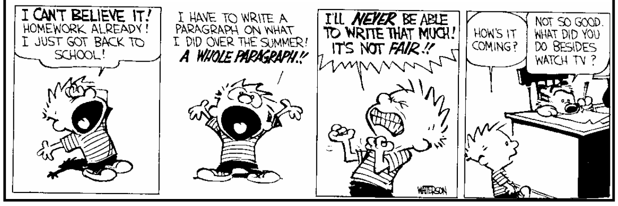
Lexical Analysis

Finite Automata

(Part 1 of 2)



Cunning Plan

- Informal Sketch of Lexical Analysis
  - Identifies tokens from input string
  - lexer : (char list) → (token list)
- Issues in Lexical Analysis
  - Lookahead
  - Ambiguity
- Specifying Lexers
  - Regular Expressions
  - Examples

#2

One-Slide Summary

- Lexical analysis turns a stream of characters into a stream of tokens.
- Regular expressions are a way to specify sets of strings. We use them to describe tokens.

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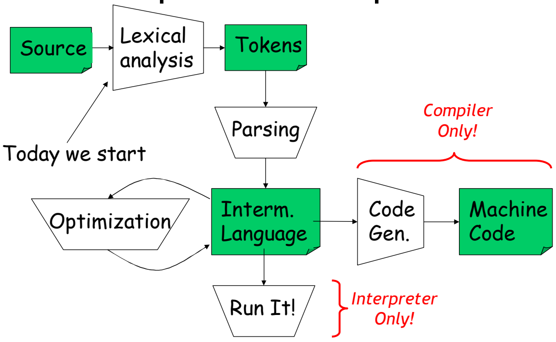
Recall: The Structure of a Compiler or Interpreter



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Lexical Analysis

- What do we want to do? Example:

```
if (i == j)
    z = 0;
else
    z = 1;
```

- The input is just a sequence of characters:

  `\tif (i == j)\n\t\tz = 0;\n\telse\n\t\tz = 1;`

- Goal: Partition input string into substrings
  - And classify them according to their role

#5

What's a Token?

- Output of lexical analysis is a list of tokens
- A token is a syntactic category
  - In English:
    noun, verb, adjective, …
  - In a programming language:
    Identifier, Integer, Keyword, Whitespace, …
- Parser relies on the token distinctions:
  - e.g., identifiers are treated differently than keywords

#6

Tokens

- Tokens correspond to sets of strings.

- Identifier: *strings of letters or digits, starting with a letter*
- Integer: *a non-empty string of digits*
- Keyword: *"else" or "if" or "begin" or …*
- Whitespace: *a non-empty sequence of blanks, newlines, and tabs*
- OpenPar: *a left-parenthesis*

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Lexical Analyzer: Implementation

- An implementation must do two things:

1. Recognize substrings corresponding to tokens

2. Return the value or lexeme of the token
   - The lexeme is the substring

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Example

- Recall:
  \tif (i == j)\n\t\tz = 0;\n\telse\n\t\tz = 1;
- Token-lexeme pairs returned by the lexer:
  - (Whitespace, "\t")
  - (Keyword, "if")
  - (OpenPar, "(")
  - (Identifier, "i")
  - (Relation, "==")
  - (Identifier, "j")
  - …

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Lexical Analyzer: Implementation

- The lexer usually *discards* "uninteresting" tokens that don't contribute to parsing.
- Examples: Whitespace, Comments
- Question: What happens if we remove all whitespace and all comments *prior* to lexing?

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Lookahead

- Two important points:
  1. The goal is to partition the string. This is implemented by reading left-to-right, recognizing one token at a time
  2. "Lookahead" may be required to decide where one token ends and the next token begins
  - Even our simple example has lookahead issues
    i vs. **if**
    = vs. **==**

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Next We Need

- A way to describe the lexemes of each token
- A way to resolve ambiguities
  - Is if two variables i and f?
  - Is == two equal signs = =?

