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Today's Cunning Plan

- Review, Truth, and Provability
- Large-Step Opsem Commentary
- Small-Step Contextual Semantics
  - Reductions, Redexes, and Contexts
- Applications and Recent Research

Summary - Semantics

- A formal semantics is a system for assigning meanings to programs.
- For now, programs are IMP commands and expressions
- In operational semantics the meaning of a program is "what it evaluates to"
- Any opsem system gives rules of inference that tell you how to evaluate programs

Summary - Judgments

- Rules of inference allow you to derive judgments ("something that is knowable") like

$$<e, \sigma> \Downarrow n$$

  - In state σ, expression e evaluates to n

$$<c, \sigma> \Downarrow \sigma'$$

  - After evaluating command c in state σ the new state will be σ'
- State σ maps variables to values ($\sigma : L \rightarrow Z$)
- Inferences equivalent up to variable renaming:

$$<c, \sigma> \Downarrow \sigma' \quad === \quad <c', \sigma_7> \Downarrow \sigma_8$$

Notation: Rules of Inference

- We express the evaluation rules as rules of inference for our judgment
  - called the derivation rules for the judgment
  - also called the evaluation rules (for operational semantics)
- In general, we have one rule for each language construct:

$$\frac{\langle e_1, \sigma \rangle \Downarrow n_1 \qquad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 + e_2, \sigma \rangle \Downarrow n_1 + n_2}$$

This is the only rule for $e_1 + e_2$

Rules of Inference

$$\frac{\text{Hypothesis}_1 \ \ldots \ \text{Hypothesis}_N}{\text{Conclusion}}$$

$$\frac{\Gamma \vdash \text{b : bool} \qquad \Gamma \vdash \text{e1} : \tau \qquad \Gamma \vdash \text{e2} : \tau}{\Gamma \vdash \text{if b then e1 else e2} : \tau}$$

- For any given proof system, a finite number of rules of inference (or schema) are listed somewhere
- Rule instances should be easily checked
- What is the definition of “NP”?

Derivation

$$\dfrac{\dfrac{\dfrac{\Gamma(x)=int}{\Gamma\vdash x:int}\,\text{var}\qquad \dfrac{}{\Gamma\vdash 3:int}\,\text{int}}{\Gamma\vdash x>3:bool}\,\text{gt}\qquad \dfrac{\dfrac{\Gamma(x)=int}{\Gamma\vdash x:int}\,\text{var}\qquad \dfrac{\dfrac{\Gamma(x)=int}{\Gamma\vdash x:int}\,\text{var}\qquad \dfrac{}{\Gamma\vdash 1:int}\,\text{int}}{\Gamma\vdash x-1:int}\,\text{sub}}{\Gamma\vdash x := x-1}\,\text{assign}}{\Gamma\vdash \texttt{while}\ x>3\ \texttt{do}\ x:=x-1\ \texttt{done}}\,\text{while}$$

- Tree-structured (conclusion at bottom)
- May include multiple sorts of rules-of-inference
- Could be constructed, typically are not
- Typically verified in polynomial time

Evaluation Rules (for Aexp)

$$\frac{}{\langle n, \sigma \rangle \Downarrow n} \qquad \frac{}{\langle x, \sigma \rangle \Downarrow \sigma(x)}$$

$$\frac{\langle e_1, \sigma \rangle \Downarrow n_1 \qquad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 + e_2, \sigma \rangle \Downarrow n_1 \textbf{ plus } n_2} \qquad \frac{\langle e_1, \sigma \rangle \Downarrow n_1 \qquad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 - e_2, \sigma \rangle \Downarrow n_1 \textbf{ minus } n_2}$$

$$\frac{\langle e_1, \sigma \rangle \Downarrow n_1 \qquad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 * e_2, \sigma \rangle \Downarrow n_1 \textbf{ times } n_2}$$

- This is called structural operational semantics
  - rules defined based on the structure of the expression
- These rules do not impose an order of evaluation!

