# CS 6120/CS4120: Natural Language Processing

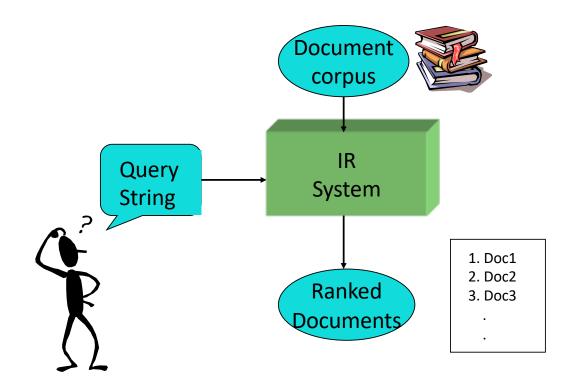
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#### Outline

- Vector Semantics
- Sparse representation
  - Pointwise Mutual Information (PMI)
- Dense representation
  - Singular Value Decomposition (SVD)
  - Neural Language Model (Word2Vec)
  - Brown cluster

# Information Retrieval System



## The Vector-Space Model

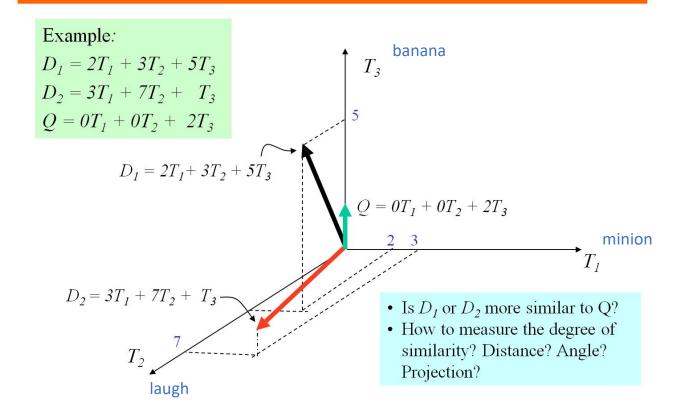
- Assume *t* distinct terms remain after preprocessing; call them index terms or the vocabulary.
- These "orthogonal" terms form a vector space.

Dimension = 
$$t = |vocabulary|$$

- Each term, i, in a document or query, j, is given a real-valued weight,  $w_{ij}$ .
- Both documents and queries are expressed as t-dimensional vectors:

$$d_{j} = (w_{1j}, w_{2j}, ..., w_{tj})$$

## **Graphic Representation**



## Term Weights: Term Frequency

• More frequent terms in a document are more important, i.e. more indicative of the topic.

$$f_{ij}$$
 = frequency of term  $i$  in document  $j$ 

• May want to normalize *term frequency* (*tf*) by dividing by the frequency of the most common term in the document:

$$tf_{ij} = f_{ij} / max_i \{f_{ij}\}$$

## Term Weights: Inverse Document Frequency

• Terms that appear in many *different* documents are *less* indicative of overall topic.

```
df_i = document frequency of term i
= number of documents containing term i
idf_i = inverse document frequency of term i,
= \log_2 (N/df_i)
(N: total number of documents)
```

- An indication of a term's discrimination power.
- Log used to dampen the effect relative to *tf*.

## TF-IDF Weighting

• A typical combined term importance indicator is *tf-idf weighting*:

$$w_{ij} = tf_{ij} idf_i = tf_{ij} \log_2 (N/df_i)$$

- A term occurring frequently in the document but rarely in the rest of the collection is given high weight.
- Many other ways of determining term weights have been proposed.
- Experimentally, *tf-idf* has been found to work well.

## Similarity Measure

- A similarity measure is a function that computes the *degree of similarity* between two vectors.
- Using a similarity measure between the query and each document:
  - It is possible to rank the retrieved documents in the order of presumed relevance.
  - It is possible to enforce a certain threshold so that the size of the retrieved set can be controlled.

## Cosine Similarity Measure

 $t_3$ 

- Cosine similarity measures the cosine of the angle between two vectors.
- Inner product normalized by the vector lengths.