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Regular Languages

- There are several formalisms for specifying tokens
- *Regular languages* are the most popular
  - Simple and useful theory
  - Easy to understand
  - Efficient implementations

#13

Languages

Def. Let Σ be a set of characters. A *language over Σ* is a set of strings of characters drawn from Σ

(Σ is called the *alphabet*)

#14

Examples of Languages

- Alphabet = English characters
- Language = English sentences
- Not every string on English characters is an English sentence

- Alphabet = ASCII
- Language = C programs
- Note: ASCII character set is different from English character set

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Notation

- Languages are sets of strings
- Need some notation for specifying which sets we want
- For lexical analysis we care about *regular languages*, which can be described using *regular expressions*.

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Regular Expressions and Regular Languages

- Each regular expression is a notation for a regular language (a set of words)
  - You'll see the exact notation in a minute!
- If A is a regular expression then we write L(A) to refer to the language denoted by A

#17

Atomic Regular Expressions

- Single character: 'c'

  L('c') = { "c" } (for any c $\in \Sigma$)

- Concatenation: AB (where A and B are reg. exp.)

  L(AB) = { ab | a $\in$ L(A) and b $\in$ L(B) }

- Example: L('i' 'f') = { "if" }

  (we will abbreviate 'i' 'f' as 'if')

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Compound Regular Expressions

- Union

  L(A | B) = { s | s ∈ L(A) *or* s ∈ L(B) }

- Examples:

  'if' | 'then' | 'else' = { "if", "then", "else"}

  '0' | '1' | … | '9' = { "0", "1", …, "9" }

  (note the … are just an abbreviation)

- Another example:

  ('0' | '1') ('0' | '1') = { "00", "01", "10", "11" }

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More Compound Regular Expressions

- So far we do not have a notation for infinite languages
- Iteration: A^*

  $L(A^*)$ = { "" } ∪ L(A) ∪ L(AA) ∪ L(AAA) ∪ …

- Examples:

  $'0'^*$ = { "", "0", "00", "000", …}

  '1' $'0'^*$ = { strings starting with 1, followed by 0's }

- Epsilon: ε

  L(ε) = { "" }

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Example: Keyword

- Keyword: *"else" or "if" or "begin" or …*

  'else' | 'if' | 'begin' | …

  (Recall: 'else' abbreviates 'e' 'l' 's' 'e')

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Example: Integers

Integer: *a non-empty string of digits*

digit = '0' | '1' | '2' | '3' | '4' | '5' | '6' | '7' | '8' | '9'

number = digit digit*

Abbreviation: A+ = A A*

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Example: Identifier

Identifier: *strings of letters or digits, starting with a letter*

letter = 'A' | … | 'Z' | 'a' | … | 'z'

identifier = letter (letter | digit) *

Is (letter* | digit*) the same ?

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Example: Whitespace

Whitespace: *a non-empty sequence of blanks, newlines, and tabs*

(' ' | '\t' | '\n')+

(Can you spot a small mistake?)

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Example: Phone Numbers

- Regular expressions are all around you!
- Consider (434) 924-1021

  Σ = { 0, 1, 2, 3, …, 9, (,), - }
  area = digit^3
  exchange = digit^3
  phone = digit^4
  number =
  '(' area ')' exchange '-' phone

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Example: Email Addresses

- Consider *weimer@cs.virginia.edu*

Σ = letters $\cup$ { ., @ }
name = letter^+
address = name '@' name ('.' name)*

#26

Summary

- Regular expressions describe many useful languages
- Next: Given a string *s* and a rexp *R*, is

$$s \in L(R)?$$

- But a yes/no answer is not enough!
- Instead: partition the input into lexemes
- We will adapt regular expressions to this goal

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Outline

- Specifying lexical structure using regular expressions
- Finite automata
  - Deterministic Finite Automata (DFAs)
  - Non-deterministic Finite Automata (NFAs)
- Implementation of regular expressions

  RegExp ⇒ NFA ⇒ DFA ⇒ Tables

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Regular Expressions => Lexical Spec. (1)