Evaluation Rules (for Bexp)

$$\frac{}{\langle \texttt{true}, \sigma\rangle \Downarrow \mathit{true}}$$

$$\frac{\langle e_1, \sigma\rangle \Downarrow n_1 \qquad \langle e_2, \sigma\rangle \Downarrow n_2}{\langle e_1 \le e_2, \sigma\rangle \Downarrow n_1 \le n_2}$$

$$\frac{}{\langle \texttt{false}, \sigma\rangle \Downarrow \mathit{false}}$$

$$\frac{\langle e_1, \sigma\rangle \Downarrow n_1 \qquad \langle e_2, \sigma\rangle \Downarrow n_2}{\langle e_1 = e_2, \sigma\rangle \Downarrow n_1 = n_2}$$

$$\frac{\langle b_1, \sigma\rangle \Downarrow \mathit{false}}{\langle b_1 \land b_2, \sigma\rangle \Downarrow \mathit{false}}$$

$$\frac{\langle b_2, \sigma\rangle \Downarrow \mathit{false}}{\langle b_1 \land b_2, \sigma\rangle \Downarrow \mathit{false}}$$

$$\frac{\langle b_1, \sigma\rangle \Downarrow \mathit{true} \qquad \langle b_2, \sigma\rangle \Downarrow \mathit{true}}{\langle b_1 \land b_2, \sigma\rangle \Downarrow \mathit{true}}$$

(show: candidate $\lor$ rule)

How to Read the Rules?

- Forward (top-down) = inference rules
  - if we know that the hypothesis judgments hold then we can infer that the conclusion judgment also holds
  - If we know that $<e_1, \sigma> \Downarrow 5$ and $<e_2, \sigma> \Downarrow 7$, then we can infer that

$$<e_1 + e_2, \sigma> \Downarrow 12$$

How to Read the Rules?

- Backward (bottom-up) = evaluation rules
  - Suppose we want to evaluate $e_1 + e_2$, i.e., find n s.t. $e_1 + e_2 \Downarrow n$ is derivable using the previous rules
  - By inspection of the rules we notice that the last step in the derivation of $e_1 + e_2 \Downarrow n$ **must be** the addition rule
    - the other rules have conclusions that would not match $e_1 + e_2 \Downarrow n$
    - this is called reasoning by inversion on the derivation rules

Evaluation By Inversion

- Thus we must find n_1 and n_2 such that $e_1 \Downarrow n_1$ and $e_2 \Downarrow n_2$ are derivable
  - This is done recursively
- If there is exactly one rule for each kind of expression we say that the rules are syntax-directed
  - At each step at most one rule applies
  - This allows a simple evaluation procedure as above (recursive tree-walk)
  - True for our Aexp but not Bexp. *Why?*

Evaluation of Commands

- The evaluation of a Com may have side effects but has no direct result
  - What is the result of evaluating a command ?
- The "result" of a Com is a new state:

$$<c, \sigma> \Downarrow \sigma'$$

  - But the evaluation of Com might not terminate! Danger Will Robinson! (huh?)

Com Evaluation Rules 1

$$\frac{}{\langle \text{skip}, \sigma\rangle \Downarrow \sigma}$$

$$\frac{\langle c_1, \sigma\rangle \Downarrow \sigma' \qquad \langle c_2, \sigma'\rangle \Downarrow \sigma''}{\langle c_1 ; c_2, \sigma\rangle \Downarrow \sigma''}$$

$$\frac{\langle b, \sigma\rangle \Downarrow \text{true} \qquad \langle c_1, \sigma\rangle \Downarrow \sigma'}{\langle \text{if } b \text{ then } c_1 \text{ else } c_2, \sigma\rangle \Downarrow \sigma'}$$

$$\frac{\langle b, \sigma\rangle \Downarrow \text{false} \qquad \langle c_2, \sigma\rangle \Downarrow \sigma'}{\langle \text{if } b \text{ then } c_1 \text{ else } c_2, \sigma\rangle \Downarrow \sigma'}$$

Com Evaluation Rules 2

$$\frac{\langle e, \sigma\rangle \Downarrow n}{\langle x := e, \sigma\rangle \Downarrow \sigma[x := n]}$$

Def: $\sigma[x:= n](x) = n$

$\sigma[x:= n](y) = \sigma(y)$

- Let's do while together

Com Evaluation Rules 3

$$\frac{\langle e, \sigma\rangle \Downarrow n}{\langle x := e, \sigma\rangle \Downarrow \sigma[x := n]}$$

Def: $\sigma[x{:=}\ n](x) = n$

$\sigma[x{:=}\ n](y) = \sigma(y)$

$$\frac{\langle b, \sigma\rangle \Downarrow \text{false}}{\langle \text{while } b \text{ do } c, \sigma\rangle \Downarrow \sigma}$$

$$\frac{\langle b, \sigma\rangle \Downarrow \text{true} \quad \langle c; \text{while } b \text{ do } c, \sigma\rangle \Downarrow \sigma'}{\langle \text{while } b \text{ do } c, \sigma\ \rangle \Downarrow \sigma'}$$

