Symbolic Execution

Wei Hu Memorial Homework Award

- Many turned in HW3 code like this:

```
let rec matches re s = match re with
 | Star(r) -> union (singleton s)
                    (matches (Concat(r,Star(r))) s)
```

- Which is a direct translation of:

$$R[\![r^*]\!]s = \{s\} \cup R[\![rr^*]\!]s$$

or, equivalently:

$$R[\![r^*]\!]s = \{s\} \cup \{\ y \mid \exists x \in R[\![r]\!]s \ \wedge \ y \in R[\![r^*]\!]x\ \}$$

- Why doesn't this work?

Today's Cunning Plan

- Symbolic Execution & Forward VCGen
- Handling Exponential Blowup
  - Invariants
  - Dropping Paths
- VCGen For Exceptions (double trouble)
- VCGen For Memory (McCarthyism)
- VCGen For Structures (have a field day)
- VCGen For "Dictator For Life"

Simple Assembly Language

- Consider the language of instructions:
  I ::= x := e | f() | if e goto L | goto L |
  L: | return | inv e
- The "inv e" instruction is an annotation
  - Says that boolean expression e holds at that point
- Each function f() comes with Pre_f and $Post_f$ annotations (pre- and post-conditions)
- New Notation (yay!): I_k is the instruction at address k

Symex States

- We set up a symbolic execution state:

Σ : Var $\rightarrow$ SymbolicExpressions

$\Sigma(x)$ = the symbolic value of x in state Σ

$\Sigma[x:=e]$ = a new state in which x's value is e

- We use states as substitutions:

$\Sigma(e)$ - obtained from e by replacing x with $\Sigma(x)$

- Much like the opsem so far …

Symex Invariants

- The symbolic executor tracks invariants passed
- A new part of symex state: $Inv \subseteq \{1 \ldots n\}$
- If $k \in Inv$ then I_k is an invariant instruction that we have already executed
- Basic idea: execute an inv instruction only <u>twice</u>:
  - The first time it is encountered
  - Once more time around an <u>arbitrary</u> iteration

Symex Rules

- Define a VC function as an interpreter:

$VC : Address \times SymbolicState \times InvariantState \rightarrow Assertion$

$VC(k, \Sigma, Inv) =$

$VC(L, \Sigma, Inv)$	if I_k = goto L
$e \Rightarrow VC(L, \Sigma, Inv) \quad \wedge$ $\neg e \Rightarrow VC(k+1, \Sigma, Inv)$	if I_k = if e goto L
$VC(k+1, \Sigma[x:=\Sigma(e)], Inv)$	if I_k = x := e
$\Sigma(Post_{current\text{-}function})$	if I_k = return
$\Sigma(Pre_f) \quad \wedge$ $\forall a_1..a_m.\Sigma'(Post_f) \Rightarrow$ $VC(k+1, \Sigma', Inv)$ (where $y_1, \ldots, y_m$ are modified by f) and $a_1, \ldots, a_m$ are fresh parameters and $\Sigma' = \Sigma[y_1 := a_1, \ldots, y_m := a_m]$	if I_k = f()

Symex Invariants (2a)

Two cases when seeing an invariant instruction:

1. We see the invariant for the first time
   - I_k = inv e
   - $k \notin Inv$ (= "not in the set of invariants we've seen")
   - Let $\{y_1, \ldots, y_m\}$ = the variables that could be modified on a path from the invariant back to itself
   - Let $a_1, \ldots, a_m$ be fresh new symbolic parameters

$VC(k, \Sigma, Inv) =$

$$\Sigma(e) \land \forall a_1 \ldots a_m.\ \Sigma'(e) \Rightarrow VC(k+1, \Sigma', Inv \cup \{k\}])$$

with $\Sigma' = \Sigma[y_1 := a_1, \ldots, y_m := a_m]$

(like a function call)

Symex Invariants (2b)

- We see the invariant for the second time
  - I_k = inv E
  - $k \in Inv$

  VC(k, Σ, Inv) = Σ(e)

  (like a function return)

- Some tools take a more simplistic approach
  - Do not require invariants
  - Iterate through the loop a fixed number of times
  - PREfix, versions of ESC (DEC/Compaq/HP SRC)
  - Sacrifice completeness for usability

