CS 6120/CS4120: Natural Language Processing

Instructor: Prof. Lu Wang
College of Computer and Information Science
Northeastern University
Webpage: www.ccs.neu.edu/home/luwang

Outline

- Vector Semantics
- Sparse representation
  - Pointwise Mutual Information (PMI)
- Dense representation
  - Singular Value Decomposition (SVD)
  - Brown Clusters
  - Neural Language Model

Sparse versus dense vectors

- PPMI vectors are
  - **long** (length |V|= 20,000 to 50,000)
  - **sparse** (most elements are zero)

Sparse versus dense vectors

- PPMI vectors are
  - **long** (length |V|= 20,000 to 50,000)
  - **sparse** (most elements are zero)
- Alternative: learn vectors which are
  - **short** (length 200-1000)
  - **dense** (most elements are non-zero)

Sparse versus dense vectors

- Why dense vectors?
  - Short vectors may be easier to use as features in machine learning (less weights to tune)
  - Dense vectors may generalize better than storing explicit counts
  - They may do better at capturing synonymy:
    - *car* and *automobile* are synonyms; but are represented as distinct dimensions; this fails to capture similarity between a word with *car* as a neighbor and a word with *automobile* as a neighbor

Three methods for getting short dense vectors

- Singular Value Decomposition (SVD)
- Brown clustering
- "Neural Language Model" – inspired by predictive models

Singular Value Decomposition (SVD)

Rank of a Matrix

- What is the rank of a matrix A?

Rank of a Matrix

- What is the rank of a matrix A?
- Number of linearly independent columns of A

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix}$$

Rank of a Matrix

- What is the rank of a matrix A?
- Number of linearly independent columns of A

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix}$$

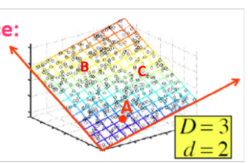
- Rank is 2
- We can rewrite A as two “basis” vectors: [1 2 1] [-2 -3 1]

Rank as “Dimensionality”

Cloud of points 3D space:

- Think of point positions as a matrix:

$$\text{1 row per point: } \begin{array}{c} \begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix} \end{array} \begin{array}{c} \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \end{array}$$

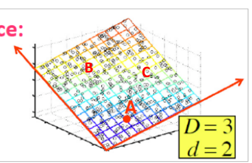


Rank as “Dimensionality”

Cloud of points 3D space:

- Think of point positions as a matrix:

$$\text{1 row per point: } \begin{array}{c} \begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix} \end{array} \begin{array}{c} \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \end{array}$$

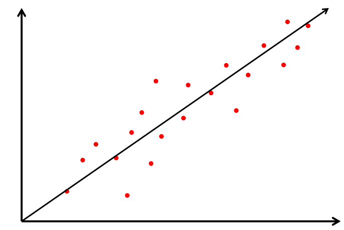


- Rewrite the coordinates in a more efficient way!
  - Old basis vectors: [1 0 0], [0 1 0], [0 0 1]
  - New basis vectors: [1 2 1], [-2 -3 1]

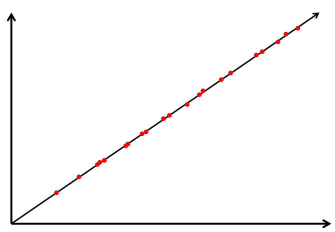
Intuition of Dimensionality Reduction

- Approximate an N-dimensional dataset using fewer dimensions
- By first rotating the axes into a new space
- In which the highest order dimension captures the most variance in the original dataset
- And the next dimension captures the next most variance, etc.

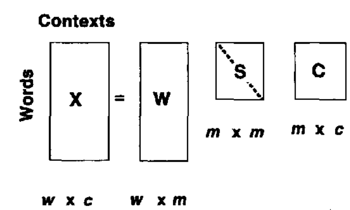
Sample Dimensionality Reduction



Sample Dimensionality Reduction



Singular Value Decomposition



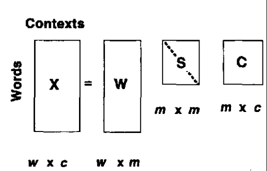
(assuming the matrix has rank m)   Landuaer and Dumais 1997

Singular Value Decomposition

*Any rectangular w x c matrix **X** equals the product of 3 matrices:*

W: rows corresponding to original but m columns represents a dimension in a new latent space, such that

- m column vectors are orthogonal to each other
- Columns are ordered by the amount of variance in the dataset each new dimension accounts for



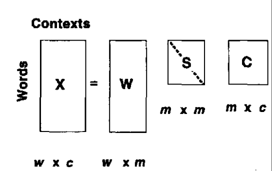
Singular Value Decomposition

*Any rectangular w x c matrix **X** equals the product of 3 matrices:*

W: rows corresponding to original but m columns represents a dimension in a new latent space, such that

- m column vectors are orthogonal to each other
- Columns are ordered by the amount of variance in the dataset each new dimension accounts for

S: diagonal *m* x *m* matrix of **singular values** expressing the importance of each dimension.