Summary - Rules

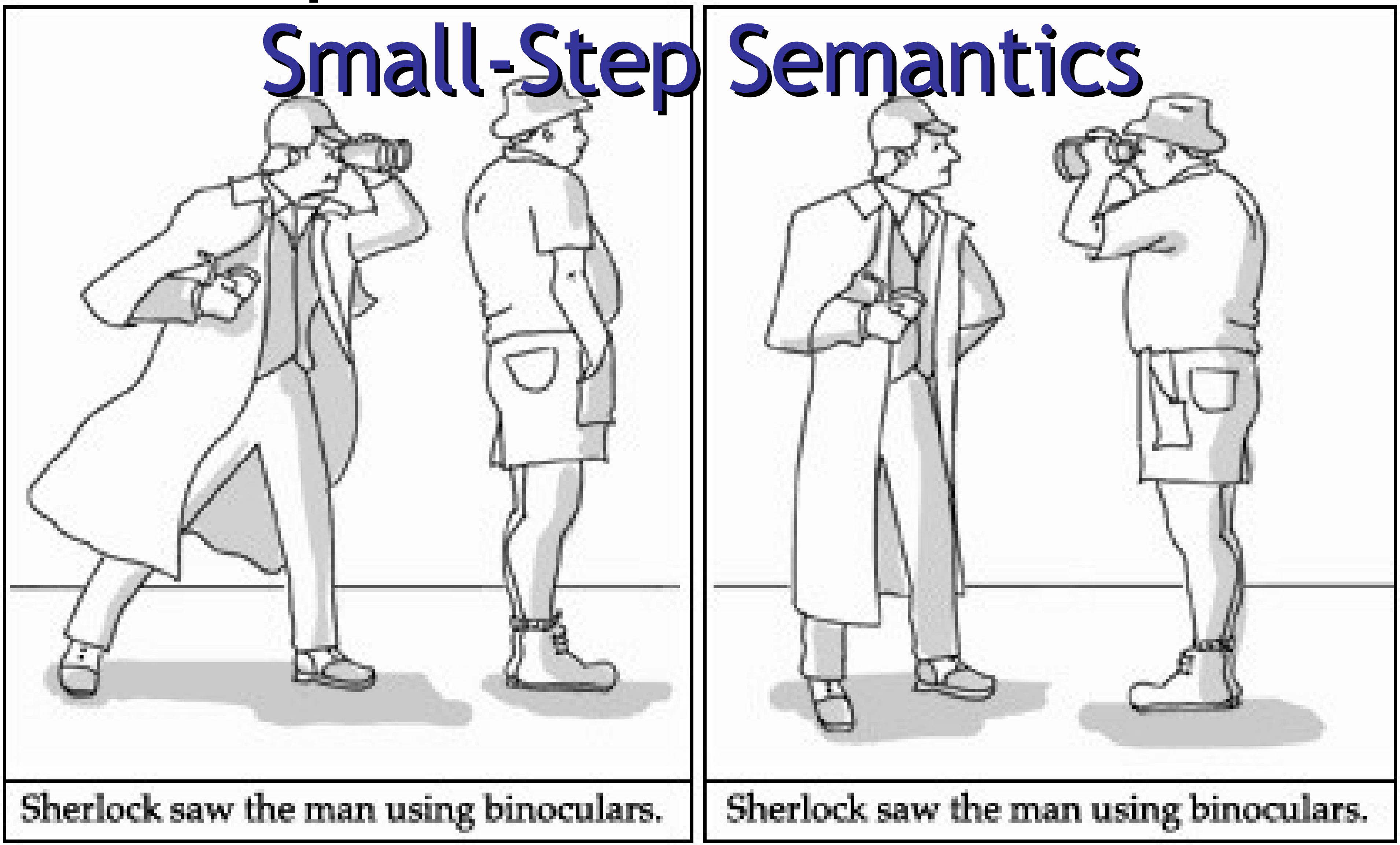
- Rules of inference list the hypotheses necessary to arrive at a conclusion

$$\frac{}{\langle x, \sigma \rangle \Downarrow \sigma(x)} \qquad \frac{\langle e_1, \sigma \rangle \Downarrow n_1 \quad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 - e_2, \sigma \rangle \Downarrow n_1 \text{ minus } n_2}$$

- A derivation involves interlocking (well-formed) instances of rules of inference

$$\frac{\dfrac{\langle 4, \sigma_3 \rangle \Downarrow 4 \quad \langle 2, \sigma_3 \rangle \Downarrow 2}{\langle 4*2, \sigma_3 \rangle \Downarrow 8} \qquad \langle 6, \sigma_3 \rangle \Downarrow 6}{\langle (4*2) - 6, \sigma_3 \rangle \Downarrow 2}$$

