Operational Semantics

One-Slide Summary

- **Operational semantics** are a precise way of specifying how to evaluate a program.
- A **formal semantics** tells you what each expression means.
- Meaning depends on **context**: a **variable environment** will map variables to memory locations and a **store** will map memory locations to values.

Lecture Outline: OpSem

- Motivation
- Notation
- **The Rules**
  - Simple Expressions
  - while
  - new
  - dispatch

Motivation

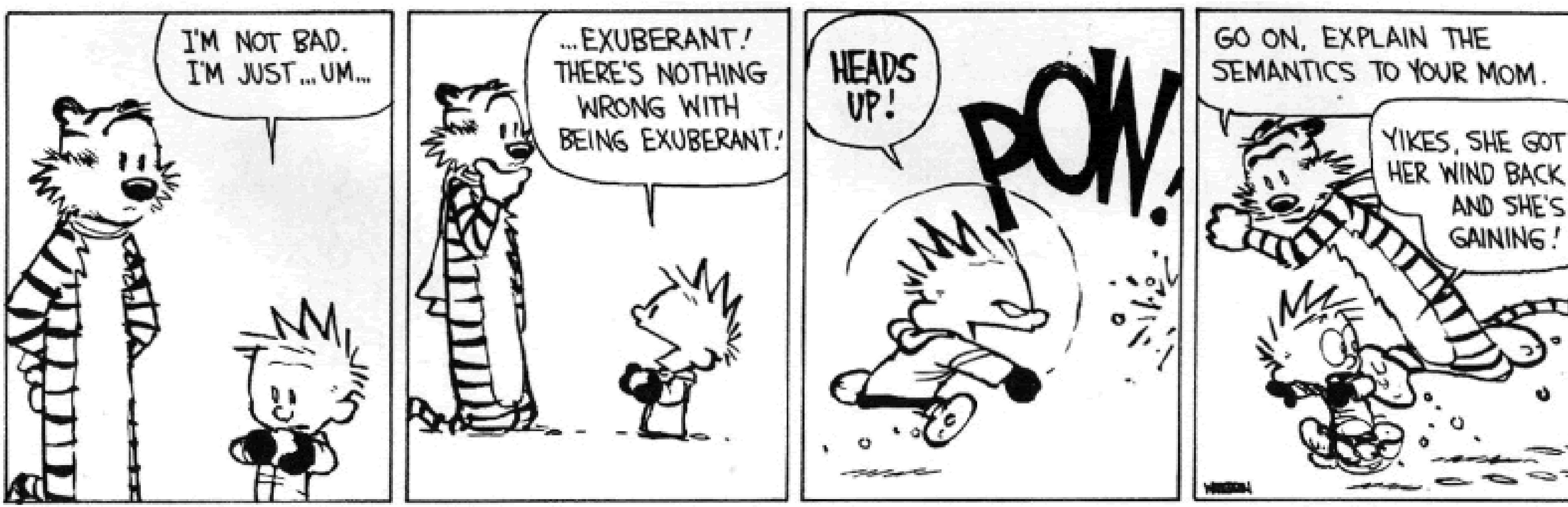
- We must specify for every Cool expression *what happens when it is evaluated*
  - This is the **meaning** of an expression

- The definition of a programming language:
  - The tokens $\Rightarrow$ lexical analysis
  - The grammar $\Rightarrow$ syntactic analysis
  - The typing rules $\Rightarrow$ semantic analysis
  - The evaluation rules $\Rightarrow$ interpretation

Evaluation Rules So Far

- So far, we specified the evaluation rules intuitively
  - We described how dynamic dispatch behaved in words (e.g., “just like Java”)
  - We talked about scoping, variables, arithmetic expressions (e.g., “they work as expected”)

- Why isn’t this description good enough?



