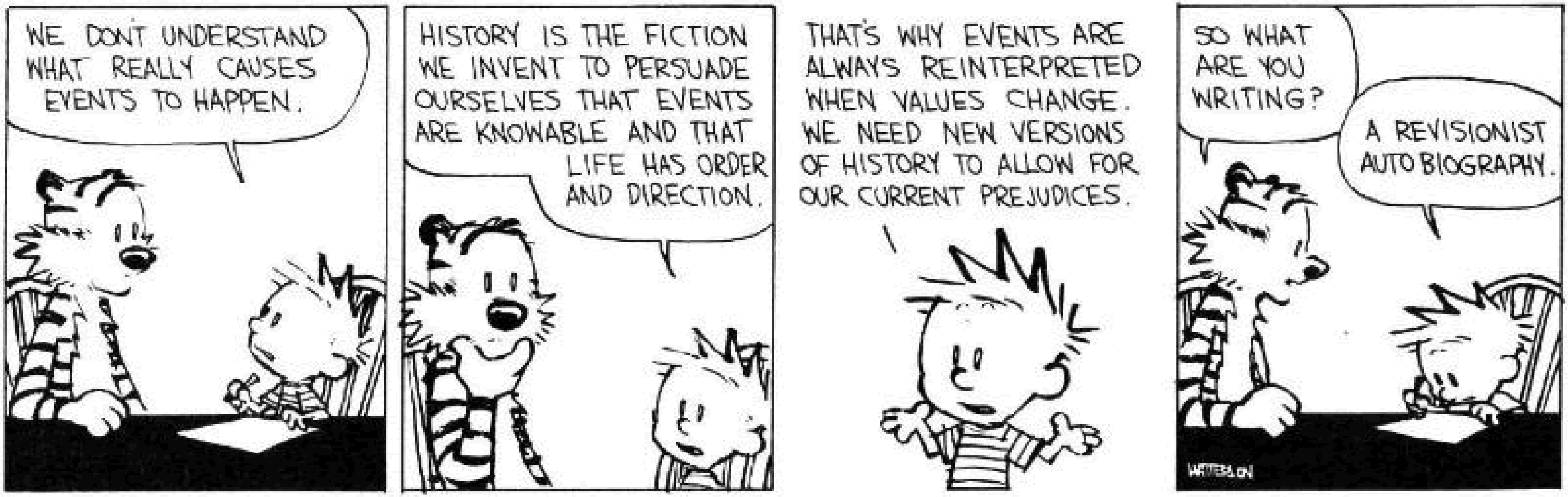
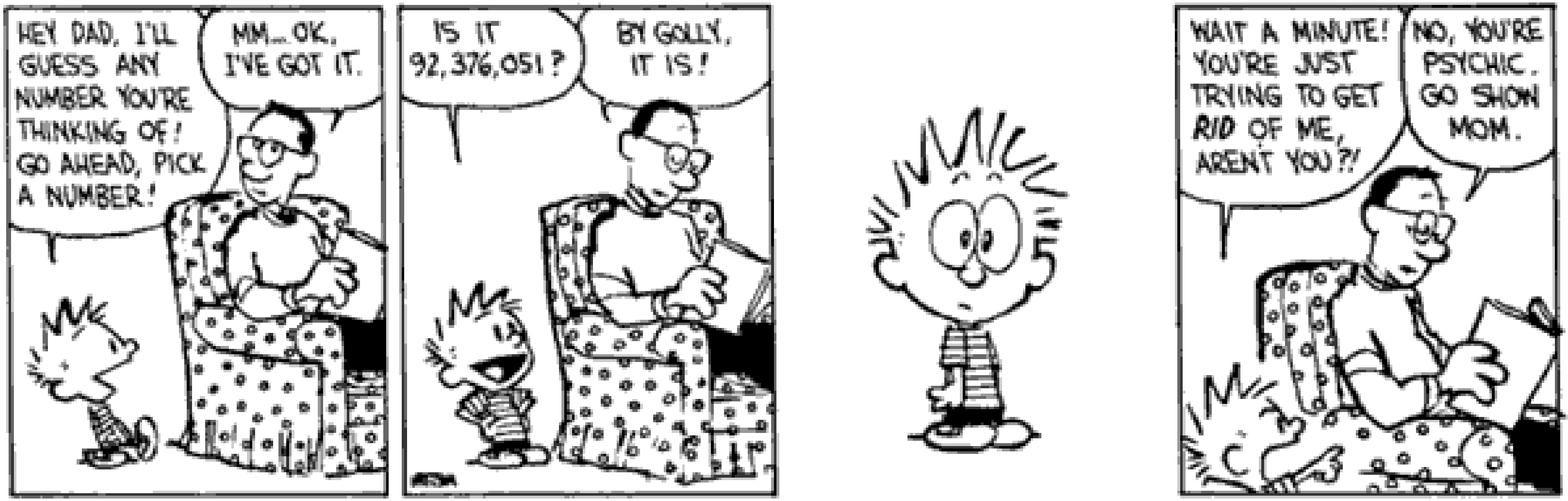