Operational Semantics
Small-Step Semantics



Provability

- Given an opsem system, $\langle e, \sigma \rangle \Downarrow n$ is <u>provable</u> *if there exists* a well-formed derivation with $\langle e, \sigma \rangle \Downarrow n$ as its conclusion
  - "well-formed" = "every step in the derivation is a valid instance of one of the rules of inference for this opsem system"
  - "$\vdash \langle e, \sigma \rangle \Downarrow n$" = "it is provable that $\langle e, \sigma \rangle \Downarrow n$"
- We would *like* truth and provability to be closely related

Truth?

- "A Vorlon said understanding is a three-edged sword. Your side, their side and the truth."
  - Sheridan, *Into The Fire*
- We will not formally define "truth" yet
- Instead we appeal to your intuition
  - $<2+2, \sigma> \Downarrow 4$ -- *should be* true
  - $<2+2, \sigma> \Downarrow 5$ -- *should be* false

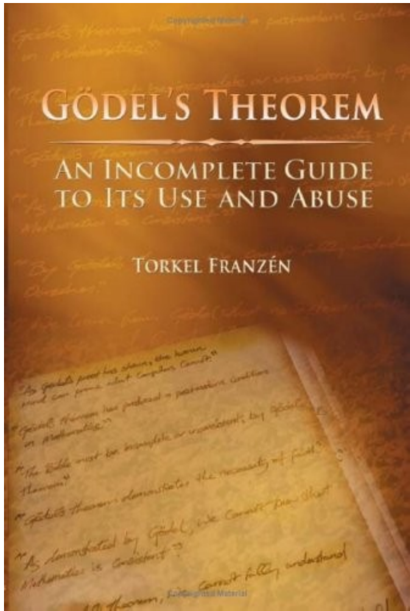
Completeness

- A proof system (like our operational semantics) is complete if every true judgment is provable.
- If we *replaced* the subtract rule with:

$$\frac{\langle e_1, \sigma\rangle \Downarrow n \qquad \langle e_2, \sigma\rangle \Downarrow 0}{\langle e_1 - e_2, \sigma\rangle \Downarrow n}$$

- Our opsem would be incomplete:

  $\langle 4\text{-}2, \sigma\rangle \Downarrow 2$ -- true but not provable



Consistency

- A proof system is consistent (or sound) if every provable judgment is true.
- If we ***replaced*** the subtract rule with:

$$\frac{\langle e_1, \sigma\rangle \Downarrow n_1 \qquad \langle e_2, \sigma\rangle \Downarrow n_2}{\langle e_1 - e_2, \sigma\rangle \Downarrow n_1 + 3}$$

- Our opsem would be inconsistent (or unsound):
  - $<6\text{-}1, \sigma> \Downarrow 9$ -- false but provable

"A foolish consistency is the hobgoblin of little minds, adored by little statesmen and philosophers and divines."
-- Ralph Waldo Emerson, *Essays. First Series. Self-Reliance.*

Desired Traits

- Typically a system (of operational semantics) is always complete (unless you forget a rule)
- If you are not careful, however, your system may be unsound
- Usually that is *very bad*
  - A paper with an unsound type system is usually rejected
  - Papers often prove (sketch) that a system is sound
  - Recent research (e.g., Engler, ESP) into useful but unsound systems exists, however
- In this class your work should be complete and consistent (e.g., on homework problems)

> **Dr. Peter Venkman**: I'm a little fuzzy on the whole "good/bad" thing here. What do you mean, "bad"?
> **Dr. Egon Spengler**: Try to imagine all life as you know it stopping instantaneously and every molecule in your body exploding at the speed of light.

With That In Mind

- We now return to opsem for IMP

$$\frac{\langle e, \sigma\rangle \Downarrow n}{\langle x := e, \sigma\rangle \Downarrow \sigma[x := n]}$$

Def: $\sigma[x:= n](x) = n$
$\sigma[x:= n](y) = \sigma(y)$

$$\frac{\langle b, \sigma\rangle \Downarrow \text{false}}{\langle \text{while } b \text{ do } c, \sigma\rangle \Downarrow \sigma}$$

$$\frac{\langle b, \sigma\rangle \Downarrow \text{true} \quad \langle c; \text{while } b \text{ do } c, \sigma\rangle \Downarrow \sigma'}{\langle \text{while } b \text{ do } c, \sigma \rangle \Downarrow \sigma'}$$

