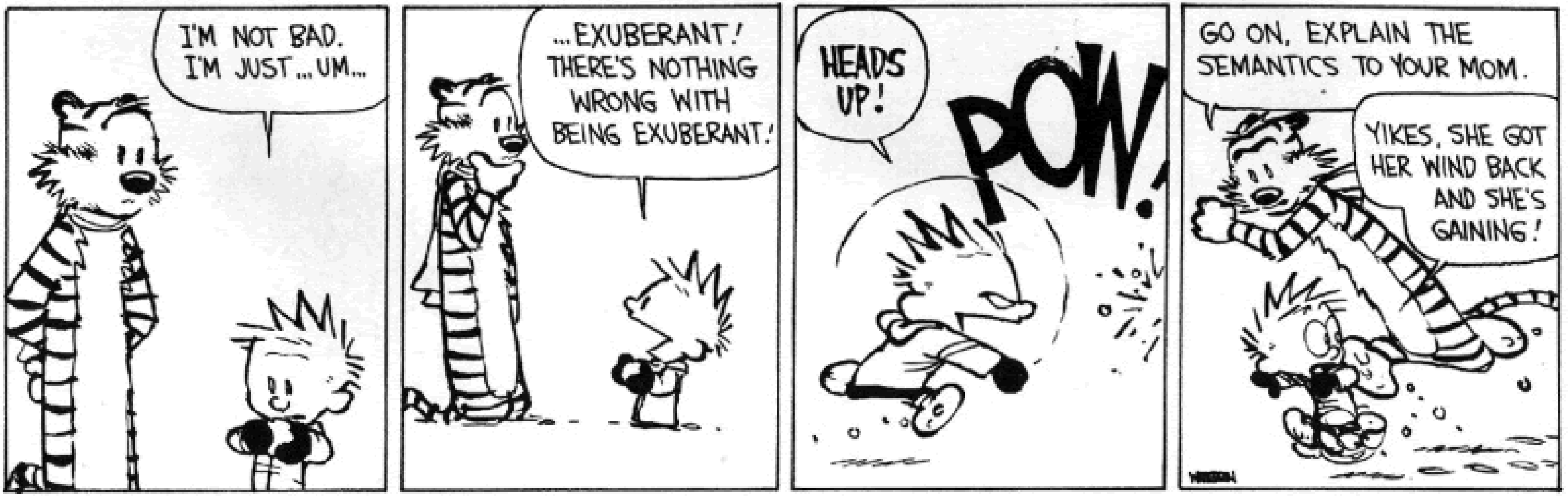
Axiomatic Semantics III

The Verification Crusade



Wei Hu Memorial Homework Award

- Many turned in HW3 code:

```
let rec matches re s =
match re with
| Star(r) -> union (singleton s)
                    (matches (Concat(r,Star(r))) s)
```

- Which is a direct translation of:

$$R[\![r^*]\!]s = \{s\} \cup R[\![rr^*]\!]s$$

or, equivalently:

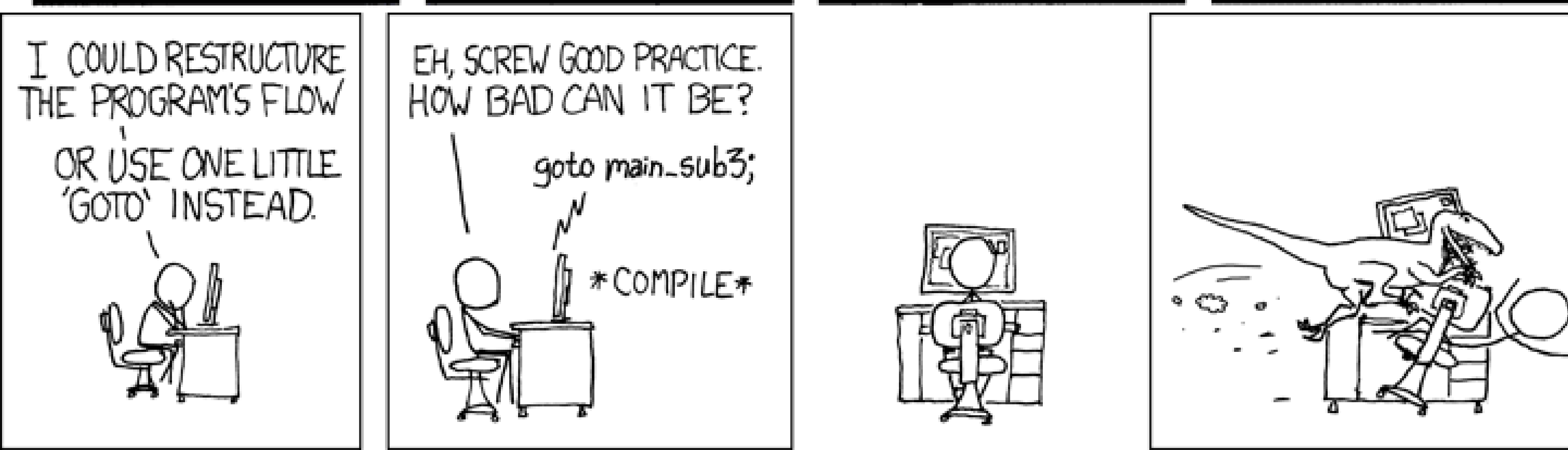
$$R[\![r^*]\!]s = \{s\} \cup \{ y \mid \exists x \in R[\![r]\!]s \ \wedge \ y \in R[\![r^*]\!]x \}$$

- Why doesn't this work?

