Proofs

"Checking proofs ain't like dustin' crops, boy!"

Proof Generation

- We want our theorem prover to **emit proofs**
  - No need to trust the prover
  - Can find bugs in the prover
  - Can be used for proof-carrying code
  - Can be used to extract invariants
  - Can be used to extract models (e.g., in SLAM)
- Implements the soundness argument
  - On every run, a soundness proof is constructed

Proof Representation

- Proofs are trees
  - Leaves are hypotheses/axioms
  - Internal nodes are inference rules
- Axiom: "true introduction"
  - Constant: truei : pf
  - pf is the type of proofs
- Inference: "conjunction introduction"
  - Constant: andi : pf → pf → pf
- Inference: "conjunction elimination"
  - Constant: andel : pf → Pf
- Problem:
  - "andel truei : pf" but does not represent a valid proof
  - Need a more powerful *type system that checks content*

$$\frac{}{\vdash \text{true}}\ \textit{truei}$$

$$\frac{\vdash A \qquad \vdash B}{\vdash A \wedge B}\ \textit{andi}$$

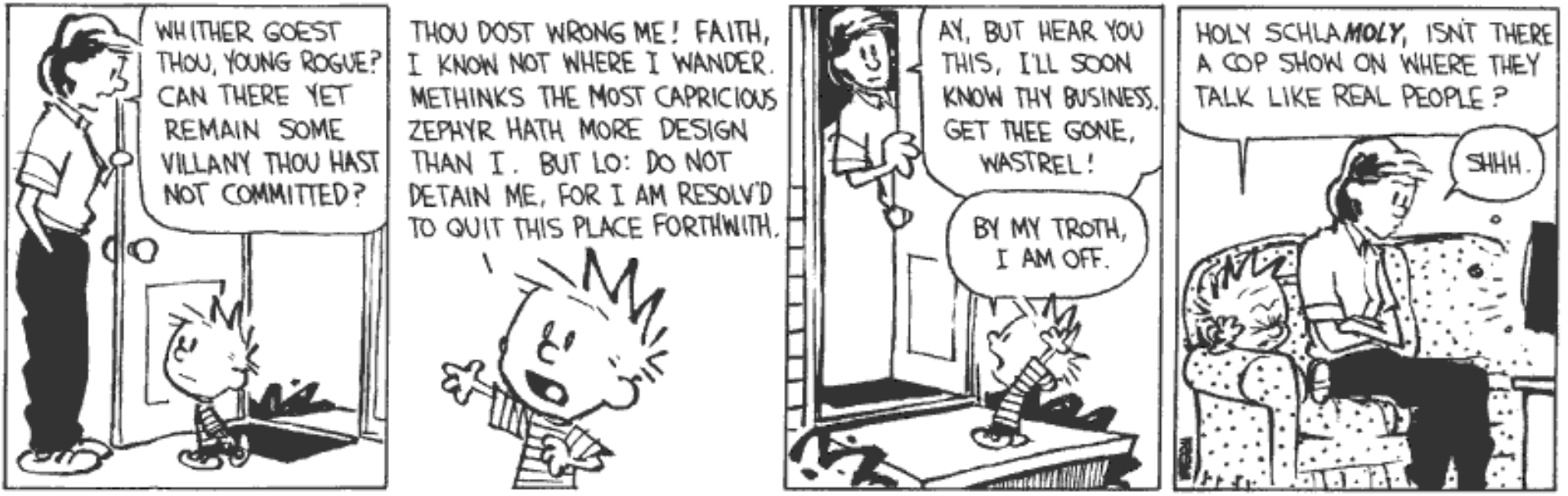
$$\frac{\vdash A \wedge B}{\vdash A}\ \textit{andel}$$