Command Evaluation Notes

- The order of evaluation is important
  - c_1 is evaluated before c_2 in $c_1; c_2$
  - c_2 is not evaluated in "if true then c_1 else c_2"
  - c is not evaluated in "while false do c"
  - b is evaluated first in "if b then c_1 else c_2"
  - this is explicit in the evaluation rules
- Conditional constructs (e.g., $b_1 \vee b_2$) have multiple evaluation rules
  - but only one can be applied at one time

Command Evaluation Trials

- The evaluation rules are not syntax-directed
  - See the rules for while, $\wedge$
  - The evaluation might not terminate
- Recall: the evaluation rules suggest an interpreter
- Natural-style semantics has two big disadvantages (continued …)

Disadvantages of Natural-Style Operational Semantics

- It is hard to talk about commands whose evaluation does not terminate
  - i.e., when there is no σ' such that <c, σ> ⇓ σ'
  - But that is true also of ill-formed or erroneous commands (in a richer language)!
- It does not give us a way to talk about intermediate states
  - Thus we cannot say that on a parallel machine the execution of two commands is interleaved (= no modeling threads)

Semantics Solution

- Small-step semantics addresses these problems
  - Execution is modeled as a (possible infinite) sequence of states
- Not quite as easy as large-step natural semantics, though
- Contextual semantics is a small-step semantics where the atomic execution step is a rewrite of the program

Contextual Semantics

- We will define a relation $<c, \sigma> \to <c', \sigma'>$
  - c' is obtained from c via an atomic rewrite step
  - Evaluation terminates when the program has been rewritten to a terminal program
    - one from which we cannot make further progress
  - For IMP the terminal command is "skip"
  - As long as the command is not "skip" we can make further progress
    - some commands *never* reduce to skip (e.g., "while true do skip")

Contextual Derivations

- In small-step contextual semantics, derivations are not tree-structured
- A contextual semantics derivation is a sequence (or list) of atomic rewrites:

$$<x+(7-3),\sigma> \rightarrow <x+(4),\sigma> \rightarrow <5+4,\sigma> \rightarrow <9,\sigma>$$

$\sigma(x)=5$

What is an Atomic Reduction?

- What is an atomic reduction step?
  - Granularity is a choice of the semantics designer
- How to select the next reduction step, when several are possible?
  - This is the order of evaluation issue

Redexes

- A redex is a syntactic expression or command that can be reduced (transformed) in one atomic step
- Redexes are defined via a grammar:

$$
\begin{aligned}
r ::= {} & x \qquad\qquad (x \in L) \\
| {} & n_1 + n_2 \\
| {} & x := n \\
| {} & \text{skip};\ c \\
| {} & \text{if true then } c_1 \text{ else } c_2 \\
| {} & \text{if false then } c_1 \text{ else } c_2 \\
| {} & \text{while } b \text{ do } c
\end{aligned}
$$

- For brevity, we mix exp and command redexes
- Note that $(1 + 3) + 2$ is not a redex, but $1 + 3$ is

Local Reduction Rules for IMP

- One for each redex: **$\langle r, \sigma \rangle \rightarrow \langle e, \sigma' \rangle$**
  - means that in state σ, the redex r can be *replaced in one step* with the expression e

$\langle x, \sigma \rangle \rightarrow \langle \sigma(x), \sigma \rangle$

$\langle n_1 + n_2, \sigma \rangle \rightarrow \langle n, \sigma \rangle$ where $n = n_1$ plus n_2

$\langle n_1 = n_2, \sigma \rangle \rightarrow \langle \text{true}, \sigma \rangle$ if $n_1 = n_2$

$\langle x := n, \sigma \rangle \rightarrow \langle \text{skip}, \sigma[x := n] \rangle$

$\langle \text{skip}; c, \sigma \rangle \rightarrow \langle c, \sigma \rangle$

$\langle \text{if true then } c_1 \text{ else } c_2, \sigma \rangle \rightarrow \langle c_1, \sigma \rangle$

$\langle \text{if false then } c_1 \text{ else } c_2, \sigma \rangle \rightarrow \langle c_2, \sigma \rangle$

$\langle \text{while } b \text{ do } c, \sigma \rangle \rightarrow$

$\langle \text{if } b \text{ then } c; \text{while } b \text{ do } c \text{ else skip}, \sigma \rangle$

Not happy? I'll explain with pictures soon!

