CS 6120/CS4120: Natural Language Processing

Instructor: Prof. Lu Wang

College of Computer and Information Science

Northeastern University

Webpage: www.ccs.neu.edu/home/luwang

Logistics

- Project proposal is due on Feb 6.
- If you haven't found a group yet, make a private post on piazza today and let me know.
- Assignment 2 is released, due on March 20th, 11:59pm.

Two views of linguistic structure: 1. Constituency (phrase structure)

- Phrase structure organizes words into nested constituents.
    - Fed raises interest rates

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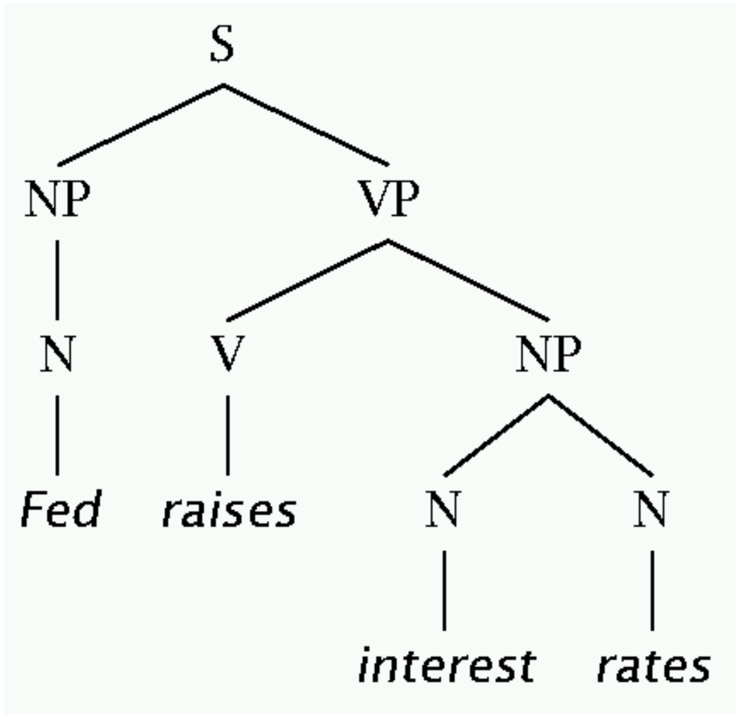
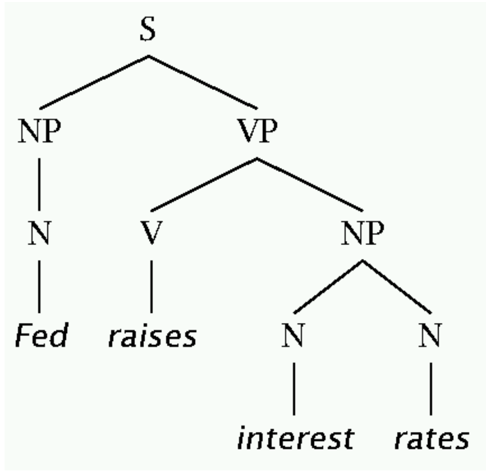
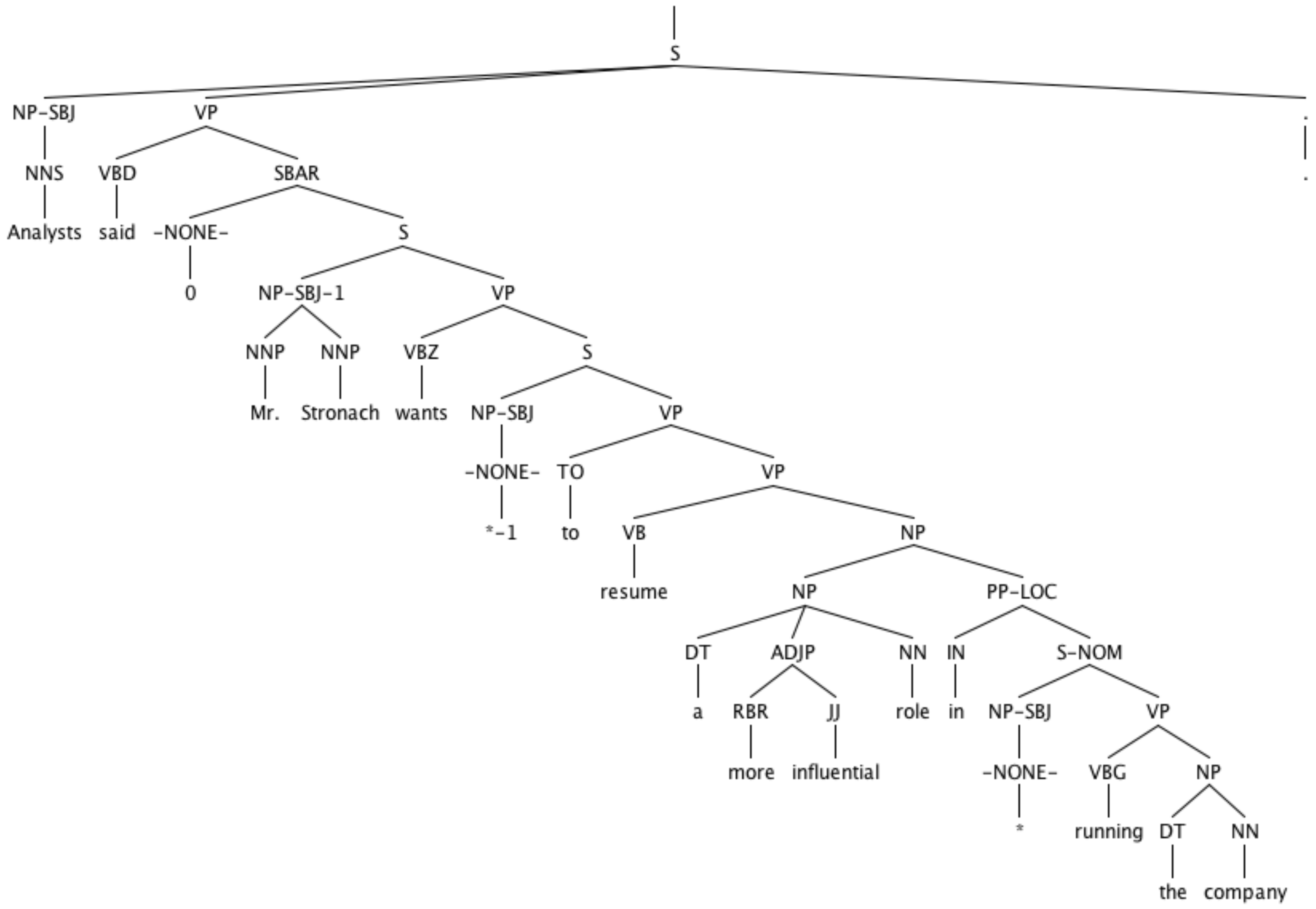
- Phrase structure organizes words into nested constituents.



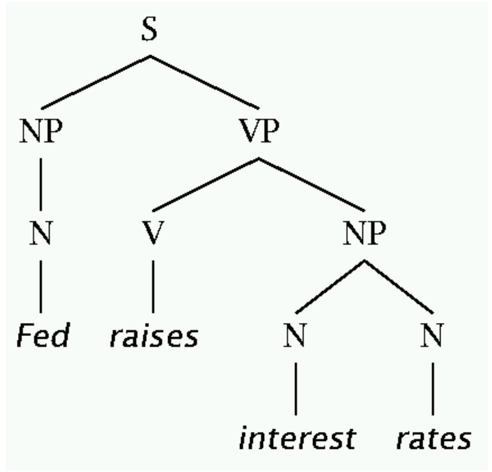
Two views of linguistic structure: 1. Constituency (phrase structure)



- Phrase structure organizes words into nested constituents.
- How do we know what is a constituent? (Not that linguists don't argue about some cases.)
  - Distribution: a constituent behaves as a unit that can appear in different places:
    - John talked [to the children] [about drugs].
    - John talked [about drugs] [to the children].
    - *John talked drugs to the children about
  - Substitution/expansion/pronoun:
    - I sat [on the box/right on top of the box/there].


S
NP-SBJ
VP
.
NNS
VBD
SBAR
.
Analysts
said
-NONE-
S
0
NP-SBJ-1
VP
NNP
NNP
VBZ
S
Mr.
Stronach
wants
NP-SBJ
VP
-NONE-
TO
VP
*-1
to
VB
NP
resume
NP
PP-LOC
DT
ADJP
NN
IN
S-NOM
a
RBR
JJ
role
in
NP-SBJ
VP
more
influential
-NONE-
VBG
NP
*
running
DT
NN
the
company

Headed phrase structure



- Context-free grammar
- VP → … VB* …
- NP → … NN* …
- ADJP → … JJ* …
- ADVP → … RB* …

- S → … NP VP …

- Plus minor phrase types:
  - QP (quantifier phrase in NP), CONJP (multi word constructions: *as well as*), INTJ (interjections), etc.

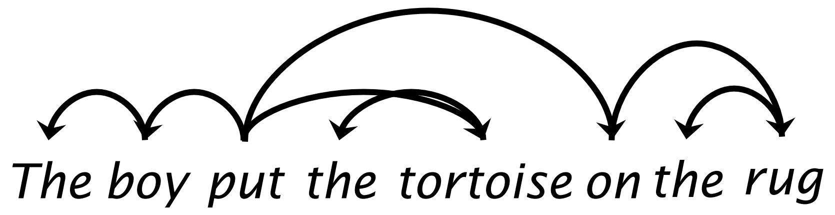
Two views of linguistic structure: 2. Dependency structure

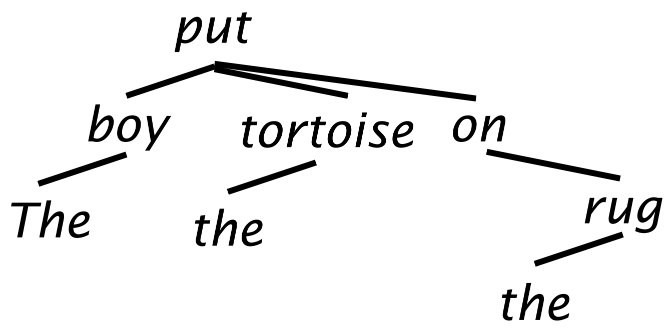
- Dependency structure shows which words depend on (modify or are arguments of) which other words.

The boy put the tortoise on the rug

Two views of linguistic structure: 2. Dependency structure

- Dependency structure shows which words depend on (modify or are arguments of) which other words.





Phrase Chunking

- Find all non-recursive noun phrases (NPs) and verb phrases (VPs) in a sentence.
  - [NP I] [VP ate] [NP the spaghetti] [PP with] [NP meatballs].
  - [NP He] [VP reckons] [NP the current account deficit] [VP will narrow] [PP to] [NP only 1.8 billion] [PP in] [NP September]

Phrase Chunking as Sequence Labeling

- Tag individual words with one of 3 tags
  - B (Begin) word starts new target phrase
  - I (Inside) word is part of target phrase but not the first word
  - O (Other) word is not part of target phrase
- Sample for NP chunking
  - He reckons the current account deficit will narrow to only 1.8 billion in September.

Begin **Inside** **Other**

Evaluating Chunking

Per token accuracy does not evaluate finding correct full chunks. Instead use:

$$\text{Precision} = \frac{\text{Number of correct chunks found}}{\text{Total number of chunks found}}$$

$$\text{Recall} = \frac{\text{Number of correct chunks found}}{\text{Total number of actual chunks}}$$

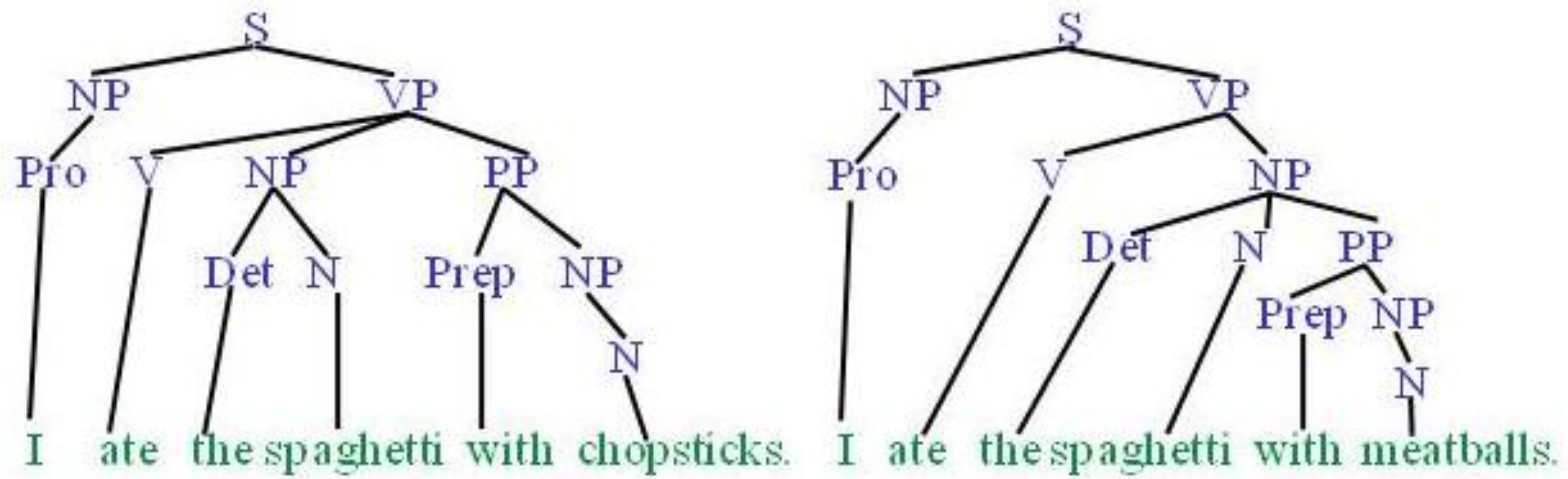
F measure: $$F_1 = \frac{1}{(\frac{1}{P} + \frac{1}{R})/2} = \frac{2PR}{P+R}$$

Current Chunking Results

- Best system for NP chunking: F_1=96%
- Typical results for finding range of chunk types (CONLL 2000 shared task: NP, VP, PP, ADV, SBAR, ADJP) is F_1=92−94%

Syntactic Parsing

- Produce the correct syntactic parse tree for a sentence.


S
NP
VP
Pro
V
NP
PP
Det
N
Prep
NP
N
I ate the spaghetti with chopsticks.
S
NP
VP
Pro
V
NP
Det
N
PP
Prep
NP
N
I ate the spaghetti with meatballs.