One-Slide Summary

- **Verification Conditions** make axiomatic semantics practical. We can compute verification conditions **forward** for use on unstructured code (= assembly language). This is sometimes called **symbolic execution**.
- We can add extra invariants or drop paths (dropping is unsound) to help verification condition generation **scale**.
- We can model **exceptions**, **memory** operations and **data structures** using verification condition generation.

Symbolic Execution



Where Are We?

- Axiomatic Semantics: the meaning of a program is what is true after it executes
- Hoare Triples: {A} c {B}
- Weakest Precondition: { WP(c,B) } c {B}
- Verification Condition: $A \Rightarrow VC(c,B) \Rightarrow WP(c,b)$
  - Requires Loop Invariants
  - Backward VC works for structured programs
  - Forward VC (Symbolic Exec) works for assembly
  - Here we are today …

Today's Cunning Plan

- Symbolic Execution & Forward VCGen
- Handling Exponential Blowup
  - Invariants
  - Dropping Paths
- VCGen For Exceptions (double trouble)
- VCGen For Memory (McCarthyism)
- VCGen For Structures (have a field day)
- VCGen For "Dictator For Life"

VC and Invariants

- Consider the Hoare triple:

  $$\{x \leq 0\} \text{ while}_{I(x)} \; x \leq 5 \text{ do } x := x + 1 \; \{x = 6\}$$

- The VC for this is:

  $$x \leq 0 \Rightarrow I(x) \;\wedge\; \forall x.\ (I(x) \Rightarrow (x > 5 \Rightarrow x = 6 \;\wedge\; x \leq 5 \Rightarrow I(x+1)\ ))$$

- Requirements on the invariant:
  - Holds on entry $\forall x.\ x \leq 0 \Rightarrow I(x)$
  - Preserved by the body $\forall x.\ I(x) \wedge x \leq 5 \Rightarrow I(x+1)$
  - Useful $\forall x.\ I(x) \wedge x > 5 \Rightarrow x = 6$
- Check that $I(x) = x \leq 6$ satisfies all constraints

Forward VCGen

- Traditionally the VC is computed <u>backwards</u>
  - That's how we've been doing it in class
  - It works well for structured code
- But it can also be computed <u>forward</u>
  - Works even for un-structured languages (e.g., assembly language)
  - Uses symbolic execution, a technique that has broad applications in program analysis
    - e.g., the PREfix tool (Intrinsa, Microsoft) does this

Forward VC Gen Intuition

- Consider the sequence of assignments

  $$x_1 := e_1;\ x_2 := e_2$$

- The $VC(c, B) = [e_1/x_1]([e_2/x_2]B)$

  $$= [e_1/x_1,\ e_2[e_1/x_1]/x_2]\ B$$

- We can compute the substitution in a forward way using <u>symbolic execution</u> (aka <u>symbolic evaluation</u>)
  - Keep a symbolic state that maps variables to expressions
  - Initially, $\Sigma_0 = \{\ \}$
  - After $x_1 := e_1$, $\Sigma_1 = \{\ x_1 \to e_1\ \}$
  - After $x_2 := e_2$, $\Sigma_2 = \{x_1 \to e_1,\ x_2 \to e_2[e_1/x_1]\ \}$
  - Note that we have applied Σ_1 as a substitution to right-hand side of assignment $x_2 := e_2$

Simple Assembly Language

- Consider the language of instructions:
  I ::= x := e | f() | if e goto L | goto L |
  L: | return | inv e
- The "inv e" instruction is an annotation
  - Says that boolean expression e holds at that point
- Each function f() comes with Pre_f and $Post_f$ annotations (pre- and post-conditions)
- New Notation (yay!): I_k is the instruction at address k

Symex States

- We set up a symbolic execution state:

Σ : Var $\rightarrow$ SymbolicExpressions

$\Sigma(x)$ = the symbolic value of x in state Σ

$\Sigma[x:=e]$ = a new state in which x's value is e

- We use states as substitutions:

$\Sigma(e)$ - obtained from e by replacing x with $\Sigma(x)$

- Much like the opsem so far ...