Symex Summary

- Let $x_1, \ldots, x_n$ be all the variables and $a_1, \ldots, a_n$ fresh parameters
- Let Σ_0 be the state $[x_1 := a_1, \ldots, x_n := a_n]$
- Let $\emptyset$ be the empty Inv set

- For all functions f in your program, prove:

$$\forall a_1 \ldots a_n.\ \Sigma_0(Pre_f) \Rightarrow VC(f_{entry}, \Sigma_0, \emptyset)$$

- If you start the program by invoking any f in a state that satisfies Pre_f, then the program will execute such that
  - At all "inv e" the e holds, and
  - If the function returns then $Post_f$ holds
- Can be proved w.r.t. a real interpreter (operational semantics)
- Or via a proof technique called co-induction (or, assume-guarantee)

Forward VCGen Example

- Consider the program

Precondition: $x \leq 0$

```
Loop: inv x ≤ 6
        if x > 5 goto End
        x := x + 1
        goto Loop
End:  return
```

Postconditon: $x = 6$

Forward VCGen Example (2)

$$\forall x.\; x \le 0 \Rightarrow x \le 6 \wedge \forall x'.\; (x' \le 6 \Rightarrow x' > 5 \Rightarrow x' = 6 \wedge x' \le 5 \Rightarrow x' + 1 \le 6)$$

- VC contains both proof obligations and assumptions about the control flow

VCs Can Be Large

- Consider the sequence of conditionals

  (if $x < 0$ then $x := -x$); (if $x \leq 3$ then $x += 3$)

  - With the postcondition $P(x)$
- The VC is

$$
\begin{array}{lll}
x < 0 \wedge -x \leq 3 & \Rightarrow P(-x + 3) & \wedge \\
x < 0 \wedge -x > 3 & \Rightarrow P(-x) & \wedge \\
x \geq 0 \wedge x \leq 3 & \Rightarrow P(x + 3) & \wedge \\
x \geq 0 \wedge x > 3 & \Rightarrow P(x\ ) &
\end{array}
$$

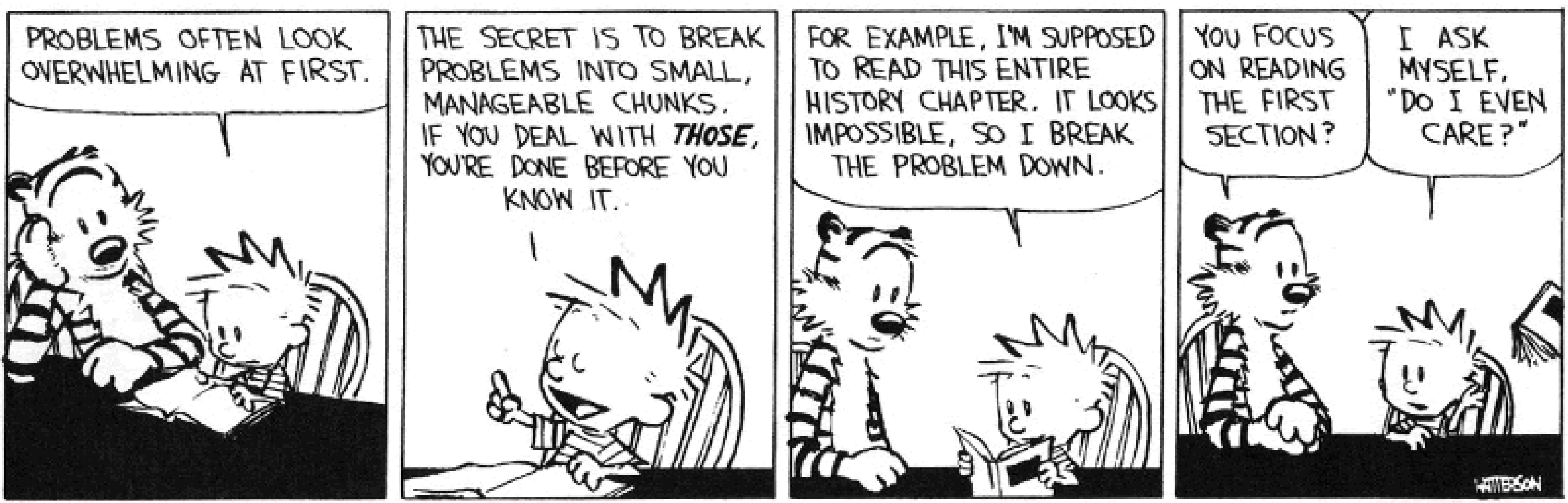
- There is one conjunct for each path

  $\Rightarrow$ exponential number of paths!