Classical NLP Parsing: The problem and its solution

- Adding constraints to grammars to limit unlikely/weird parses for sentences
  - But the attempt make the grammars not robust
    - In traditional systems, commonly 30% of sentences in even an edited text would have *no* parse.
- A less constrained grammar can parse more sentences
  - But simple sentences end up with ever more parses with no way to choose between them
- We need mechanisms that allow us to find the most likely parse(s) for a sentence
  - Statistical parsing lets us work with very loose grammars that admit millions of parses for sentences but still quickly find the best parse(s)

The rise of annotated data: The Penn Treebank

[Marcus et al. 1993, *Computational Linguistics*]

```
( (S
  (NP-SBJ (DT The) (NN move))
  (VP (VBD followed)
   (NP
    (NP (DT a) (NN round))
    (PP (IN of)
     (NP
      (NP (JJ similar) (NNS increases))
      (PP (IN by)
       (NP (JJ other) (NNS lenders)))
      (PP (IN against)
       (NP (NNP Arizona) (JJ real) (NN estate) (NNS loans))))))
   (, ,)
   (S-ADV
    (NP-SBJ (-NONE- *))
    (VP (VBG reflecting)
     (NP
      (NP (DT a) (VBG continuing) (NN decline))
      (PP-LOC (IN in)
       (NP (DT that) (NN market)))))))
  (. .)))
```

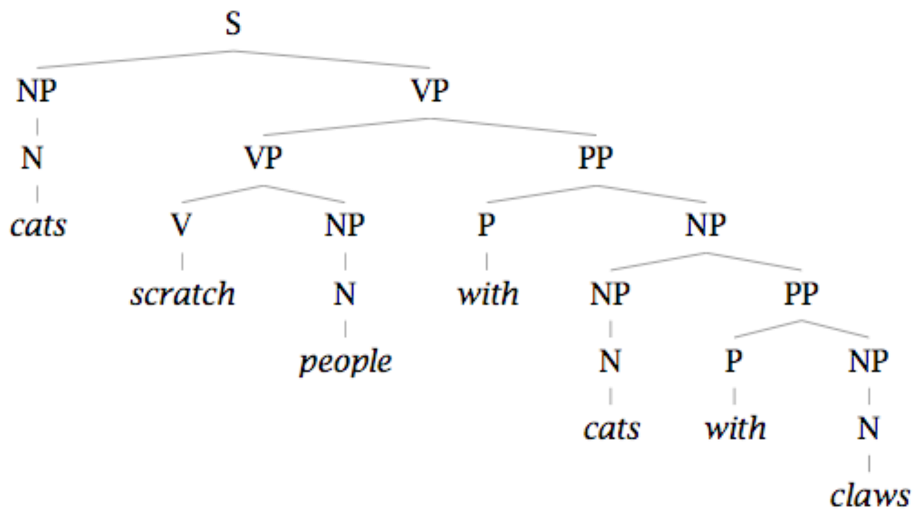
The rise of annotated data

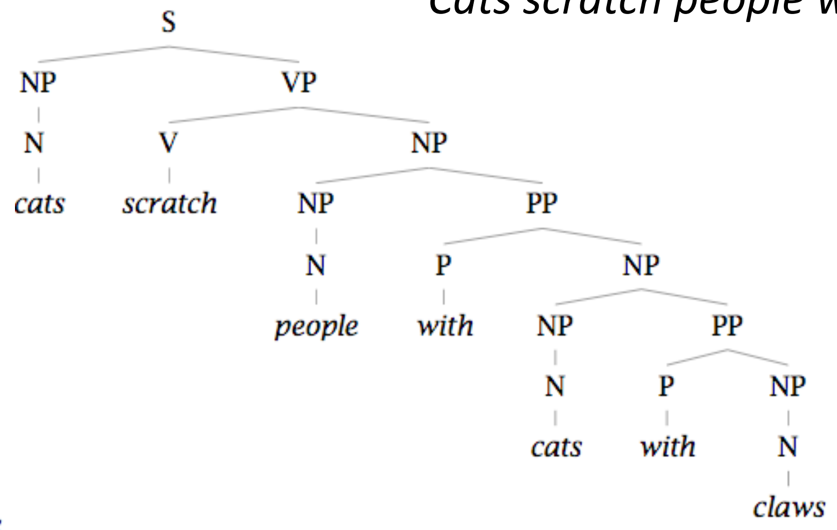
- Starting off, building a treebank seems a lot slower and less useful than building a grammar

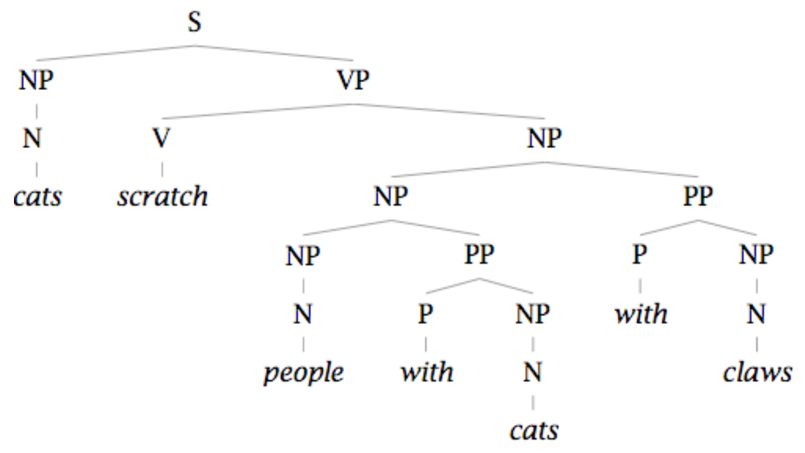
- But a treebank gives us many things
    - Reusability of the labor
        - Many parsers, POS taggers, etc.
        - Valuable resource for linguistics
    - Broad coverage
    - Frequencies and distributional information
    - A way to evaluate systems

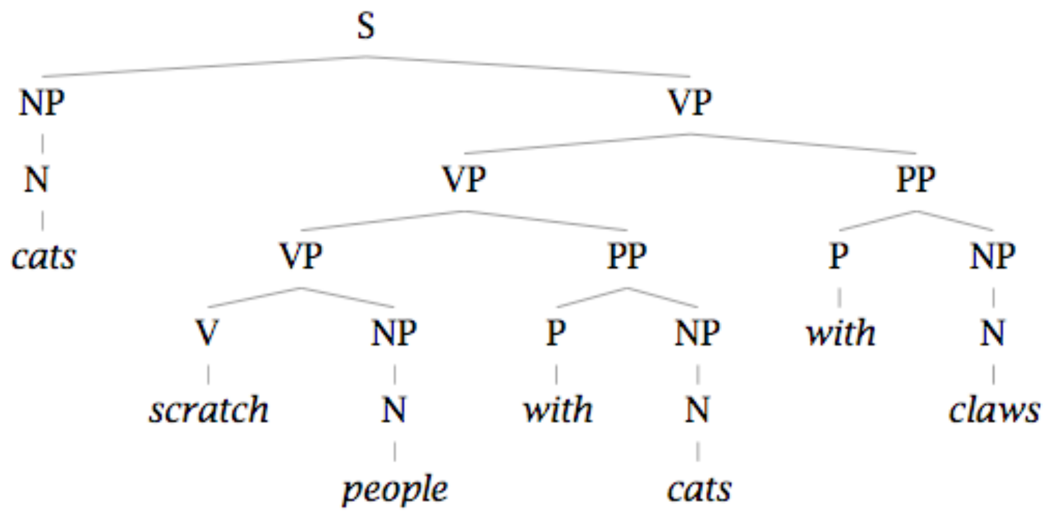
Two problems to solve for parsing:
1. Repeated work…

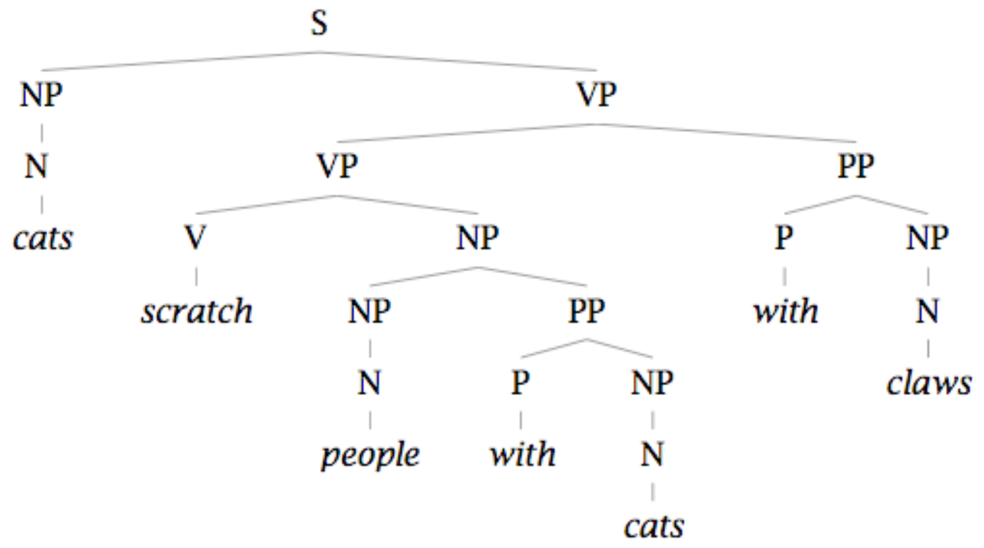
“Cats scratch people with cats with claws”


S NP VP N VP PP cats V NP P NP scratch N with NP PP people N P NP cats with N claws


S NP VP N V NP cats scratch NP PP N P NP people with NP PP N P NP cats with N claws

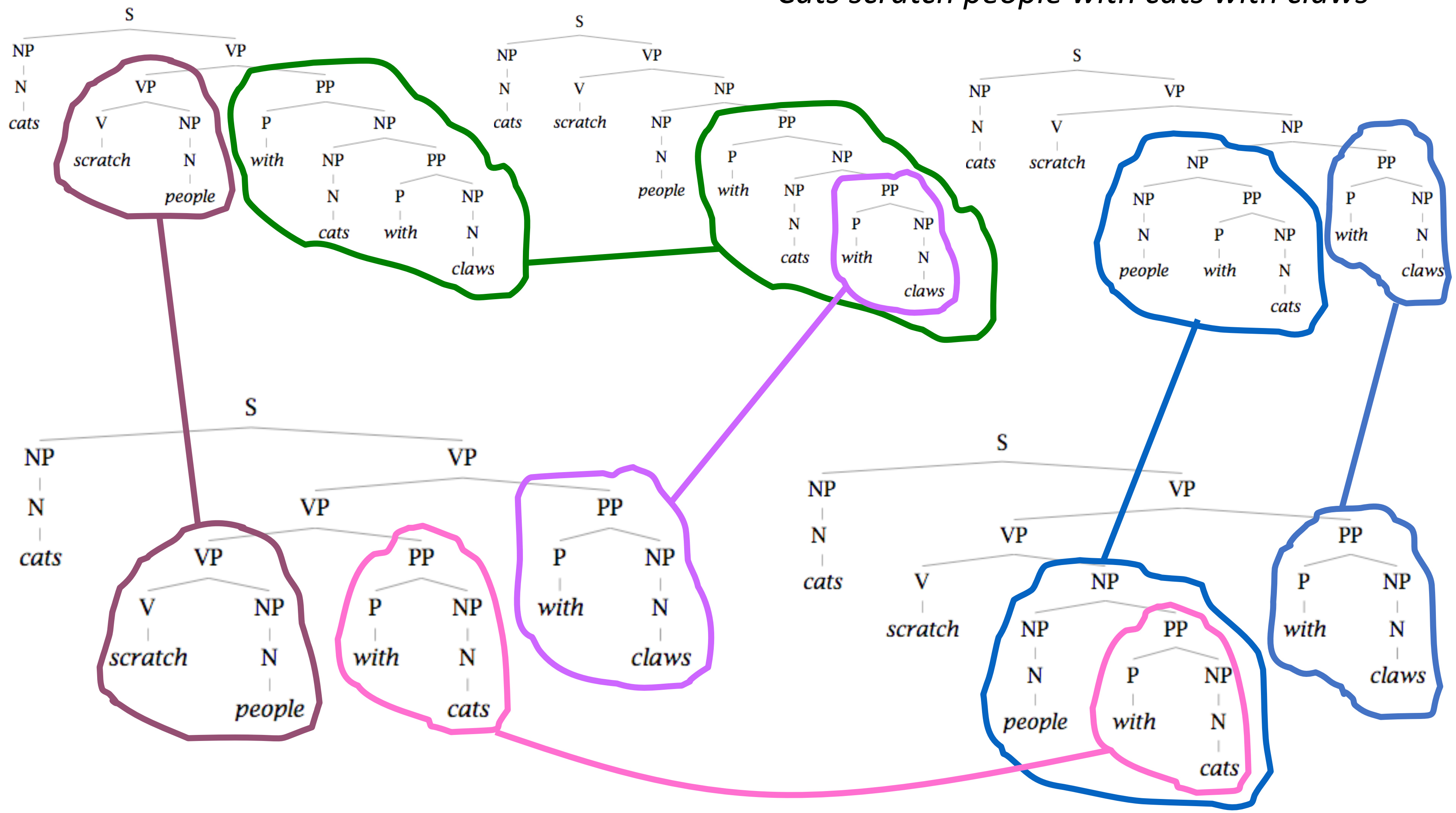

S NP VP N V NP cats scratch NP PP NP PP P NP N P NP with N people with N claws cats


S NP VP N VP PP cats VP PP P NP V NP P NP with N scratch N with N claws people cats


S NP VP N VP PP cats V NP P NP scratch NP PP with N N P NP claws people with N cats

Two problems to solve for parsing:
1. Repeated work…

"Cats scratch people with cats with claws"



Two problems to solve for parsing: 2. Choosing the correct parse

- How do we work out the correct attachment:
  - She saw the man with a telescope
- Words are good predictors of attachment, even absent full understanding
  - Moscow **sent** more than 100,000 soldiers **into** Afghanistan …
  - Sydney Water breached an **agreement with** NSW Health …
- Our statistical parsers will try to exploit such statistics.

Statistical parsing applications

Statistical parsers are now robust and widely used in larger NLP applications:

- High precision question answering [Pasca and Harabagiu SIGIR 2001]
- Improving biological named entity finding [Finkel et al. JNLPBA 2004]
- Syntactically based sentence compression [Lin and Wilbur 2007]
- Extracting opinions about products [Bloom et al. NAACL 2007]
- Improved interaction in computer games [Gorniak and Roy 2005]
- Helping linguists find data [Resnik et al. BLS 2005]
- Source sentence analysis for machine translation [Xu et al. 2009]
- Relation extraction systems [Fundel et al. Bioinformatics 2006]

(Probabilistic) Context-Free Grammars

- CFG
- PCFG

Phrase structure grammars = context-free grammars (CFGs)

- G = (T, N, S, R)
  - T is a set of terminal symbols
  - N is a set of nonterminal symbols
  - S is the start symbol ($S \in N$)
  - R is a set of rules/productions of the form $X \rightarrow \gamma$
    - $X \in N$ and $\gamma \in (N \cup T)^*$

A phrase structure grammar

S $\rightarrow$ NP VP
VP $\rightarrow$ V NP
VP $\rightarrow$ V NP PP
NP $\rightarrow$ NP NP
NP $\rightarrow$ NP PP
NP $\rightarrow$ N
NP $\rightarrow$ *e*
PP $\rightarrow$ P NP

people fish tanks
people fish with rods

N $\rightarrow$ *people*
N $\rightarrow$ *fish*
N $\rightarrow$ *tanks*
N $\rightarrow$ *rods*
V $\rightarrow$ *people*
V $\rightarrow$ *fish*
V $\rightarrow$ *tanks*
P $\rightarrow$ *with*

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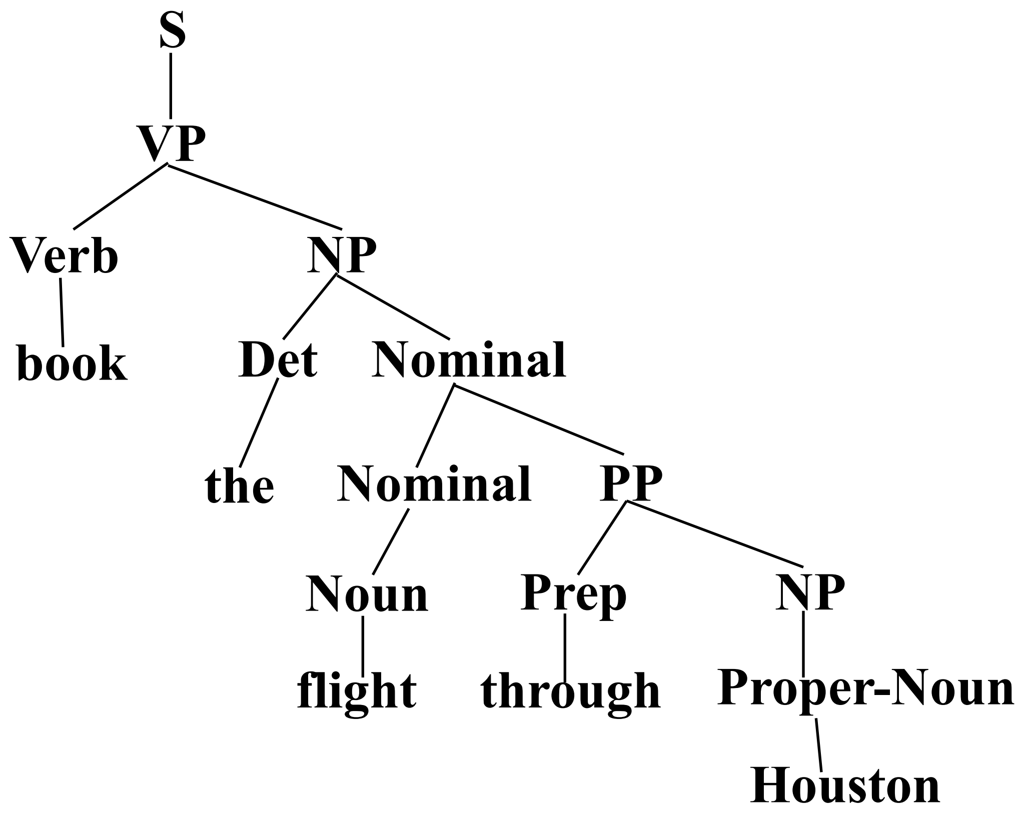
- A grammar G generates a language L.