Dependent Types

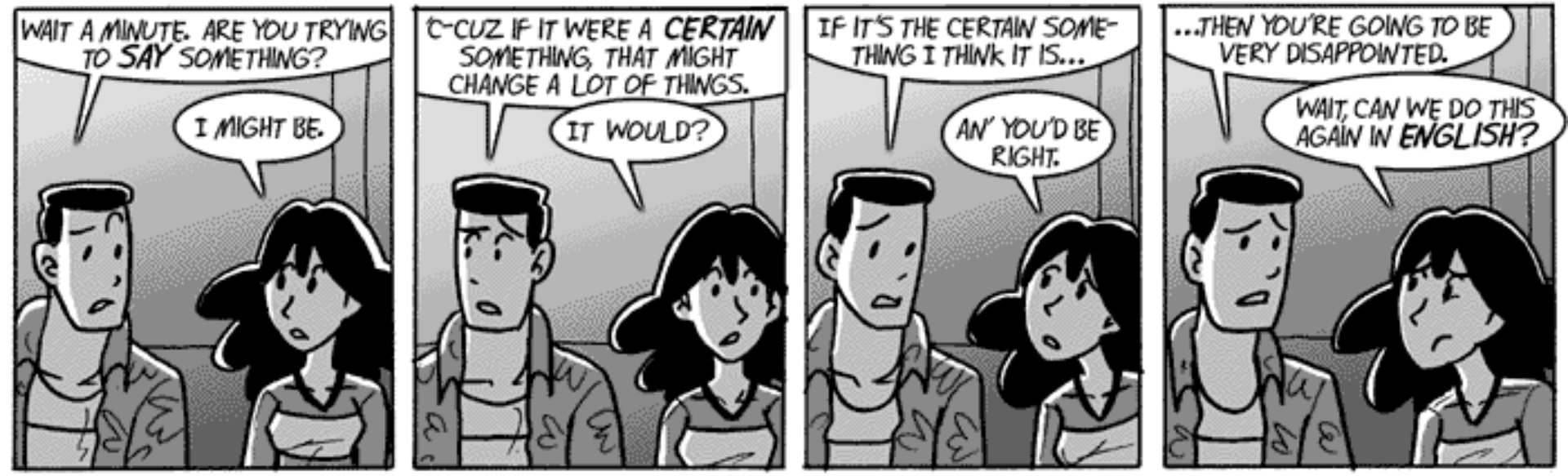
- Make pf a family of types indexed by formulas
  - f : Type (type of encodings of formulas)
  - e : Type (type of encodings of expressions)
  - pf : f $\rightarrow$ Type (the type of proofs indexed by formulas: it is a proof ***that f is true***)
- Examples:
  - true : f
  - and : f $\rightarrow$ f $\rightarrow$ f
  - truei : pf true
  - andi : pf A $\rightarrow$ pf B $\rightarrow$ pf (and A B)
  - **andi : ΠA:f. ΠB:f. pf A $\rightarrow$ pf B $\rightarrow$ pf (and A B)**

Proof Checking

- Validate proof trees by type-checking them
- Given a proof tree X claiming to prove $A \wedge B$
- Must check X : pf (and A B)
- We use "expression tree equality", so
  - andel (andi "1+2=3" "x=y") does *not* have type pf (3=3)
  - This is already a proof system! If the proof-supplier wants to use the fact that $1+2=3 \Leftrightarrow 3=3$, she can include a proof of it somewhere!
- Thus Type Checking = Proof Checking
  - And it's quite easily *decidable*! □



Communication and Concurrency



Preliminary Definition

- A calculus is a *method or system of calculation*
- The early Greeks used pebbles arranged in patterns to learn arithmetic and geometry
- The Latin word for pebble is "calculus" (diminutive of calx/calcis)
- Popular flavors:
  - differential, integral, propositional, predicate, lambda, pi, join, of communicating systems

Cunning Plan

- Types of Concurrency
- Modeling Concurrency
- Pi Calculus
- Channels and Scopes
- Semantics
- Security
- Real Languages

Take-Home Message

- The pi calculus is a formal system for modeling concurrency in which "communication channels" take center stage.
- Key concerns include non-determinism and security. The pi calculus models synchronous communication. Can someone eavesdrop on my channel?

