Global Optimizations

One-Slide Summary

- A **global optimization** changes an entire method (consisting of multiple basic blocks).
- We must be **conservative** and only apply global optimizations when they preserve the semantics.
- We use **global flow analyses** to determine if it is OK to apply an optimization.
- Flow analyses are built out of simple **transfer functions** and can work **forwards** or **backwards**.

#2

Lecture Outline

- Global flow analysis
- Global constant propagation
- Liveness analysis

#3

Local Optimization

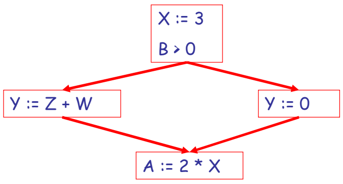
Recall the simple basic-block optimizations
- Constant propagation
- Dead code elimination

```
X := 3          X := 3
Y := Z * W  →   Y := Z * W  →   Y := Z * W
Q := X + Y      Q := 3 + Y      Q := 3 + Y
```

#4

Global Optimization

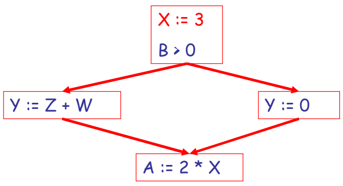
These optimizations can be extended to an entire control-flow graph



#5

Global Optimization

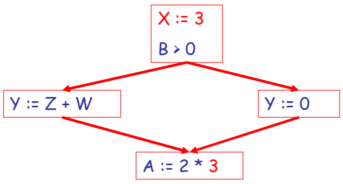
These optimizations can be extended to an entire control-flow graph



#6

Global Optimization

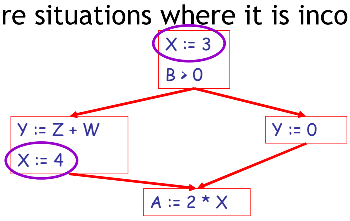
These optimizations can be extended to an entire control-flow graph



#7

Correctness

- How do we know it is OK to globally propagate constants?
- There are situations where it is incorrect:



#8

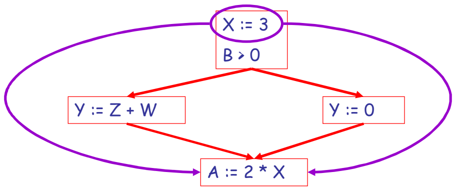
Correctness (Cont.)

To replace a use of x by a constant k we must know this **correctness condition**:

On every path to the use of x, the last assignment to x is x := k **

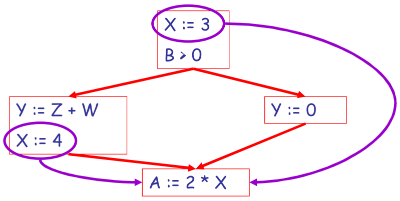
#9

Example 1 Revisited



#10

Example 2 Revisited



#11

Discussion

- The correctness condition is not trivial to check
- "**All paths**" includes paths around loops and through branches of conditionals
- Checking the condition requires **global analysis**
  - Global = an analysis of the entire control-flow graph for *one* method body

#12

Global Analysis

Global optimization tasks share several traits:

- The optimization depends on knowing a property P at a particular point in program execution
- Proving P at any point requires knowledge of the entire method body
- Property P is typically *undecidable*!

#13

Undecidability of Program Properties

- **Rice's Theorem**: Most interesting dynamic properties of a program are undecidable:
  - Does the program halt on all (some) inputs?
    - This is called the **halting problem**
  - Is the result of a function F always positive?
    - Assume we can answer this question precisely
    - Take function H and find out if it halts by testing function F(x) { H(x); return 1; } whether it has positive result
- Syntactic properties are decidable!
  - e.g., How many occurrences of "x" are there?
- Programs without looping are also decidable!