Model Checking

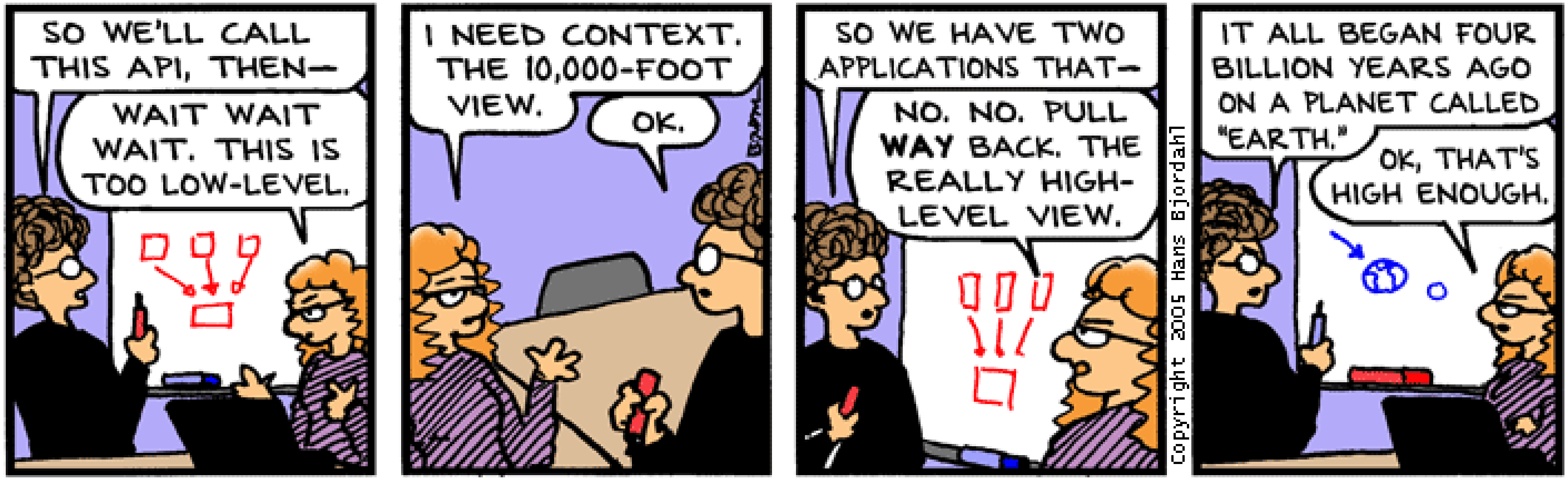


Double Header

- Two Lectures
  - Model Checking
  - Software Model Checking
  - SLAM and BLAST
- "Flying Boxes"
  - It is traditional to describe this stuff (especially SLAM and BLAST) with high-gloss animation.
- Some Key Players:
  - Model Checking: Ed Clarke, Ken McMillan, Amir Pnueli
  - SLAM: Tom Ball, Sriram Rajamani
  - BLAST: Ranjit Jhala, Rupak Majumdar, Tom Henzinger

Who are we again?

- We're going to find critical bugs in important bits of software
  - using PL techniques!
- You will be enthusiastic about this
  - and thus want to learn the gritty details



Bug Bash by Hans Bjordahl http://www.bugbash.net/

Take-Home Message

- **Model checking** is the exhaustive exploration of the **state space** of a system, typically to see if an error state is **reachable**. It produces concrete **counter-examples**.
- The state **explosion problem** refers to the large number of states in the model.
- **Temporal logic** allows you to specify properties with concepts like “eventually” and “always”.

Overarching Plan

- **Model Checking** *(Today)*
  - Transition Systems (Models)
  - Temporal Properties
  - LTL and CTL
  - (Explicit State) Model Checking
  - Symbolic Model Checking
- **Counterexample Guided Abstraction Refinement**
  - Safety Properties
  - Predicate Abstraction ("c2bp")
  - Software Model Checking ("bebop")
  - Counterexample Feasibility ("newton", "hw 5")
  - Abstraction Refinement (weakest pre, thrm prvr)

Spoiler Space

- This stuff really works!
  - This is not ESC or PCC or Denotational Semantics
- Symbolic Model Checking is a massive success in the model-checking field
  - I know people who think Ken McMillan walks on water in a “ha-ha-ha only serious” way
- SLAM took the PL world by storm
  - Spawned multiple copycat projects
  - Incorporated into Windows DDK as “static driver verifier”

Topic:
(Generic) Model Checking

- There are complete courses in model checking; I will skim.
  - *Model Checking* by Edmund C. Clarke, Orna Grumberg, and Doron A. Peled, MIT press
  - *Symbolic Model Checking* by Ken McMillan

Model Checking

- Model checking is an *automated* technique
- Model checking verifies *transition systems*
- Model checking verifies *temporal properties*
- Model checking can be also used for falsification by generating *counter-examples*
- Model Checker: A program that checks if a (transition) system satisfies a (temporal) property

Verification vs. Falsification

- An automated verification tool
  - can report that the system is verified (with a **proof**)
  - or that the system was not verified (with **???**)
- When the system was not verified it would be helpful to explain why
  - Model checkers can output an error counter-example: a concrete execution scenario that demonstrates the error
- Can view a model checker as a falsification tool
  - The main goal is to find bugs
- OK, so what can we verify or falsify?