Sentence Generation

- Sentences are generated by recursively rewriting the start symbol using the productions until only terminals symbols remain.



Phrase structure grammars in NLP

- G = (T, C, N, S, L, R)
  - T is a set of terminal symbols
  - C is a set of preterminal symbols
  - N is a set of nonterminal symbols
  - S is the start symbol ($S \in N$)
  - L is the lexicon, a set of items of the form $X \rightarrow x$
    - $X \in C$ and $x \in T$
  - R is the grammar, a set of items of the form $X \rightarrow \gamma$
    - $X \in N$ and $\gamma \in (N \cup C)^*$
- By usual convention, S is the start symbol, but in statistical NLP, we usually have an extra node at the top (ROOT, TOP)
- We usually write *e* for an empty sequence, rather than nothing

A phrase structure grammar

S $\rightarrow$ NP VP
VP $\rightarrow$ V NP
VP $\rightarrow$ V NP PP
NP $\rightarrow$ NP NP
NP $\rightarrow$ NP PP
NP $\rightarrow$ N
NP $\rightarrow$ *e*
PP $\rightarrow$ P NP

people fish tanks
people fish with rods

N $\rightarrow$ *people*
N $\rightarrow$ *fish*
N $\rightarrow$ *tanks*
N $\rightarrow$ *rods*
V $\rightarrow$ *people*
V $\rightarrow$ *fish*
V $\rightarrow$ *tanks*
P $\rightarrow$ *with*

Probabilistic – or stochastic – context-free grammars (PCFGs)

- G = (T, N, S, R, P)
  - T is a set of terminal symbols
  - N is a set of nonterminal symbols
  - S is the start symbol (S ∈ N)
  - R is a set of rules/productions of the form $X \to \gamma$
  - P is a probability function
    - P: R $\to$ [0,1]
    - $\forall X \in N, \sum_{X \to \gamma \in R} P(X \to \gamma) = 1$
- A grammar G generates a language model L.

A PCFG

S $\rightarrow$ NP VP	1.0
VP $\rightarrow$ V NP	0.6
VP $\rightarrow$ V NP PP	0.4
NP $\rightarrow$ NP NP	0.1
NP $\rightarrow$ NP PP	0.2
NP $\rightarrow$ N	0.7
PP $\rightarrow$ P NP	1.0

N $\rightarrow$ *people*	0.5
N $\rightarrow$ *fish*	0.2
N $\rightarrow$ *tanks*	0.2
N $\rightarrow$ *rods*	0.1
V $\rightarrow$ *people*	0.1
V $\rightarrow$ *fish*	0.6
V $\rightarrow$ *tanks*	0.3
P $\rightarrow$ *with*	1.0

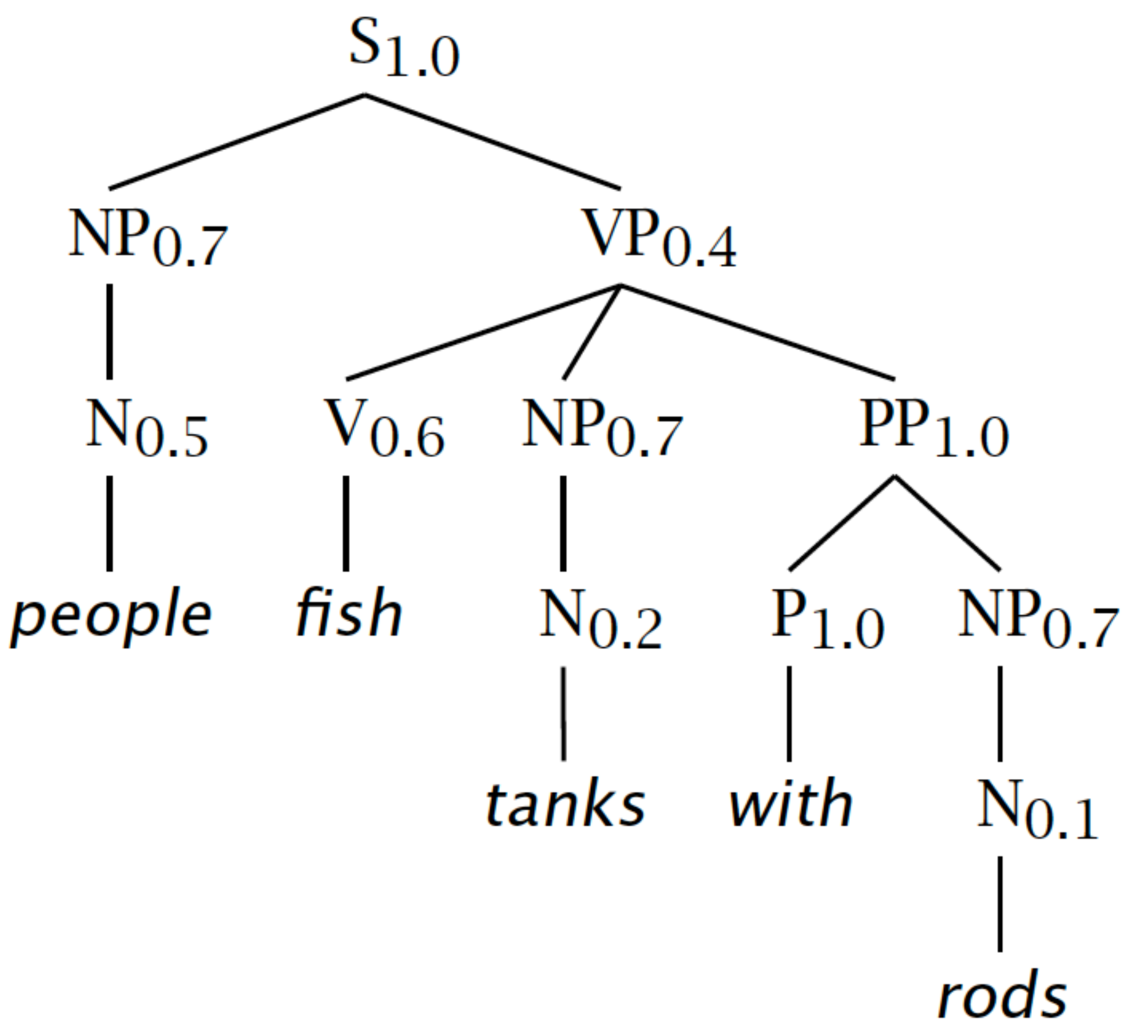
[With empty NP removed so less ambiguous]

The probability of trees and strings

- P(*t*) – The probability of a tree *t* is the product of the probabilities of the rules used to generate it.
- P(*s*) – The probability of the string *s* is the sum of the probabilities of the trees which have that string as their yield

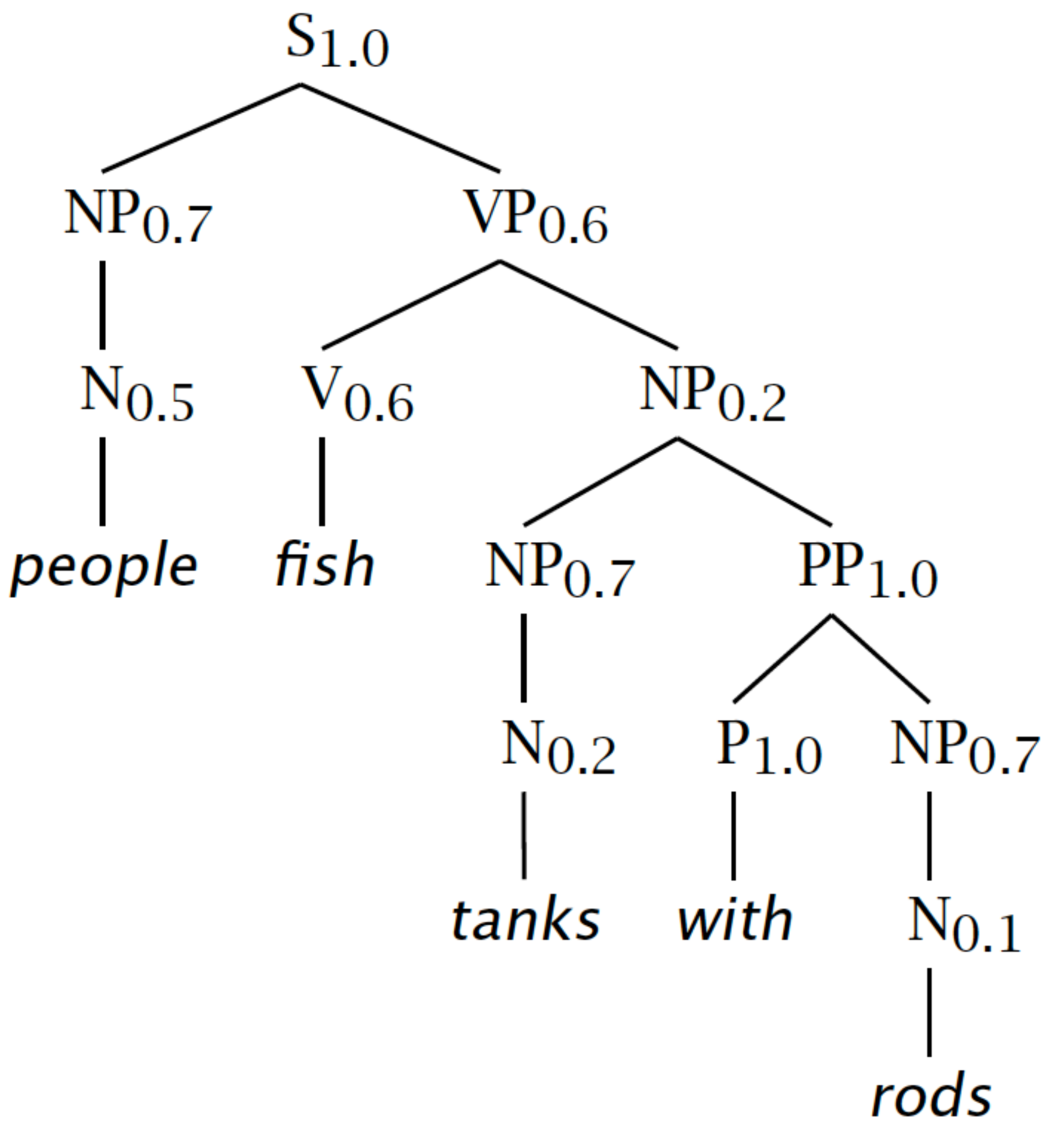
$$P(s) = \Sigma_t P(s, t) \quad \text{where } t \text{ is a parse of } s$$
$$= \Sigma_t P(t)$$

t_1:



S1.0
NP0.7
VP0.4
N0.5
V0.6
NP0.7
PP1.0
people
fish
N0.2
P1.0
NP0.7
tanks
with
N0.1
rods

t_2:



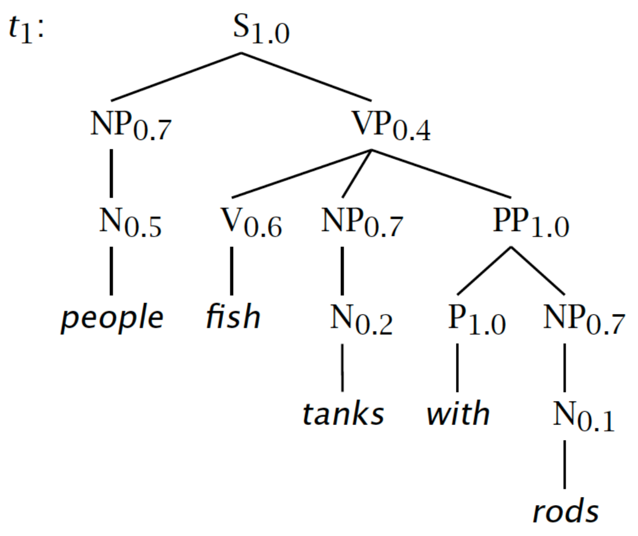
S1.0
NP0.7
VP0.6
N0.5
V0.6
NP0.2
people
fish
NP0.7
PP1.0
N0.2
P1.0
NP0.7
tanks
with
N0.1
rods

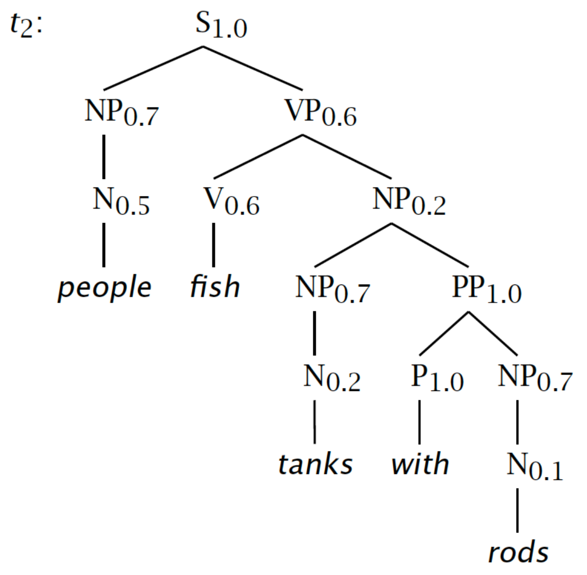
Tree and String Probabilities

- s = *people fish tanks with rods*
- $P(t_1) = 1.0 \times 0.7 \times 0.4 \times 0.5 \times 0.6 \times 0.7 \times 1.0 \times 0.2 \times 1.0 \times 0.7 \times 0.1 = 0.0008232$
- $P(t_2) = 1.0 \times 0.7 \times 0.6 \times 0.5 \times 0.6 \times 0.2 \times 0.7 \times 1.0 \times 0.2 \times 1.0 \times 0.7 \times 0.1 = 0.00024696$
- $P(s) = P(t_1) + P(t_2) = 0.0008232 + 0.00024696 = 0.00107016$

Verb attach

Noun attach





Chomsky Normal Form

- All rules are of the form $X \rightarrow Y\ Z$ or $X \rightarrow w$
  - $X, Y, Z \in N$ and $w \in T$
- A transformation to this form doesn't change the generative capacity of a CFG
  - That is, it recognizes the same language
    - But maybe with different trees
- Empties and unaries are removed recursively
- n-ary rules are divided by introducing new nonterminals ($n > 2$)

A phrase structure grammar

S $\rightarrow$ NP VP	N $\rightarrow$ *people*
VP $\rightarrow$ V NP	N $\rightarrow$ *fish*
VP $\rightarrow$ V NP PP	N $\rightarrow$ *tanks*
NP $\rightarrow$ NP NP	N $\rightarrow$ *rods*
NP $\rightarrow$ NP PP	V $\rightarrow$ *people*
NP $\rightarrow$ N	V $\rightarrow$ *fish*
NP $\rightarrow$ *e*	V $\rightarrow$ *tanks*
PP $\rightarrow$ P NP	P $\rightarrow$ *with*