1. Select a set of tokens
   - Number, Keyword, Identifier, ...
2. Write a R.E. for the lexemes of each token
   - **Number** = **digit⁺**
   - **Keyword** = **'if' | 'else' | …**
   - **Identifier** = **letter (letter | digit)***
   - **OpenPar** = **'('**
   - …

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Regular Expressions => Lexical Spec. (2)

3. Construct R, matching all lexemes for all tokens

$$R = \text{Keyword} \mid \text{Identifier} \mid \text{Number} \mid \ldots$$
$$= R_1 \mid R_2 \mid R_3 \mid \ldots$$

Fact: If $s \in L(R)$ then s is a lexeme

- Furthermore $s \in L(R_j)$ for some "j"
- This "j" determines the token that is reported

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Regular Expressions => Lexical Spec. (3)

4. Let the input be $x_1 \ldots x_n$
   ($x_1 \ldots x_n$ are characters in the language alphabet Σ)
   - For $1 \le i \le n$ check
     $x_1 \ldots x_i \in L(R)$?
5. It must be that
   $x_1 \ldots x_i \in L(R_j)$ for some i and j
6. Remove $x_1 \ldots x_i$ from input and go to step (4.)

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Lexing Example

R = Whitespace | Integer | Identifier | '+'

- Parse "f +3 +g"
  - "f" matches R, more precisely Identifier
  - "+" matches R, more precisely '+'
  - ...
  - The token-lexeme pairs are
    (Identifier, "f"), ('+', "+"), (Integer, "3")
    (Whitespace, " "), ('+', "+"), (Identifier, "g")
- We would like to drop the Whitespace tokens
  - after matching Whitespace, continue matching

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Ambiguities (1)

- There are *ambiguities* in the algorithm
- Example:
  R = Whitespace | Integer | Identifier | '+'
- Parse "foo+3"
  - "f" matches R, more precisely Identifier
  - But also "fo" matches R, and "foo", but not "foo+"
- How much input is used? What if
  - $x_1 \ldots x_i \in L(R)$ and also $x_1 \ldots x_K \in L(R)$
  - **"Maximal munch"** rule: Pick the longest possible substring that matches R

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More Ambiguities

R = Whitespace | 'new' | Integer | Identifier

- Parse "new foo"
  - "new" matches R, more precisely 'new'
  - but also Identifier, which one do we pick?
- In general, if $x_1 \ldots x_i \in L(R_j)$ and $x_1 \ldots x_i \in L(R_k)$
  - Rule: use rule listed first (j if $j < k$)
- We must list 'new' before Identifier

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Error Handling

R = Whitespace | Integer | Identifier | '+'

- Parse "=56"
  - No prefix matches R: not "=", nor "=5", nor "=56"
- Problem: Can't just get stuck …
- Solution:
  - Add a rule matching all "bad" strings; and put it last
- Lexer tools allow the writing of:

  $R = R_1 \mid \ldots \mid R_n \mid$ Error

  - Token Error matches if nothing else matches

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Summary

- Regular expressions provide a concise notation for string patterns
- Use in lexical analysis requires small extensions
  - To resolve ambiguities
  - To handle errors
- Good algorithms known (next)
  - Require only single pass over the input
  - Few operations per character (table lookup)

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Finite Automata

- Regular expressions = specification
- Finite automata = implementation

- A finite automaton consists of
  - An input alphabet Σ
  - A set of states **S**
  - A start state **n**
  - A set of accepting states $\mathbf{F} \subseteq \mathbf{S}$
  - A set of transitions **state** $\rightarrow^{\text{input}}$ **state**

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Finite Automata

- Transition

$$s_1 \rightarrow^{a} s_2$$

- Is read

  In state s_1 on input "a" go to state s_2

- If end of input (or no transition possible)
  - If in accepting state ⇒ accept
  - Otherwise ⇒ reject