  - Conjuncts for infeasible paths have un-satisfiable guards!
- Try with $P(x) = x \geq 3$

VCs Can Be Exponential

- VCs are exponential in the size of the source because they attempt relative completeness:
  - Perhaps the correctness of the program must be argued independently for each path
- Unlikely that the programmer wrote a program by considering an exponential number of cases
  - But possible. Any examples? Any solutions?



VCs Can Be Exponential

- VCs are exponential in the size of the source because they attempt relative completeness:
  - Perhaps the correctness of the program must be argued independently for each path
- Standard Solutions:
  - Allow invariants even in straight-line code
  - And thus do not consider all paths independently!

Invariants in Straight-Line Code

- Purpose: modularize the verification task
- Add the command "after c establish Inv"
  - Same semantics as c (Inv is only for VC purposes)

$$VC(\text{after } c \text{ establish } Inv, P) =_{def}$$

$$VC(c, Inv) \wedge \forall x_i.\ Inv \Rightarrow P$$

  - where x_i are the ModifiedVars(c)
- Use when c contains many paths

after if $x < 0$ then $x := - x$ establish $x \geq 0$;

if $x \leq 3$ then x += 3 { $P(x)$ }

- VC is now:

$$(x < 0 \Rightarrow - x \geq 0) \wedge (x \geq 0 \Rightarrow x \geq 0) \wedge$$

$$\forall x.\ x \geq 0 \Rightarrow (x \leq 3 \Rightarrow P(x+3) \wedge x > 3 \Rightarrow P(x))$$

Dropping Paths

- In absence of annotations, we can drop some paths
- VC(if E then c_1 else c_2, P) = choose one of
  - $E \Rightarrow VC(c_1, P) \wedge \neg E \Rightarrow VC(c_2, P)$ (drop no paths)
  - $E \Rightarrow VC(c_1, P)$ (drops "else" path!)

    $\neg E \Rightarrow VC(c_2, P)$ (drops "then" path!)
- We sacrifice soundness! (we are now unsound)
  - No more guarantees
  - Possibly still a good debugging aid
- Remarks:
  - A recent trend is to sacrifice soundness to increase usability (e.g., Metal, ESP, even ESC)
  - The PREfix tool considers only 50 non-cyclic paths through a function (almost at random)

VCGen for Exceptions

- We extend the source language with exceptions without arguments (cf. HW2):
  - throw throws an exception
  - try c_1 catch c_2 executes c_2 if c_1 throws
- Problem:
  - We have non-local transfer of control
  - What is VC(throw, P) ?

VCGen for Exceptions

- We extend the source language with exceptions without arguments (cf. HW2):
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- Problem:
  - We have non-local transfer of control
  - What is VC(throw, P) ?
- Standard Solution: use 2 postconditions
  - One for normal termination
  - One for exceptional termination

VCGen for Exceptions (2)

- VC(c, P, Q) is a precondition that makes c either not terminate, or terminate normally with P or throw an exception with Q
- Rules

$$VC(skip, P, Q) = P$$

$$VC(c_1; c_2, P, Q) = VC(c_1, VC(c_2, P, Q), Q)$$

$$VC(throw, P, Q) = Q$$

$$VC(try\ c_1\ catch\ c_2, P, Q) = VC(c_1, P, VC(c_2, P, Q))$$

$$VC(try\ c_1\ finally\ c_2, P, Q) = ?$$

VCGen Finally

- Given these:

  $VC(c_1; c_2, P, Q) = VC(c_1, VC(c_2, P, Q), Q)$

  $VC(\text{try } c_1 \text{ catch } c_2, P, Q) = VC(c_1, P, VC(c_2, P, Q))$

- Finally is somewhat like "if":

  $VC(\text{try } c_1 \text{ finally } c_2, P, Q) =$

  $VC(c_1, VC(c_2, P, Q), \text{true}) \quad \wedge$

  $VC(c_1, \text{true}, VC(c_2, Q, Q))$

- Which reduces to:

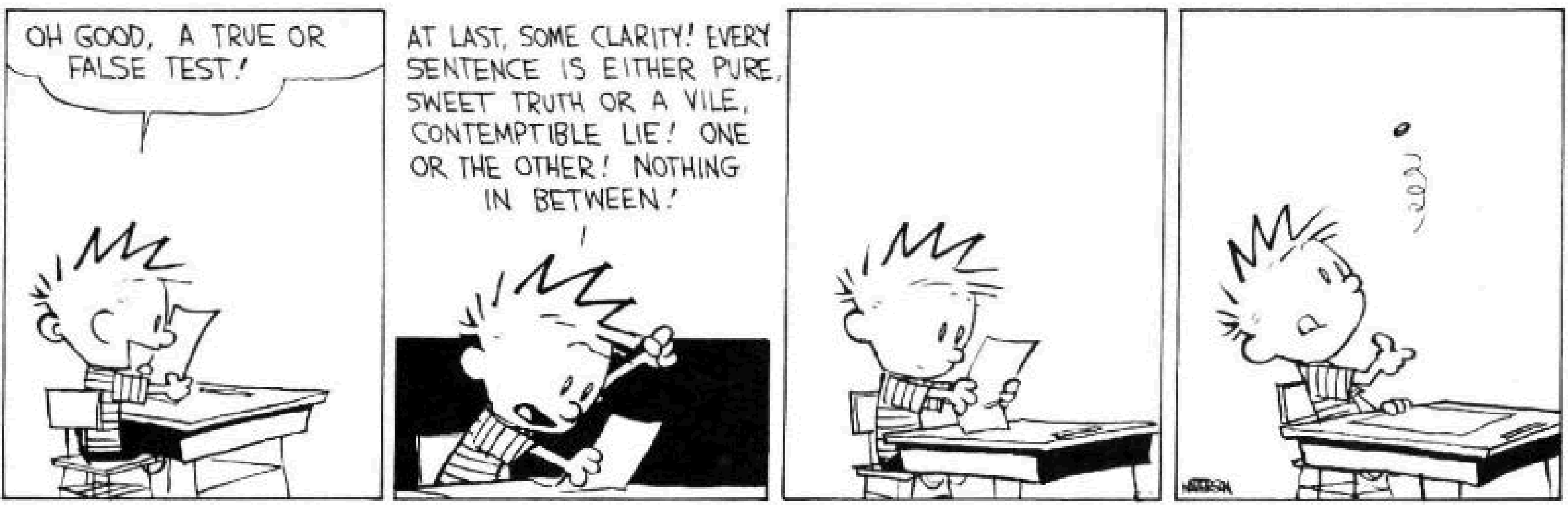
$$VC(c_1, VC(c_2, P, Q), VC(c_2, Q, Q))$$

Hoare Rules and the Heap

- When is the following Hoare triple valid?

$$\{ A \}\ {*x} := 5\ \{ {*x} + {*y} = 10 \}$$

- A *should be* "$*y = 5$ or $x = y$"
- The Hoare rule for assignment would give us:
  - $[5/{*x}]({*x} + {*y} = 10) = 5 + {*y} = 10 =$
  - $*y = 5$ (we lost one case)
- Why didn't this work?



Handling The Heap

- We do not yet have a way to talk about memory (the heap, pointers) in assertions
- Model the state of memory as a symbolic mapping from addresses to values:
  - If A denotes an address and M is a memory state then:
  - sel(M,A) denotes the contents of the memory cell
  - upd(M,A,V) denotes a new memory state obtained from M by writing V at address A

More on Memory

- We allow variables to range over memory states
  - We can quantify over all possible memory states
- Use the special pseudo-variable μ (mu) in assertions to refer to the current memory
- Example:

$$\forall i.\ i \geq 0 \wedge i < 5 \Rightarrow \mathrm{sel}(\mu, A + i) > 0$$

says that entries $0..4$ in array A are positive

Hoare Rules: Side-Effects

- To model writes we use memory expressions
  - A memory write changes the value of memory

$$\frac{}{\{\ B[upd(\mu, A, E)/\mu]\ \}\ \mathbf{*A := E}\ \{B\}}$$

- Important technique: treat memory as a whole
- And reason later about memory expressions with inference rules such as (McCarthy Axioms, ~'67):

$$sel(upd(M, A_1, V), A_2) = \begin{cases} V & \text{if } A_1 = A_2 \\ sel(M, A_2) & \text{if } A_1 \neq A_2 \end{cases}$$