Chomsky Normal Form steps

S $\rightarrow$ NP VP
S $\rightarrow$ VP
VP $\rightarrow$ V NP
VP $\rightarrow$ V
VP $\rightarrow$ V NP PP
VP $\rightarrow$ V PP
NP $\rightarrow$ NP NP
NP $\rightarrow$ NP
NP $\rightarrow$ NP PP
NP $\rightarrow$ PP
NP $\rightarrow$ N
PP $\rightarrow$ P NP
PP $\rightarrow$ P

N $\rightarrow$ *people*
N $\rightarrow$ *fish*
N $\rightarrow$ *tanks*
N $\rightarrow$ *rods*
V $\rightarrow$ *people*
V $\rightarrow$ *fish*
V $\rightarrow$ *tanks*
P $\rightarrow$ *with*

Chomsky Normal Form steps

S → NP VP
VP → V NP
S → V NP
VP → V
S → V
VP → V NP PP
S → V NP PP
VP → V PP
S → V PP
NP → NP NP
NP → NP
NP → NP PP
NP → PP
NP → N
PP → P NP
PP → P

N → *people*
N → *fish*
N → *tanks*
N → *rods*
V → *people*
V → *fish*
V → *tanks*
P → *with*

Chomsky Normal Form steps

S → NP VP
VP → V NP
S → V NP
VP → V
VP → V NP PP
S → V NP PP
VP → V PP
S → V PP
NP → NP NP
NP → NP
NP → NP PP
NP → PP
NP → N
PP → P NP
PP → P

N → *people*
N → *fish*
N → *tanks*
N → *rods*
V → *people*
S → *people*
V → *fish*
S → *fish*
V → *tanks*
S → *tanks*
P → *with*

Chomsky Normal Form steps

S → NP VP

VP → V NP

S → V NP

VP → V NP PP

S → V NP PP

VP → V PP

S → V PP

NP → NP NP

NP → NP

NP → NP PP

NP → PP

NP → N

PP → P NP

PP → P

N → *people*

N → *fish*

N → *tanks*

N → *rods*

V → *people*

S → *people*

VP → *people*

V → *fish*

S → *fish*

VP → *fish*

V → *tanks*

S → *tanks*

VP → *tanks*

P → *with*

Chomsky Normal Form steps

S → NP VP

VP → V NP

S → V NP

VP → V NP PP

S → V NP PP

VP → V PP

S → V PP

NP → NP NP

NP → NP PP

NP → P NP

PP → P NP

NP → *people*

NP → *fish*

NP → *tanks*

NP → *rods*

V → *people*

S → *people*

VP → *people*

V → *fish*

S → *fish*

VP → *fish*

V → *tanks*

S → *tanks*

VP → *tanks*

P → *with*

PP → *with*

Chomsky Normal Form steps

S $\rightarrow$ NP VP

VP $\rightarrow$ V NP

S $\rightarrow$ V NP

VP $\rightarrow$ V @VP_V

@VP_V $\rightarrow$ NP PP

S $\rightarrow$ V @S_V

@S_V $\rightarrow$ NP PP

VP $\rightarrow$ V PP

S $\rightarrow$ V PP

NP $\rightarrow$ NP NP

NP $\rightarrow$ NP PP

NP $\rightarrow$ P NP

PP $\rightarrow$ P NP

NP $\rightarrow$ *people*

NP $\rightarrow$ *fish*

NP $\rightarrow$ *tanks*

NP $\rightarrow$ *rods*

V $\rightarrow$ *people*

S $\rightarrow$ *people*

VP $\rightarrow$ *people*

V $\rightarrow$ *fish*

S $\rightarrow$ *fish*

VP $\rightarrow$ *fish*

V $\rightarrow$ *tanks*

S $\rightarrow$ *tanks*

VP $\rightarrow$ *tanks*

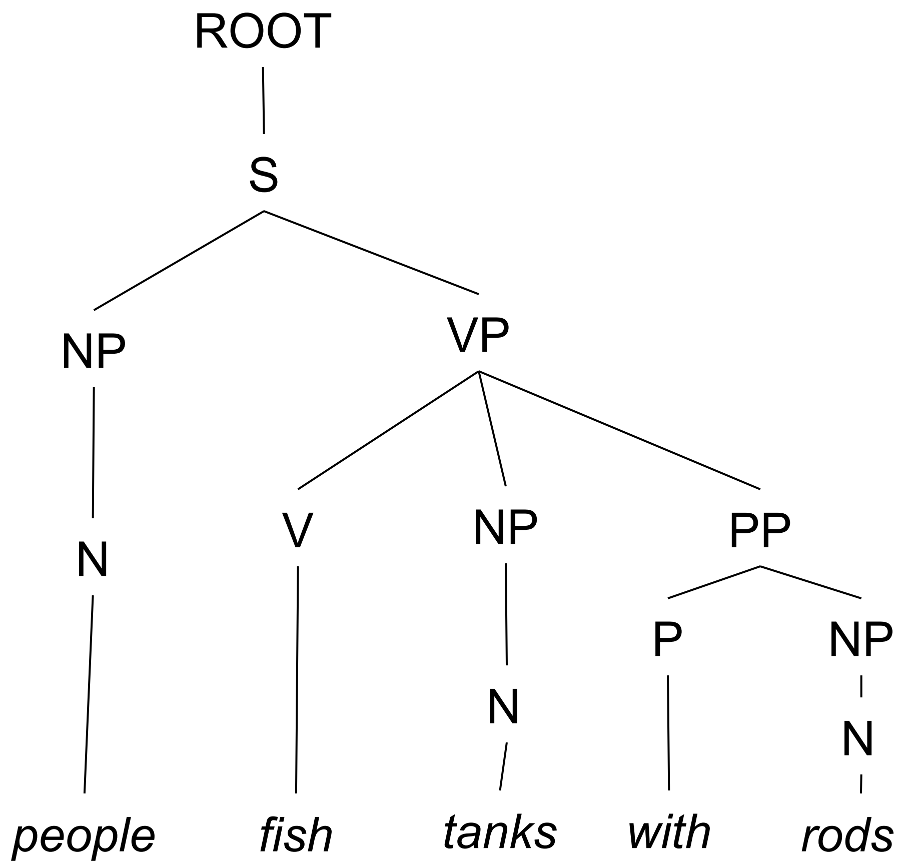
P $\rightarrow$ *with*

PP $\rightarrow$ *with*

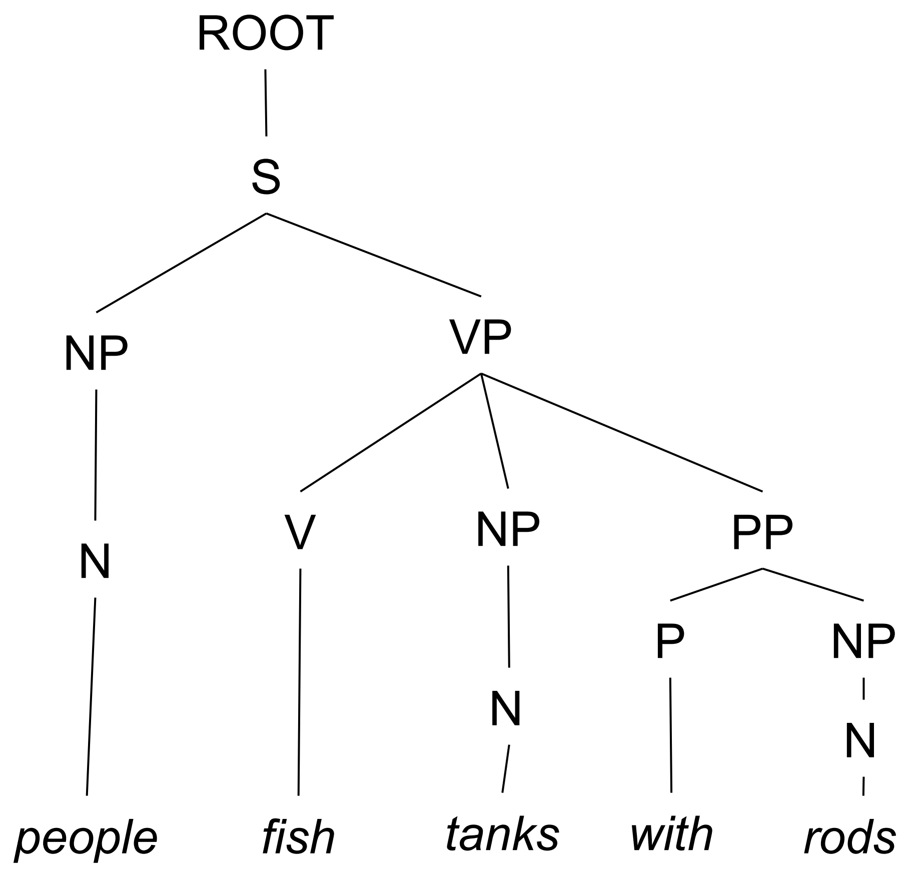
Chomsky Normal Form

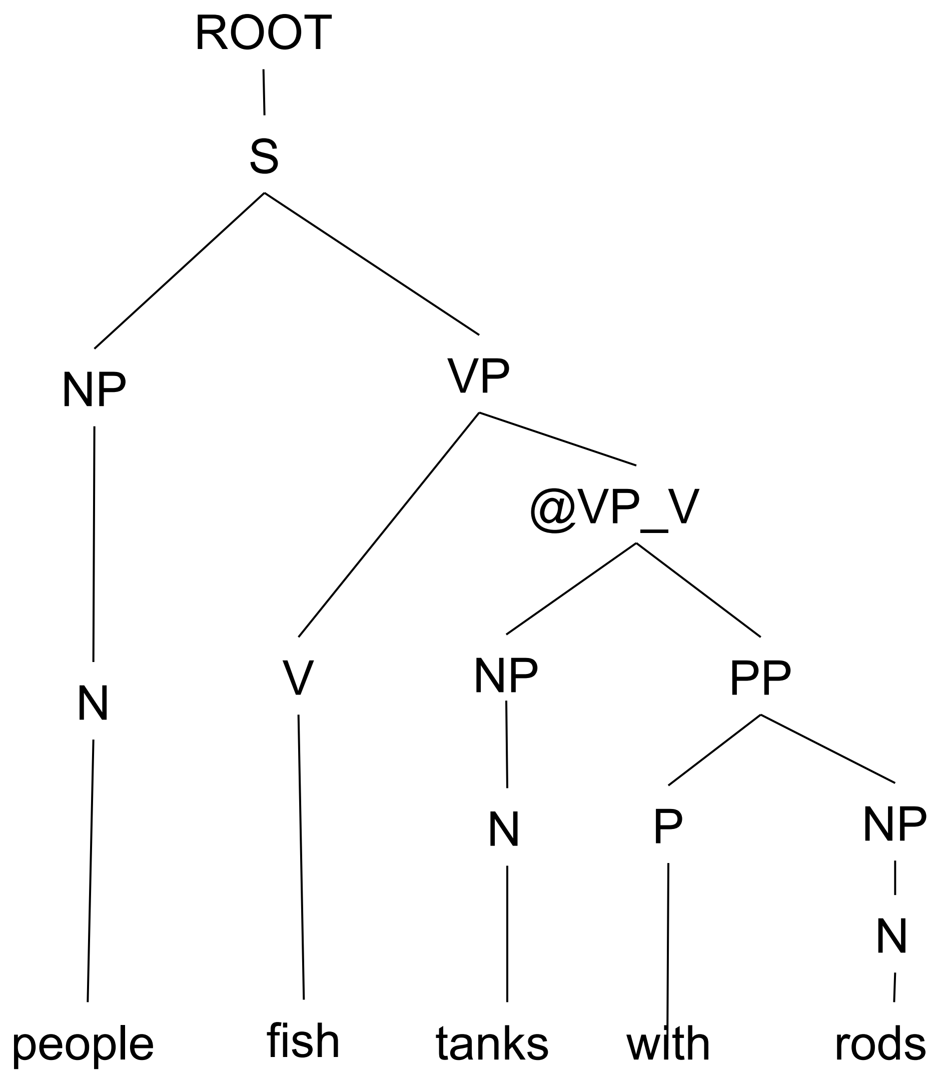
- You should think of this as a transformation for efficient parsing
- **Binarization** is crucial for cubic time CFG parsing
- The rest isn’t necessary; it just makes the algorithms cleaner and a bit quicker

An example: before binarization…



Before and After binarization on VP

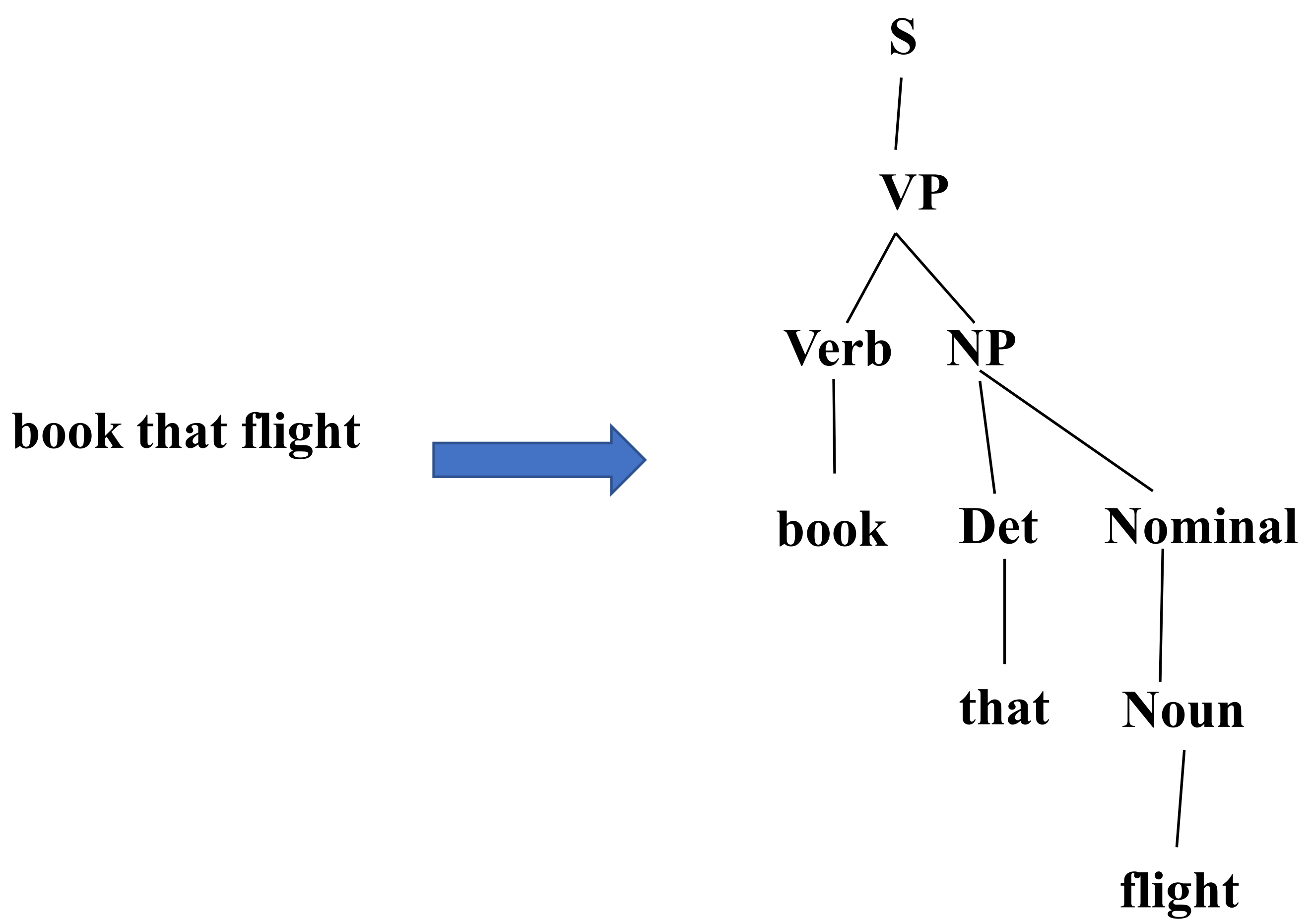




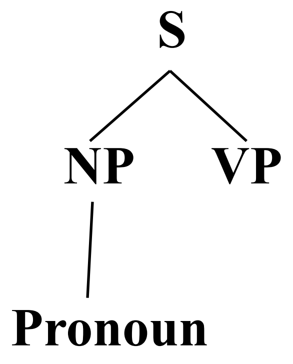
Parsing

- Given a string of terminals (e.g. sentences) and a CFG, determine if the string can be generated by the CFG.
  - Also return a parse tree for the string
  - Also return all possible parse trees for the string
- Must search space of derivations for one that derives the given string.
  - **Top-Down Parsing**: Start searching space of derivations for the start symbol.
  - **Bottom-up Parsing**: Start search space of reverse derivations from the terminal symbols in the string.