Temporal Properties

- Temporal Property: A property with time-related operators such as "invariant" or "eventually"
- Invariant(*p*): is true in a state if property *p* is true in every state on all execution paths starting at that state
  - The Invariant operator has different names in different temporal logics:
    - **G, AG, □ ("goal" or "box" or "forall")**
- Eventually(*p*): is true in a state if property *p* is true at some state on every execution path starting from that state
    - **F, AF, ◇ ("diamond" or "future" or "exists")**

An Example Concurrent Program

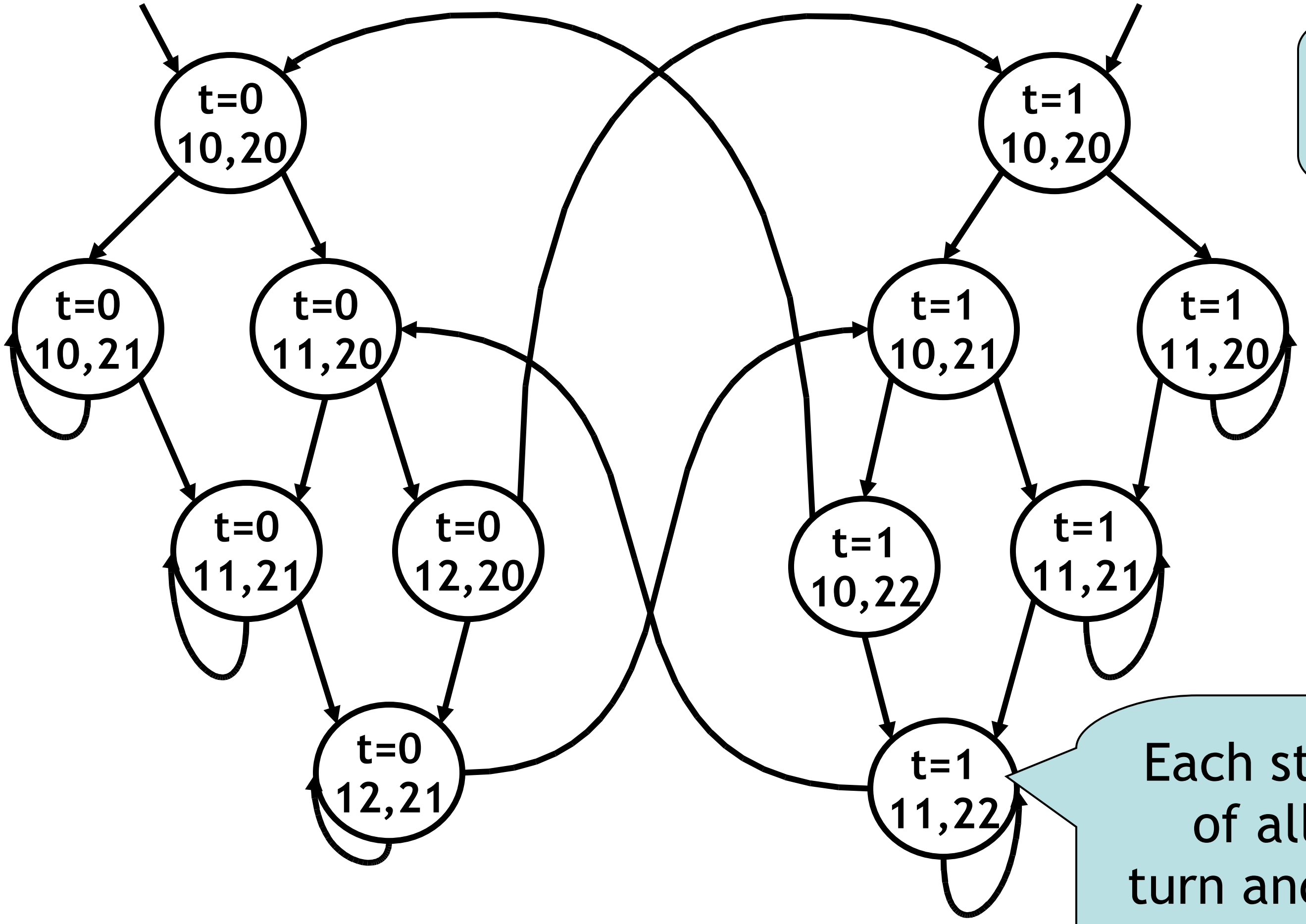
- A simple concurrent mutual exclusion program
- Two processes execute asynchronously
- There is a shared variable **`turn`**
- Two processes use the shared variable to ensure that they are not in the critical section at the same time
- Can be viewed as a "fundamental" program: any bigger concurrent one would include this one

```
10: while True do
11:       wait(turn = 0);
      // critical section
12:       work(); turn := 1;
13:   end while;

|| // concurrently with

20: while True do
21:     wait(turn = 1);
      // critical section
22:     work(); turn := 0;
23: end while
```

Reachable States of the Example Program



Next: formalize this intuition …

Each state is a valuation of all the variables: turn and the two program counters for two processes

Transition Systems

- In model checking the system being analyzed is represented as a labeled transition system

$$T = (S, I, R, L)$$

  - Also called a Kripke Structure
  - S = Set of states // standard FSM
  - $I \subseteq S$ = Set of initial states // standard FSM
  - $R \subseteq S \times S$ = Transition relation // standard FSM
  - $L: S \rightarrow \mathcal{P}(AP)$ = Labeling function // this is new!
- *AP*: Set of atomic propositions (e.g., "x=5"$\in$AP)
  - Atomic propositions capture basic properties
  - For software, atomic props depend on variable values
  - The labeling function labels each state with the set of propositions true in that state

Properties of the Program

- Example: “In all the reachable states (configurations) of the system, the two processes are *never in the critical section at the same time*”
  - Equivalently, we can say that
    - *Invariant*($\neg$(pc1=12 $\wedge$ pc2=22))
- Also: “*Eventually the first process enters* the critical section”
    - *Eventually*(pc1=12)
- “pc1=12”, “pc2=22” are atomic properties