Memory Aliasing

- Consider again: { A } *x := 5 { *x + *y = 10 }
- We obtain:

A = [upd(μ, x, 5)/μ] (*x + *y = 10)

= [upd(μ, x, 5)/μ] (sel(μ, x) + sel(μ, y) = 10)

(1) = sel(upd(μ, x, 5), x) + sel(upd(μ, x, 5), y) = 10

= 5 + sel(upd(μ, x, 5), y) = 10

= if x = y then 5 + 5 = 10 else 5 + sel(μ, y) = 10

(2) = x = y or *y = 5

- Up to (1) is theorem generation
- From (1) to (2) is theorem proving

Alternative Handling for Memory

- Reasoning about aliasing can be expensive
  - It is NP-hard (and/or undecideable)
- Sometimes completeness is sacrificed with the following (approximate) rule:

$$\mathrm{sel}(\mathrm{upd}(M, A_1, V), A_2) = \begin{cases} V & \text{if } A_1 = \text{(obviously) } A_2 \\ \mathrm{sel}(M, A_2) & \text{if } A_1 \neq \text{(obviously) } A_2 \\ P & \text{otherwise (p is a fresh new parameter)} \end{cases}$$

- The meaning of "obviously" varies:
  - The addresses of two distinct globals are ≠
  - The address of a global and one of a local are ≠
- PREfix and GCC use such schemes

VCGen Overarching Example

- Consider the program
  - Precondition: *B : bool* $\wedge$ *A : array(bool, L)*

```
1: I := 0
   R := B
3: inv I ≥ 0 ∧ R : bool
   if I ≥  L goto 9
   assert saferd(A + I)
   T := *(A + I)
   I := I + 1
   R := T
   goto 3
9: return R
```

  - Postcondition: *R : bool*

VCGen Overarching Example

$$
\begin{array}{l}
\forall A.\ \forall B.\ \forall L.\ \forall \mu \\
\quad B : \text{bool} \wedge A : \text{array}(\text{bool}, L) \Rightarrow \\
\quad\quad 0 \geq 0 \wedge B : \text{bool} \wedge \\
\quad\quad\quad \forall I.\ \forall R. \\
\quad\quad\quad\quad I \geq 0 \wedge R : \text{bool} \Rightarrow \\
\quad\quad\quad\quad\quad I \geq L \Rightarrow R : \text{bool} \\
\quad\quad\quad\quad\quad\quad \wedge \\
\quad\quad\quad\quad\quad I < L \Rightarrow \text{saferd}(A + I) \wedge \\
\quad\quad\quad\quad\quad\quad I + 1 \geq 0 \wedge \\
\quad\quad\quad\quad\quad\quad \text{sel}(\mu, A + I) : \text{bool}
\end{array}
$$

- VC contains both proof obligations and assumptions about the control flow

Mutable Records - Two Models

- Let r : RECORD { f1 : T1; f2 : T2 } END
- For us, records are reference types
- Method 1: one "memory" for each record
  - One index constant for each field
  - r.f1 is sel(r,f1) and r.f1 := E is r := upd(r,f1,E)
- Method 2: one "memory" for each field
  - The record address is the index
  - r.f1 is sel(f1,r) and r.f1 := E is f1 := upd(f1,r,E)
- Only works in strongly-typed languages like Java
  - Fails in C where &r.f2 = &r + sizeof(T1)

VC as a “Semantic Checksum”

- Weakest preconditions are an expression of the program’s semantics:
  - Two equivalent programs have logically equivalent WPs
  - No matter how different their syntax is!

