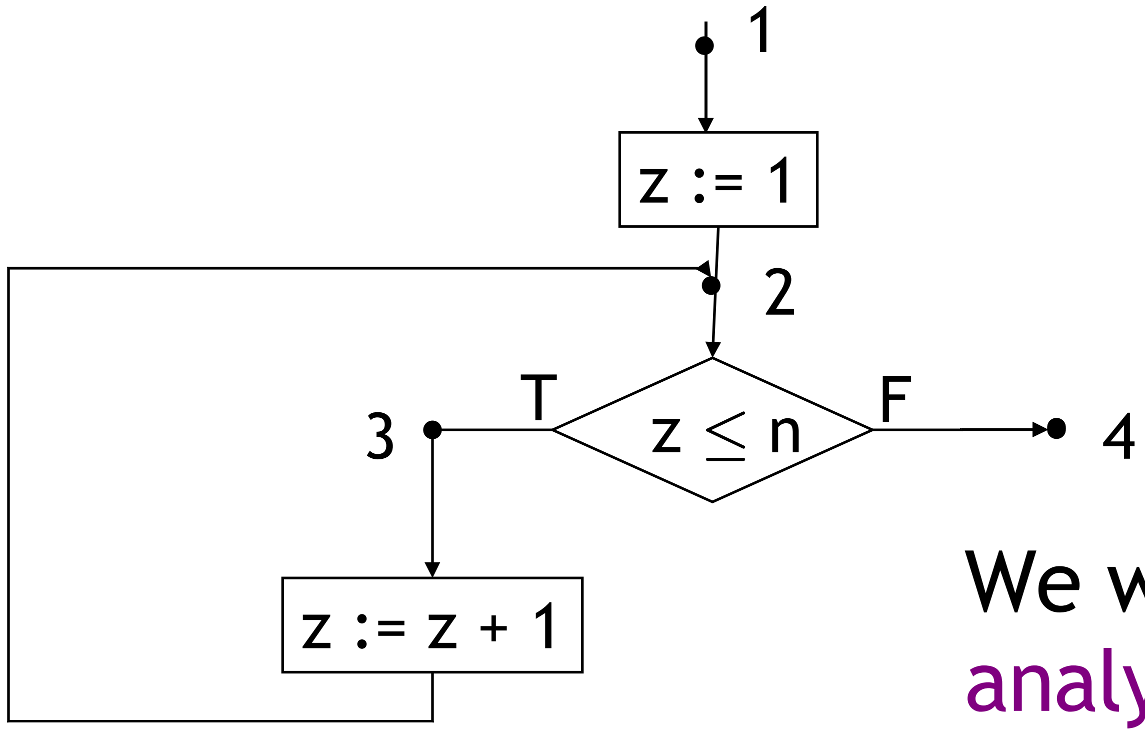
$$A = \mathcal{P}(\{\bot, -, 0, +, \top\} \times \{\bot, -, 0, +, \top\})$$

Other Abstract Domains

- Range analysis
  - Lattice of ranges: R ={ $\bot$, [n..m], ($-\infty$, m], [n, $+\infty$), $\top$ }
  - It is a complete lattice
    - [n..m] $\sqcup$ [n'..m'] = [min(n, n')..max(m,m')]
    - [n..m] $\sqcap$ [n'..m'] = [max(n, n')..min(m, m')]
    - With appropriate care in dealing with ∞
  - $\beta : \mathbb{Z} \to$ R such that β(n) = [n..n]
  - $\alpha : \mathcal{P}(\mathbb{Z}) \to$ R such that α(S) = lub {β(n) | n $\in$ S} = [min(S)..max(S)]
  - γ : R $\to \mathcal{P}$(Z) such that γ(r) = { n | n $\in$ r }
- This lattice has infinite-height chains
  - So the abstract interpretation might not terminate!

Example of Non-Termination

- Consider this (common) program fragment



We want to do range analysis on it.