Symex Invariants

- The symbolic executor tracks invariants passed
- A new part of symex state: $Inv \subseteq \{1 \ldots n\}$
- If $k \in Inv$ then I_k is an invariant instruction that we have already executed
- Basic idea: execute an inv instruction only twice:
  - The first time it is encountered
  - Once more time around an arbitrary iteration

Symex Rules

- Define a VC function as an interpreter:

VC : Address $\times$ SymbolicState $\times$ InvariantState $\to$ Assertion

$\mathrm{VC}(k, \Sigma, \mathrm{Inv}) =$

$\mathrm{VC}(L, \Sigma, \mathrm{Inv})$	if I_k = goto L
$e \Rightarrow \mathrm{VC}(L, \Sigma, \mathrm{Inv}) \quad \wedge$ $\neg\, e \Rightarrow \mathrm{VC}(k+1, \Sigma, \mathrm{Inv})$	if I_k = if e goto L
$\mathrm{VC}(k+1, \Sigma[x{:=}\Sigma(e)], \mathrm{Inv})$	if I_k = x := e
$\Sigma(\mathrm{Post}_{\text{current-function}})$	if I_k = return
$\Sigma(\mathrm{Pre}_f) \quad \wedge$ $\forall a_1..a_m.\Sigma'(\mathrm{Post}_f) \Rightarrow$ $\mathrm{VC}(k+1, \Sigma', \mathrm{Inv})$ (where $y_1, \ldots, y_m$ are modified by f) and $a_1, \ldots, a_m$ are fresh parameters and $\Sigma' = \Sigma[y_1 := a_1, \ldots, y_m := a_m]$	if I_k = f()

Symex Invariants (2a)

Two cases when seeing an invariant instruction:

1. We see the invariant for the first time
   - I_k = inv e
   - $k \notin$ Inv (= "not in the set of invariants we've seen")
   - Let $\{y_1, \ldots, y_m\}$ = the variables that could be modified on a path from the invariant back to itself
   - Let $a_1, \ldots, a_m$ be fresh new symbolic parameters

$\mathrm{VC}(k, \Sigma, \mathrm{Inv}) =$

$$\Sigma(e) \wedge \forall a_1 \ldots a_m.\ \Sigma'(e) \Rightarrow \mathrm{VC}(k+1, \Sigma', \mathrm{Inv} \cup \{k\}])$$

with $\Sigma' = \Sigma[y_1 := a_1, \ldots, y_m := a_m]$

(like a function call)

Symex Invariants (2b)

- We see the invariant for the second time
  - I_k = inv E
  - $k \in Inv$

  $VC(k, \Sigma, Inv) = \Sigma(e)$

  (like a function return)
- Some tools take a more simplistic approach
  - Do not require invariants
  - Iterate through the loop a fixed number of times
  - PREfix, versions of ESC (DEC/Compaq/HP SRC)
  - Sacrifice completeness for usability

Symex Summary

- Let $x_1, \ldots, x_n$ be all the variables and $a_1, \ldots, a_n$ fresh parameters
- Let Σ_0 be the state $[x_1 := a_1, \ldots, x_n := a_n]$
- Let $\emptyset$ be the empty Inv set

- For all functions f in your program, prove:

$$\forall a_1 \ldots a_n.\ \Sigma_0(\mathrm{Pre}_f) \Rightarrow \mathrm{VC}(f_{entry}, \Sigma_0, \emptyset)$$

- If you start the program by invoking any f in a state that satisfies Pre_f, then the program will execute such that
  - At all "inv e" the e holds, and
  - If the function returns then Post_f holds
- Can be proved w.r.t. a real interpreter (operational semantics)
- Or via a proof technique called co-induction (or, assume-guarantee)

Forward VCGen Example

- Consider the program

Precondition: $x \leq 0$

```
Loop: inv x ≤ 6
        if x > 5 goto End
        x := x + 1
        goto Loop
End:  return
```

Postconditon: $x = 6$

Forward VCGen Example (2)

$$
\begin{array}{l}
\forall x. \\
\quad x \leq 0 \Rightarrow \\
\quad\quad x \leq 6 \wedge \\
\quad\quad\quad \forall x'. \\
\quad\quad\quad\quad (x' \leq 6 \Rightarrow \\
\quad\quad\quad\quad\quad x' > 5 \Rightarrow x' = 6 \\
\quad\quad\quad\quad\quad\quad\quad \wedge \\
\quad\quad\quad\quad\quad x' \leq 5 \Rightarrow x' + 1 \leq 6 \;)
\end{array}
$$

- VC contains both proof obligations and assumptions about the control flow

VCs Can Be Large

- Consider the sequence of conditionals

  (if $x < 0$ then $x := - x$); (if $x \leq 3$ then $x += 3$)

  – With the postcondition $P(x)$

- The VC is

$$
\begin{array}{lll}
x < 0 \land -x \leq 3 & \Rightarrow P(-x + 3) & \land \\
x < 0 \land -x > 3 & \Rightarrow P(-x) & \land \\
x \geq 0 \land x \leq 3 & \Rightarrow P(x + 3) & \land \\
x \geq 0 \land x > 3 & \Rightarrow P(x\ ) &
\end{array}
$$

- There is one conjunct for each path

  $\Rightarrow$ exponential number of paths!

  – Conjuncts for infeasible paths have un-satisfiable guards!