Singular Value Decomposition

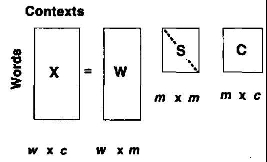
*Any rectangular w x c matrix **X** equals the product of 3 matrices:*

W: rows corresponding to original but m columns represents a dimension in a new latent space, such that

- m column vectors are orthogonal to each other
- Columns are ordered by the amount of variance in the dataset each new dimension accounts for

S: diagonal *m* x *m* matrix of **singular values** expressing the importance of each dimension.

C: columns corresponding to original but m rows corresponding to singular values



Singular Value Decomposition

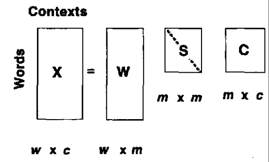
*Any rectangular w x c matrix **X** equals the product of 3 matrices:*

W: rows corresponding to original but m columns represents a dimension in a new latent space, such that

- m column vectors are orthogonal to each other
- Columns are ordered by the amount of variance in the dataset each new dimension accounts for

S: diagonal *m* x *m* matrix of **singular values** expressing the importance of each dimension.

C: columns corresponding to original but m rows corresponding to singular values

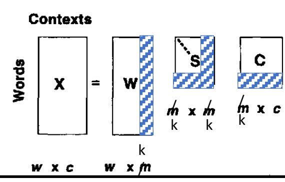


Existing tools from Python, MATLAB, R, etc, for SVD

SVD applied to term-document matrix: Latent Semantic Analysis

Deerwester et al (1988)

- If instead of keeping all m dimensions, we just keep the top k singular values. Let's say 300.
- Each row of W (keeping k columns of the original W):
  - A k-dimensional vector
  - Representing word w



SVD on Term-Document Matrix: Example

- The matrix X

	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

Matrix **W**

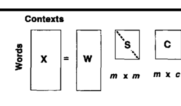
	1	2	3	4	5
ship	−0.44	−0.30	0.57	0.58	0.25
boat	−0.13	−0.33	−0.59	0.00	0.73
ocean	−0.48	−0.51	−0.37	0.00	−0.61
wood	−0.70	0.35	0.15	−0.58	0.16
tree	−0.26	0.65	−0.41	0.58	−0.09

Matrix **S**

	1	2	3	4	5
1	2.16	0.00	0.00	0.00	0.00
2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	1.28	0.00	0.00
4	0.00	0.00	0.00	1.00	0.00
5	0.00	0.00	0.00	0.00	0.39

Matrix **C**

	d_1	d_2	d_3	d_4	d_5	d_6
1	−0.75	−0.28	−0.20	−0.45	−0.33	−0.12
2	−0.29	−0.53	−0.19	0.63	0.22	0.41
3	0.28	−0.75	0.45	−0.20	0.12	−0.33
4	0.00	0.00	0.58	0.00	−0.58	0.58
5	−0.53	0.29	0.63	0.19	0.41	−0.22



Matrix **W**

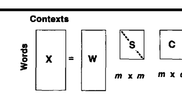
	1	2	3	4	5
ship	−0.44	−0.30	0.57	0.58	0.25
boat	−0.13	−0.33	−0.59	0.00	0.73
ocean	−0.48	−0.51	−0.37	0.00	−0.61
wood	−0.70	0.35	0.15	−0.58	0.16
tree	−0.26	0.65	−0.41	0.58	−0.09

Matrix **S**

	1	2	3	4	5
1	2.16	0.00	0.00	0.00	0.00
2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	1.28	0.00	0.00
4	0.00	0.00	0.00	1.00	0.00
5	0.00	0.00	0.00	0.00	0.39

Matrix **C**

	d_1	d_2	d_3	d_4	d_5	d_6
1	−0.75	−0.28	−0.20	−0.45	−0.33	−0.12
2	−0.29	−0.53	−0.19	0.63	0.22	0.41
3	0.28	−0.75	0.45	−0.20	0.12	−0.33
4	0.00	0.00	0.58	0.00	−0.58	0.58
5	−0.53	0.29	0.63	0.19	0.41	−0.22



Reduce dimension: The Matrix W

	1	2	3	4	5
ship	−0.44	−0.30	0.57	0.58	0.25
boat	−0.13	−0.33	−0.59	0.00	0.73
ocean	−0.48	−0.51	−0.37	0.00	−0.61
wood	−0.70	0.35	0.15	−0.58	0.16
tree	−0.26	0.65	−0.41	0.58	−0.09

	1	2	3	4	5
ship	−0.44	−0.30	0.00	0.00	0.00
boat	−0.13	−0.33	0.00	0.00	0.00
ocean	−0.48	−0.51	0.00	0.00	0.00
wood	−0.70	0.35	0.00	0.00	0.00
tree	−0.26	0.65	0.00	0.00	0.00