Assembly Language Description of Semantics

- We might just tell you how to compile it
- But assembly-language descriptions of language implementation have too many irrelevant details
  - Which way the stack grows
  - How integers are represented on a particular machine
  - The particular instruction set of the architecture
- We need a complete but not overly restrictive specification

Programming Language Semantics

- There are many ways to specify programming language semantics
- They are all equivalent but some are more suitable to various tasks than others
- **Operational semantics**
  - Describes the evaluation of programs on an abstract machine
  - Most useful for specifying implementations
  - This is what we will use for Cool

Other Kinds of Semantics

- **Denotational semantics**
  - The meaning of a program is expressed as a mathematical object
  - Elegant but quite complicated
- **Axiomatic semantics**
  - Useful for checking that programs satisfy certain correctness properties
    - e.g., that the quick sort function sorts an array
  - The foundation of many program verification systems

Introduction to Operational Semantics

- Once, again we introduce a formal notation
  - Using logical rules of inference, just like typing
- Recall the typing judgment

$$\text{Context} \vdash e : T$$

(in the given context, expression e has type T)

- We try something similar for evaluation

$$\text{Context} \vdash e : v$$

(in the given context, expression e evaluates to value v)

Example Operational Semantics Inference Rule

$$\frac{\textbf{Context} \vdash e_1 : 5 \qquad \textbf{Context} \vdash e_2 : 7}{\textbf{Context} \vdash e_1 + e_2 : 12}$$

- In general the result of evaluating an expression *depends on* the result of evaluating its subexpressions
- The logical rules specify everything that is needed to evaluate an expression

Aside

- The operational semantics inference rules for Cool will become quite complicated
  - i.e., many hypotheses
- This may initially look daunting
- Until you realize that the opsem rules specify exactly how to build an interpreter
- That is, every rule of inference in this lecture is pseudocode for something in PA5
  - So by walking through the opsem is just like walking through the project.

- It might be tempting to protest this excursion into Theory
- But I assert it will come in handy very soon!

What Contexts Are Needed?

- Contexts are needed to handle variables
- Consider the evaluation of $y \leftarrow x + 1$
  - We need to keep track of values of variables
  - We need to allow variables to change their values during the evaluation
- We track variables and their values with:
  - An **environment** : tells us at what address in memory is the value of a variable stored
  - A **store** : tells us what is the contents of a memory location

What Contexts Are Needed?

- Contexts are needed to handle variabl

Remind me – why do we need a separate *store* and *environment*? We're just building an interpreter. Aren't those compiler notions?

s

values

An environment : tells us at what address in memory is the value of a variable stored

– A store : tells us what is the contents of a memory location

Variable Environments

- A variable **environment** is a map from variable names to **locations**
- Tells in what memory location the value of a variable is stored
  - Locations = Memory Addresses
- Environment tracks in-scope variables only
- Example environment:

$$E = [a : l_1,\ \ b : l_2]$$

- To lookup a variable a in environment E we write $E(a)$

Lost?

- Environments may seem hostile and unforgiving
- But soon they'll feel just like home!

- Names → Locations

Stores

- A **store** maps memory locations to values
- Example store:

$$S = [l_1 \to 5,\ \ l_2 \to 7]$$

- To lookup the contents of a location l_1 in store S we write $S(l_1)$
- To perform an assignment of 12 to location l_1 we write $S[12/l_1]$
  - This denotes a new store S' such that

$$S'(l_1) = 12 \quad \text{and} \quad S'(l) = S(l) \text{ if } l \neq l_1$$

- Avoid mistakes in your stores!
- Locations → Values

Cool Values

- All **values** in Cool are objects
  - All objects are instances of some class (the dynamic type of the object)
- To denote a Cool object we use the notation $X(a_1 = l_1, \ldots, a_n = l_n)$ where
  - X is the dynamic type of the object
  - a_i are the attributes (including those inherited)
  - l_i are the locations where the values of attributes are stored

Cool Values (Cont.)