- Try with $P(x) = x \geq 3$

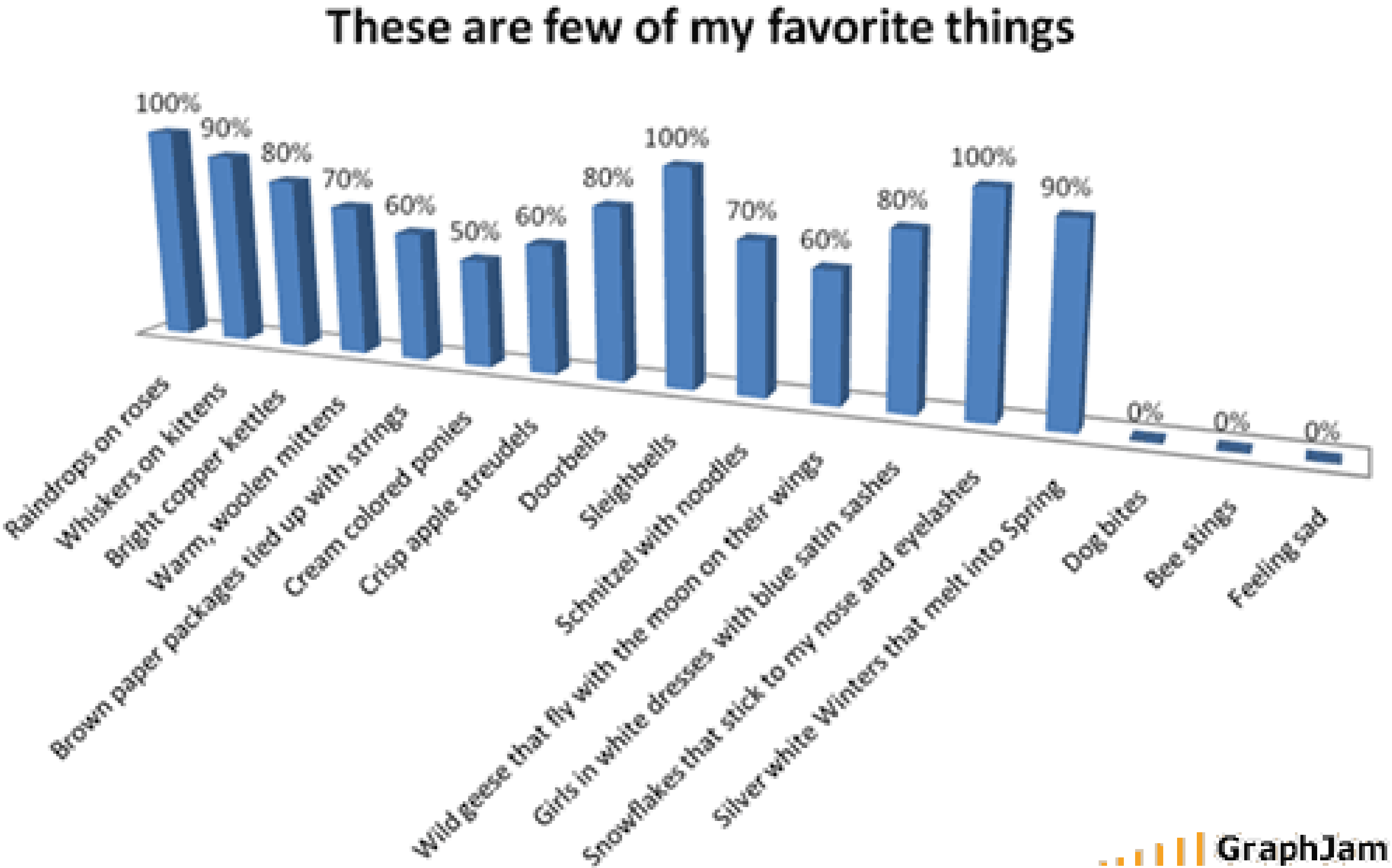
Q: Theatre (019 / 842)

- Name the composer or the title of the 1937 musical that includes the lyrics: *"O Fortuna, velut luna statu variabilis, semper crescis aut decrescis; vita detestabilis nunc obdurat et tunc curat ludo mentis aciem, egestatem, potestatem dissolvit ut glaciem."*

Q: Music (142 / 842)

- Give the next line in 2 of the following 4 song lyrics:
  - "Hello darkness, my old friend"
  - "I'm gonna lay down my sword and shield"
  - "Make new friends but keep the old"
  - "Raindrops on roses and whiskers on kittens"

Answer Key?


These are few of my favorite things
100%
90%
80%
70%
60%
50%
60%
80%
100%
70%
60%
80%
100%
90%
0%
0%
0%
Raindrops on roses
Whiskers on kittens
Bright copper kettles
Warm, woolen mittens
Brown paper packages tied up with strings
Cream colored ponies
Crisp apple streudels
Doorbells
Sleighbells
Schnitzel with noodles
Wild geese that fly with the moon on their wings
Girls in white dresses with blue satin sashes
Snowflakes that stick to my nose and eyelashes
Silver white Winters that melt into Spring
Dog bites
Bee stings
Feeling sad
GraphJam