CosSim(
$$\mathbf{d}_{j}$$
,  $\mathbf{q}$ ) = 
$$\frac{\vec{d}_{j} \cdot \vec{q}}{\left|\vec{d}_{j}\right| \cdot \left|\vec{q}\right|} = \frac{\sum_{i=1}^{t} (w_{ij} \cdot w_{iq})}{\sqrt{\sum_{i=1}^{t} w_{ij}^{2} \cdot \sum_{i=1}^{t} w_{iq}^{2}}}$$
  $\theta_{2}$ 

$$\begin{array}{ll} D_1 = 2T_1 + 3T_2 + 5T_3 & \operatorname{CosSim}(D_1 \ , \ Q) = 10 \ / \ \sqrt{(4+9+25)(0+0+4)} = 0.81 \\ D_2 = 3T_1 + 7T_2 + 1T_3 & \operatorname{CosSim}(D_2 \ , \ Q) = 2 \ / \ \sqrt{(9+49+1)(0+0+4)} = 0.13 \\ Q = 0T_1 + 0T_2 + 2T_3 & \end{array}$$

 $D_I$  is 6 times better than  $D_2$  using cosine similarity but only 5 times better using inner product.

# Why vector models of meaning? computing the similarity between words

```
"fast" is similar to "rapid"
```

"tall" is similar to "height"

#### Question answering:

Q: "How tall is Mt. Everest?"

Candidate A: "The official height of Mount Everest is 29029 feet"

## Beyond Dead Parrots

• Automatically constricted clusters of semantically similar words (Charniak, 1997):

Friday Monday Thursday Wednesday Tuesday Saturday Sunday

People guys folks fellows CEOs commies blocks

water gas cola liquid acid carbon steam shale

that the theat

head body hands eyes voice arm seat eye hair mouth

## Smoothing for statistical language models

Two alternative guesses of speech recognizer:

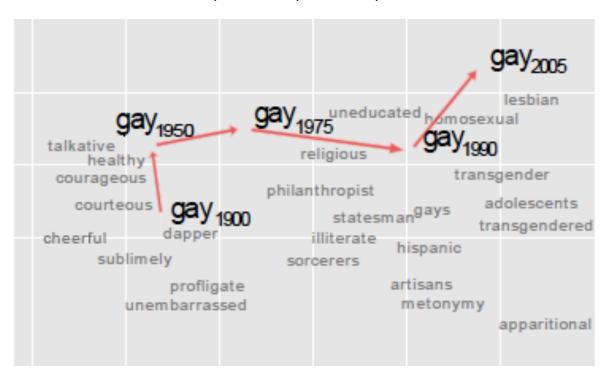
For breakfast, she ate durian.

For breakfast, she ate Dorian.

- Our corpus contains neither "ate durian" nor "ate Dorian"
- But, our corpus contains "ate orange", "ate banana"

# Word similarity for historical linguistics: semantic change over time

Kulkarni, Al-Rfou, Perozzi, Skiena 2015



## Distributional models of meaning

- = vector-space models of meaning
- = vector semantics

#### **Intuitions**: Zellig Harris (1954):

- "oculist and eye-doctor ... occur in almost the same environments"
- "If A and B have almost identical environments we say that they are synonyms."

#### Firth (1957):

"You shall know a word by the company it keeps!"

## Intuition of distributional word similarity

#### • Example:

A bottle of **tesgüino** is on the table Everybody likes **tesgüino Tesgüino** makes you drunk We make **tesgüino** out of corn.

- From context words humans can guess *tesgüino* means
  - an alcoholic beverage like **beer**
- Intuition for algorithm:
  - Two words are similar if they have similar word contexts.