- Special cases (classes without attributes)

Int(5)	the integer 5
Bool(true)	the boolean true
String(4, “Cool”)	the string “Cool” of length 4

- There is a special value void that is a member of all types
  - No operations can be performed on it
  - Except for the test isvoid
  - Concrete implementations might use NULL here

Operational Rules of Cool

- The evaluation judgment is

$$so, E, S \vdash e : v, S'$$

read:

  - Given so the current value of the self object
  - And E the current variable environment
  - And S the current store
  - If the evaluation of e terminates then
  - The returned value is v
  - And the new store is S'

Notes

- The “result” of evaluating an expression is both a value and a new store
- Changes to the store model side-effects
  - side-effects = assignments to variables
- The variable environment does not change
- Nor does the value of “self”
- The operational semantics allows for non-terminating evaluations
- We define one rule for each kind of expression

Operational Semantics for Base Values

$$\frac{}{\text{so, E, S} \vdash \textbf{true} : \text{Bool(true), S}}$$

$$\frac{}{\text{so, E, S} \vdash \textbf{false} : \text{Bool(false), S}}$$

$$\frac{\text{i is an integer literal}}{\text{so, E, S} \vdash \textbf{i} : \text{Int(i), S}}$$

$$\frac{\text{s is a string literal} \quad \text{n is the length of s}}{\text{so, E, S} \vdash \textbf{s} : \text{String(n,s), S}}$$

- No side effects in these cases

  (the store does not change)

Operational Semantics of Variable References

$$\frac{E(id) = l_{id} \qquad S(l_{id}) = v}{so, E, S \vdash id : v, S}$$

- Note the double lookup of variables
  - First from name to location
  - Then from location to value
- The store does not change
- A special case:

$$\frac{}{so, E, S \vdash self : so, S}$$

Operational Semantics of Assignment

$$\frac{\begin{array}{c}so, E, S \vdash e : v, S_1 \\ E(id) = l_{id} \\ S_2 = S_1[v/l_{id}]\end{array}}{so, E, S \vdash id \leftarrow e : v, S_2}$$

- A three step process
  - Evaluate the right hand side
    - $\Rightarrow$ a value v and a new store S_1
  - Fetch the location of the assigned variable
  - The result is the value v and an updated store
- The environment does not change

Operational Semantics of Conditionals

$$\frac{\begin{array}{c}so, E, S \vdash e_1 : Bool(true), S_1 \\ so, E, S_1 \vdash e_2 : v, S_2\end{array}}{so, E, S \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : v, S_2}$$

- The "threading" of the store enforces an evaluation sequence
  - e_1 must be evaluated first to produce S_1
  - Then e_2 can be evaluated
- The result of evaluating e_1 is a boolean object
  - The typing rules ensure this
  - There is another, similar, rule for Bool(false)

Operational Semantics of Sequences

$$\frac{\begin{array}{c} so, E, S \vdash e_1 : v_1, S_1 \\ so, E, S_1 \vdash e_2 : v_2, S_2 \\ \ldots \\ so, E, S_{n-1} \vdash e_n : v_n , S_n \end{array}}{so, E, S \vdash \{\ e_1;\ \ldots;\ e_n;\ \} : v_n, S_n}$$

- Again the threading of the store expresses the intended evaluation sequence
- Only the last value is used
- But all the side-effects are collected (how?)

Q: Music (198 / 842)

- Give both of the other place names that occur in the song **Istanbul (Not Constantinople)**. It was originally performed in 1953 by **The Four Lads** and was covered by **They Might Be Giants** in 1990.

Q: Games (516 / 842)

- This 1988 entry in the King's Quest series of games was the first to feature a female protagonist. The quest involved finding the magical healing fruit and defeating an evil fairy to recover a talisman.