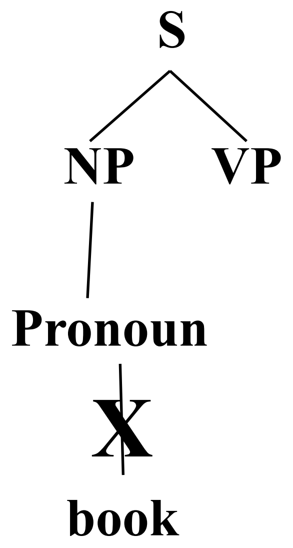
Parsing Example



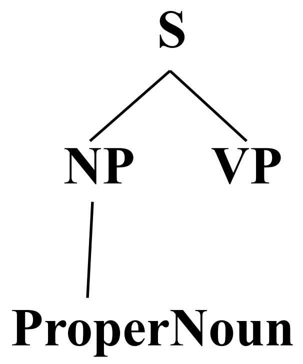
Top Down Parsing



Top Down Parsing



Top Down Parsing



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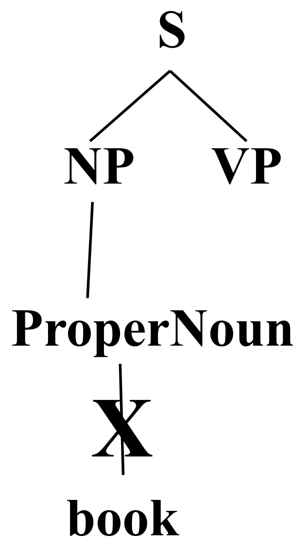

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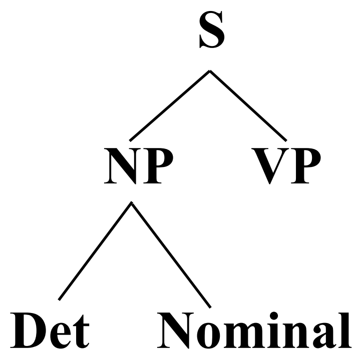

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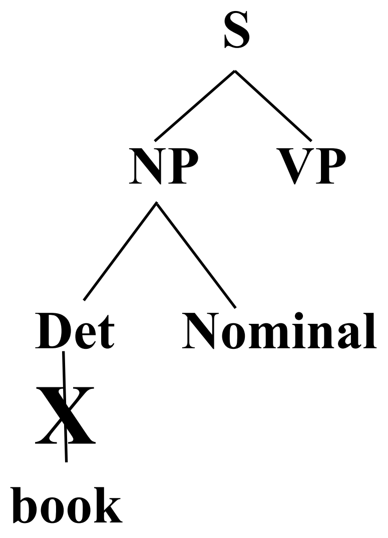

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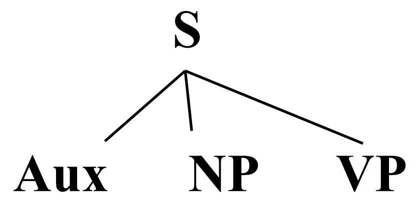

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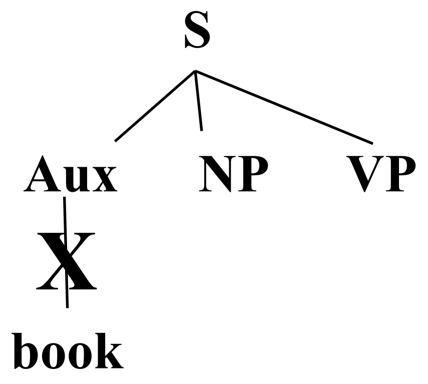

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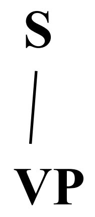

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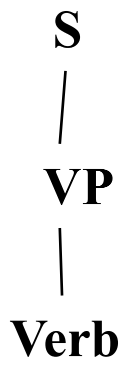

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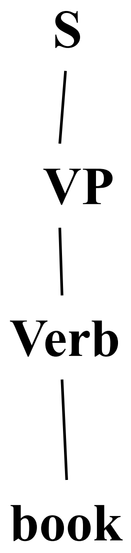

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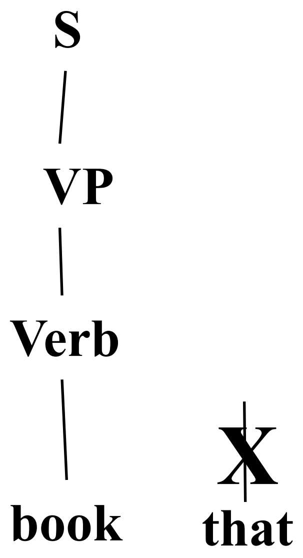

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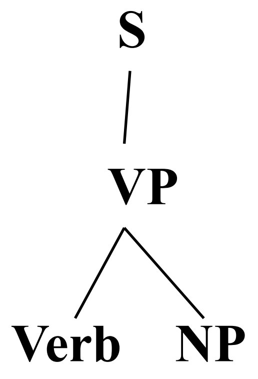

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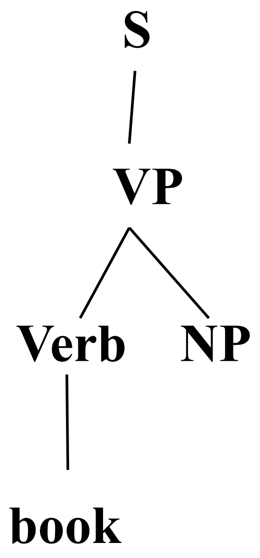

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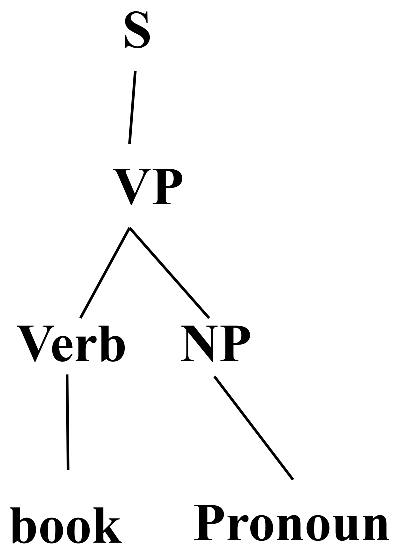

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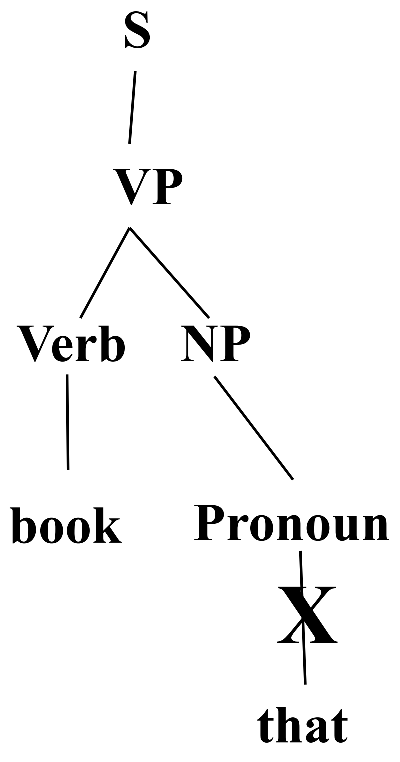

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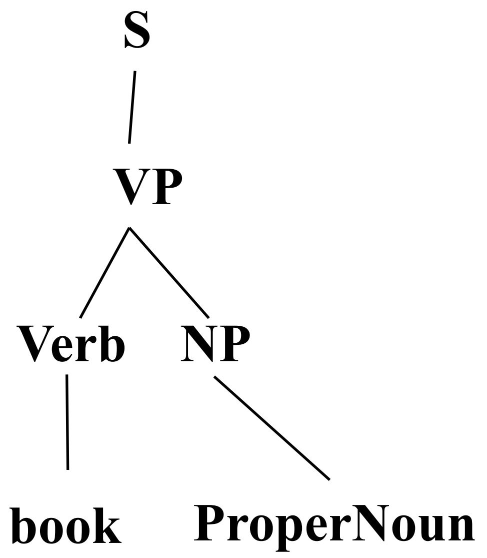

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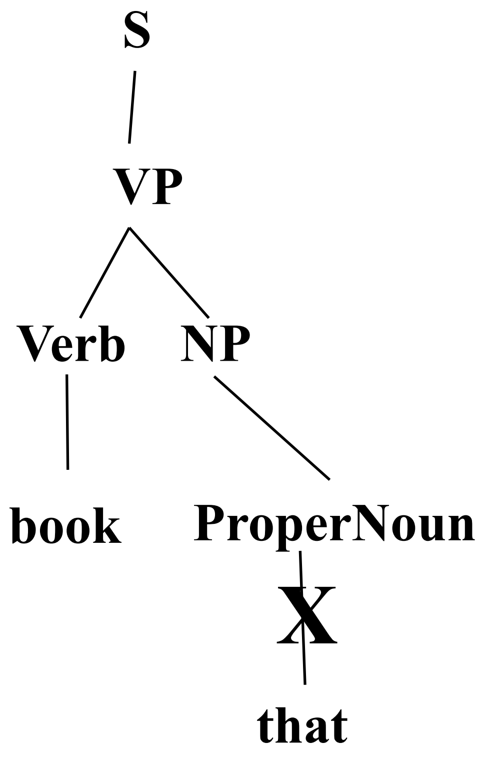

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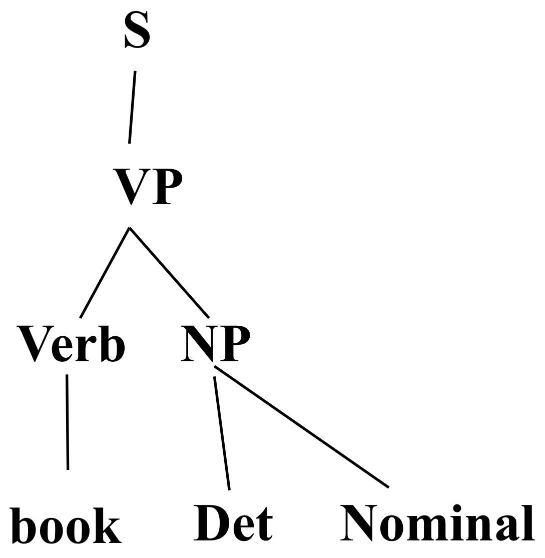

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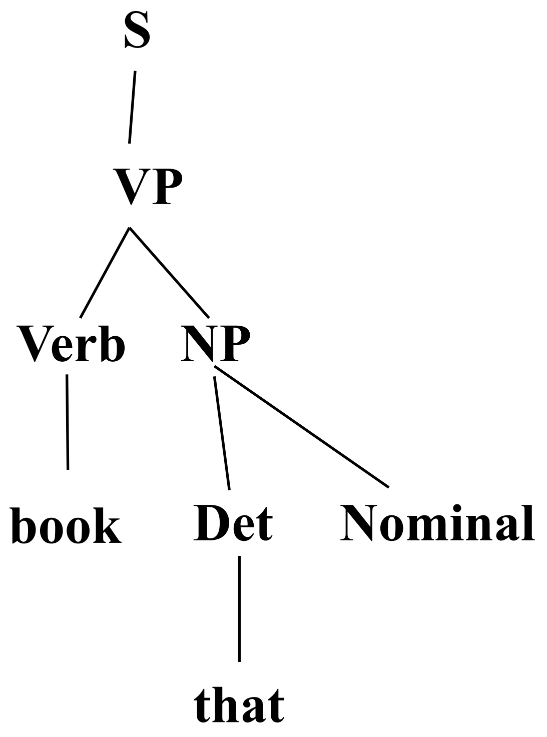

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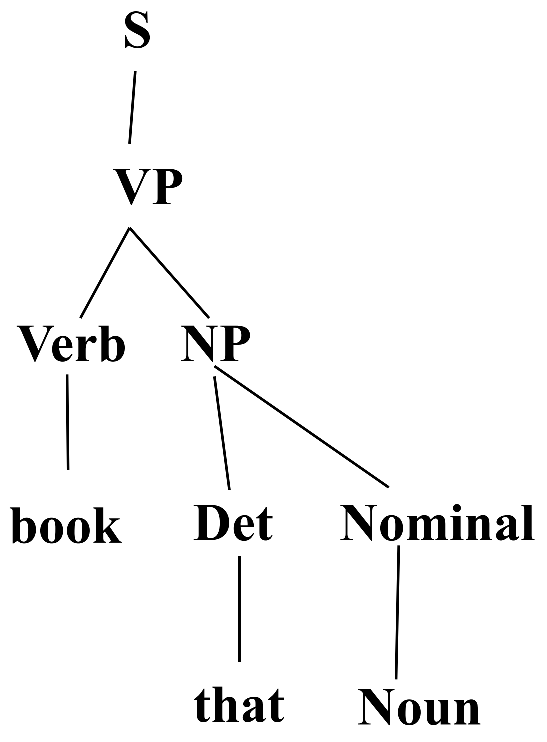

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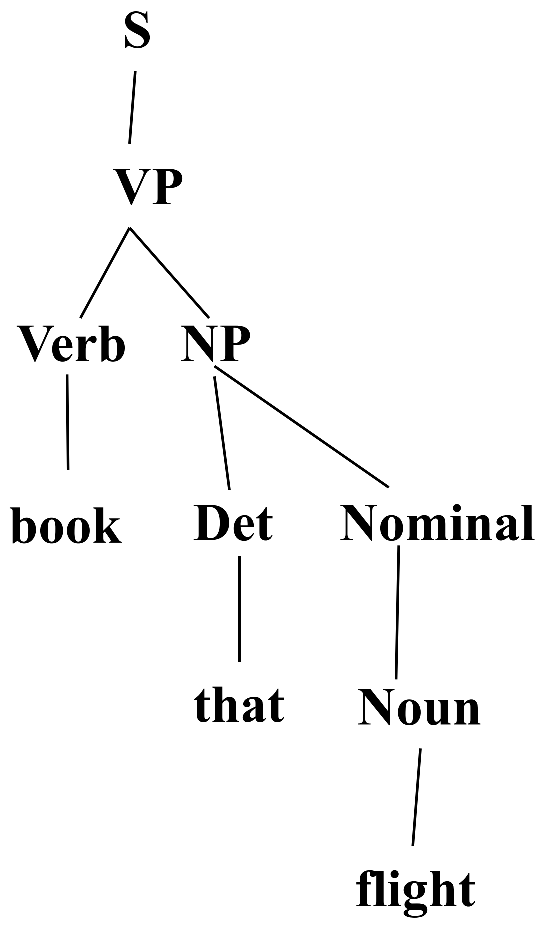
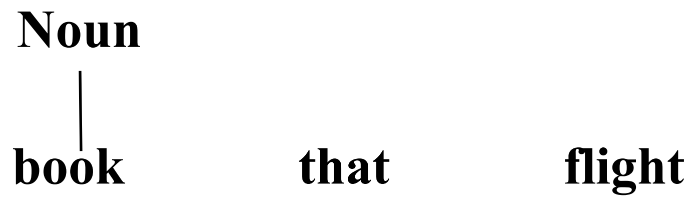