Example of Non-Termination

- Consider the sequence of abstract states at point 2
  - [1..1], [1..2], [1..3], …
  - The analysis never terminates
  - Or terminates very late if the loop bound is known statically
- It is time to approximate even more: widening
- We redefine the join (lub) operator of the lattice to ensure that from [1..1] upon union with [2..2] the result is [1..+∞) and not [1..2]
- Now the sequence of states is
  - [1..1], [1, +∞), [1, +∞) Done (no more infinite chains)

Formal Definition of Widening
(Cousot 16.399 “Abstract Interpretation”, 2005)

- A widening $\triangledown : (P \times P) \rightarrow P$ on a poset $\langle P, \sqsubseteq \rangle$ satisfies:
  - $\forall\, x, y \in P .\quad x \sqsubseteq (x \triangledown y) \quad \wedge \quad y \sqsubseteq (x \triangledown y)$
  - For all increasing chains $x^0 \sqsubseteq x^1 \sqsubseteq \ldots$ the increasing chain $y^0 =^{def} x^0, \ldots, y^{n+1} =^{def} y^n \triangledown x^{n+1}, \ldots$ is not strictly increasing.
- Two different main uses:
  - Approximate missing lubs. *(Not for us.)*
  - Convergence acceleration. *(This is the real use.)*
    - A widening operator can be used to effectively compute an upper approximation of the least fixpoint of $F \in L \triangledown L$ starting from below when L is computer-representable but does not satisfy the ascending chain condition.

Formal Widening Example

$[1,1] \triangledown [1,2] = [1,+\infty)$

- Range Analysis on z:

```
L0:    z := 1 ;
L1:    while z<99 do
L2:           z := z+1
L3:    done /* z ≥ 99 */
L4:
```

$x^{Li}_j =^{def}$ the jth iterative attempt to compute an abstract value for z at label Li

Recall lub S = [min(S)..max(S)]
lub $\{[2,+\infty),[1,+\infty)\} = \{[1,+\infty)\}$

Original x^i	Widened y^i
$x^{L0}_0 = \perp$	$y^{L0}_0 = \perp$
$x^{L1}_0 = [1,1]$	$y^{L1}_0 = [1,1]$
$x^{L2}_0 = [1,1]$	$y^{L2}_0 = [1,1]$
$x^{L3}_0 = [2,2]$	$y^{L3}_0 = [2,2]$
$x^{L2}_1 = [1,2]$	$y^{L2}_1 = [1,+\infty)$
$x^{L3}_1 = [2,+\infty)$	$y^{L3}_1 = [2,+\infty)$
$x^{L4}_0 = [99,+\infty)$	$y^{L4}_0 = [99,+\infty)$
stable (fewer than 99 iterations!)	

Other Abstract Domains

- Linear relationships between variables
  - A convex polyhedron is a subset of $\mathbb{Z}^k$ whose elements satisfy a number of inequalities:

    $$a_1x_1 + a_2x_2 + \ldots + a_kx_k \geq c_i$$

  - This is a complete lattice; linear programming methods compute lubs
- Linear relationships with at most two variables
  - Convex polyhedra but with $\leq$ 2 variables per constraint
  - Octagons ($x \pm y \geq c$) have efficient algorithms
- Modulus constraints (e.g. even and odd)

Abstract Chatter

- AI, Dataflow and Software Model Checking
  - The big three (aside from flow-insensitive type systems) for program analyses
- Are in fact quite related:
  - David Schmidt. *Data flow analysis is model checking of abstract interpretation*. POPL '98.
- AI is usually flow-sensitive (per-label answer)
- AI can be path-sensitive (if your abstract domain includes $\vee$, for example), which is just where model checking uses BDD's
- Metal, SLAM, ESP, … can all be viewed as AI

Abstract Interpretation Conclusions

- AI is a very powerful technique that underlies a large number of program analyses
- AI can also be applied to functional and logic programming languages
- There are a few success stories
  - Strictness analysis for lazy functional languages
  - PolySpace for linear constraints
- In most other cases however AI is still slow
- When the lattices have infinite height and widening heuristics are used the result becomes unpredictable