Q: Movies (403 / 842)

- In this 1989 comedy also starring George Carlin, the title duo collect historical figures to avoid flunking out of San Dimas High School. An indicative exchange: *"Take them to the iron maiden. / Excellent! / Execute them. / Bogus!"*

Q: Books (711 / 842)

- In this 1943 Antoine de Saint-Exupery novel the title character lives on an asteroid with a rose but eventually travels to Earth.

Operational Semantics of while (1)

$$\frac{so, E, S \vdash e_1 : Bool(false), S_1}{so, E, S \vdash \text{while } e_1 \text{ loop } e_2 \text{ pool} : void, S_1}$$

- If e_1 evaluates to Bool(false) then the loop terminates immediately
  - With the side-effects from the evaluation of e_1
  - And with (arbitrary) result value void
- The typing rules ensure that e_1 evaluates to a boolean object

Operational Semantics of while (2)

$$\frac{\begin{array}{c}\mathbf{so, E, S} \vdash \mathbf{e_1 : Bool(true), S_1} \\ \mathbf{so, E, S_1} \vdash \mathbf{e_2 : v, S_2} \\ \mathbf{so, E, S_2} \vdash \mathbf{while\ e_1\ loop\ e_2\ pool : void, S_3}\end{array}}{\mathbf{so, E, S} \vdash \mathbf{while\ e_1\ loop\ e_2\ pool : void, S_3}}$$

- Note the sequencing ($S \to S_1 \to S_2 \to S_3$)
- Note how looping is expressed
  - Evaluation of "while …" is expressed in terms of the evaluation of itself in another state
- The result of evaluating e_2 is discarded
  - Only the side-effect is preserved

Operational Semantics of let Expressions (1)

$$\frac{\begin{array}{c} so, E, S \vdash e_1 : v_1, S_1 \\ so, ?, ? \vdash e_2 : v, S_2 \end{array}}{so, E, S \vdash \text{let } id : T \leftarrow e_1 \text{ in } e_2 : v_2, S_2}$$

- What is the context in which e_2 must be evaluated?
  - Environment like E but with a new binding of id to a fresh location l_{new}
  - Store like S_1 but with l_{new} mapped to v_1

Operational Semantics of let Expressions (II)

- We write l_{new} = newloc(S) to say that l_{new} is a location that is not already used in S
  - Think of newloc as the dynamic memory allocation function
- The operational rule for let:

$$\frac{\begin{array}{c} so, E, S \vdash e_1 : v_1, S_1 \\ l_{new} = newloc(S_1) \\ so, E[l_{new}/id]\ , S_1[v_1/l_{new}] \vdash e_2 : v_2, S_2 \end{array}}{so, E, S \vdash \text{let } id : T \leftarrow e_1 \text{ in } e_2 : v_2, S_2}$$

Balancing Act

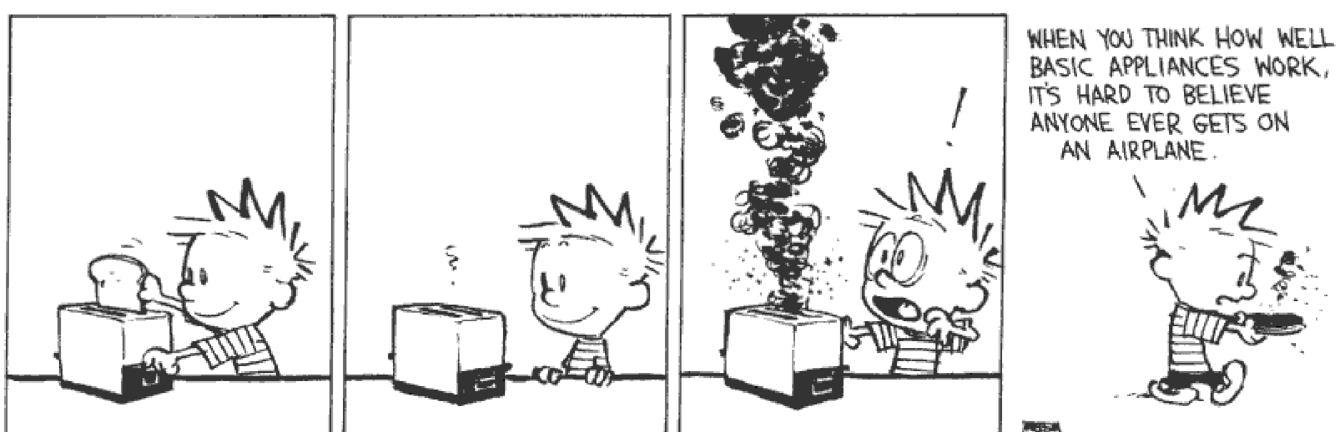
- Now we're going to do some very difficult rules
  - new, dispatch
- This may initially seem tricky
  - How could that possibly work?
  - What's going on here?
- With time, these rules can actually be elegant!