Temporal Logics

- There are four basic temporal operators:
- *X p* = Next p, p holds in the next state
- *G p* = Globally p, p holds in every state, p is an invariant
- *F p* = Future p, p will hold in a future state, p holds eventually
- *p U q* = p Until q, assertion p will hold until q holds
- Precise meaning of these temporal operators are defined on execution paths

Execution Paths

- A path in a transition system is an infinite sequence of states

  $(s_0, s_1, s_2, \ldots)$, such that $\forall i \geq 0.\ (s_i, s_{i+1}) \in R$
- A path $(s_0, s_1, s_2, \ldots)$ is an execution path if $s_0 \in I$
- Given a path $x = (s_0, s_1, s_2, \ldots)$
  - x_i denotes the ith state s_i
  - x^i denotes the ith suffix $(s_i, s_{i+1}, s_{i+2}, \ldots)$

- In some temporal logics one can quantify the paths starting from a state using path quantifiers
  - A : for all paths
  - E : there exists a path

Being Judgmental

- We write

$$x \models p$$

- "the path x makes the predicate p true"
  - x is a path in a transition system
  - p is a temporal logic predicate
- Example:

$$A\ x.\quad x \models G\ (\neg(pc1=12 \wedge pc2=22))$$

Linear Time Logic (LTL)

- LTL properties are constructed from atomic propositions in AP; logical operators $\wedge$, $\vee$, $\neg$; and temporal operators X, G, F, U.
- The semantics of LTL properties is defined on paths:

Given a path x:

$x \models p$ iff $L(x_0, p)$ // atomic prop

$x \models X\ p$ iff $x^1 \models p$ // next

$x \models F\ p$ iff $\exists i \geq 0.\ x^i \models p$ // future

$x \models G\ p$ iff $\forall i \geq 0.\ x^i \models p$ // globally

$x \models p\ U\ q$ iff $\exists i \geq 0.\ x^i \models q$ and $\forall j < i.\ x^j \models p$ // until

Satisfying Linear Time Logic

- Given a transition system T = (S, I, R, L) and an LTL property p, T satisfies p if all paths starting from all initial states I satisfy p
- Example LTL formulas:
  - *Invariant*($\neg$(pc1=12 $\wedge$ pc2=22)):

    $$G(\neg(pc1=12 \wedge pc2=22))$$
  - *Eventually*(pc1=12):

    $$F(pc1=12)$$

Computation Tree Logic (CTL)

- In CTL temporal properties use path quantifiers
  - A : for all paths
  - E : there exists a path
- The semantics of CTL properties is defined on states:

Given a path x

$s \models p \quad \text{iff} \quad L(s, p)$

$s_0 \models EX\ p \quad \text{iff} \quad \exists \text{ a path } (s_0, s_1, s_2, \ldots).\ s_1 \models p$

$s_0 \models AX\ p \quad \text{iff} \quad \forall \text{ paths } (s_0, s_1, s_2, \ldots).\ s_1 \models p$

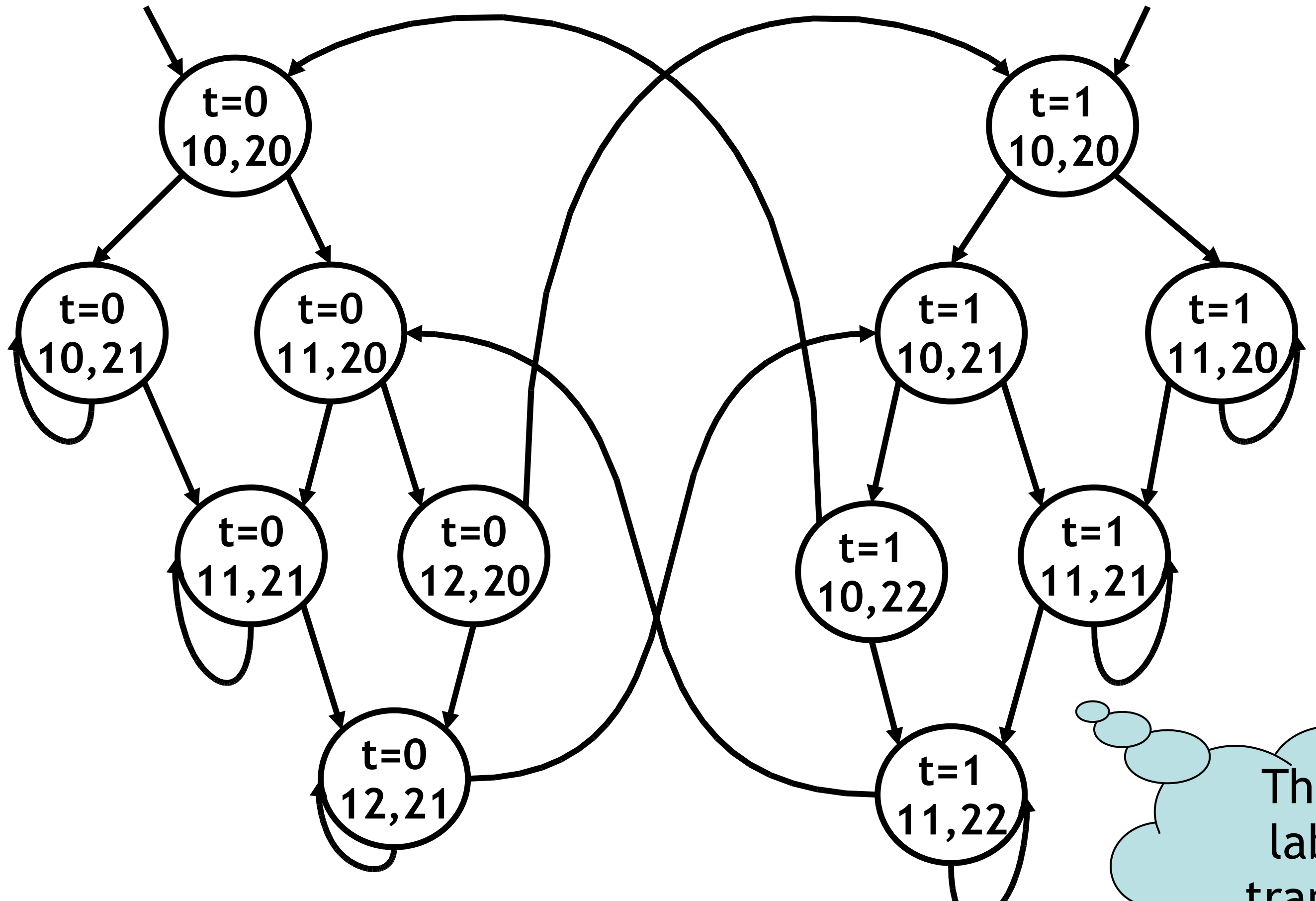
$s_0 \models EG\ p \quad \text{iff} \quad \exists \text{ a path } (s_0, s_1, s_2, \ldots).\ \forall i \geq 0.\ s_i \models p$