#### Four kinds of vector models

#### Sparse vector representations

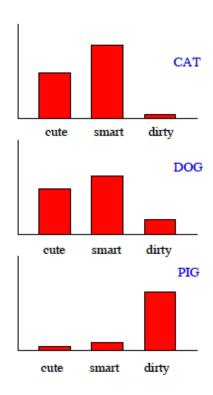
1. Mutual-information weighted word co-occurrence matrices

#### Dense vector representations:

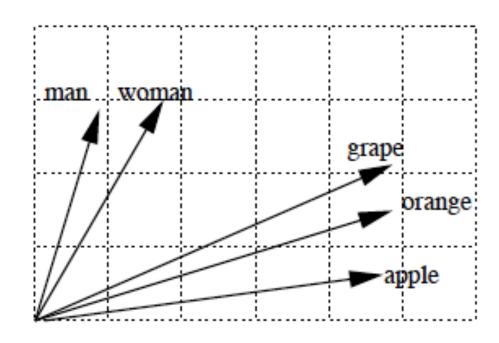
- 2. Singular value decomposition (and Latent Semantic Analysis)
- 3. Neural-network-inspired models (skip-grams, CBOW)
- 4. Brown clusters

#### Shared intuition

- Model the meaning of a word by "embedding" in a vector space.
- The meaning of a word is a vector of numbers
  - Vector models are also called "embeddings".



# Sample Lexical Vector Space



#### Term-document matrix

- Each cell: count of term t in a document d:  $tf_{t,d}$ :
  - Each document is a count vector in  $\mathbb{N}^{\mathsf{v}}$ : a column below

	As You Lik	e It	Twelfth Night	Julius Caesar	Henry V
battle		1	1	8	15
soldier		2	2	12	36
fool		37	58	1	5
clown		6	117	0	0

## Reminder: Term-document matrix

• Two documents are similar if their vectors are similar

J	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	1	8	15
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fool	37	58	1	5
clown	6	117	0	0

## The words in a term-document matrix

• Each word is a count vector in  $\mathbb{N}^{\mathbb{D}}$ : a row below

	As You l	ike It	Twelfth Night	Julius Caesar	Henry V
battle		1	1	8	15
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## Term-context matrix for word similarity

• Two words are similar in meaning if their context vectors are similar

sugar, a sliced lemon,	a tablespoonful of	apricot	preserve or	iam, a	pinch ea	ach of
sugar, a sinced terriori,	a tablespoonful of	αρτιουι	preserve or	jairi, a	pincii ce	ich o

	aardvark	computer	data	pinch	result	sugar	
apricot	0	0	0	1	0	1	
pineapple	0	0	0	1	0	1	
digital	0	2	1	0	1	0	
information	0	1	6	0	4	0	

#### The word-word or word-context matrix

- Instead of entire documents, use smaller contexts
  - Paragraph
  - Window of  $\pm$  4 words
- A word is now defined by a vector over counts of context words
- Instead of each vector being of length D
- Each vector is now of length |V|
- The word-word matrix is |V|x|V|

# Word-Word matrix Sample contexts $\pm$ 7 words

sugar, a sliced lemon, a tablespoonful of apricot their enjoyment. Cautiously she sampled her first **pineapple** well suited to programming on the digital **computer**.

preserve or jam, a pinch each of, and another fruit whose taste she likened In finding the optimal R-stage policy from for the purpose of gathering data and information necessary for the study authorized in the

# Sample Word-Word matrix

	aardvark	computer	data	pinch	result	sugar	•••
apricot	0	0	0	1	0	1	
pineapple	0	0	0	1	0	1	
digital	0	2	1	0	1	0	
information	0	1	6	0	4	0	

...

#### Word-word matrix

- We showed only 4x6, but the real matrix is 50,000 x 50,000
  - So it's very **sparse** 
    - Most values are 0.
  - That's OK, since there are lots of efficient algorithms for sparse matrices.

#### Word-word matrix

- We showed only 4x6, but the real matrix is 50,000 x 50,000
  - So it's very **sparse** 
    - Most values are 0.
  - That's OK, since there are lots of efficient algorithms for sparse matrices.
- The size of windows depends on your goals
  - The shorter the windows , the more **syntactic** the representation
    - $\pm$  1-3 very syntacticy
    - You may see playing is similar to cooking or singing, played is similar to cooked or sang
  - The longer the windows, the more **semantic** the representation
    - $\pm$  4-10 more semanticy

Positive Pointwise Mutual Information (PPMI)