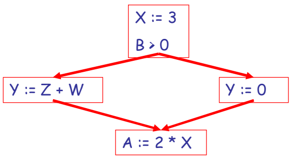
#14

Conservative Program Analyses

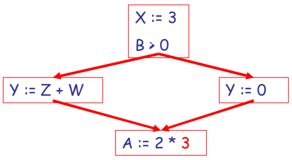
- So, we cannot tell for sure that "x" is always 3
  - Then, how can we apply constant propagation?
- It is OK to be **conservative**. If the optimization requires P to be true, then want to know either
  - P is definitely true
  - Don't know if P is true
- It is always correct to say "don't know"
  - We try to say don't know as rarely as possible
- All program analyses are conservative

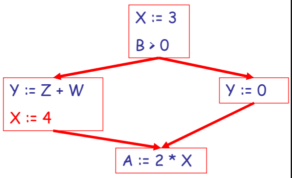
#15

Global Optimization: Review

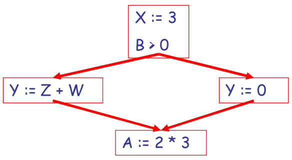


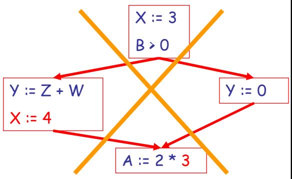
Global Optimization: Review





Global Optimization: Review





- To replace a use of x by a constant k we must know that:

  On every path to the use of x, the last assignment to x is x := k **

Review

- The correctness condition is not trivial to check
- Checking the condition requires global analysis
  - An analysis of the entire control-flow graph for one method body

#19

Global Analysis

- **Global dataflow analysis** is a standard technique for solving problems with these characteristics
- Global constant propagation is one example of an optimization that requires global dataflow analysis

#20

Global Constant Propagation

- Global constant propagation can be performed at any point where ** holds
- Consider the case of computing ** for a single variable X at all program points

#21

It's rarely this easy to find key locations …

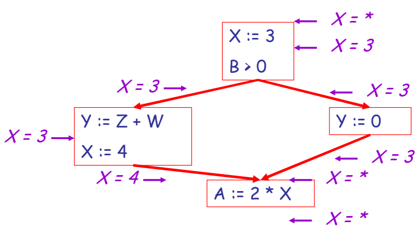
Global Constant Propagation (Cont.)

- To make the problem precise, we associate one of the following values with X *at every program point*

value	*interpretation*
#	This statement is not reachable
c	X = constant c
*	Don't know if X is a constant

#23

Example



#24

Using the Information

- Given global constant information, it is easy to perform the optimization
  - Simply inspect the x = ? associated with a statement using x
  - If x is constant at that point replace that use of x by the constant
- But how do we compute the properties x = ?

#25

The Idea

The analysis of a complicated program can be expressed as a combination of simple rules relating the change in information between adjacent statements

#26

Explanation

- The idea is to "push" or "**transfer**" information from one statement to the next
- For each statement s, we compute information about the value of x immediately before and after s

$C_{in}(x,s)$ = value of x before s

$C_{out}(x,s)$ = value of x after s

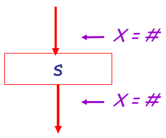
#27

Transfer Functions

- Define a **transfer function** that transfers information from one statement to another

#28

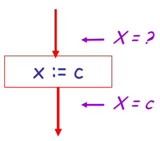
Rule 1



$C_{out}(x, s) = \#$ if $C_{in}(x, s) = \#$

#29

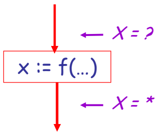
Rule 2



$C_{out}(x, x := c) = c$ if c is a constant

#30

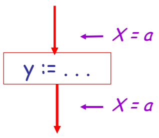
Rule 3



$C_{out}(x, x := f(\ldots)) = *$

#31

Rule 4



$C_{out}(x, y := \ldots) = C_{in}(x, y := \ldots)$ if $x \neq y$

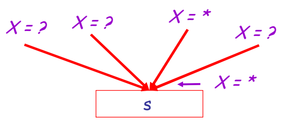
#32

The Other Half

- Rules 1-4 relate the *in* of a statement to the *out* of the same statement
  - they propagate information across statements
- Now we need rules relating the *out* of one statement to the *in* of the successor statement
  - to propagate information forward across CFG edges
- In the following rules, let statement s have immediate predecessor statements $p_1,\ldots,p_n$