$s_0 \models AG\ p \quad \text{iff} \quad \forall \text{ paths } (s_0, s_1, s_2, \ldots).\ \forall i \geq 0.\ s_i \models p$

Linear vs. Branching Time

- LTL is a linear time logic
  - When determining if a path satisfies an LTL formula we are only concerned with a single path
- CTL is a branching time logic
  - When determining if a state satisfies a CTL formula we are concerned with multiple paths
  - In CTL the computation is not viewed as a single path but as a computation tree which contains all the paths
  - The computation tree is obtained by unrolling the transition relation
- The expressive powers of CTL and LTL are incomparable (LTL $\subseteq$ CTL*, CTL $\subseteq$ CTL*)
  - Basic temporal properties can be expressed in both logics
  - Not in this lecture, sorry! (Take a class on Modal Logics)

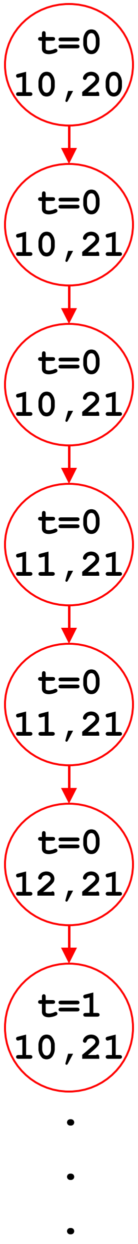
Remember the Example



This is a labeled transition system.

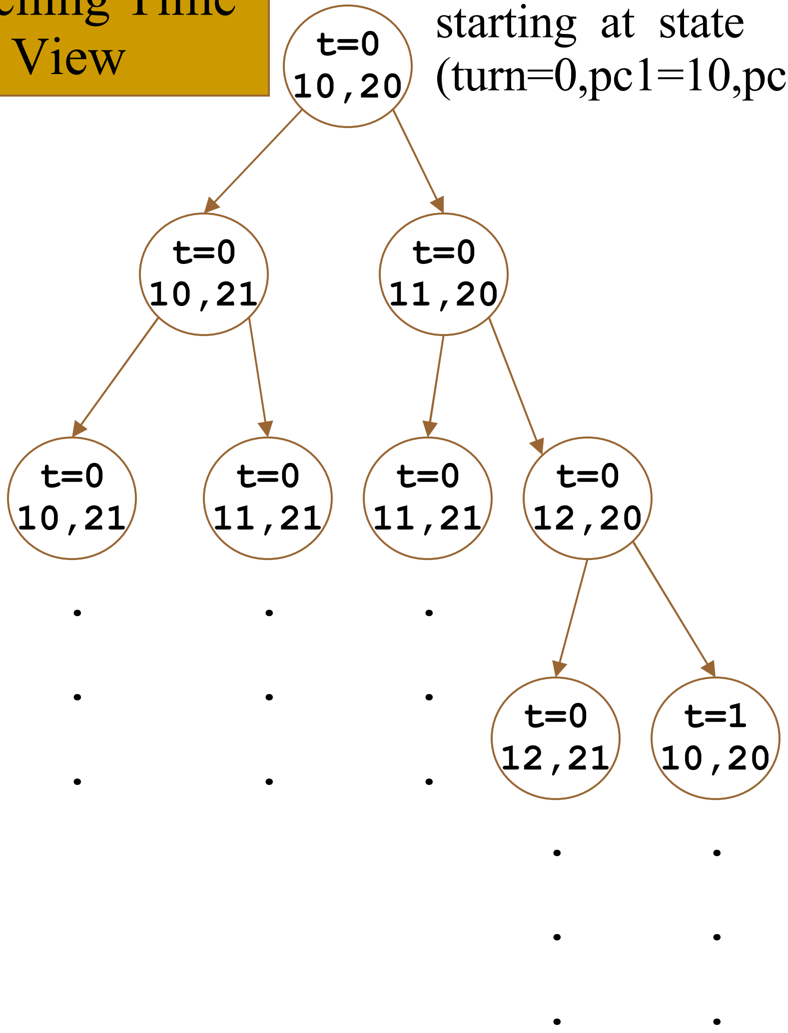
Linear vs. Branching Time

One path starting at state
(turn=0,pc1=10,pc2=20)