#### Problem with raw counts

- Raw word frequency is not a great measure of association between words
  - It's very skewed
    - "the" and "of" are very frequent, but maybe not the most discriminative
- We'd rather have a measure that asks whether a context word is **particularly informative** about the target word.
  - Positive Pointwise Mutual Information (PPMI)

#### Problem with raw counts

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## Pointwise Mutual Information

#### **Pointwise mutual information:**

Do events x and y co-occur more than if they were independent?

$$PMI(X,Y) = \log_2 \frac{P(x,y)}{P(x)P(y)}$$

#### Pointwise Mutual Information

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Do events x and y co-occur more than if they were independent?

$$PMI(X,Y) = \log_2 \frac{P(x,y)}{P(x)P(y)}$$

PMI between two words: (Church & Hanks 1989)

Do words x and y co-occur more than if they were independent?

$$PMI(word_1, word_2) = \log_2 \frac{P(word_1, word_2)}{P(word_1)P(word_2)}$$

#### Positive Pointwise Mutual Information

- PMI ranges from  $-\infty$  to  $+\infty$
- But the negative values are problematic
  - Things are co-occurring less than we expect by chance
  - Unreliable without enormous corpora
    - Imagine w1 and w2 whose probability is each 10<sup>-6</sup>
    - Hard to be sure p(w1,w2) is significantly different than 10<sup>-12</sup>
  - Plus it's not clear people are good at "unrelatedness"
- So we just replace negative PMI values by 0
- Positive PMI (PPMI) between word1 and word2:

$$PPMI(word_1, word_2) = \max \left( \log_2 \frac{P(word_1, word_2)}{P(word_1)P(word_2)}, 0 \right)$$

## Computing PPMI on a term-context matrix

apricot

digital information

- Matrix F with W rows (words) and C columns (contexts, e.g. in the form of words)
- f<sub>ij</sub> is number of times w<sub>i</sub> occurs in context c<sub>j</sub>

$f_{\cdot \cdot}$	$\sum^C f_{ij}$	$\sum_{ij}^{W}f_{ij}$
$p_{ij} = \frac{J_{ij}}{\sum_{C} C}$	$p_{i^*} = \frac{\overline{j=1}}{W C}$	$p_{*j} = \frac{\overline{i=1}}{W C}$
$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} f_{ij}$	$\sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij}$	$\sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij}$

rdvark	compute	r d	ata	pinch	result	sugar
C	) (	)	0	1	0	1
C	) (	)	0	1	0	1
C	)	2	1	0	1	0
C	)	1	6	0	4	0

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*}p_{*j}} \qquad ppmi_{ij} = \begin{cases} pmi_{ij} & \text{if } pmi_{ij} > 0\\ 0 & \text{otherwise} \end{cases}$$

#### Count(w,context)

$$p_{ij} = \frac{f_{ij}}{N} - \frac{computer}{N} - \frac{data}{N} - \frac{computer}{N} - \frac{data}{N} - \frac{computer}{N} - \frac{data}{N} - \frac{computer}{N} - \frac{compute$$

• pmi(information,data) =  $\log_2(.32 / (.37*.58)) = .57$ 

#### PPMI(w,context)

	computer	data	pinch	result	sugar
apricot	-	-	2.25	-	2.25
pineapple	_	_	2.25	-	2.25
digital	1.66	0.00	-	0.00	-
information	0.00	0.57	-	0.47	-

#### Weighting PMI

- PMI is biased toward infrequent events
  - Very rare words have very high PMI values
- Two solutions:
  - Give rare words slightly higher probabilities
  - Use add-one smoothing (which has a similar effect)

## Weighting PMI: Giving rare context words slightly higher probability

• Raise the context probabilities to  $\alpha = 0.75$ :

$$PPMI_{\alpha}(w,c) = \max(\log_2 \frac{P(w,c)}{P(w)P_{\alpha}(c)}, 0)$$

$$P_{\alpha}(c) = \frac{count(c)^{\alpha}}{\sum_{c} count(c)^{\alpha}}$$

- This helps because  $P_{\alpha}(c) > P(c)$  for rare c
- Consider two events, P(a) = .99 and P(b)=.01 (here we use probability to show the effect)