Bottom Up Parsing

book **that** **flight**

Bottom Up Parsing



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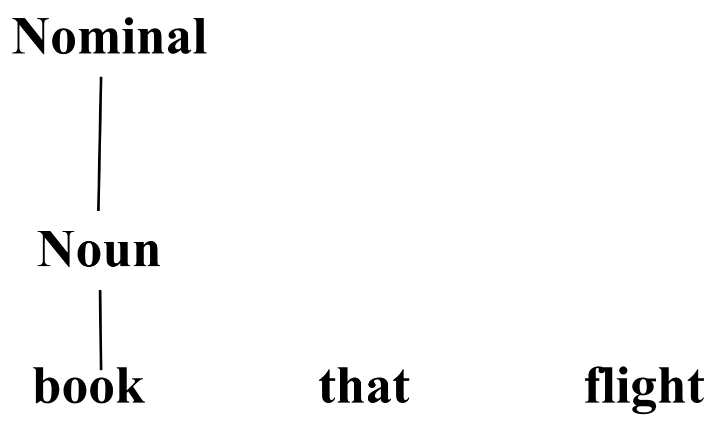

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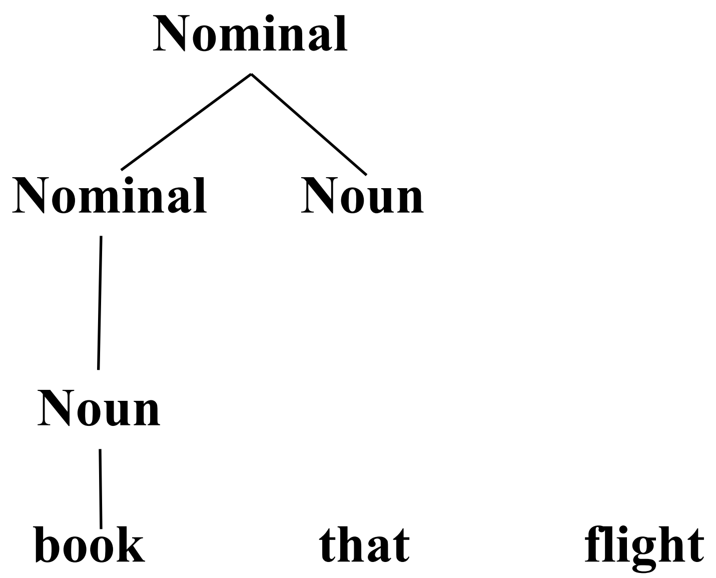

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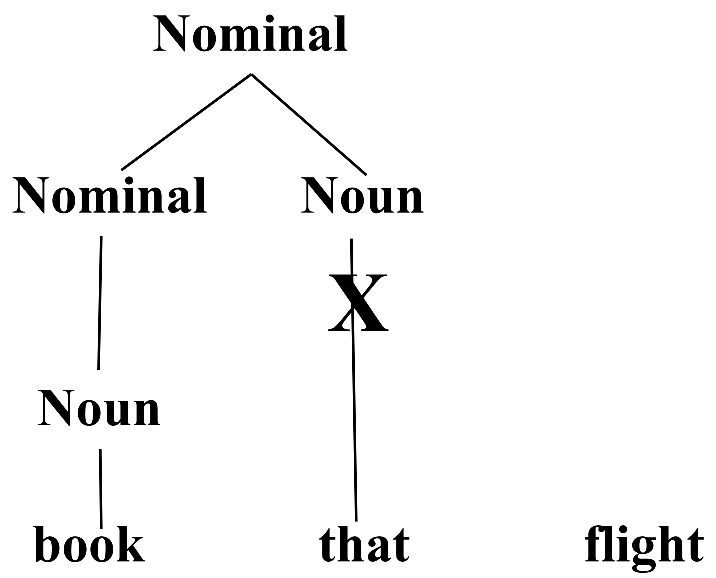

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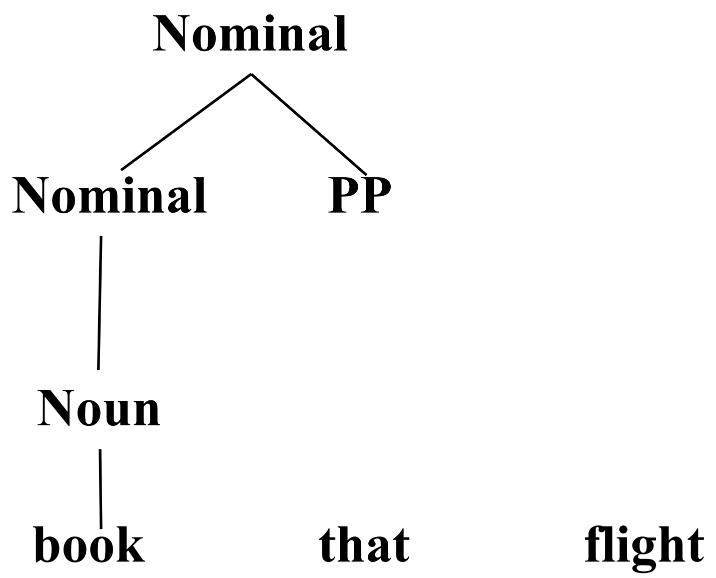

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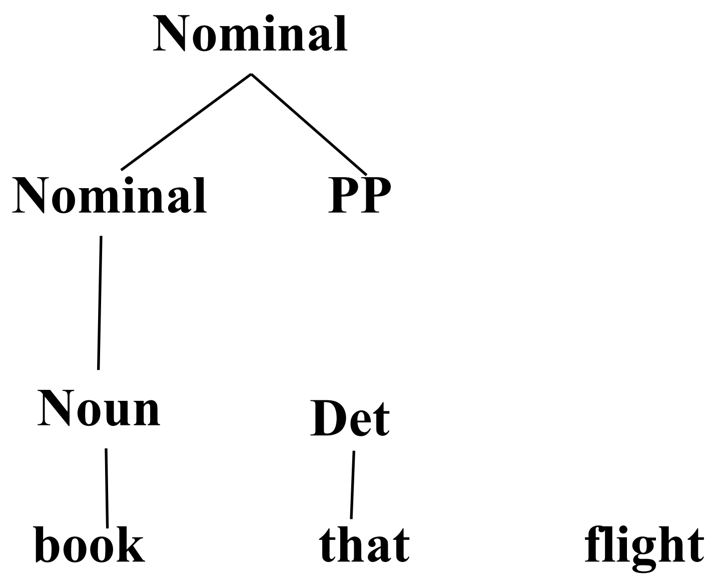

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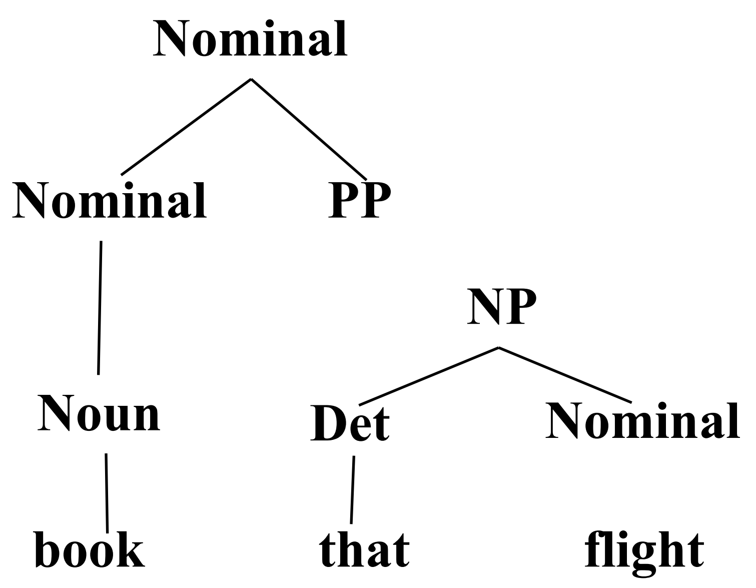

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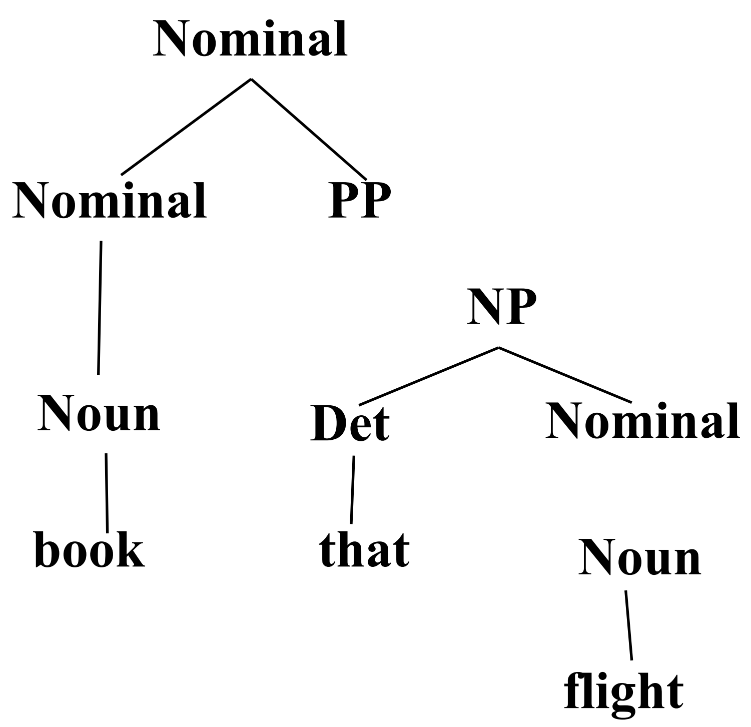

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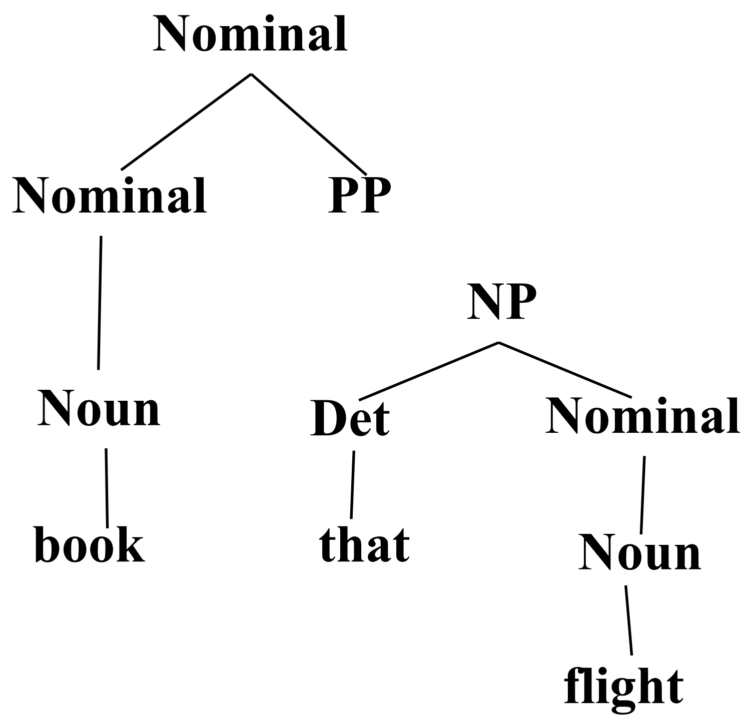

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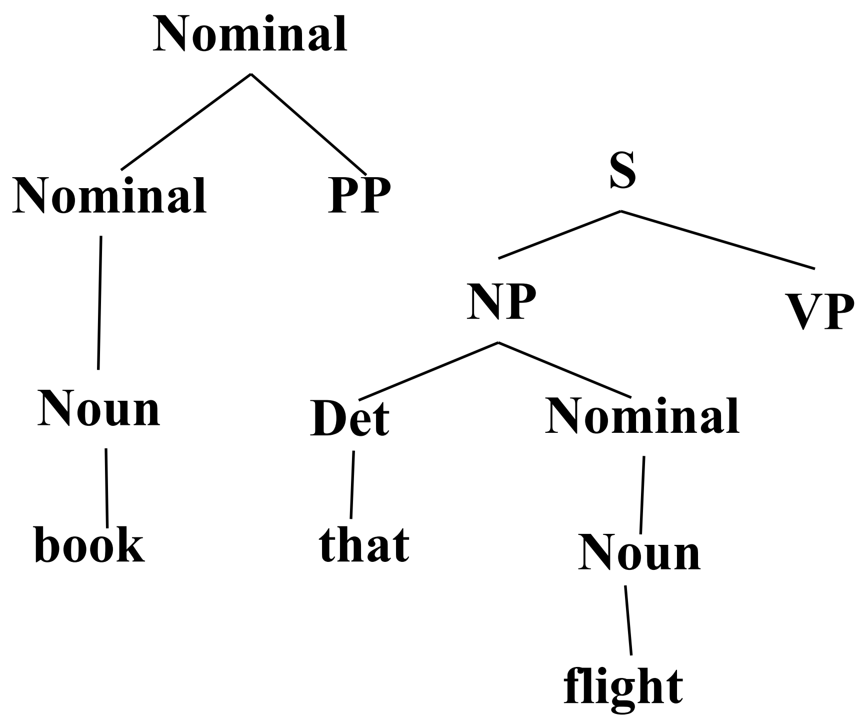

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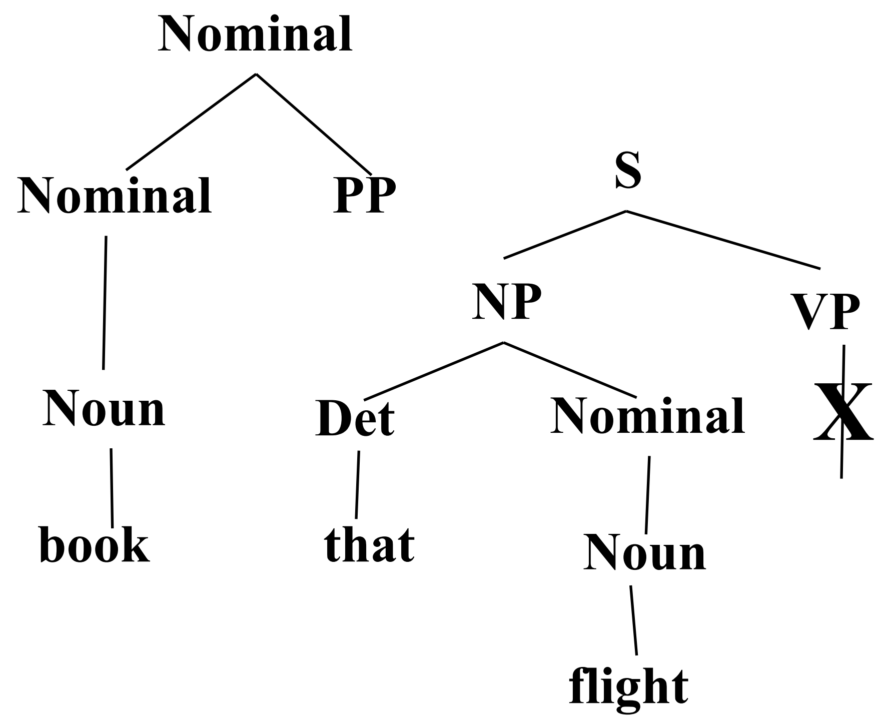