#33

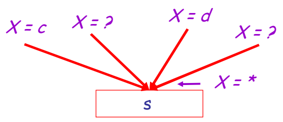
Rule 5



if $C_{out}(x, p_i) = *$ for some i, then $C_{in}(x, s) = *$

#34

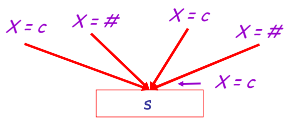
Rule 6



if $C_{out}(x, p_i) = c$ and $C_{out}(x, p_j) = d$ and $d \neq c$
then $C_{in}(x, s) = *$

#35

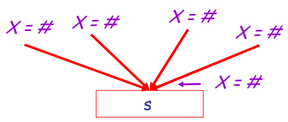
Rule 7



if $C_{out}(x, p_i) = c$ or # for all i,
then $C_{in}(x, s) = c$

#36

Rule 8



if $C_{out}(x, p_i) = \#$ for all i,
then $C_{in}(x, s) = \#$

#37

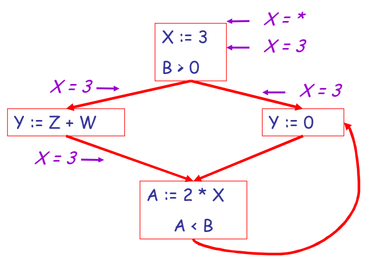
An Algorithm

1. For every entry s to the program, set $C_{in}(x, s) = *$
2. Set $C_{in}(x, s) = C_{out}(x, s) = \#$ everywhere else
3. Repeat until all points satisfy 1-8:
   Pick s not satisfying 1-8 and update using the appropriate rule

#38

The Value

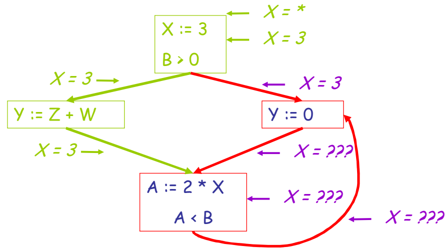
- To understand why we need #, look at a loop



#39

The Value

- To understand why we need #, look at a loop



#40

The Value # (Cont.)

- Because of cycles, all points must have values at all times during the analysis
- Intuitively, assigning some initial value allows the analysis to break cycles
- The initial value # means "so far as we know, control never reaches this point"

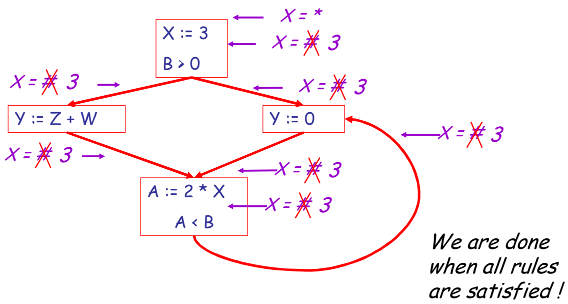
#41

Sometimes all paths lead to the same place.

Thus you need #.

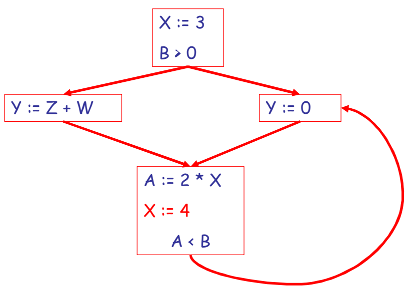
#42

Example



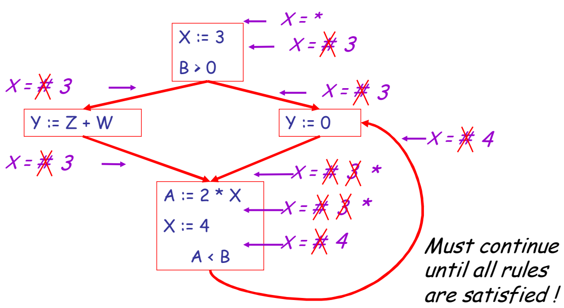
#43

Another Example



#44

Another Example



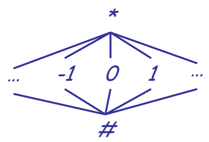
#45

Orderings

- We can simplify the presentation of the analysis by ordering the values

< c < *

Drawing a picture with "lower" values drawn lower, we get



Orderings (Cont.)