• 
$$P_{\alpha}(a) = \frac{.99^{.75}}{.99^{.75} + .01^{.75}} = .97 \ P_{\alpha}(b) = \frac{.01^{.75}}{.99^{.75} + .01^{.75}} = .03$$

## Add-n smoothing

#### **Add-2 Smoothed Count**

	computer	data	pinch	result	sugar
apricot	2	2	3	2	3
pineapple	2	2	3	2	3
digital	4	3	2	3	2
information	3	8	2	6	2

	p(w)					
	computer	data	pinch	result	sugar	
apricot	0.03	0.03	0.05	0.03	0.05	0.20
pineapple	0.03	0.03	0.05	0.03	0.05	0.20
digital	0.07	0.05	0.03	0.05	0.03	0.24
information	0.05	0.14	0.03	0.10	0.03	0.36
p(context)	0.19	0.25	0.17	0.22	0.17	

#### PPMI versus add-2 smoothed PPMI

#### PPMI(w,context)

	computer	data	pinch	result	sugar
apricot	-	-	2.25	-	2.25
pineapple	_	-	2.25	-	2.25
digital	1.66	0.00	-	0.00	-
information	0.00	0.57	-	0.47	-

#### PPMI(w,context) [add-2]

	computer	data	pinch	result	sugar
apricot	0.00	0.00	0.56	0.00	0.56
pineapple	0.00	0.00	0.56	0.00	0.56
digital	0.62	0.00	0.00	0.00	0.00
information	0.00	0.58	0.00	0.37	0.00

## Measuring similarity

- Given 2 target words v and w
- We'll need a way to measure their similarity.
- Most measure of vectors similarity are based on the:
- Dot product or inner product from linear algebra (raw counts)

dot-product
$$(\vec{v}, \vec{w}) = \vec{v} \cdot \vec{w} = \sum_{i=1}^{N} v_i w_i = v_1 w_1 + v_2 w_2 + \dots + v_N w_N$$

- High when two vectors have large values in same dimensions.
- Low (in fact 0) for orthogonal vectors with zeros in complementary distribution

#### Problem with dot product

dot-product
$$(\vec{v}, \vec{w}) = \vec{v} \cdot \vec{w} = \sum_{i=1}^{N} v_i w_i = v_1 w_1 + v_2 w_2 + \dots + v_N w_N$$

• Dot product is longer if the vector is longer. Vector length:

$$|\vec{v}| = \sqrt{\sum_{i=1}^{N} v_i^2}$$

- Vectors are longer if they have higher values in each dimension
- That means more frequent words will have higher dot products
- That's bad: we don't want a similarity metric to be sensitive to word frequency

#### Solution: cosine

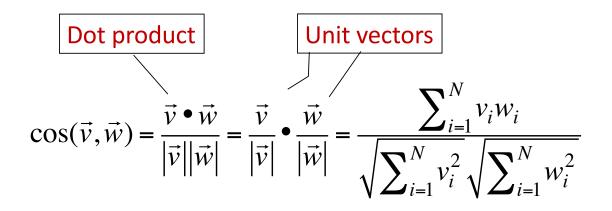
• Just divide the dot product by the length of the two vectors!

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

• This turns out to be the cosine of the angle between them!

$$ec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$
 $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \cos \theta$ 

## Cosine for computing similarity



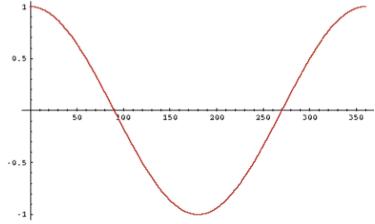
 $v_i$  is the PPMI value for word v in context i  $w_i$  is the PPMI value for word w in context i.