English Prose

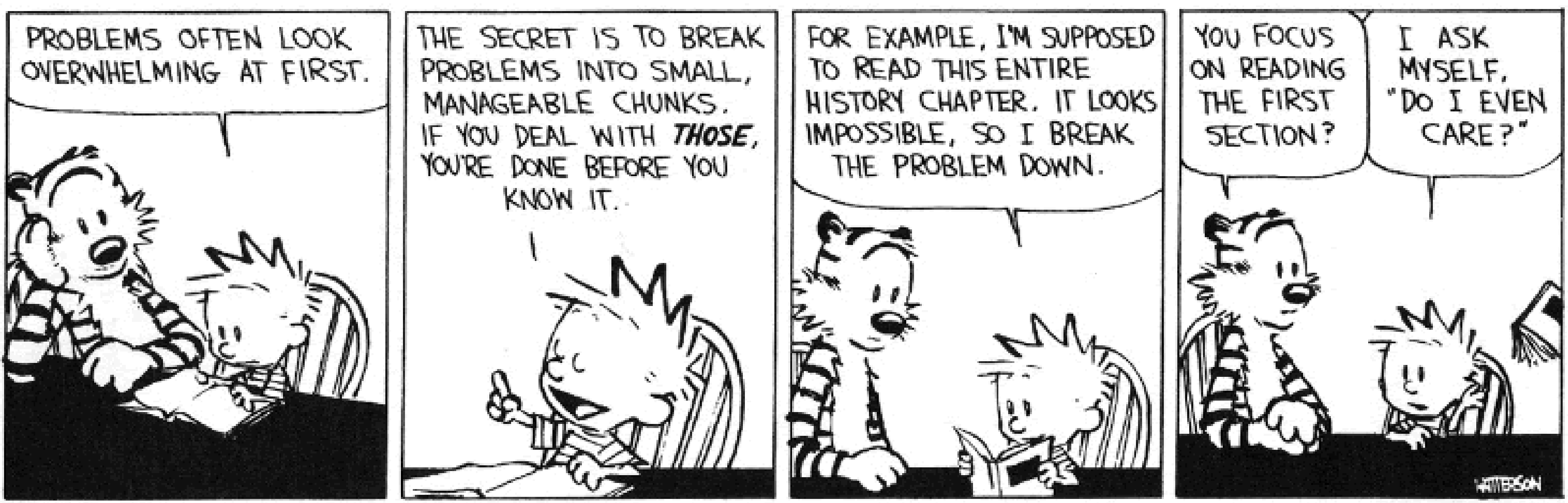
341. Van and Hitomi walked an inaudible distance from those guy's Van was hanging out with.
253. However, when he got into his chamber and sat down with a blank canvas propped up on its easel, his vision vanished as if it were nothing but a floating dust moat.
352. "Good evening my league." He picked her up by the wrist. "I think that you and I have some talking to do, actually I have a preposition"

Q: Movie Music (437 / 842)

- This most common word in the 1991 Disney song **Belle** remains the same in the French localization of the movie.

VCs Can Be Exponential

- VCs are exponential in the size of the source because they attempt relative completeness:
  - Perhaps the correctness of the program must be argued independently for each path
- Unlikely that the programmer wrote a program by considering an exponential number of cases
  - But possible. Any examples? Any solutions?



VCs Can Be Exponential

- VCs are exponential in the size of the source because they attempt relative completeness:
  - Perhaps the correctness of the program must be argued independently for each path
- Standard Solutions:
  - Allow invariants even in straight-line code
  - And thus do not consider all paths independently!

Invariants in Straight-Line Code

- Purpose: modularize the verification task
- Add the command "after c establish Inv"
  - Same semantics as c (Inv is only for VC purposes)

$$\mathrm{VC}(\text{after } c \text{ establish Inv}, P) =_{\text{def}}$$

$$\mathrm{VC}(c, \mathrm{Inv}) \wedge \forall x_i.\ \mathrm{Inv} \Rightarrow P$$

  - where x_i are the ModifiedVars(c)

- Use when c contains many paths

after if $x < 0$ then $x := - x$ establish $x \geq 0$;

if $x \leq 3$ then x += 3 { P(x) }

- VC is now:

$$(x < 0 \Rightarrow - x \geq 0) \wedge (x \geq 0 \Rightarrow x \geq 0) \wedge$$

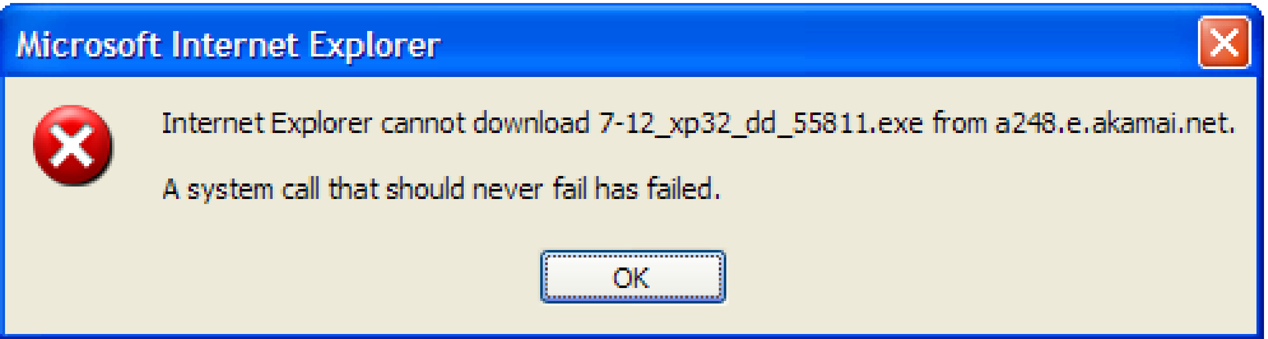
$$\forall x.\ x \geq 0 \Rightarrow (x \leq 3 \Rightarrow P(x+3) \wedge x > 3 \Rightarrow P(x))$$