The Global Reduction Rule

- General idea of contextual semantics
  - Decompose the current expression into the redex-to-reduce-next and the remaining program
    - The remaining program is called a context
  - Reduce the redex "r" to some other expression "e"
  - The resulting (reduced) expression consists of "e" with the original context

As A Picture (1)

(Context)

…

x := 2+2

…

Step 1: Find The Redex

As A Picture (2)

(Context)

…

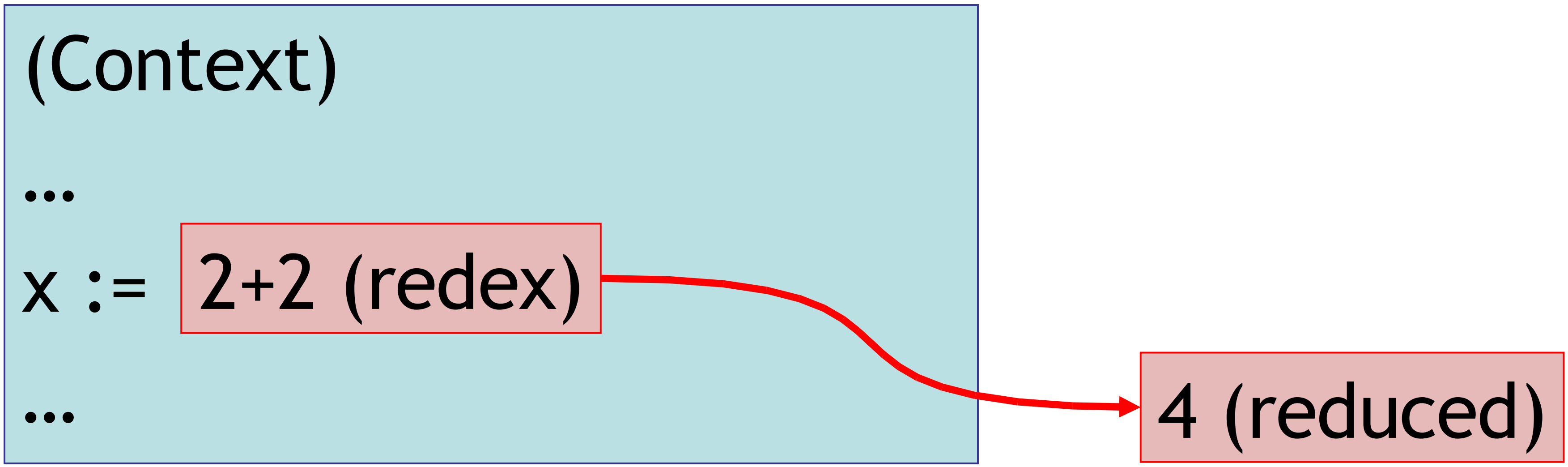
x := 2+2 (redex)

…

Step 1: Find The Redex

Step 2: Reduce The Redex

As A Picture (3)



Step 1: Find The Redex

Step 2: Reduce The Redex

As A Picture (4)

(Context)

…

x := 4

…

Step 1: Find The Redex

Step 2: Reduce The Redex

Step 3: Replace It In The Context

Contextual Analysis

- We use H to range over contexts
- We write H[r] for the expression obtained by placing redex r in context H
- Now we can define a small step

If $\langle r, \sigma \rangle \to \langle e, \sigma' \rangle$

then $\langle H[r], \sigma \rangle \to \langle H[e], \sigma' \rangle$

Contexts

- A context is like an expression (or command) with a marker • in the place where the redex goes
- Examples:
  - To evaluate "(1 + 3) + 2" we use the redex 1 + 3 and the context "• + 2"
  - To evaluate "if x > 2 then c_1 else c_2" we use the redex x and the context "if • > 2 then c_1 else c_2"

Context Terminology

- A context is also called an "expression with a hole"
- The marker • is sometimes called a hole
- H[r] is the expression obtained from H by replacing • with the redex r

"Avoid context and specifics; generalize and keep repeating the generalization."
-- Jack Schwartz

Contextual Semantics Example

- x := 1 ; x := x + 1 with initial state [x:=0]

<Comm, State>	**Redex •**	**Context**
<x := 1; x := x+1, [x := 0]>	x := 1	•; x := x+1
<skip; x := x+1, [x := 1]>	skip; x := x+1	•
<x := x+1, [x := 1]>	x	x := • + 1
What happens next?		