Possible Concurrency

- No Concurrency
- Threads and Shared Variables
  - A language mechanism for specifying interleaving computations; often run on a single processor
- Parallel (SIMD)
  - A single program with simultaneous operations on multiple data (high-perf physics, science, …)
- Distributed processes
  - Code running at multiple sites (e.g., internet agents, DHT, Byzantine fault tolerance, Internet routing)
- Different research communities $\Rightarrow$ different notions

(There Must Be) Fifty Ways to Describe Concurrency

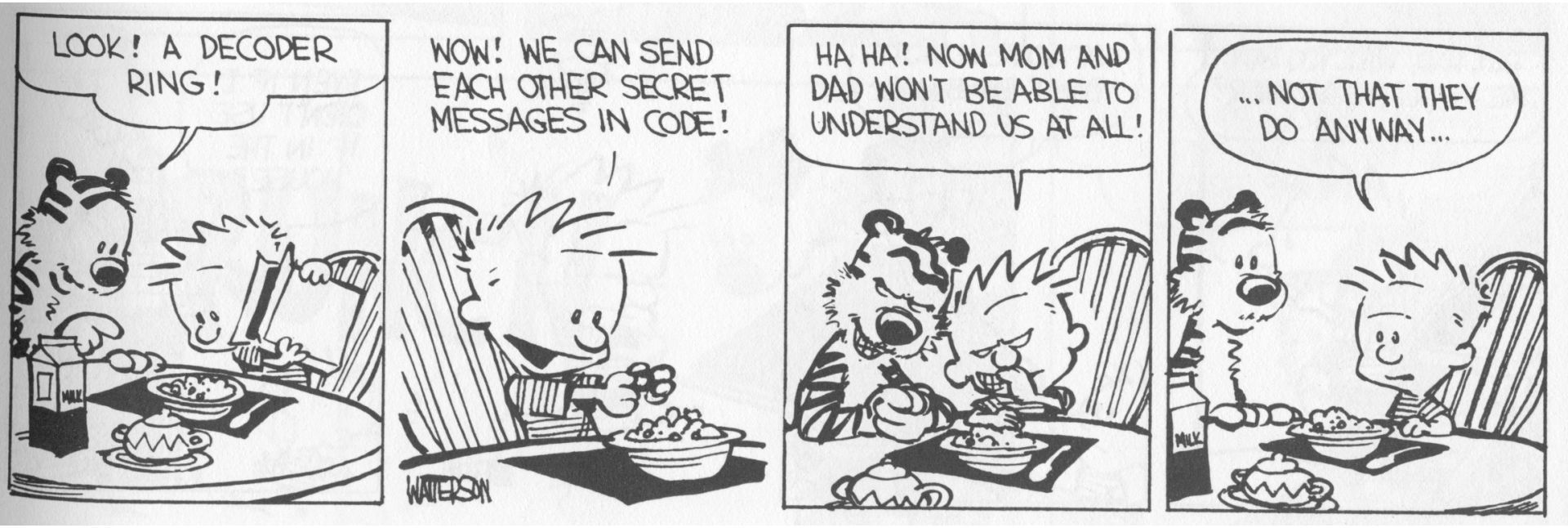
- No Concurrency
  - Sequential processes are modeled by the λ-calculus. Natural way to observe an algorithm: examine its output for various inputs ⇒ functions
- Threads and Shared Variables
  - Small-step opsem with contextual semantics (e.g., callcc), or special type systems (e.g., [FF00])
- Parallel (SIMD)
  - Not in this class (e.g., Titanium, etc.)
- Distributed processes
  - ???

Modeling Concurrency

- Concurrent systems are naturally non-deterministic
  - Interleaving of atomic actions from different processes
  - New concurrent scheduling possibly yields new result
- Concurrent processes can be observed in many ways
  - When are two concurrent systems equivalent?
  - Intra-process behavior vs. inter-process behavior
- Concurrency can be described in many ways
  - **Process creation**: fork/wait, cobegin/coend, data parallelism
  - **Process communication**: shared memory, message passing
  - **Process synchronization**: monitors, semaphores, transactions

Message Passing

- These "many ways" lead to a variety of process calculi
- We will focus on message passing!