- VC are almost as powerful

VC as a "Semantic Checksum" (2)

- Consider the "assembly language" program to the right

```
x := 4
x := (x == 5)
   assert x : bool
x := not x
   assert x
```

- High-level type checking is not appropriate here
- The VC is: $((4 == 5) : \text{bool}) \wedge (\text{not } (4 == 5))$
- No confusion from reuse of x with different types

Invariance of VC Across Optimizations

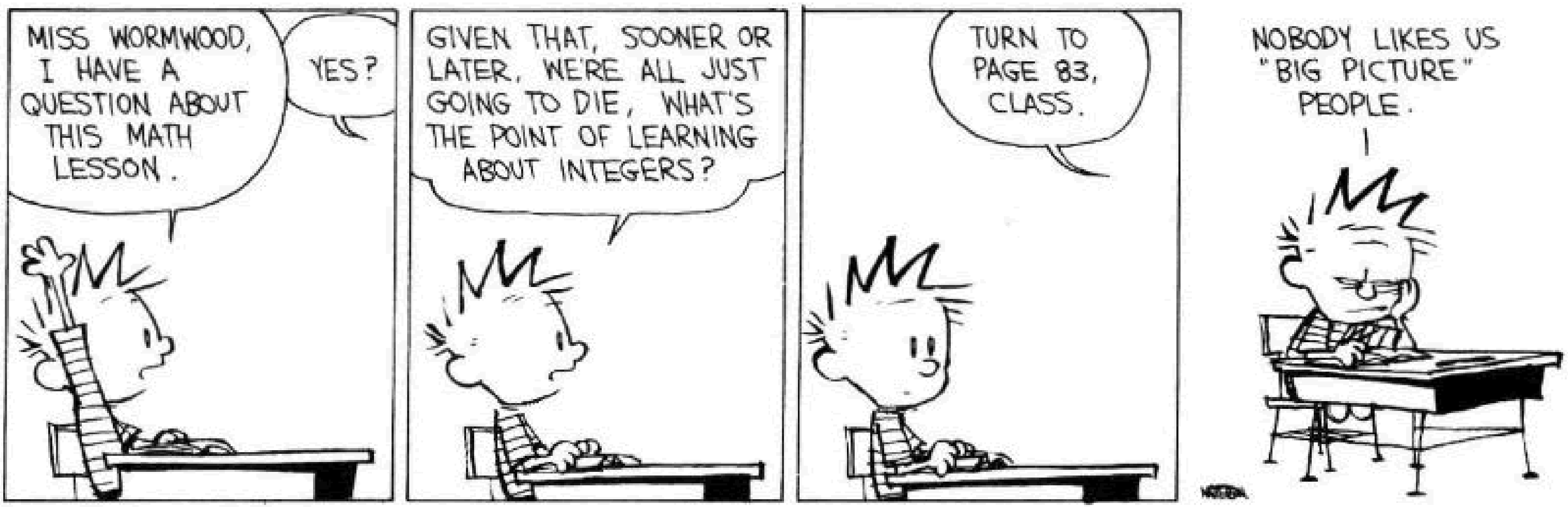
- VC is so good at abstracting syntactic details that it is *syntactically preserved* by many common optimizations
  - Register allocation, instruction scheduling
  - Common subexp elim, constant and copy propagation
  - Dead code elimination
- We have *identical* VCs whether or not an optimization has been performed
  - Preserves syntactic form, not just semantic meaning!
- This can be used to verify correctness of compiler optimizations (Translation Validation)

VC Characterize a Safe Interpreter

- Consider a fictitious "safe" interpreter
  - As it goes along it performs checks (e.g. "safe to read from this memory addr", "this is a null-terminated string", "I have not already acquired this lock")
  - Some of these would actually be hard to implement
- The VC describes all of the checks to be performed
  - Along with their context (assumptions from conditionals)
  - Invariants and pre/postconditions are used to obtain a finite expression (through induction)
- VC is valid $\Rightarrow$ interpreter *never fails*
  - We enforce same level of "correctness"
  - But better (static + more powerful checks)

VC Big Picture

- Verification conditions
  - Capture the semantics of code + specifications
  - Language independent
  - Can be computed backward/forward on structured/unstructured code
  - Make Axiomatic Semantics practical



Invariants Are Not Easy

- Consider the following code from QuickSort

```
int partition(int *a, int L0, int H0, int pivot) {
    int L = L0, H = H0;
    while(L < H) {
        while(a[L] < pivot) L ++;
        while(a[H] > pivot) H --;
        if(L < H) { swap a[L] and a[H] }
    }
    return L
}
```

- Consider verifying only memory safety
- What is the loop invariant for the outer loop ?

Done!