- * is the greatest value, # is the least
  - All constants are in between and incomparable
- Let *lub* be the least-upper bound in this ordering
- Rules 5-8 can be written using lub:

$C_{in}(x, s)$ = lub { $C_{out}(x, p)$ | p is a predecessor of s }

Termination

- Simply saying "repeat until nothing changes" doesn't guarantee that eventually nothing changes
- The use of lub explains why the algorithm **terminates**
  - Values start as # and only *increase*
  - # can change to a constant, and a constant to *
  - Thus, C_(x, s) can change at most twice

Termination (Cont.)

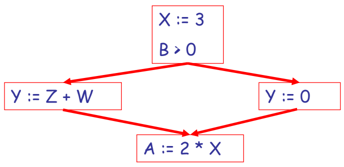
Thus the algorithm is linear in program size

Number of steps =
Number of C_(….) values computed * 2 =
Number of program statements * 4

#49

Liveness Analysis

Once constants have been globally propagated, we would like to eliminate dead code

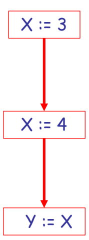


After constant propagation, X := 3 is dead ?
(assuming this is the entire CFG)

#50

Live and Dead

- The first value of x is **dead** (never used)
- The second value of x is **live** (may be used)
- Liveness is an important concept



#51

Liveness

A variable x is live at statement s if

- There exists a statement s' that uses x
- There is a path from s to s'
- That path has no intervening assignment to x

#52

Global Dead Code Elimination

- A statement x := … is dead code if x is dead after the assignment
- Dead statements can be deleted from the program
- But we need liveness information first . . .

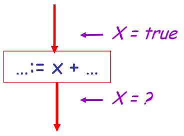
#53

Computing Liveness

- We can express liveness in terms of information transferred between adjacent statements, just as in constant propagation
- Liveness is simpler than constant propagation, since it is a boolean property (true or false)

#54

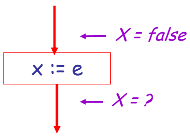
Liveness Rule 1



$L_{in}(x, s)$ = true if s refers to x on the rhs

#55

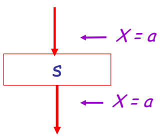
Liveness Rule 2



$L_{in}(x, x := e)$ = false if e does not refer to x

#56

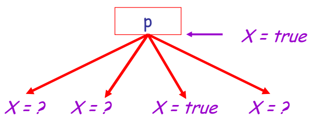
Liveness Rule 3



$L_{in}(x, s) = L_{out}(x, s)$ if s does not refer to x

#57

Liveness Rule 4



$L_{out}(x, p) = \text{Ú} \{ L_{in}(x, s) \mid s \text{ a successor of } p \}$

#58

Algorithm

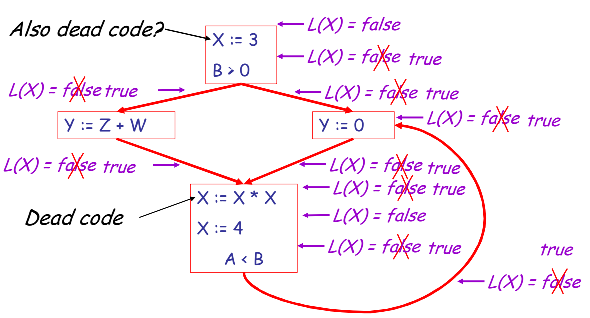
1. Let all L_(...) = false initially

2. Repeat process until all statements s satisfy rules 1-4 :

   Pick s where one of 1-4 does not hold and update using the appropriate rule

#59

Another Example



#60

Termination

- A value can change from false to true, but not the other way around
- Each value can change only once, so termination is guaranteed
- Once the analysis is computed, it is simple to eliminate dead code

#61

Forward vs. Backward Analysis

We've seen two kinds of analysis:

Constant propagation is a **forwards** analysis: information is pushed from inputs to outputs

Liveness is a **backwards** analysis: information is pushed from outputs back towards inputs

#62

Analysis Analysis

- There are many other global flow analyses
- Most can be classified as either forward or backward
- Most also follow the methodology of local rules relating information between adjacent program points

#63

Homework

- PA4 due this Friday March 30th (tomorrow)
- For Tuesday – Read Chapter 7.7
  - Optional David Bacon article
- Midterm 2 – Thursday April 12 (15 days)

#64