Communication and Messages

- Communication is a fundamental concept
  - But not for everything (e.g., not much about parallel or scientific computing in this lecture)
- Communication through message passing
  - synchronous or asynchronous
  - static or dynamic communication topology
  - first-order or high-order data
- Historically: Weak treatment of communication
  - I/O often not considered part of the language
- Even “modern” languages have primitive I/O
  - First-class messages are rare
  - Higher-level remote procedure call is rare

Calculi and Languages

- Many calculi and languages use message-passing
  - Communicating Sequential Processes (CSP) (Hoare, 1978)
  - Occam (Jones)
  - Calculus of Communicating Systems (CCS) (Milner, 1980)
  - The Pi Calculus (Milner, 1989 and others)
  - Pict (Pierce and Turner)
  - Concurrent ML (Reppy)
  - Java RMI
- Messaging is built in some higher-level primitives
  - Remote procedure call
  - Remote method invocation

The Pi Calculus

- The pi calculus is a process algebra
  - Each process runs a different program
  - Processes run concurrently
  - But they can communicate
- Communication happens on channels
  - channels are first-class objects
    - channel names can be sent on channels
  - can have access restrictions for channels
- In λ-calculus everything is a function
- In Pi calculus everything is a process

Pi Calculus Grammar

- Processes communicate on channels
  - **c<M>** *send message M on channel c*
  - **c(x)** *receives message value x from channel c*
- Sequencing
  - **c<M>.p** *sends message M on c, then does p*
  - **c(x).p** *receives x on c, then does p with x (x is bound in p)*
- Concurrency
  - **p | q** *is the parallel composition of p and q*
- Replication
  - **! p** *creates an infinite number of replicas of p*

Examples

- For example we might define

```
Speaker      = air<M>              // send msg M over air
Phone        = air(x).wire<x>      // copy air to wire
ATT          = wire(x).fiber<x>    // copy wire to fiber
System       = Speaker | Phone | ATT
```

- Communication between processes is modeled by reduction:

```
Speaker | Phone → wire<M>          // send msg M to wire
wire<M> | ATT → fiber<M>          // send msg M to fiber
```

- Composing these reductions we get

```
Speaker | Phone | ATT  → fiber<M>  // send msg M to fiber
```

Channel Visibility

- Anybody can monitor an unrestricted channel!
- Modeling such snooping:

  WireTap = wire(x).wire<x>.NSA<x>

  - Copies the messages from the wire to NSA
  - Possible since the name “wire” is globally visible
- Now the composition:

  WireTap | wire<M> | ATT $\rightarrow$

  wire<M>.NSA<M> | ATT $\rightarrow$

  NSA<M> | fiber<M> *// OOPS !*

Restriction

- The restriction operator (νc) p makes a fresh channel c within process p
  - ν is the Greek letter "nu"
  - The name c is local (bound) in p
  - c is not known outside of p
- Restricted channels *cannot be monitored*

  wire(x) … | (ν wire)(wire<M> | ATT) →

  wire(x) … | fiber<M>
- The scope of the name wire is restricted
- There is no conflict with the global wire

Restriction and Scope

- Restriction
  - is a binding construct (like λ, $\forall$, $\exists$, ...)
  - is lexically scoped
  - allocates a new object (a new channel)
  - somewhat like Unix pipe(2) system call

$(\nu c)p$ is like let c = new Channel() in p

- c can be sent outside its initial scope
  - But only if p decides so (intentional leak)

First-Class Channels

- Channel c can leave its scope of declaration
  - via a message d<c> from within p
  - d is some other channel known to p
  - Intentional with “friend” processes (e.g., send my IM handle=c to a buddy via email=d)
- Allowing channels to be sent as messages means communication topology is dynamic
  - If channels are not sent as messages (or stored in the heap) then the communication topology is static
  - This differentiates Pi-calculus from CCS

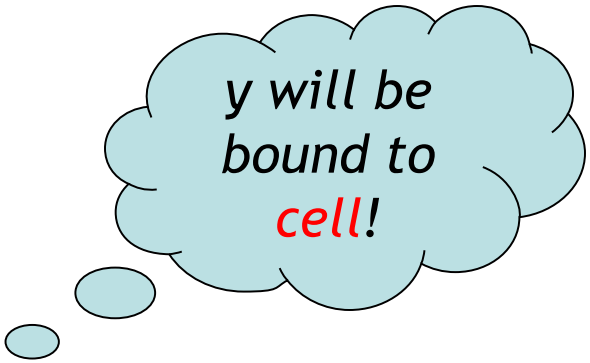
Example of First-Class Channels

Consider:

MobilePhone= air(x).cell<x>

ATT1 = wire<cell>

ATT2 = wire(y).y(x).fiber<x>



in

(ν cell)(MobilePhone | ATT1) | ATT2

- ATT1 passes cell out of the static scope of the restriction ν cell

Scope Extrusion

- A channel is just a name
  - First-class names must be usable in any scope
- The pi calculus restrictions to distribute:

  $((\nu\ c)\ p) \mid q = (\nu\ c)(p \mid q)$ *if c not free in q*
- Renaming is needed in general:

  $((\nu\ c)\ p) \mid q = ((\nu\ d)\ [d/c]\ p) \mid q$

  $= (\nu\ d)([d/c]\ p \mid q)$

  where "d" is fresh (does not appear in p or q)
- This scope extrusion distinguishes the pi calculus from other process calculi

Syntax of the Pi Calculus

There are many versions of the Pi calculus

A basic version:

p,q ::=		*(p and q are processes)*
	nil	*nil process (sometimes written 0)*
	x<y>.p	*sending data y on channel x*
	x(y).p	*receiving data y from channel x*
	p | q	*parallel composition*
	!p	*replication*
	(ν x)p	*restriction (new channel x used in p)*

- Note that only variables can be channels and messages

Operational Semantics

- One basic rule of computation: data transfer

$$\frac{}{x\langle y\rangle.p \mid x(z).q \;\rightarrow\; p \mid [y/z]q}$$

  - Synchronous communication: 1 sender, 1 receiver
  - Both the sender and the receiver proceed afterwards
- Rules for local (non-communicating) progress:

$$\frac{p \rightarrow p'}{p \mid q \rightarrow p' \mid q} \qquad \frac{p \rightarrow p'}{(\nu x)p \rightarrow (\nu x)p'}$$

$$\frac{p \equiv p' \quad p' \rightarrow q' \quad q' \equiv q}{p \rightarrow q}$$

Structural Congruence

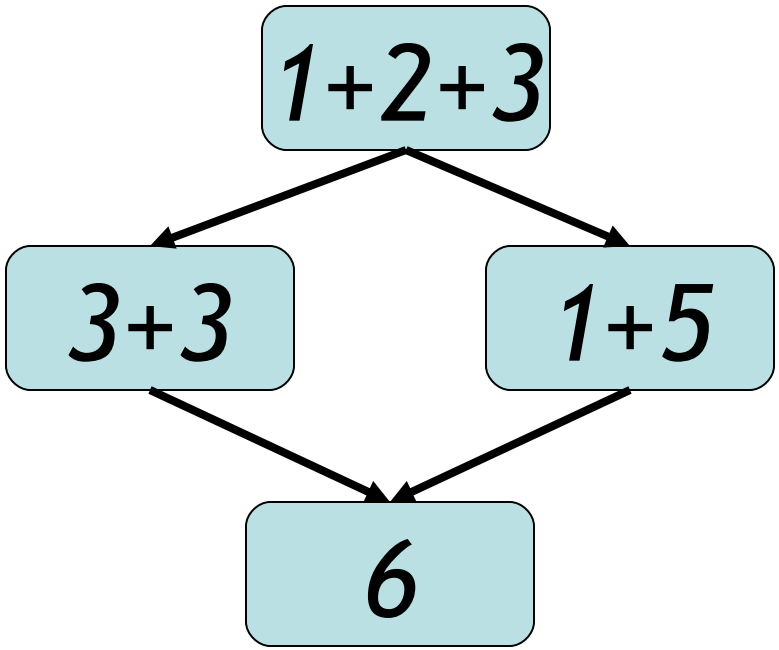
$$\frac{}{p \equiv p} \qquad \frac{q \equiv p}{p \equiv q} \qquad \frac{p \equiv q \quad q \equiv r}{p \equiv r}$$

$$\frac{p \equiv p'}{p \mid q \equiv p' \mid q} \qquad \frac{p \equiv p'}{(\nu x)p \equiv (\nu x)p'}$$

$$\begin{aligned}
!p &\equiv p \mid !p \\
p \mid \mathtt{nil} &\equiv p \\
p \mid q &\equiv q \mid p \\
(\nu x)(\nu y)p &\equiv (\nu y)(\nu x)p \\
(\nu x)\mathtt{nil} &\equiv \mathtt{nil} \\
(\nu x)(p \mid q) &\equiv (\nu x)p \mid q \qquad x \text{ not free in } q
\end{aligned}$$

Semantics and Evaluation

- IMP opsem has the "diamond property"
- Does the Pi Calculus? Why or why not?