Linear Time
View

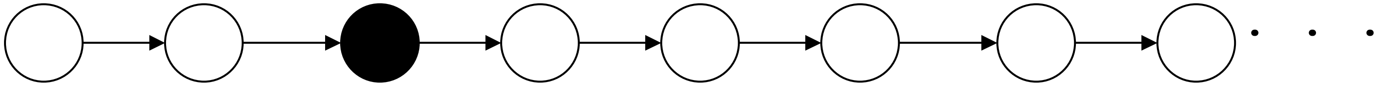
Branching Time
View



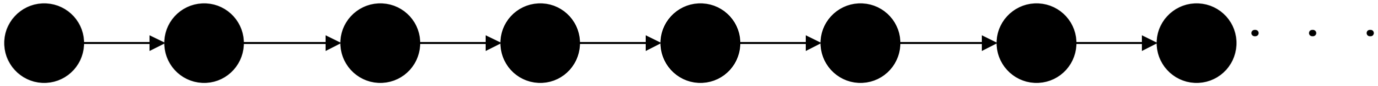
A computation tree
starting at state
(turn=0,pc1=10,pc2=20)

LTL Satisfiability Examples

○ p does not hold ● p holds



On this path: F p holds, G p does not hold, p does not hold, X p does not hold, X (X p) holds, X (X (X p)) does not hold

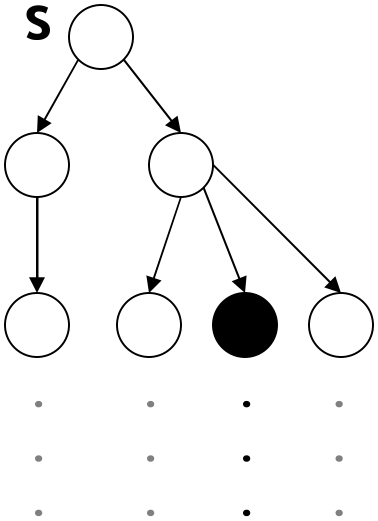


On this path: F p holds, G p holds, p holds, X p holds, X (X p) holds, X (X (X p))) holds

○ p does not hold

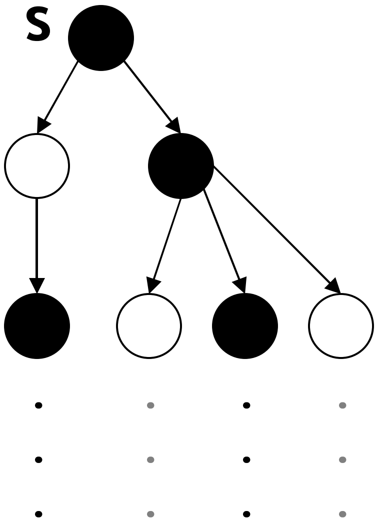
● p holds

CTL Examples



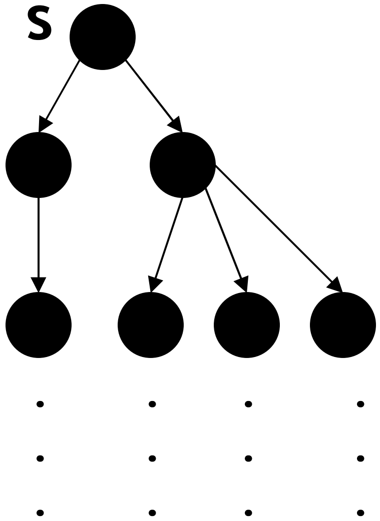
At state s:
EF p, EX (EX p),
AF (¬p), ¬p holds

AF p, AG p,
AG (¬p), EX p,
EG p, p does not hold



At state s:
EF p, AF p,
EX (EX p),
EX p, EG p, p holds

AG p, AG (¬p),
AF (¬p) does not hold



At state s:
EF p, AF p,
AG p, EG p,
Ex p, AX p, p holds

EG (¬ p), EF (¬p),
does not hold

Q: Music (155 / 842)

- Give the age of the *"Dancing Queen"* in ABBA's 1976 #1 hit.

Q: General (468 / 842)

- This country's automobile stickers use the abbreviation **CH** (Confederatio Helvetica). The 1957 Max Miedinger typeface **Helvetica** is also named for this country.

Q: Music (163 / 842)

- This 1980's Hong Kong singer and film star was dubbed "the God of Songs" and one of the "Four Heavenly Kings" and "skilled class" singers of Cantopop. He is considered to be one of the successors of Alan Tam. His most famous songs include "Everyday Loving You More", "Only Thinking Of Going Through Life With You" and "**Goodbye Kiss**". "**Goodbye Kiss**" was one of the best-selling albums of all time with 3 million copies sold in 1993 alone.