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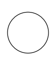
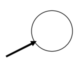
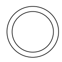
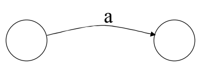
Finite Automata State Graphs

- A state 
  - The start state 
  - An accepting state 
  - A transition 

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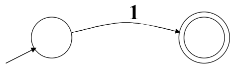
A Simple Example

- A finite automaton that accepts only "1"

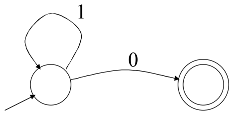


- A finite automaton <u>accepts</u> a string if we can follow transitions labeled with the characters in the string from the start to some accepting state

#40

Another Simple Example

- A finite automaton accepting any number of 1's followed by a single 0
- Alphabet $\Sigma = \{0,1\}$

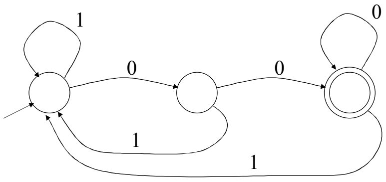


- Check that "**1110**" is accepted but "**110…**" is not

#41

And Another Example

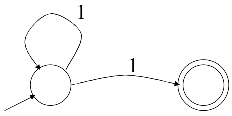
- Alphabet $\Sigma = \{0,1\}$
- What language does this recognize?



#42

And Another Example

- Alphabet still $\Sigma = \{ 0, 1 \}$



- The operation of the automaton is not completely defined by the input
  - On input "11" the automaton could be in either state

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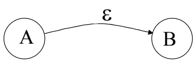
Epsilon Moves

- Another kind of transition: ε-moves



- Machine can move from state A to state B *without reading input*

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Deterministic and Nondeterministic Automata

- Deterministic Finite Automata (DFA)
  - One transition per input per state
  - No ε-moves
- Nondeterministic Finite Automata (NFA)
  - Can have multiple transitions for one input in a given state
  - Can have ε-moves
- Finite automata have finite memory
  - Need only to encode the current state

#45

Execution of Finite Automata

- A DFA can take only one path through the state graph
  - Completely determined by input
- NFAs can choose
  - Whether to make ε-moves
  - Which of multiple transitions for a single input to take

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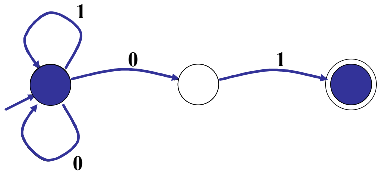
Acceptance of NFAs

- An NFA can get into multiple states



- Input: 1 0 1
- Rule: NFA accepts if it can get in a final state

#47

NFA vs. DFA (1)

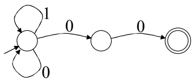
- NFAs and DFAs recognize the *same* set of languages (regular languages)
  - They have the same expressive power
- DFAs are easier to implement
  - There are no choices to consider

#48

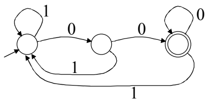
NFA vs. DFA (2)

- For a given language the NFA can be simpler than the DFA

NFA



DFA



- DFA can be *exponentially* larger than NFA

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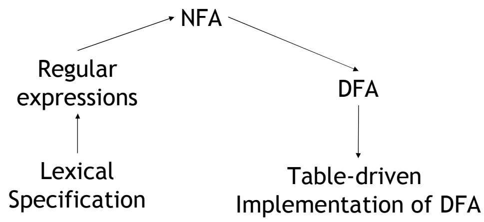
Regular Expressions to Finite Automata

- High-level sketch



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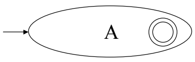
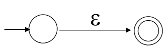
Regular Expressions to NFA (1)

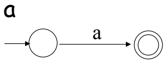
- For each kind of rexp, define an NFA
  - Notation: NFA for rexp A



- For ε



- For input a



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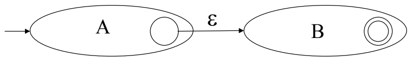
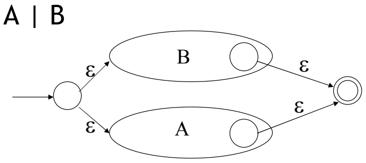
Regular Expressions to NFA (2)