Operational Semantics of new

- Consider the expression new T
- Informal semantics
  - Allocate new locations to hold the values for all attributes of an object of class T
    - Essentially, allocate space for a new object
  - Initialize those locations with the default values of attributes
  - Evaluate the initializers and set the resulting attribute values
  - Return the newly allocated object

Default Values

- For each class A there is a default value denoted by D_A
  - D_{int} = Int(0)
  - D_{bool} = Bool(false)
  - D_{string} = String(0, “”)
  - D_A = void (for all others classes A)



More Notation

- For a class A we write

  $$\text{class}(A) = (a_1 : T_1 \leftarrow e_1, \ldots, a_n : T_n \leftarrow e_n)$$

where

  - a_i are the attributes (including inherited ones)
  - T_i are their declared types
  - e_i are the initializers

- This is the **class map** from PA4!

Operational Semantics of new

- Observation: new SELF_TYPE allocates an object with the same dynamic type as self

$$\frac{\begin{array}{l} T_0 = \text{if } T == \text{SELF\_TYPE and } so = X(\ldots) \text{ then } X \text{ else } T \\ class(T_0) = (a_1 : T_1 \leftarrow e_1, \ldots, a_n : T_n \leftarrow e_n) \\ l_i = newloc(S) \text{ for } i = 1,\ldots,n \\ \left.\begin{array}{l} v = T_0(a_1 = l_1,\ldots,a_n = l_n) \\ E' = [a_1 : l_1, \ldots, a_n : l_n] \\ S_1 = S[D_{T1}/l_1,\ldots,D_{Tn}/l_n] \\ v, E', S_1 \vdash \{ a_1 \leftarrow e_1; \ldots; a_n \leftarrow e_n; \} : v_n, S_2 \end{array}\right\} \textit{Initialize new object} \end{array}}{so, E, S \vdash \text{new } T : v, S_2}$$

Operational Semantics of new

- The first three lines allocate the object
- The rest of the lines initialize it
  - By evaluating a sequence of assignments
- State in which the initializers are evaluated:
  - Self is the current object
  - Only the attributes are in scope (same as in typing)
  - Starting value of attributes are the default ones
- Side-effects of initialization are kept (in s_2)

Operational Semantics of Method Dispatch

- Consider the expression $e_0.f(e_1,\ldots,e_n)$
- Informal semantics:
  - Evaluate the arguments in order $e_1,\ldots,e_n$
  - Evaluate e_0 to the target object
  - Let X be the dynamic type of the target object
  - Fetch from X the definition of f (with n args)
  - Create n new locations and an environment that maps f's formal arguments to those locations
  - Initialize the locations with the actual arguments
  - Set self to the target object and evaluate f's body

More Notation

- For a class A and a method f of A (possibly inherited) we write:

$$\mathrm{imp}(A, f) = (x_1, \ldots, x_n, e_{body})$$

where

- x_i are the names of the formal arguments
- e_{body} is the body of the method

- This is the imp map from PA4!