Theory of Pi Calculus

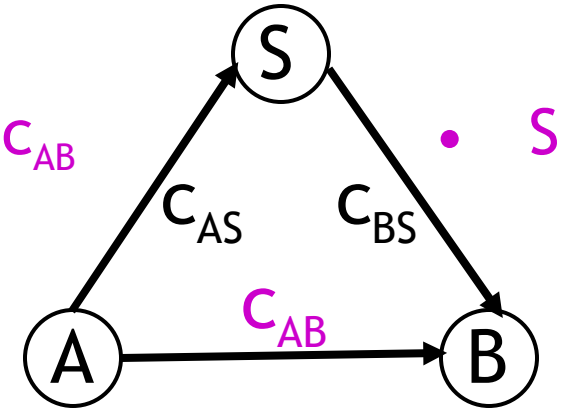
- The Pi calculus does ***not*** have the Church-Rosser property
  - Recall: WireTap | wire<M> | ATT $\rightarrow^*$ NSA<M> | fiber<M>
  - Also: WireTap | wire<M> | ATT $\rightarrow^*$ WireTap | fiber<M>
  - This captures the *non-deterministic nature* of concurrency
- For Pi-calculus there are
  - Type systems
  - Equivalences and logics
  - Expressiveness results, through encodings of numbers, lists, procedures, objects

Pi Calculus Applications

- A number of languages are based on Pi
  - e.g., Pict (Pierce and Turner)
- Specification and verification
  - mobile phone protocols, security protocols
- Pi channels have nice built-in properties, such as:
  - integrity
  - confidentiality (with ν)
  - exactly-once semantics
  - mobility (channels as first-class values)
- These properties are useful in high-level descriptions of security protocols
- More detailed descriptions are possible in the spi calculus (= pi calculus + cryptography)

A Typical Security Protocol

- Establishment and use of a secret channel:



- New channel c_{AB}
- Same new channel c_{AB}

- A and B are two clients
- S is an authentication server
- c_{AS} and c_{BS} are existing private channels with server
- c_{AB} is a new channel for the clients

That Security Protocol in Pi

- That protocol is described as follows:

  $A(M) = (\nu\ c_{AB})\ c_{AS}\langle c_{AB}\rangle.\ c_{AB}\langle M\rangle$

  $S = !\ (c_{AS}(x).\ c_{BS}\langle x\rangle \mid c_{BS}(x).\ c_{AS}\langle x\rangle)$

  $B = c_{BS}(x).\ x(y).\ Work(y)$

  $System(M) = (\nu\ c_{AS})(\nu\ c_{BS})\ A(M) \mid S \mid B$

  - Where Work(y) represents what B does with the message M (bound to y) that it receives
  - The $\mid c_{BS}(x).\ c_{AS}\langle x\rangle$ makes the server symmetric

Some Security Properties

- An authenticity property
  - For all N, if B receives N then A sent N to B
- A secrecy property
  - An outsider cannot tell System(M) apart from System(N), unless B reveals some part of A's message
- Both of these properties can be formalized and proved in the Pi calculus
- The secrecy property can be treated via a simple type system

Mainstream Languages

- Communication channels are not found in popular languages
  - sockets in C are reminiscent of channels
  - STREAMS (never used) are even closer
  - ML has exactly what we've described (surprise)

- More popular is *remote procedure call* or (for OO languages) *remote method invocation*

Concurrent ML

- Concurrent ML (CML) extends of ML with:
  - threads
  - typed channels
  - pre-emptive scheduling
  - garbage collection for threads and channels
  - synchronous communication
  - events as first-class values
- OCaml has it (Event, Thread), etc.
  - "**First-class synchronous communication**. This module implements synchronous inter-thread communications over channels. As in John Reppy's Concurrent ML system, the communication events are first-class values: they can be built and combined independently before being offered for communication."