Dropping Paths

- In absence of annotations, we can drop some paths
- VC(if E then c_1 else c_2, P) = choose one of
  - $E \Rightarrow VC(c_1, P) \wedge \neg E \Rightarrow VC(c_2, P)$ (drop no paths)
  - $E \Rightarrow VC(c_1, P)$ (drops "else" path!)
    $\neg E \Rightarrow VC(c_2, P)$ (drops "then" path!)
- We sacrifice soundness! (we are now unsound)
  - No more guarantees
  - Possibly still a good debugging aid
- Remarks:
  - A recent trend is to sacrifice soundness to increase usability (e.g., Metal, ESP, even ESC)
  - The PREfix tool considers only 50 non-cyclic paths through a function (almost at random)

VCGen for Exceptions

- We extend the source language with exceptions without arguments (cf. HW2):
  - throw — throws an exception
  - try c_1 catch c_2 — executes c_2 if c_1 throws
- Problem:
  - We have non-local transfer of control
  - What is VC(throw, P) ?



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- Problem:
  - We have non-local transfer of control
  - What is VC(throw, P) ?
- Standard Solution: use 2 postconditions
  - One for normal termination
  - One for exceptional termination

VCGen for Exceptions (2)

- $VC(c, P, Q)$ is a precondition that makes c either not terminate, or terminate normally with P or throw an exception with Q
- Rules

$VC(\text{skip}, P, Q) = P$

$VC(c_1; c_2, P, Q) = VC(c_1, VC(c_2, P, Q), Q)$

$VC(\text{throw}, P, Q) = Q$

$VC(\text{try } c_1 \text{ catch } c_2, P, Q) = VC(c_1, P, VC(c_2, P, Q))$

$VC(\text{try } c_1 \text{ finally } c_2, P, Q) = ?$

VCGen Finally

- Given these:

  $VC(c_1; c_2, P, Q) = VC(c_1, VC(c_2, P, Q), Q)$

  $VC(\text{try } c_1 \text{ catch } c_2, P, Q) = VC(c_1, P, VC(c_2, P, Q))$

- Finally is somewhat like "if":

  $VC(\text{try } c_1 \text{ finally } c_2, P, Q) =$

  $VC(c_1, VC(c_2, P, Q), \text{true}) \quad \wedge$

  $VC(c_1, \text{true}, VC(c_2, Q, Q))$

- Which reduces to:

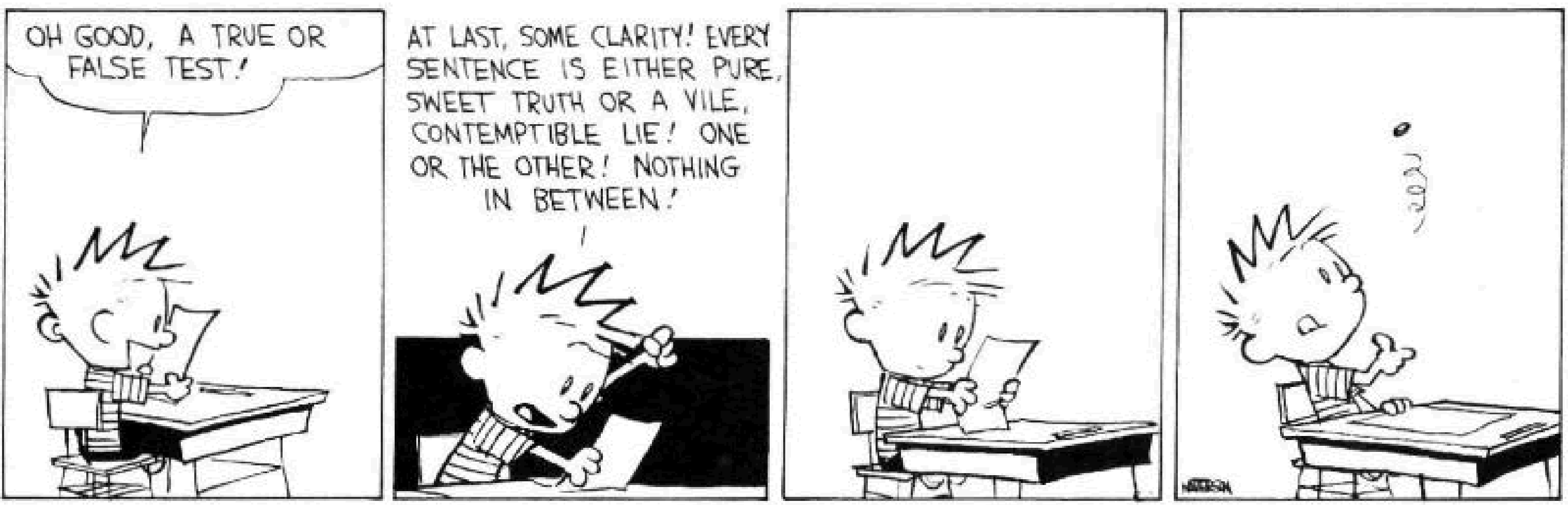
  $$VC(c_1, VC(c_2, P, Q), VC(c_2, Q, Q))$$

Hoare Rules and the Heap

- When is the following Hoare triple valid?