Contextual Semantics Example

- x := 1 ; x := x + 1 with initial state [x:=0]

<Comm, State>	**Redex •**	**Context**
<x := 1; x := x+1, [x := 0]>	x := 1	•; x := x+1
<skip; x := x+1, [x := 1]>	skip; x := x+1	•
<x := x+1, [x := 1]>	x	x := • + 1
<x := 1 + 1, [x := 1]>	1 + 1	x := •
<x := 2, [x := 1]>	x := 2	•
<skip, [x := 2]>		

More On Contexts

- Contexts are defined by a grammar:

  $H ::= \bullet \mid n + H$

  $\mid H + e$

  $\mid x := H$

  $\mid \text{if } H \text{ then } c_1 \text{ else } c_2$

  $\mid H;\ c$
- A context has exactly one • marker
- A redex is never a value

What's In A Context?

- Contexts specify precisely how to find the next redex
  - Consider e_1 + e_2 and its decomposition as H[r]
  - If e_1 is n_1 and e_2 is n_2 then H = • and r = $n_1 + n_2$
  - If e_1 is n_1 and e_2 is not n_2 then H = $n_1 + H_2$ and $e_2 = H_2[r]$
  - If e_1 is not n_1 then H = $H_1 + e_2$ and $e_1 = H_1[r]$
  - In the last two cases the decomposition is done recursively
  - Check that in each case the solution is unique

Unique Next Redex: Proof By Handwaving Examples

- e.g. c = "c_1; c_2" – either
  - c_1 = skip and then c = H[skip; c_2] with H = •
  - or $c_1 \neq$ skip and then c_1 = H[r]; so c = H'[r] with H' = H; c_2
- e.g. c = "if b then c_1 else c_2"
  - either b = true or b = false and then c = H[r] with H = •
  - or b is not a value and b = H[r]; so c = H'[r] with H' = if H then c_1 else c_2

Context Decomposition

- Decomposition theorem:

  If c is not "skip" then there exist unique H and r such that c is H[r]

  – "Exist" means progress

  – "Unique" means determinism

Short-Circuit Evaluation

- What if we want to express short-circuit evaluation of $\wedge$?
  - Define the following contexts, redexes and local reduction rules

$$H ::= \ldots \mid H \wedge b_2$$

$$r ::= \ldots \mid \text{true} \wedge b \mid \text{false} \wedge b$$

$$<\text{true} \wedge b, \sigma> \to <b, \sigma>$$

$$<\text{false} \wedge b, \sigma> \to <\text{false}, \sigma>$$

  - the local reduction kicks in before b_2 is evaluated

Contextual Semantics Summary

- Can view • as representing the program counter
- The advancement rules for • are non-trivial
  - At each step the entire command is decomposed
  - This makes contextual semantics inefficient to implement directly

- The major advantage of contextual semantics: it allows a mix of local and global reduction rules
  - For IMP we have only local reduction rules: only the redex is reduced
  - Sometimes it is useful to work on the context too
  - We'll do that when we study memory allocation, etc.

Reading Real-World Examples

- Cobbe and Felleisen, POPL 2005
- Small-step contextual opsem for Java
- Their rule for object field access:

$$P \vdash \langle \mathsf{E}[obj.fd], \mathcal{S} \rangle \hookrightarrow \langle \mathsf{E}[\mathcal{F}(fd)], \mathcal{S} \rangle$$
$$\text{where } \mathcal{F} = \mathit{fields}(\mathcal{S}(obj)) \text{ and } fd \in \mathrm{dom}(\mathcal{F})$$

$P \vdash$ <E[obj.fd],S> $\rightarrow$ <E[F(fd)],S>

– where F=fields(S(obj)) and fd $\in$ dom(F)

- They use “E” for context, we use “H”
- They use “S” for state, we use “σ”