Model Checking Complexity

- Given a transition system T = (S, I, R, L) and a CTL formula f
  - One can check if a state of the transition system satisfies the temporal logic formula f in $O(|f| \times (|S| + |R|))$ time
- Given a transition system T = (S, I, R, L) and an LTL formula f
  - One can check if the transition system satisfies the temporal logic formula f in $O(2^{|f|} \times (|S| + |R|))$ time
- Model checking procedures can generate counter-examples without increasing the complexity of verification (= "for free")

Which is slower?

State Space Explosion

- The complexity of model checking increases linearly with respect to the size of the transition system (|S| + |R|)
- However, the size of the transition system (|S| + |R|) is ***exponential*** in the number of variables and number of concurrent processes
- This exponential increase in the state space is called the state space explosion
  - Dealing with it is one of the major challenges in model checking research

Explicit-State Model Checking

- One can show the complexity results using depth first search algorithms
  - The transition system is a directed graph
  - CTL model checking is multiple depth first searches (one for each temporal operator)
  - LTL model checking is one nested depth first search (i.e., two interleaved depth-first-searches)
  - Such algorithms are called explicit-state model checking algorithms *(details on next slides)*

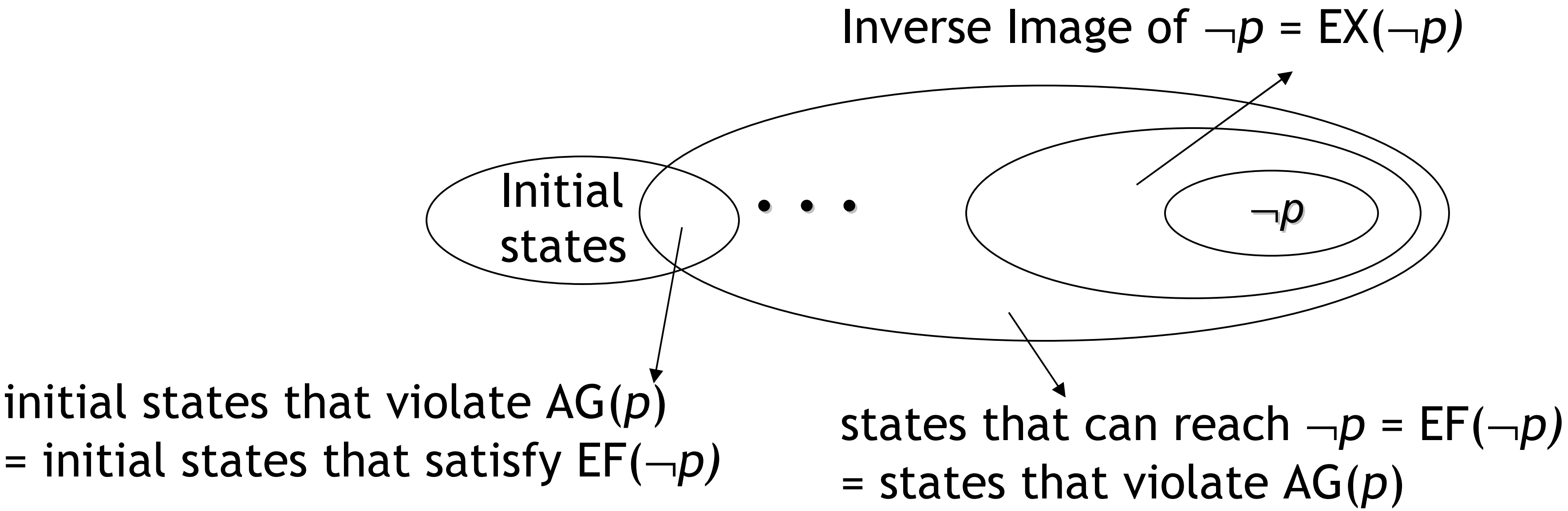
Temporal Properties $\equiv$ Fixpoints

- States that satisfy AG(p) are all the states which are not in EF($\neg p$) (= the states that can reach $\neg p$)
- Compute EF($\neg p$) as the **fixpoint** of Func: $2^S \rightarrow 2^S$
- Given $Z \subseteq S$,
  - $\text{Func}(Z) = \neg p \cup$ **reach-in-one-step**(Z)
  - or $\text{Func}(Z) = \neg p \cup \text{EX}(Z)$

This is called the inverse image of Z

- Actually, EF($\neg p$) is the ***least*-fixpoint** of Func
  - smallest set Z such that $Z = \text{Func}(Z)$
  - to compute the least fixpoint, start the iteration from $Z=\emptyset$, and apply the Func until you reach a fixpoint
  - This can be computed (unlike most other fixpoints)

Pictoral Backward Fixpoint



This fixpoint computation can be used for:

- verification of EF(¬p)
- or falsification of AG(p)

… and a similar forward fixpoint handles the other cases

Symbolic Model Checking

- Symbolic Model Checking represent state sets and the transition relation as *Boolean logic formulas*
  - Fixpoint computations manipulate sets of states rather than individual states
  - Recall: we needed to compute EX(Z), but $Z \subseteq S$
- Forward and backward fixpoints can be computed by iteratively manipulating these formulas
  - Forward, inverse image: Existential variable elimination
  - Conjunction (intersection), disjunction (union) and negation (set difference), and equivalence check
- Use an efficient data structure for manipulation of Boolean logic formulas
  - Binary Decision Diagrams (BDDs)

Binary Decision Diagrams (BDDs)