 $Cos(\overrightarrow{v,w})$  is the cosine similarity of  $\overrightarrow{v}$  and  $\overrightarrow{w}$ 

#### Cosine as a similarity metric

- -1: vectors point in opposite directions
- +1: vectors point in same directions
- 0: vectors are orthogonal





$$\cos(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}||\vec{w}|} = \frac{\vec{v}}{|\vec{v}|} \cdot \frac{\vec{w}}{|\vec{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2} \sqrt{\sum_{i=1}^{N} w_i^2}}$$

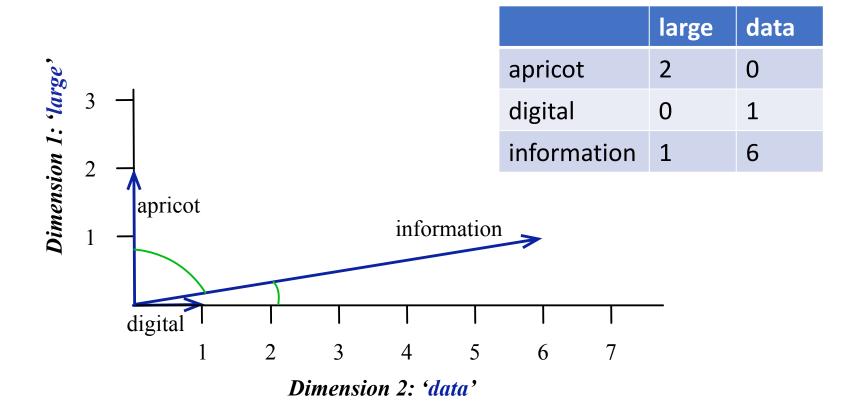
	large	data	computer
apricot	2	0	0
digital	0	1	2
information	1	6	1

Which pair of words is more similar?  $2+0+0 \over \cos(2+0+1) = \sqrt{2+0+0} \sqrt{1+36+1} = \frac{2}{\sqrt{2}\sqrt{38}} = .23$ 

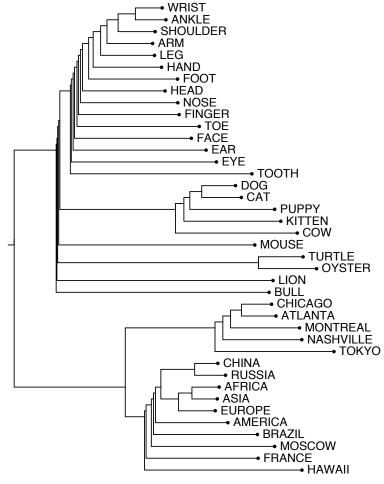
cosine(digital,information) = 
$$\frac{0+6+2}{\sqrt{0+1+4} \sqrt{1+36+1}} = \frac{8}{\sqrt{38}\sqrt{5}} = .58$$

cosine(apricot,digital) = 
$$\frac{0+0+0}{\sqrt{1+0+0}} = 0$$

## Visualizing vectors and angles



Clustering vectors to visualize similarity in co-occurrence matrices



Rohde et al. (2006)

## Other possible similarity measures

$$\begin{aligned} & \text{sim}_{\text{cosine}}(\vec{v}, \vec{w}) &= \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|} = \frac{\sum_{i=1}^{N} v_i \times w_i}{\sqrt{\sum_{i=1}^{N} v_i^2} \sqrt{\sum_{i=1}^{N} w_i^2}} \\ & \text{sim}_{\text{Jaccard}}(\vec{v}, \vec{w}) &= \frac{\sum_{i=1}^{N} \min(v_i, w_i)}{\sum_{i=1}^{N} \max(v_i, w_i)} \\ & \text{sim}_{\text{Dice}}(\vec{v}, \vec{w}) &= \frac{2 \times \sum_{i=1}^{N} \min(v_i, w_i)}{\sum_{i=1}^{N} (v_i + w_i)} \end{aligned}$$

## Using syntax to define a word's context

Zellig Harris (1968)

"The meaning of entities, and the meaning of grammatical relations among them, is related to the restriction of combinations of these entities relative to other entities"