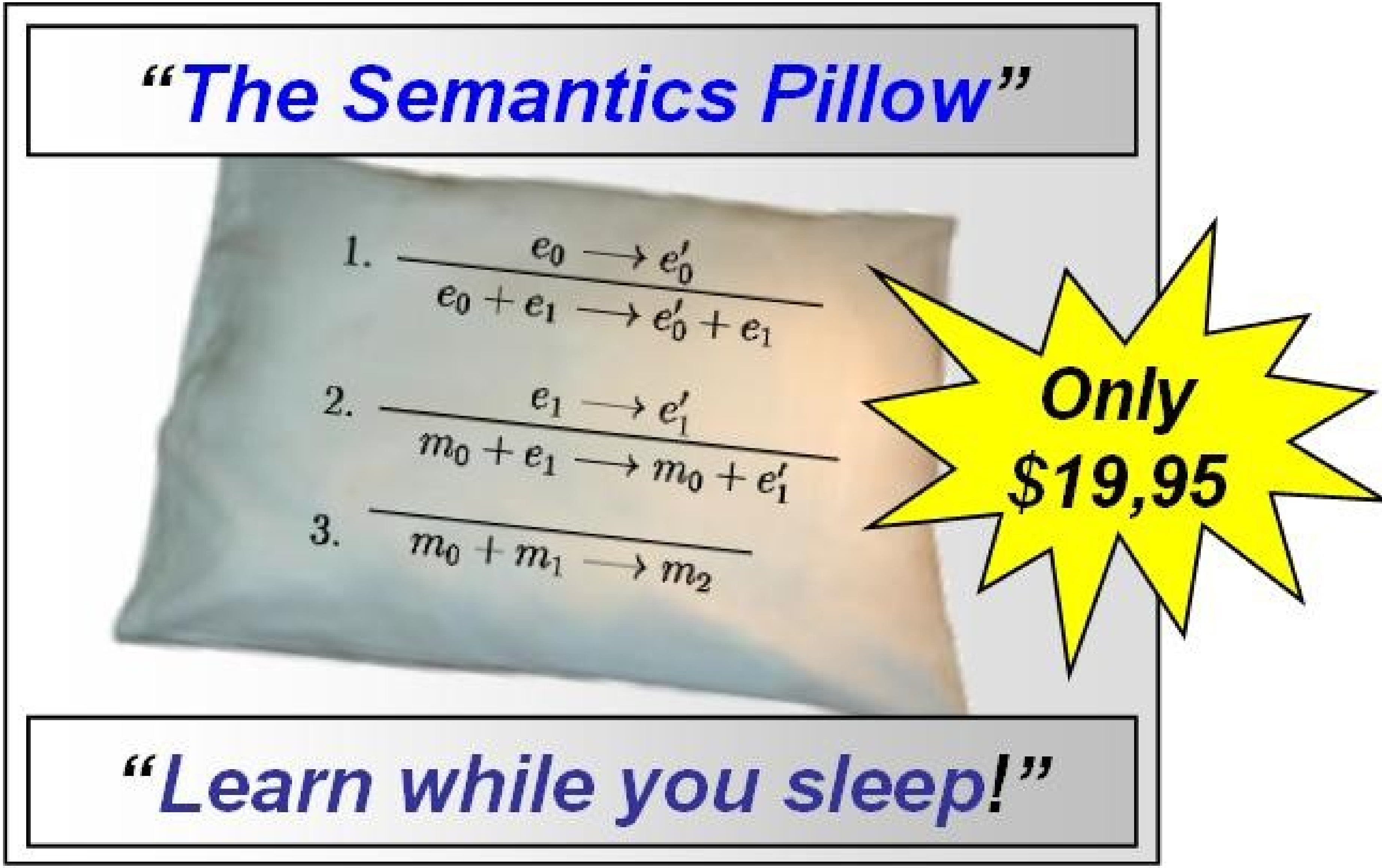
Lost In Translation

- $P \vdash$ <H[obj.fd],σ> $\rightarrow$ <H[F(fd)],σ>
  - Where F=fields(σ(obj)) and fd $\in$ dom(F)
- They have "P $\vdash$", but that just means "it can be proved in our system given P"
- <H[obj.fd],σ> $\rightarrow$ <H[F(fd)],σ>
  - Where F=fields(σ(obj)) and fd $\in$ dom(F)

Lost In Translation 2

- $<H[obj.fd],\sigma> \to <H[F(fd)],\sigma>$
  - Where $F = fields(\sigma(obj))$ and $fd \in dom(F)$
- They model objects (like obj), but we do not (yet) – let's just make fd a variable:
- $<H[fd],\sigma> \to <H[F(fd)],\sigma>$
  - Where $F = \sigma$ and $fd \in L$
- Which is just our variable-lookup rule:
- $<H[fd],\sigma> \to <H[\sigma(fd)],\sigma>$ (when $fd \in L$)

“Sleep On It”


“The Semantics Pillow”
1. $\dfrac{e_0 \longrightarrow e_0'}{e_0 + e_1 \longrightarrow e_0' + e_1}$
2. $\dfrac{e_1 \longrightarrow e_1'}{m_0 + e_1 \longrightarrow m_0 + e_1'}$
3. $\dfrac{}{m_0 + m_1 \longrightarrow m_2}$
Only
$19,95
“Learn while you sleep!”

Homework

- Homework 2 Out Today
  - Due Next Week
- Read Winskel Chapter 3
- Want an extra opsem review?
  - *Natural deduction* article
  - Plotkin Chapter 2
- Optional Philosophy of Science article