- For AB



- For A | B



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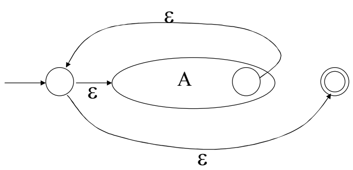
Regular Expressions to NFA (3)

- For A*



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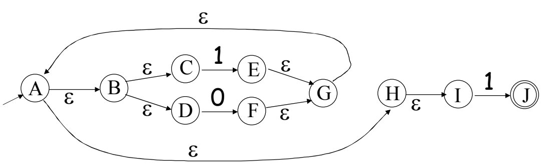
Example of RegExp -> NFA conversion

- Consider the regular expression

(1 | 0)*1

- The NFA is



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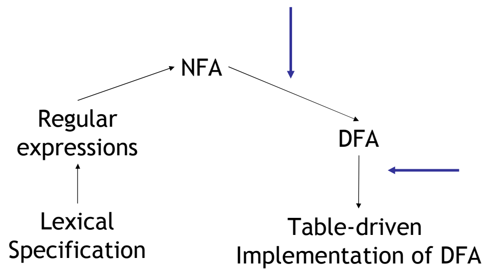
Next



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NFA to DFA: The Trick

- Simulate the NFA
- Each state of DFA
  = a non-empty *subset of states* of the NFA
- Start state
  = the set of NFA states reachable through ε-moves from NFA start state
- Add a transition S $\rightarrow^{a}$ S' to DFA iff
  - S' is the set of NFA states reachable from the states in S after seeing the input a
    - considering ε-moves as well

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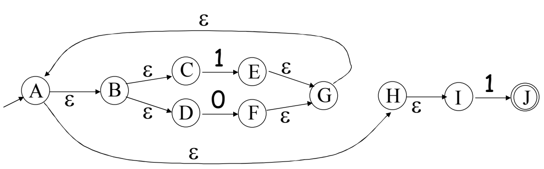
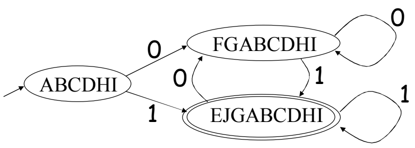
NFA → DFA Example





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NFA → DFA: Remark

- An NFA may be in many states at any time
- How many different states?
- If there are N states, the NFA must be in some subset of those N states
- How many non-empty subsets are there?
  - 2^N - 1 = finitely many

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Implementation

- A DFA can be implemented by a 2D table T
  - One dimension is “states”
  - Other dimension is “input symbols”
  - For every transition $S_i \to^a S_k$ define T[i,a] = k
- DFA “execution”
  - If in state S_i and input a, read T[i,a] = k and skip to state S_k
  - Very efficient

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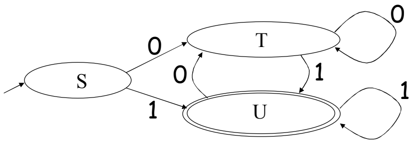
Table Implementation of a DFA



	0	1
S	T	U
T	T	U
U	T	U

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Implementation (Cont.)

- NFA → DFA conversion is at the heart of tools such as flex or ocamllex
- But, DFAs can be huge
- In practice, flex-like tools trade off speed for space in the choice of NFA and DFA representations

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PA1: Lexical Analysis

- Correctness is job #1.
  - And job #2 and #3!
- Tips on building large systems:
  - Keep it simple
  - Design systems that can be tested
  - Don't optimize prematurely
  - It is easier to modify a working system than to get a system working

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Homework

- Thursday: Chapter 2.4 - 2.4.1
  - 13 CD - 15 CD on the web
- Friday: PA1 due
- Next Tuesday: Chapters 2.3 - 2.3.2
  - Optional Wikipedia article

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