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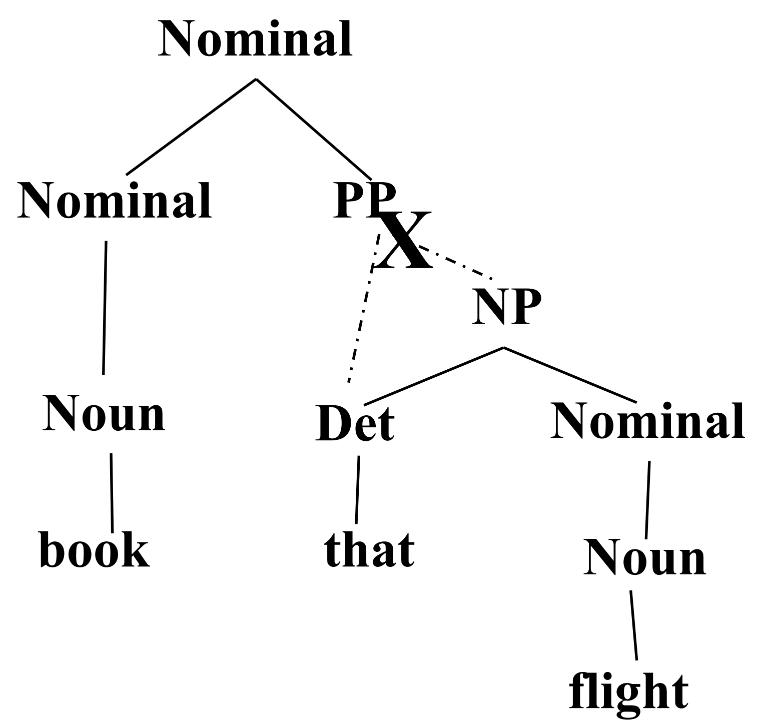

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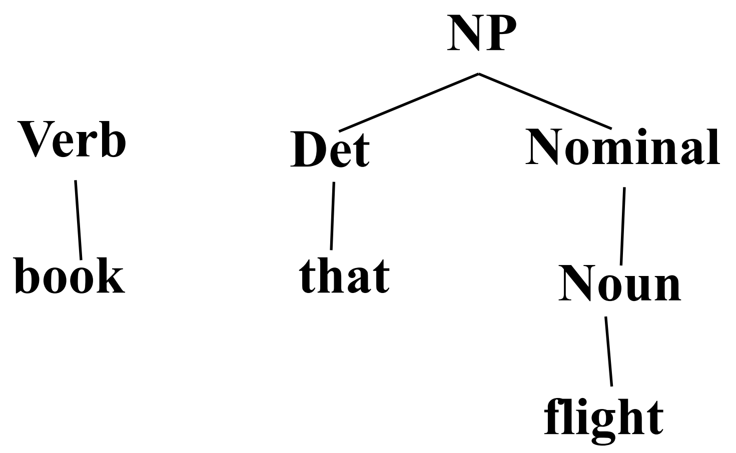

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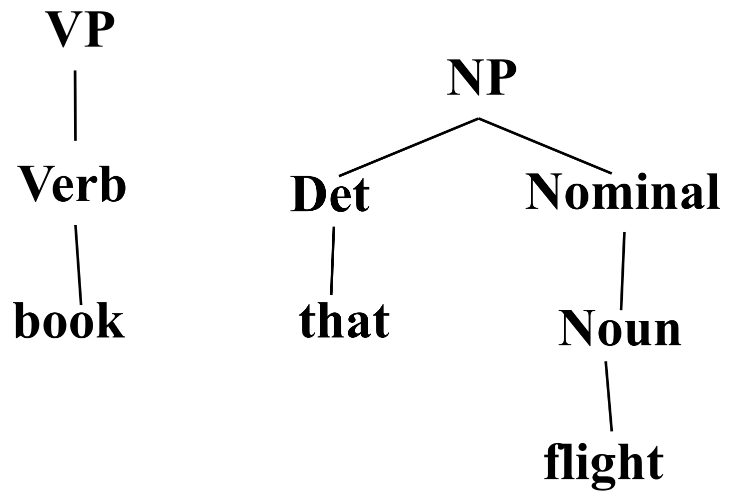

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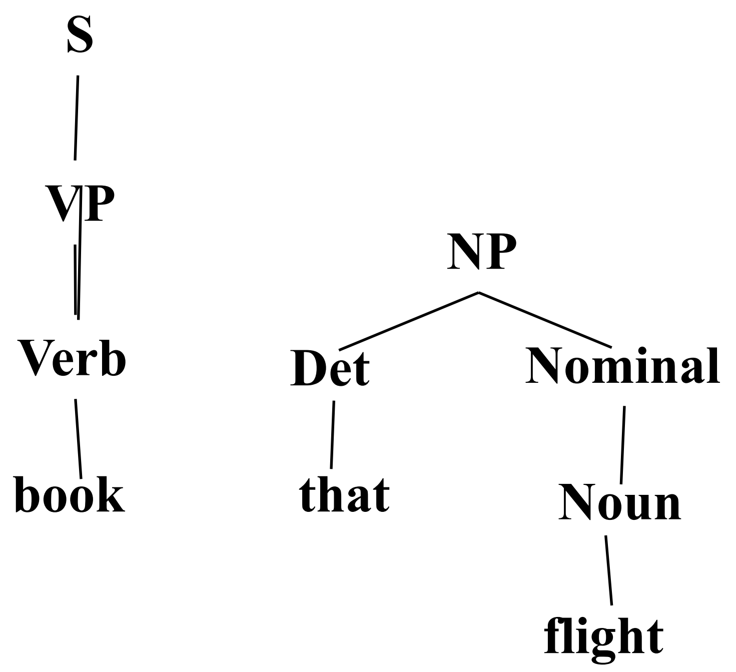

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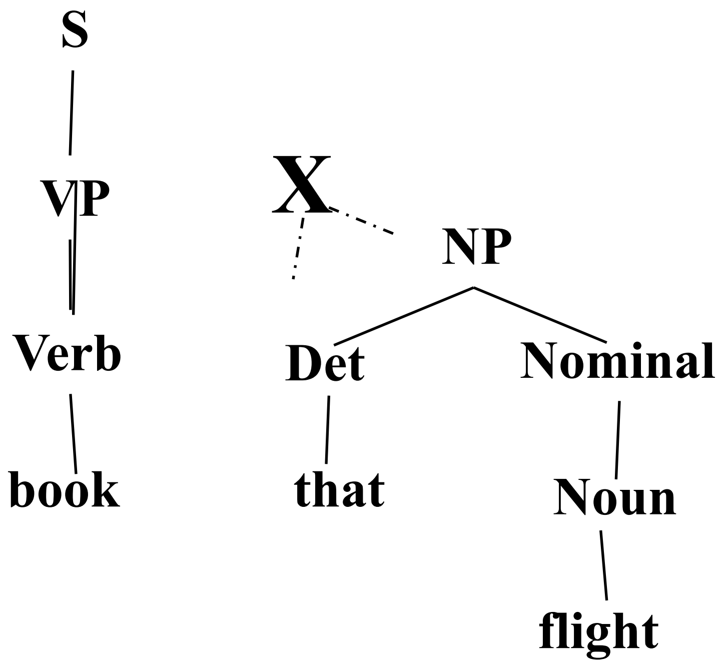

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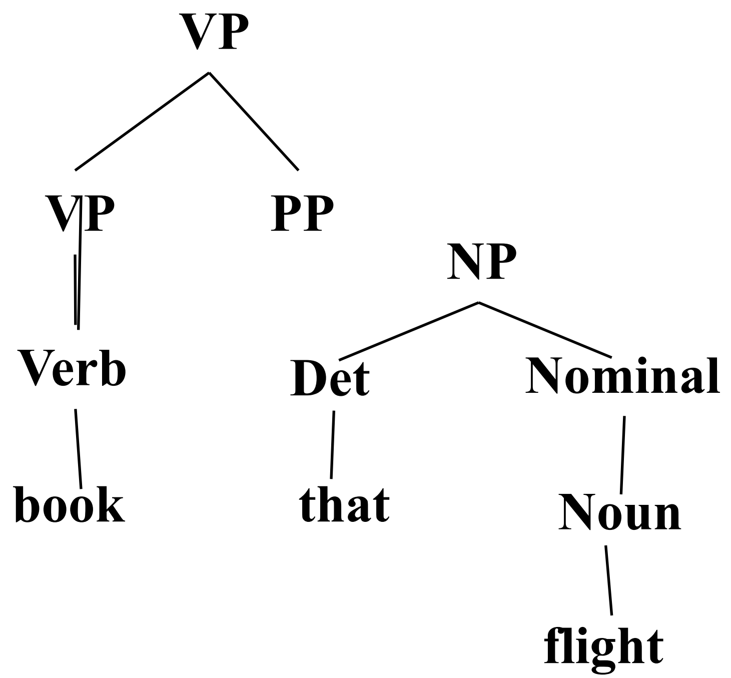

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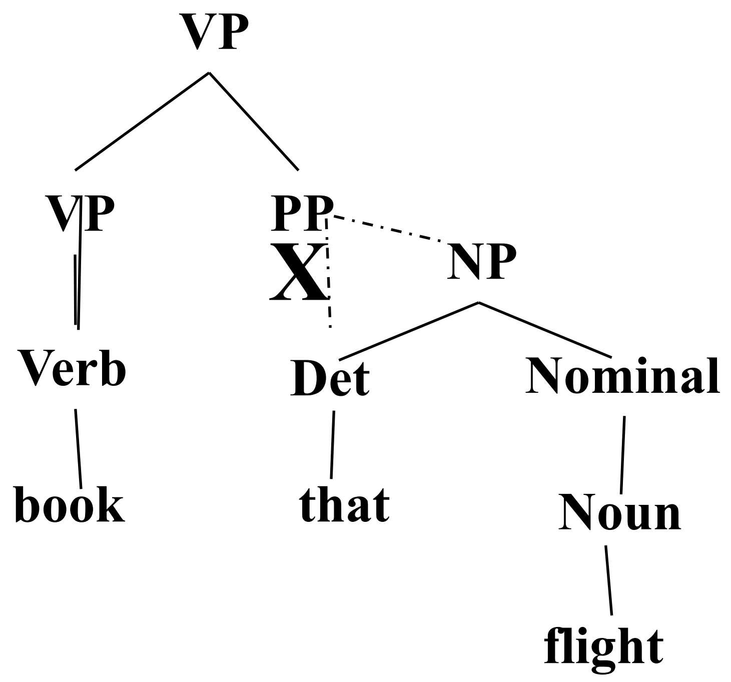

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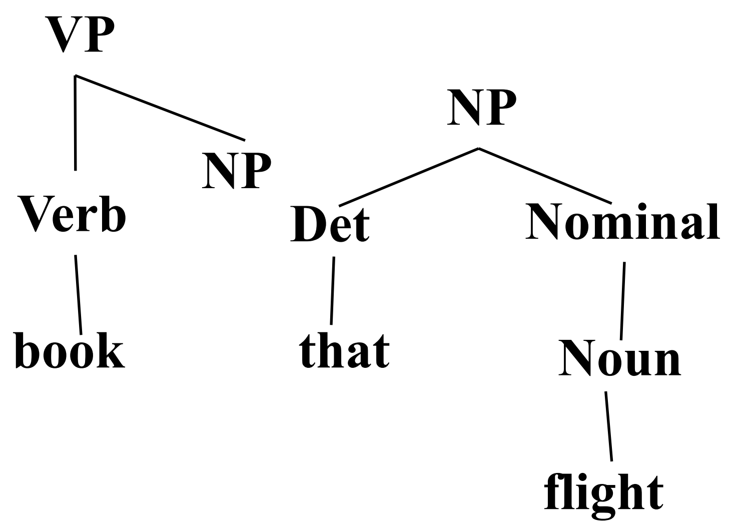

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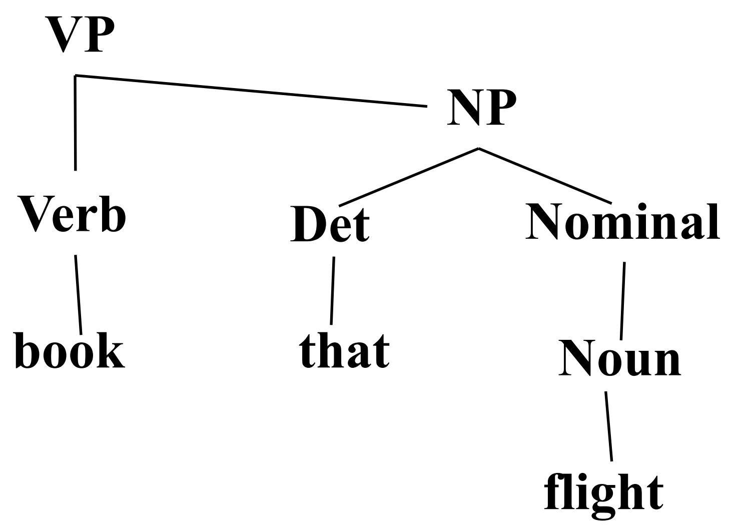

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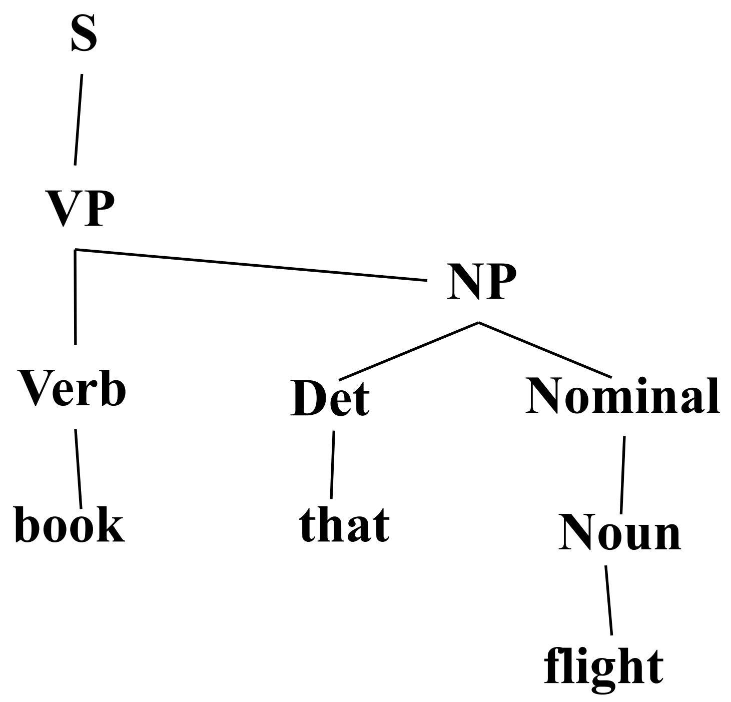
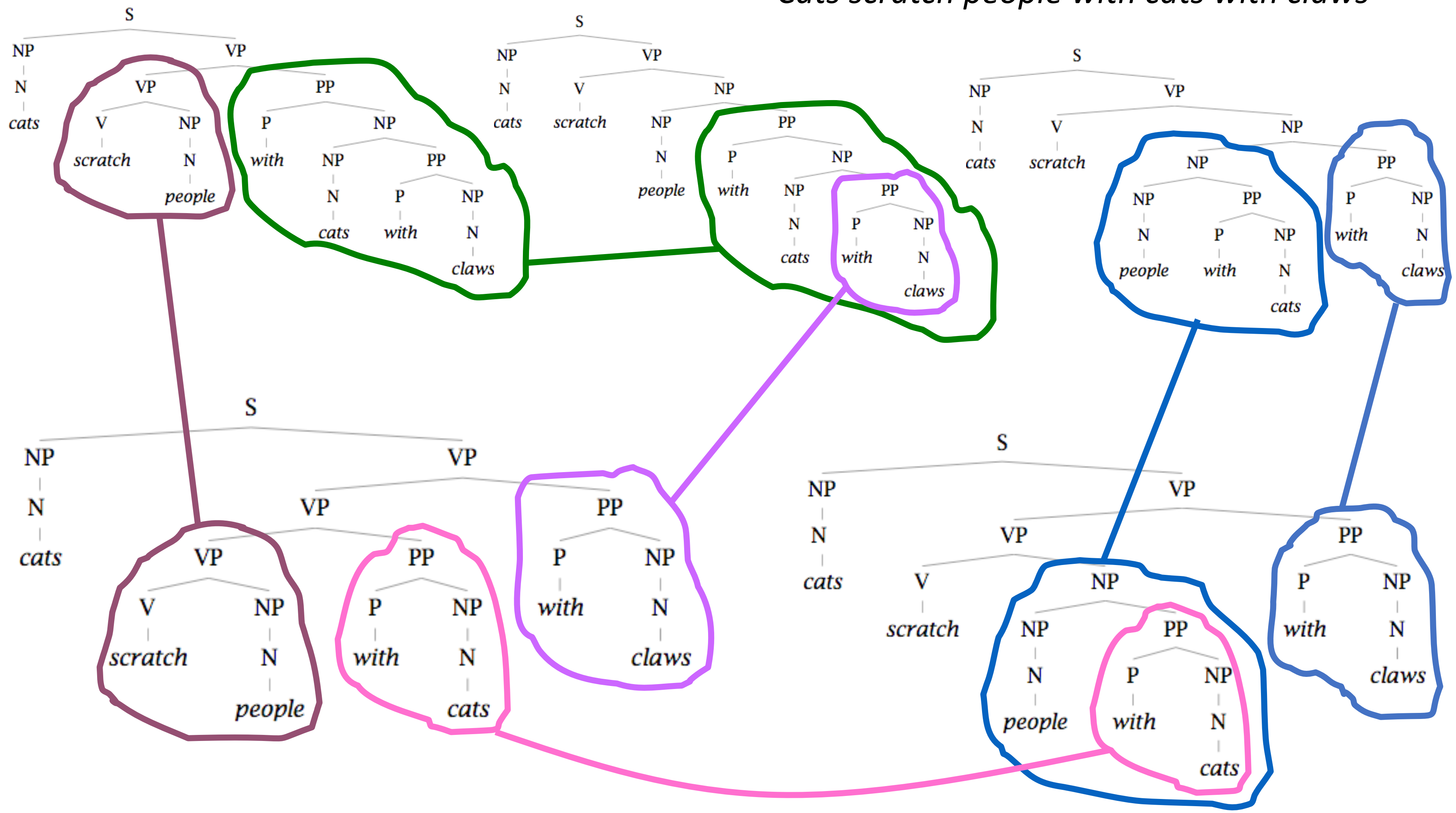