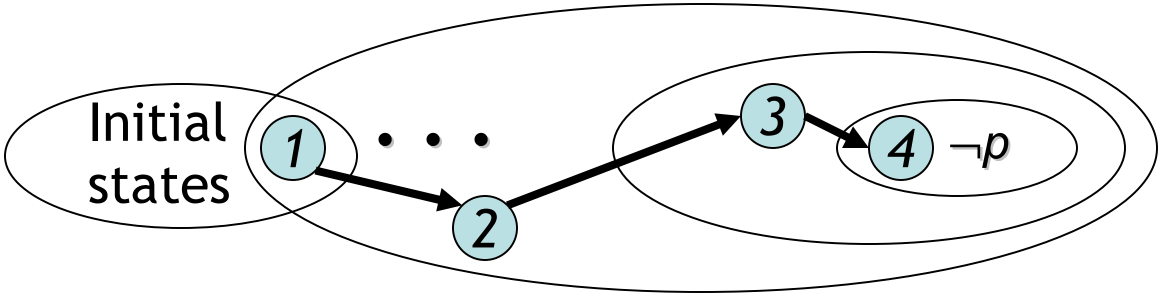
- Efficient representation for boolean functions (a set can be viewed as a function)
- Disjunction, conjunction complexity: at most quadratic
- Negation complexity: constant
- Equivalence checking complexity: constant or linear
- Image computation complexity: can be exponential

Symbolic Model Checking Using BDDs

- SMV (Symbolic Model Verifier) was the first CTL model checker to use a BDD representation
- It has been successfully used in verification
  - of hardware specifications, software specifications, protocols, etc.
- SMV verifies finite state systems
  - It supports both synchronous and asynchronous composition
  - It can handle boolean and enumerated variables
  - It can handle bounded integer variables using a binary encoding of the integer variables
    - It is not very efficient in handling integer variables although this can be fixed

Where's the Beef

- To produce the explicit counter-example, use the "onion-ring method"
  - A counter-example is a valid execution path
  - For each Image Ring (= set of states), find a state and link it with the concrete transition relation R
  - Since each Ring is "reached in one step from previous ring" (e.g., Ring#3 = EX(Ring#4)) this works
  - Each state z comes with L(z) so you know what is true at each point (= what the values of variables are)



Building Up To: Software Model Checking via Counter-Example Guided Abstraction Refinement

- There are easily two dozen SLAM/BLAST/MAGIC papers; I will skim.

Key Terms

- CEGAR = Counterexample guided abstraction refinement. A successful software model-checking approach. Sometimes called "Iterative Abstraction Refinement".
- SLAM = The first CEGAR project/tool. Developed at MSR.
- Lazy Abstraction = A CEGAR optimization used in the BLAST tool from Berkeley.
- Other terms: c2bp, bebop, newton, npackets++, MAGIC, flying boxes, etc.

So … what *is* Counterexample Guided Abstraction Refinement?

- Theorem Proving?
- Dataflow Analysis?
- Model Checking?

Verification by Theorem Proving

```
Example ( ) {
1:  do{
        lock();
        old = new;
        q = q->next;
2:      if (q != NULL){
3:          q->data = new;
            unlock();
            new ++;
        }
4:  } while(new != old);
5:    unlock ();
      return;
}
```

1. Loop Invariants
2. Logical formula
3. Check Validity

Invariant:

$$lock \wedge new = old$$

$$\vee$$

$$\neg\, lock \wedge new \neq old$$

Verification by Theorem Proving

```
Example ( ) {
1: do{
         lock();
         old = new;
         q = q->next;
2:       if (q != NULL){
3:          q->data = new;
            unlock();
            new ++;
         }
4: } while(new != old);
5:    unlock ();
      return;
}
```

1. Loop Invariants
2. Logical formula
3. Check Validity

- Loop Invariants
- Multithreaded Programs
+ Behaviors encoded in logic
+ Decision Procedures

Precise [ESC, PCC]

Verification by Program Analysis

```
Example ( ) {
1: do{○
      lock();●
      old = new;●
      q = q->next;●
2:    if (q != NULL){●
3:       q->data = new;●
         unlock();○
         new ++;○
      }○
4: } while(new != old);○
5:  unlock ();🔴
    return;
}
```

1. Dataflow Facts
2. Constraint System
3. Solve constraints

- Imprecision due to fixed facts

+ Abstraction
+ Type/Flow Analyses

Scalable [CQUAL, ESP, MC]

Verification by Model Checking

```
Example ( ) {
1: do{
         lock();
         old = new;
         q = q->next;
2:       if (q != NULL){
3:            q->data = new;
              unlock();
              new ++;
         }
4: } while(new != old);
5:    unlock ();
      return;
}
```

1. (Finite State) Program
2. State Transition Graph
3. Reachability

- Pgm → Finite state model
- State explosion

\+ State Exploration

\+ Counterexamples

Precise [SPIN, SMV, Bandera, JPF]

One Ring To Rule Them All?

Combining Strengths

Theorem Proving

- Need loop invariants

(will find automatically)

\+ Behaviors encoded in logic

(used to refine abstraction)

\+ Theorem provers

(used to compute successors, refine abstraction)

SLAM

Program Analysis

- Imprecise

(will be precise)

\+ Abstraction

(will shrink the state space we must explore)

Model Checking

- Finite-state model, state explosion

(will find small good model)

\+ State Space Exploration

(used to get a path sensitive analysis)

\+ Counterexamples

(used to find relevant facts, refine abstraction)

Homework

- Read *Lazy Abstraction*
- Optionally read *TAR*