  { A } *x := 5 { *x + *y = 10 }

- A *should be* "*y = 5 or x = y"
- The Hoare rule for assignment would give us:
  - [5/*x](*x + *y = 10) = 5 + *y = 10 =
  - *y = 5   (we lost one case)
- Why didn't this work?



Handling The Heap

- We do not yet have a way to talk about memory (the heap, pointers) in assertions
- Model the state of memory as a symbolic mapping from addresses to values:
  - If A denotes an address and M is a memory state then:
  - sel(M,A) denotes the contents of the memory cell
  - upd(M,A,V) denotes a new memory state obtained from M by writing V at address A

More on Memory

- We allow variables to range over memory states
  - We can quantify over all possible memory states
- Use the special pseudo-variable μ (mu) in assertions to refer to the current memory
- Example:

$$\forall i.\ i \geq 0 \wedge i < 5 \Rightarrow sel(\mu, A + i) > 0$$

says that entries $0..4$ in array A are positive

Hoare Rules: Side-Effects

- To model writes we use memory expressions
  - A memory write changes the value of memory

$$\frac{}{\{ B[upd(\mu, A, E)/\mu] \}\ \mathbf{{*}A := E}\ \{B\}}$$

- Important technique: treat memory as a whole
- And reason later about memory expressions with inference rules such as (McCarthy Axioms, ~'67):

$$sel(upd(M, A_1, V), A_2) = \begin{cases} V & \text{if } A_1 = A_2 \\ sel(M, A_2) & \text{if } A_1 \neq A_2 \end{cases}$$

Memory Aliasing

- Consider again: $\{ A \}$ *x := 5 $\{$ *x + *y = 10 $\}$
- We obtain:

A = [upd(μ, x, 5)/μ] (*x + *y = 10)

= [upd(μ, x, 5)/μ] (sel(μ, x) + sel(μ, y) = 10)

(1) = sel(upd(μ, x, 5), x) + sel(upd(μ, x, 5), y) = 10

= 5 + sel(upd(μ, x, 5), y) = 10

= if x = y then 5 + 5 = 10 else 5 + sel(μ, y) = 10

(2) = x = y or *y = 5

- Up to (1) is theorem generation
- From (1) to (2) is theorem proving

Alternative Handling for Memory

- Reasoning about aliasing can be expensive
  - It is NP-hard (and/or undecideable)
- Sometimes completeness is sacrificed with the following (approximate) rule:

$$\mathrm{sel}(\mathrm{upd}(M, A_1, V), A_2) = \begin{cases} V & \text{if } A_1 = \text{(obviously) } A_2 \\ \mathrm{sel}(M, A_2) & \text{if } A_1 \neq \text{(obviously) } A_2 \\ P & \text{otherwise (p is a fresh new parameter)} \end{cases}$$

- The meaning of "obviously" varies:
  - The addresses of two distinct globals are ≠
  - The address of a global and one of a local are ≠
- PREfix and GCC use such schemes

VCGen Overarching Example

- Consider the program
  - Precondition: *$B : bool \wedge A : array(bool, L)$*

```
1: I := 0
   R := B
3: inv I ≥ 0 ∧ R : bool
   if I ≥  L goto 9
   assert saferd(A + I)
   T := *(A + I)
   I := I + 1
   R := T
   goto 3
9: return R
```

  - Postcondition: *$R : bool$*

VCGen Overarching Example

$$
\begin{array}{l}
\forall A.\ \forall B.\ \forall L.\ \forall \mu \\
\quad B : \mathrm{bool} \wedge A : \mathrm{array}(\mathrm{bool}, L) \Rightarrow \\
\quad\quad 0 \geq 0 \wedge B : \mathrm{bool} \wedge \\
\quad\quad\quad \forall I.\ \forall R. \\
\quad\quad\quad\quad I \geq 0 \wedge R : \mathrm{bool} \Rightarrow \\
\quad\quad\quad\quad\quad I \geq L \Rightarrow R : \mathrm{bool} \\
\quad\quad\quad\quad\quad\quad \wedge \\
\quad\quad\quad\quad\quad I < L \Rightarrow \mathrm{saferd}(A + I) \wedge \\
\quad\quad\quad\quad\quad\quad I + 1 \geq 0 \wedge \\
\quad\quad\quad\quad\quad\quad \mathrm{sel}(\mu, A + I) : \mathrm{bool}
\end{array}
$$

- VC contains both proof obligations and assumptions about the control flow

Mutable Records - Two Models

- Let r : RECORD { f1 : T1; f2 : T2 } END
- For us, records are reference types
- Method 1: one "memory" for each record
  - One index constant for each field
  - r.f1 is sel(r,f1) and r.f1 := E is r := upd(r,f1,E)
- Method 2: one "memory" for each field
  - The record address is the index
  - r.f1 is sel(f1,r) and r.f1 := E is f1 := upd(f1,r,E)
- Only works in strongly-typed languages like Java
  - Fails in C where &r.f2 = &r + sizeof(T1)

VC as a “Semantic Checksum”

- Weakest preconditions are an expression of the program’s semantics:
  - Two equivalent programs have logically equivalent WPs
  - No matter how different their syntax is!