Two words are similar if they have similar syntactic contexts

**Duty** and **responsibility** have similar syntactic distribution:

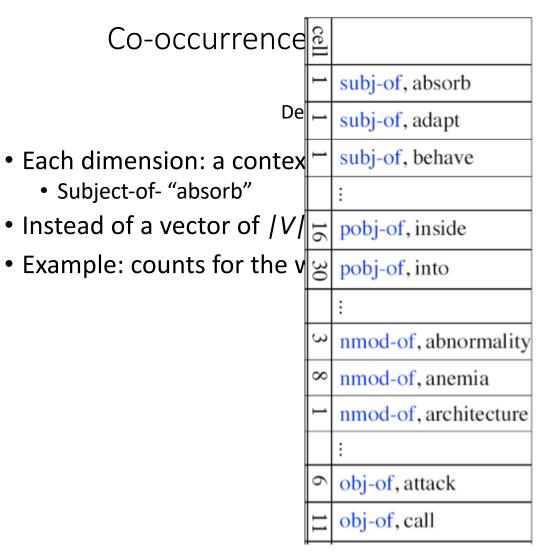
Modified by adjectives	additional, administrative, assumed, collective, congressional, constitutional
Objects of verbs	assert, assign, assume, attend to, avoid, become, breach

#### Co-occurrence vectors based on syntactic dependencies

Dekang Lin, 1998 "Automatic Retrieval and Clustering of Similar Words"

- Each dimension: a context word in one of R grammatical relations
  - Subject-of- "absorb"
- Instead of a vector of |V| features, a vector of R|V|
- Example: counts for the word *cell*:

	subj-of, absorb	subj-of, adapt	subj-of, behave	 pobj-of, inside	pobj-of, into	 nmod-of, abnormality	nmod-of, anemia	nmod-of, architecture	 obj-of, attack	obj-of, call	obj-of, come from	obj-of, decorate	 nmod, bacteria	nmod, body	nmod, bone marrow	
cell	1	1	1	16	30	3	8	1	6	11	3	2	3	2	2	



#### 1 syntactic dependencies

Retrieval and Clustering of Similar Words"

ammatical relations

R/V/

#### Syntactic dependencies for dimensions

- Alternative (Padó and Lapata 2007):
  - Instead of having a |V| x R|V| matrix
  - Have a |V| x |V| matrix
  - But the co-occurrence counts aren't just counts of words in a window
  - But counts of words that occur in one of R dependencies (subject, object, etc).
  - So M("cell","absorb") = count(subj(cell,absorb)) + count(obj(cell,absorb)) + count(pobj(cell,absorb)), etc.

#### PMI applied to dependency relations

Hindle, Don. 1990. Noun Classification from Predicate-Argument Structure. ACL

Object of "drink"	Count	PMI
tea	2	11.8
liquid	2	10.5
wine	2	9.3
anything	3	5.2
it	3	1.3

- "Drink it" more common than "drink wine"
- But "wine" is a better "drinkable" thing than "it"

#### Alternative to PPMI for measuring association

- Recall that we studied tf-idf...
- The combination of two factors
  - Term frequency (Luhn 1957): frequency of the word (can be logged)
  - Inverse document frequency (IDF) (Spark Jones 1972)
    - N is the total number of documents
    - N is the total number of documents
        $df_i$  = "document frequency of word i"
       = number of documents with word i• an und i in document i

• 
$$w_{ij}$$
: for word  $i$  in document  $j$ 

$$w_{ij} = tf_{ij} \cdot idf_i$$

# tf-idf not generally used for word-word similarity

• But is by far the most common weighting when we are considering the relationship of words to documents

## Evaluating similarity (Revisit)

- Extrinsic (task-based, end-to-end) Evaluation:
  - Question Answering
  - Spell Checking
  - Essay grading
- Intrinsic Evaluation:
  - Correlation between algorithm and human word similarity ratings
    - Wordsim353: 353 noun pairs rated 0-10. sim(plane,car)=5.77
  - Taking TOEFL multiple-choice vocabulary tests
    - <u>Levied</u> is closest in meaning to: imposed, believed, requested, correlated

#### Summary

- Distributional (vector) models of meaning
  - Sparse (PPMI-weighted word-word co-occurrence matrices)
  - Dense:
    - Word-word SVD (50-2000 dimensions)
    - Skip-grams and CBOW (100-1000 dimensions)