Dispatch OpSem

$$
\begin{array}{ll}
\left.\begin{array}{l}
so, E, S \vdash e_1 : v_1, S_1 \\
so, E, S_1 \vdash e_2 : v_2, S_2 \\
\ldots \\
so, E, S_{n-1} \vdash e_n : v_n, S_n
\end{array}\right\} & \textit{Evaluate arguments} \\
\left. so, E, S_n \vdash e_0 : v_0, S_{n+1} \right\} & \textit{Evaluate receiver object} \\
\left. v_0 = X(a_1 = l_1, \ldots, a_m = l_m) \right\} & \textit{Find type and attributes} \\
\left. imp(X, f) = (x_1, \ldots, x_n, e_{body}) \right\} & \textit{Find formals and body} \\
l_{xi} = newloc(S_{n+1}) \text{ for } i = 1, \ldots, n & \\
\left. E' = [x_1 : l_{x1}, \ldots, x_n : l_{xn}, a_1 : l_1, \ldots, a_m : l_m] \right\} & \textit{New environment} \\
\left. S_{n+2} = S_{n+1}[v_1/l_{x1}, \ldots, v_n/l_{xn}] \right\} & \textit{New store} \\
\left. v_0, E', S_{n+2} \vdash e_{body} : v, S_{n+3} \right\} & \textit{Evaluate body} \\
\hline
\multicolumn{2}{c}{so, E, S \vdash e_0.f(e_1, \ldots, e_n) : v, S_{n+3}}
\end{array}
$$

Operational Semantics of Dispatch

- The body of the method is invoked with
  - E mapping formal arguments and self's attributes
  - S like the caller's except with actual arguments bound to the locations allocated for formals
- The notion of the activation frame is implicit
  - New locations are allocated for actual arguments
- The semantics of static dispatch is similar except the implementation of f is taken from the specified class

Runtime Errors

Operational rules do not cover all cases

Consider for example the rule for dispatch:

$$\frac{\begin{array}{l} \ldots \\ so, E, S_n \vdash e_0 \;:\; v_0, S_{n+1} \\ v_0 = X(a_1 = l_1, \ldots, a_m = l_m) \\ imp(X, f) = (x_1, \ldots, x_n, e_{body}) \\ \ldots \end{array}}{so, E, S \vdash e_0.f(e_1, \ldots, e_n) \;:\; v, S_{n+3}}$$

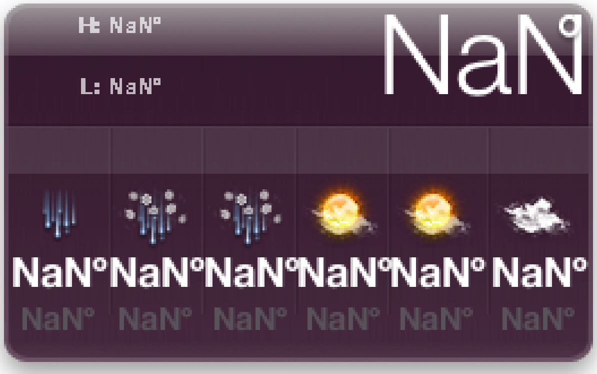
What happens if **imp(X, f)** is not defined?

Cannot happen in a well-typed program

(because of the Type Safety Theorem)

Runtime Errors

- There are some runtime errors that the type checker does not try to prevent
  - A dispatch on void
  - **Division by zero**
  - Substring out of range
  - Heap overflow



- In such case the execution must abort gracefully
  - With an error message and not with a segfault

Conclusions

- Operational rules are very precise
  - Nothing is left unspecified
- Operational rules contain a lot of details
  - Read them carefully
- Most languages do not have a well specified operational semantics
- When portability is important an operational semantics becomes essential
  - But not always using the exact notation we used for Cool

Homework

- WA4 (Semantics Checkpoint) due Tomorrow
- WA5 due Tuesday March 25
- PA4 due Wednesday March 26
- For Thursday:
  - Nothing?