Reduce dimension: The Matrix S

1	2	3	4	5
2.16	0.00	0.00	0.00	0.00
0.00	1.59	0.00	0.00	0.00
0.00	0.00	1.28	0.00	0.00
0.00	0.00	0.00	1.00	0.00
0.00	0.00	0.00	0.00	0.39

1	2	3	4	5
2.16	0.00	0.00	0.00	0.00
0.00	1.59	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00

Reduce dimension: The Matrix C

d_1	d_2	d_3	d_4	d_5	d_6
−0.75	−0.28	−0.20	−0.45	−0.33	−0.12
−0.29	−0.53	−0.19	0.63	0.22	0.41
0.28	−0.75	0.45	−0.20	0.12	−0.33
0.00	0.00	0.58	0.00	−0.58	0.58
−0.53	0.29	0.63	0.19	0.41	−0.22

d_1	d_2	d_3	d_4	d_5	d_6
−0.75	−0.28	−0.20	−0.45	−0.33	−0.12
−0.29	−0.53	−0.19	0.63	0.22	0.41
0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00

Reduce dimension: The Matrix W

	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

	1	2	3	4	5
ship	−0.44	−0.30	0.00	0.00	0.00
boat	−0.13	−0.33	0.00	0.00	0.00
ocean	−0.48	−0.51	0.00	0.00	0.00
wood	−0.70	0.35	0.00	0.00	0.00
tree	−0.26	0.65	0.00	0.00	0.00

Reduce dimension: The Matrix W

	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

	1	2	3	4	5
ship	−0.44	−0.30	0.00	0.00	0.00
boat	−0.13	−0.33	0.00	0.00	0.00
ocean	−0.48	−0.51	0.00	0.00	0.00
wood	−0.70	0.35	0.00	0.00	0.00
tree	−0.26	0.65	0.00	0.00	0.00

Similarity between ***ship*** and ***boat*** ***vs*** ***ship*** and ***wood*** ?

Reduce dimension: The Matrix W

	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

	1	2	3	4	5
ship	−0.44	−0.30	0.00	0.00	0.00
boat	−0.13	−0.33	0.00	0.00	0.00
ocean	−0.48	−0.51	0.00	0.00	0.00
wood	−0.70	0.35	0.00	0.00	0.00
tree	−0.26	0.65	0.00	0.00	0.00

More details

- 300 dimensions are commonly used
- The cells are commonly weighted by a product of two weights (TF-IDF)
  - Local weight: term frequency (or log version)
  - Global weight: idf

Let's return to PPMI word-word matrices

- Can we apply to SVD to them?

SVD applied to term-term matrix

$$\underset{|V|\times|V|}{\begin{bmatrix} \\ X \\ \\ \end{bmatrix}} = \underset{|V|\times|V|}{\begin{bmatrix} \\ W \\ \\ \end{bmatrix}} \underset{|V|\times|V|}{\begin{bmatrix} \sigma_1 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sigma_V \end{bmatrix}} \underset{|V|\times|V|}{\begin{bmatrix} \\ C \\ \\ \end{bmatrix}}$$

(assuming the matrix has rank |V|, may not be true)

SVD applied to term-term matrix

$$\underset{|V|\times|V|}{\begin{bmatrix} \\ X \\ \\ \end{bmatrix}} = \underset{|V|\times|V|}{\begin{bmatrix} \\ W \\ \\ \end{bmatrix}} \underset{|V|\times|V|}{\begin{bmatrix} \sigma_1 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sigma_V \end{bmatrix}} \underset{|V|\times|V|}{\begin{bmatrix} \\ C \\ \\ \end{bmatrix}}$$

(assuming the matrix has rank |V|, may not be true)

Truncated SVD on term-term matrix

$$\underset{|V|\times|V|}{\begin{bmatrix} \\ X \\ \\ \end{bmatrix}} = \underset{|V|\times k}{\begin{bmatrix} \\ W \\ \\ \end{bmatrix}} \underset{k\times k}{\begin{bmatrix} \sigma_1 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sigma_k \end{bmatrix}} \begin{bmatrix} C \\ k\times|V| \end{bmatrix}$$

Truncated SVD produces embeddings

- Each row of W matrix is a k-dimensional representation of each word *w*
- K might range from 50 to 1000
- Generally we keep the top k dimensions, but some experiments suggest that getting rid of the top 1 dimension or even the top 50 dimensions is helpful (Lapesa and Evert 2014).

embedding for word i

$$\underset{|V|\times k}{\begin{bmatrix} \\ W \\ \\ \end{bmatrix}}$$

Embeddings versus sparse vectors

- Dense SVD embeddings sometimes work better than sparse PPMI matrices at tasks like word similarity
  - Denoising: low-order dimensions may represent unimportant information
  - Truncation may help the models generalize better to unseen data.
  - Having a smaller number of dimensions may make it easier for classifiers to properly weight the dimensions for the task.
  - Dense models may do better at capturing higher order co-occurrence.