- VC are almost as powerful

VC as a "Semantic Checksum" (2)

- Consider the "assembly language" program to the right

```
x := 4
x := (x == 5)
   assert x : bool
x := not x
   assert x
```

- High-level type checking is not appropriate here
- The VC is: $((4 == 5) : bool) \wedge (not\ (4 == 5))$
- No confusion from reuse of x with different types

Invariance of VC Across Optimizations

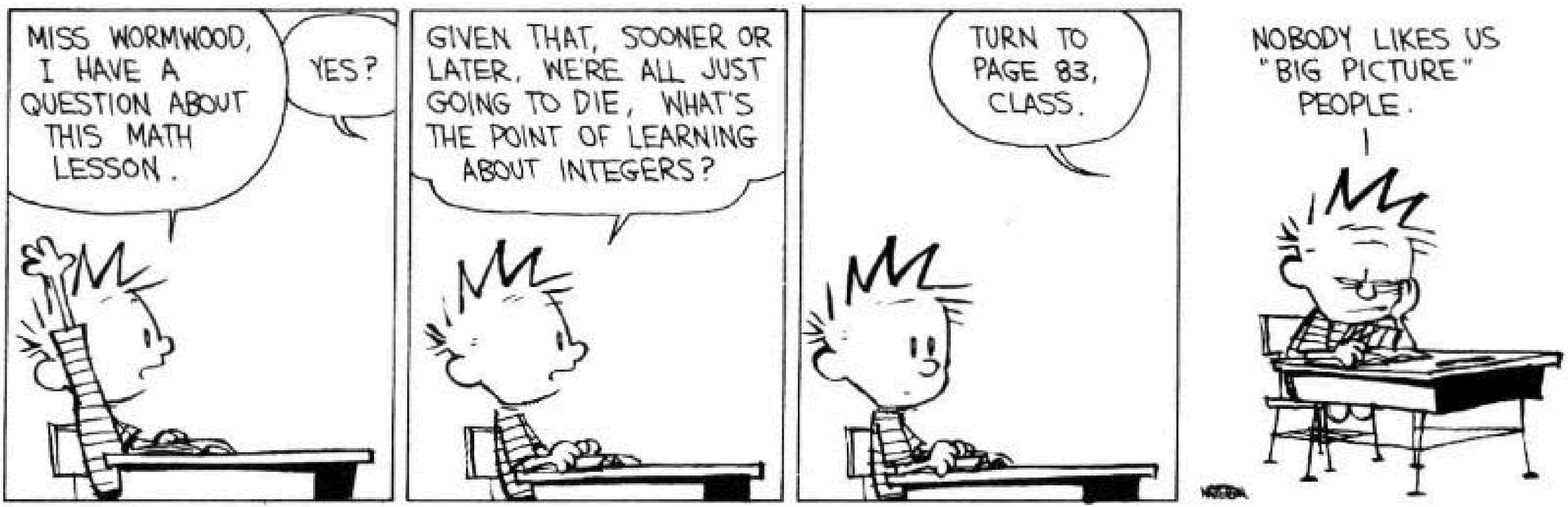
- VC is so good at abstracting syntactic details that it is *syntactically preserved* by many common optimizations
  - Register allocation, instruction scheduling
  - Common subexp elim, constant and copy propagation
  - Dead code elimination
- We have *identical* VCs whether or not an optimization has been performed
  - Preserves syntactic form, not just semantic meaning!
- This can be used to verify correctness of compiler optimizations (Translation Validation)

VC Characterize a Safe Interpreter

- Consider a fictitious "safe" interpreter
  - As it goes along it performs checks (e.g. "safe to read from this memory addr", "this is a null-terminated string", "I have not already acquired this lock")
  - Some of these would actually be hard to implement
- The VC describes all of the checks to be performed
  - Along with their context (assumptions from conditionals)
  - Invariants and pre/postconditions are used to obtain a finite expression (through induction)
- VC is valid $\Rightarrow$ interpreter *never fails*
  - We enforce same level of "correctness"
  - But better (static + more powerful checks)

VC Big Picture

- Verification conditions
  - Capture the semantics of code + specifications
  - Language independent
  - Can be computed backward/forward on structured/unstructured code
  - Make Axiomatic Semantics practical



Invariants Are Not Easy

- Consider the following code from QuickSort

```
int partition(int *a, int L0, int H0, int pivot) {
    int L = L0, H = H0;
    while(L < H) {
        while(a[L] < pivot) L ++;
        while(a[H] > pivot) H --;
        if(L < H) { swap a[L] and a[H] }
    }
    return L
}
```

- Consider verifying only memory safety
- What is the loop invariant for the outer loop ?

Done!

- Questions?