Threads and Channels in CML

```
val spawn : (unit → unit) → thread (* create a new thread *)
val channel : unit → 'a chan (* create a new typed channel *)
val accept : 'a chan → 'a (* message passing operations *)
val send : ('a chan * 'a) → unit
```

So one can write, for example:

```
fun serverLoop () =  let request = accept recCh in
                         send (replyCh, workOn request);
                         serverLoop ()
```

Basic Events in Concurrent ML

val sync : 'a event → 'a *(* force synchronization on an event, block until this communication succeeds *)*

val transmit : ('a chan * 'a) → unit event *(* nonblocking; promises to do the send at some point *)*

val receive : 'a chan → 'a event *(* sets up the rendezvous, but you don't actually get the value until you sync *)*

val choose : 'a event list → 'a event *(* succeeds when one of the events in the list succeeds *)*

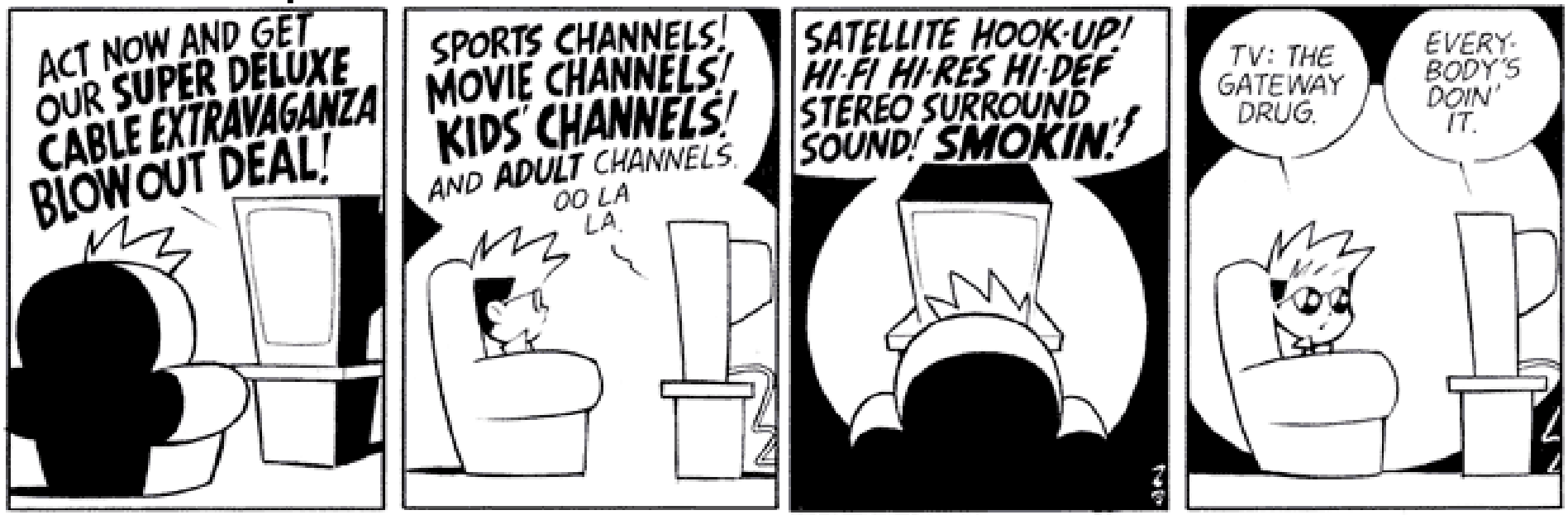
val wrap : ('a event * ('a → 'b)) → 'b event *(* do an action after synchronization on an event *)*

So you can write, as in Unix syscall select(2):

```
select (mylist : 'a event list) : 'a = sync (choose mylist)
```

Java Remote Method Invocation

- Java RMI is a Java extension with
  - Java method invocation syntax
  - similar semantics
  - static checks
  - distributed garbage collection
  - exceptions for failures



RMI notes

- Compare RMI with pure message passing
  - RMI is weaker, but OK for many purposes
- RMI not a perfect fit into Java:
  - non-remote objects are passed by copy in RMI
  - clients use remote interfaces, not remote classes
  - clients must handle RemoteException
  - using same syntax for MI and RMI leads to hidden performance costs
- But it is not an unreasonable design!

Homework

- Project
  - Need help? Stop by my office or send email.