Top Down vs. Bottom Up

- Top down never explores options that will not lead to a full parse, but can explore many options that never connect to the actual sentence.
- Bottom up never explores options that do not connect to the actual sentence but can explore options that can never lead to a full parse.
- Relative amounts of wasted search depend on how much the grammar branches in each direction.

Two problems to solve for parsing:
1. Repeated work

"Cats scratch people with cats with claws"



Dynamic Programming Parsing

- To avoid extensive repeated work, must cache intermediate results, i.e. completed phrases.
- Caching (memorizing) is critical to obtaining a polynomial time parsing (recognition) algorithm for CFGs.

(Probabilistic) CKY Parsing

Constituency Parsing

Input: a PCFG, and a sentence

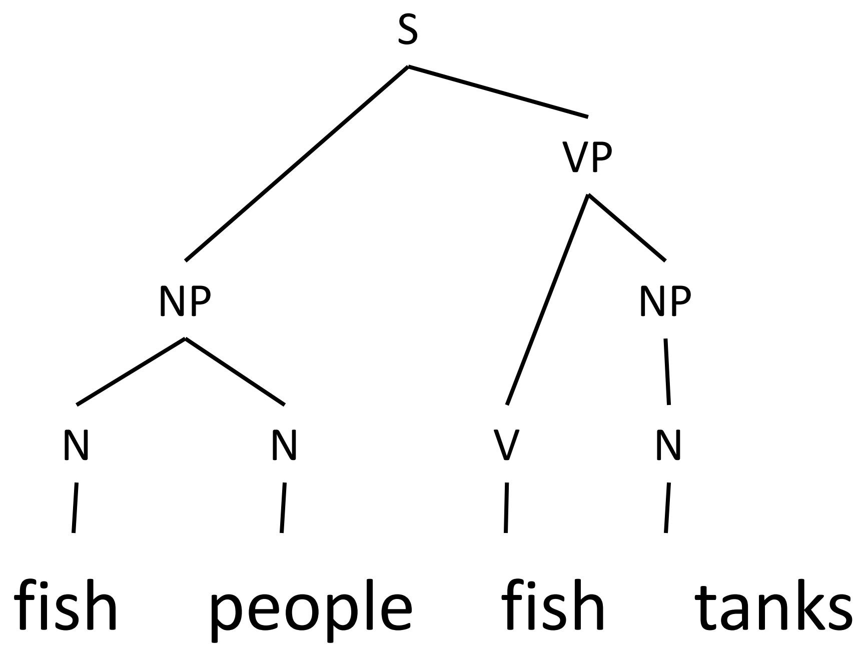
fish people fish tanks

PCFG

Rule	**Prob θ_i**
S $\rightarrow$ NP VP	θ_0
NP $\rightarrow$ NP NP	θ_1
…	
N $\rightarrow$ fish	θ_{42}
N $\rightarrow$ people	θ_{43}
V $\rightarrow$ fish	θ_{44}
…	

Constituency Parsing

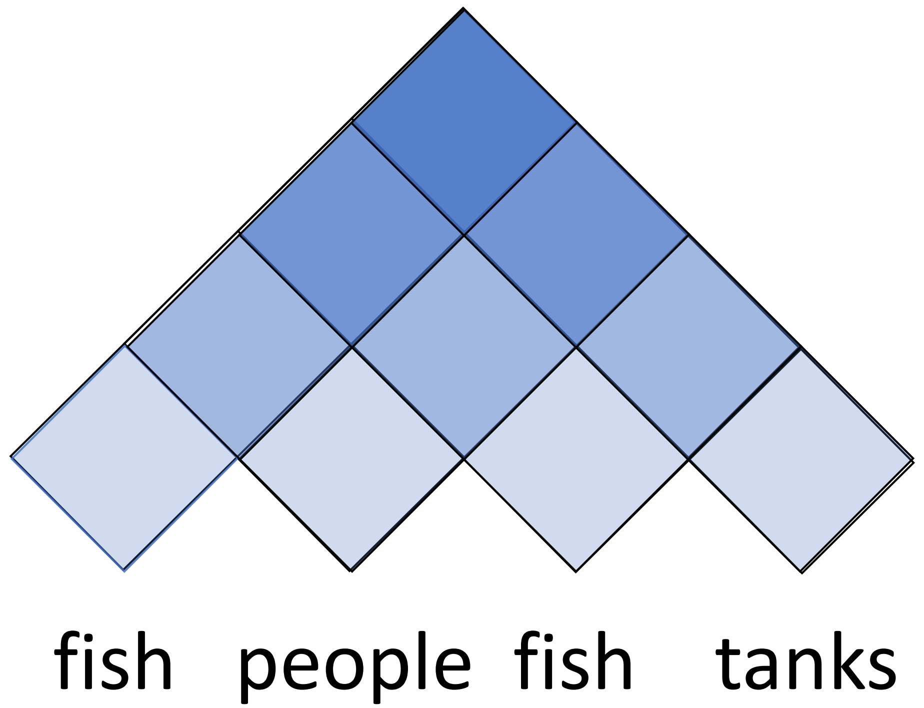
Output: a parsing tree

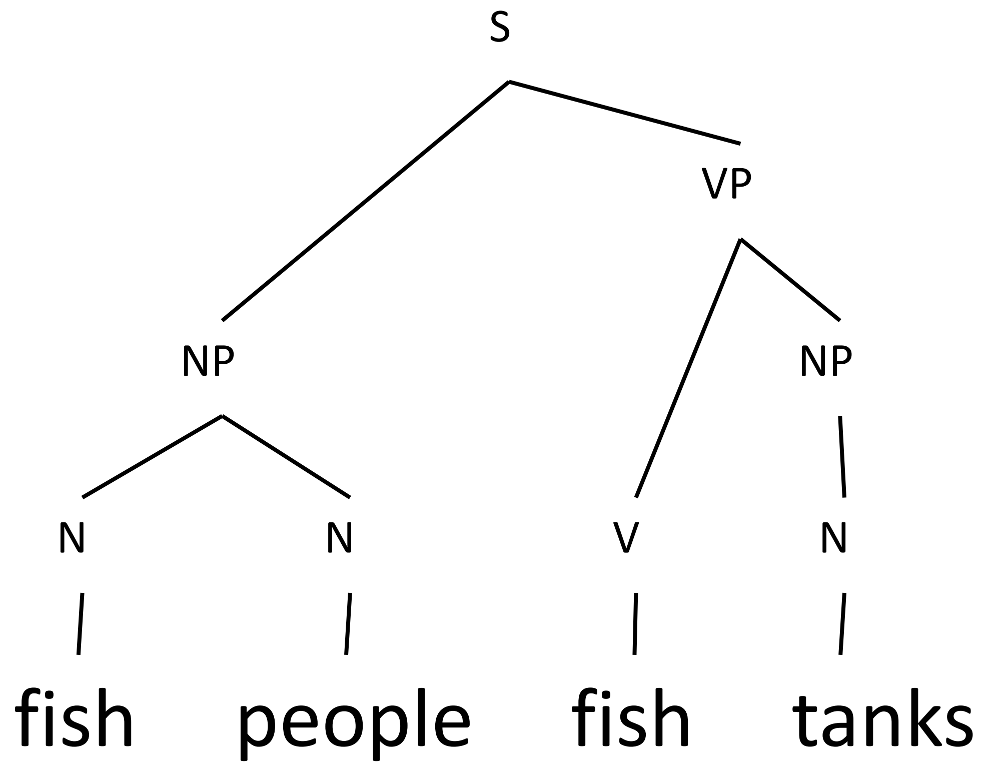


PCFG

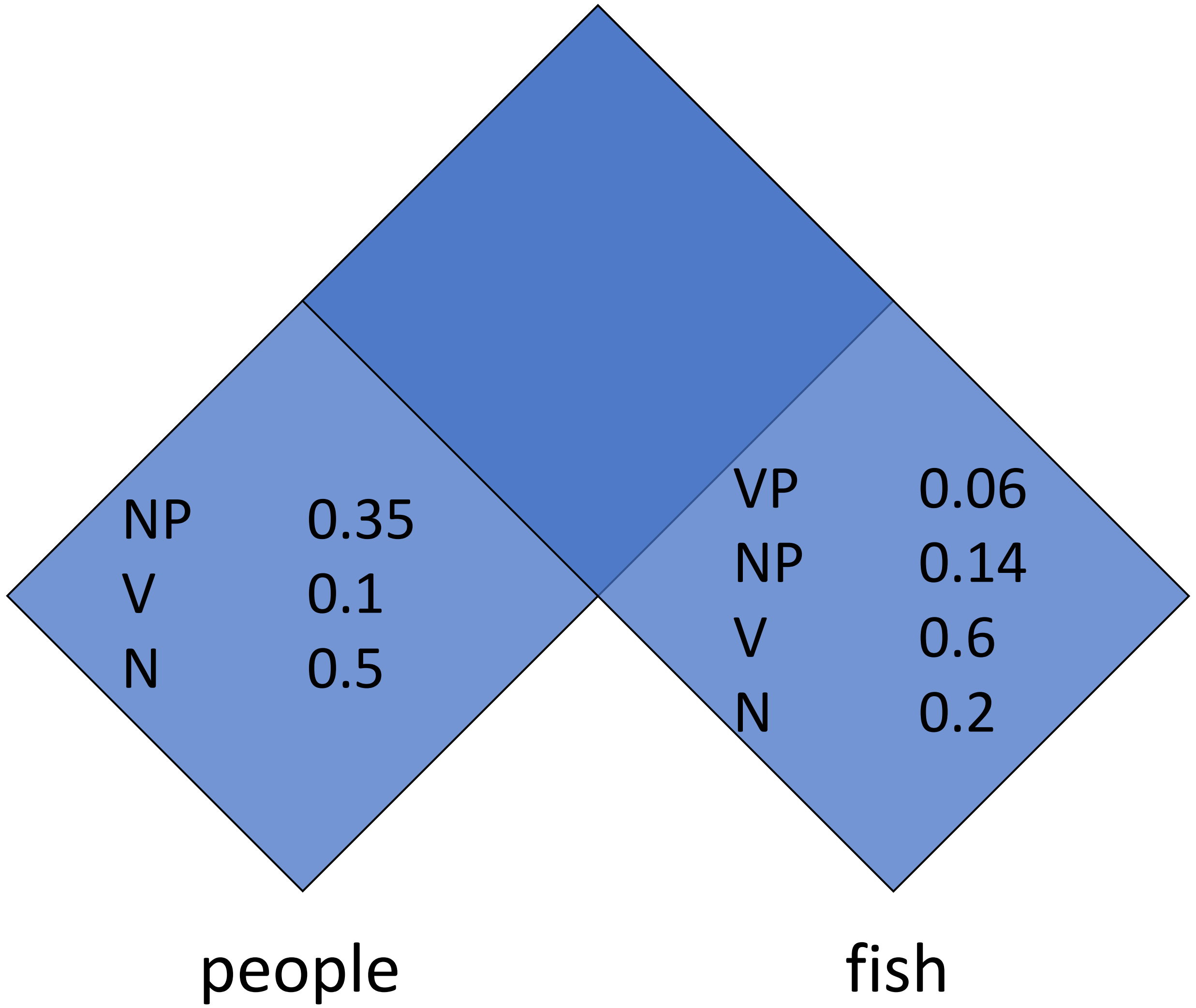
Rule	**Prob θ_i**
$S \rightarrow NP\ VP$	θ_0
$NP \rightarrow NP\ NP$	θ_1
…	
$N \rightarrow$ fish	θ_{42}
$N \rightarrow$ people	θ_{43}
$V \rightarrow$ fish	θ_{44}
…	

Cocke-Kasami-Younger (CKY) Constituency Parsing





Viterbi (Max) Scores



S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0

Extended CKY parsing

- Unaries can be incorporated into the algorithm
  - Messy, but doesn't increase algorithmic complexity
- Empties can be incorporated
  - Doesn't increase complexity; essentially like unaries
- Binarization is *vital*
  - Without binarization, you don't get parsing cubic in the length of the sentence and in the number of nonterminals in the grammar

The CKY algorithm (1960/1965) … extended to unaries

```
function CKY(words, grammar) returns [most_probable_parse,prob]
  score = new double[#(words)+1][#(words)+1][#(nonterms)]
  back = new Pair[#(words)+1][#(words)+1][#nonterms]]
  for i=0; i<#(words); i++
    for A in nonterms
      if A -> words[i] in grammar
        score[i][i+1][A] = P(A -> words[i])
    //handle unaries
    boolean added = true
    while added
      added = false
      for A, B in nonterms
        if score[i][i+1][B] > 0 && A->B in grammar
          prob = P(A->B)*score[i][i+1][B]
          if prob > score[i][i+1][A]
            score[i][i+1][A] = prob
            back[i][i+1][A] = B
            added = true
```

The CKY algorithm (1960/1965) … extended to unaries

```
for span = 2 to #(words)
  for begin = 0 to #(words)- span
    end = begin + span
    for split = begin+1 to end-1
      for A,B,C in nonterms
        prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
        if prob > score[begin][end][A]
          score[begin]end][A] = prob
          back[begin][end][A] = new Triple(split,B,C)
    //handle unaries
    boolean added = true
    while added
      added = false
      for A, B in nonterms
        prob = P(A->B)*score[begin][end][B];
        if prob > score[begin][end][A]
          score[begin][end][A] = prob
          back[begin][end][A] = B
          added = true
return buildTree(score, back)
```