CS 6120/CS4120: Natural Language Processing

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Outline

- Vector Semantics
- Sparse representation
  - Pointwise Mutual Information (PMI)
- Dense representation
  - Singular Value Decomposition (SVD)
  - Neural Language Model (Word2Vec)

Sparse versus dense vectors

- Why dense vectors?
  - Short vectors may be easier to use as features in machine learning (less weights to tune)
  - Dense vectors may generalize better than storing explicit counts (or variations)
  - They may do better at capturing synonymy:
    - *car* and *automobile* are synonyms; but are represented as distinct dimensions; this fails to capture similarity between a word with *car* as a neighbor and a word with *automobile* as a neighbor

Three methods for getting short dense vectors

- Singular Value Decomposition (SVD) (this lecture)

- “Neural Language Model” – inspired by predictive models

- Brown clustering

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- Singular Value Decomposition (SVD)
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Singular Value Decomposition (SVD)

Rank of a Matrix

- What is the rank of a matrix A?

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- Number of linearly independent columns of A

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix}$$

Rank of a Matrix

- What is the rank of a matrix A?
- Number of linearly independent columns of A

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix}$$

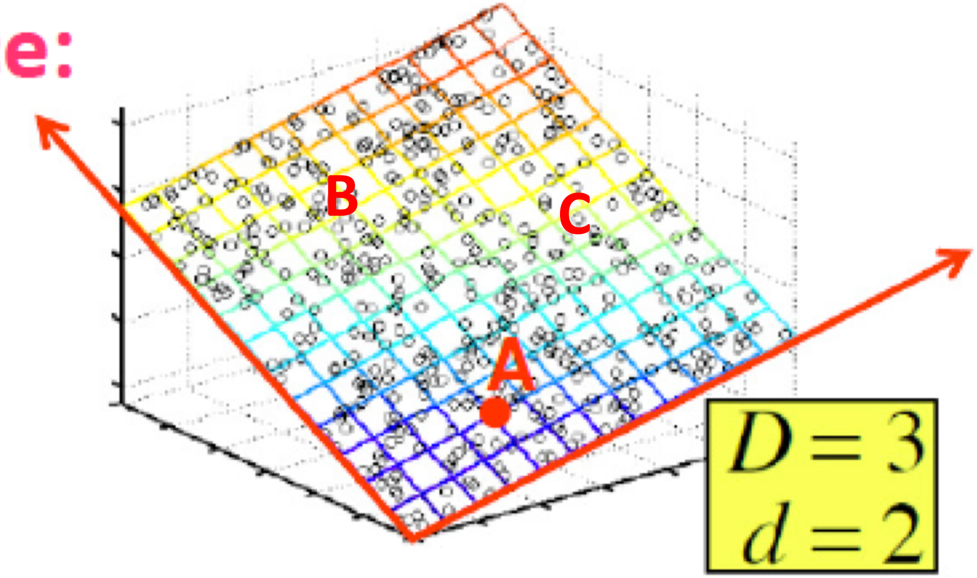
- Rank is 2
- We can rewrite A as two “basis” vectors: [1 2 1] [-2 -3 1]

Rank as “Dimensionality”

Cloud of points 3D space:

- Think of point positions as a matrix:

$$
\text{1 row per point:}\quad
\begin{array}{c}
\begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix}
\end{array}
\begin{array}{l} A \\ B \\ C \end{array}
$$



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1 row per point:
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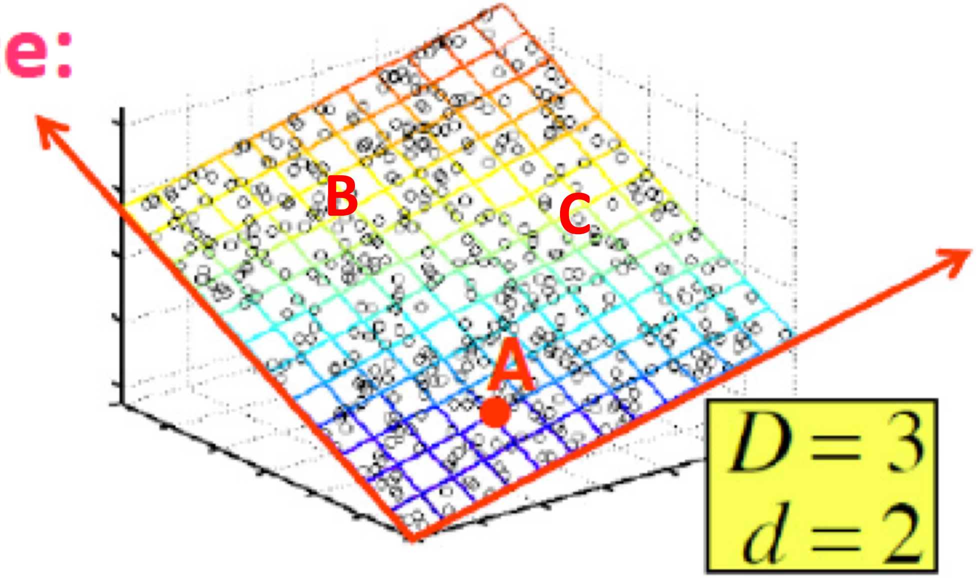
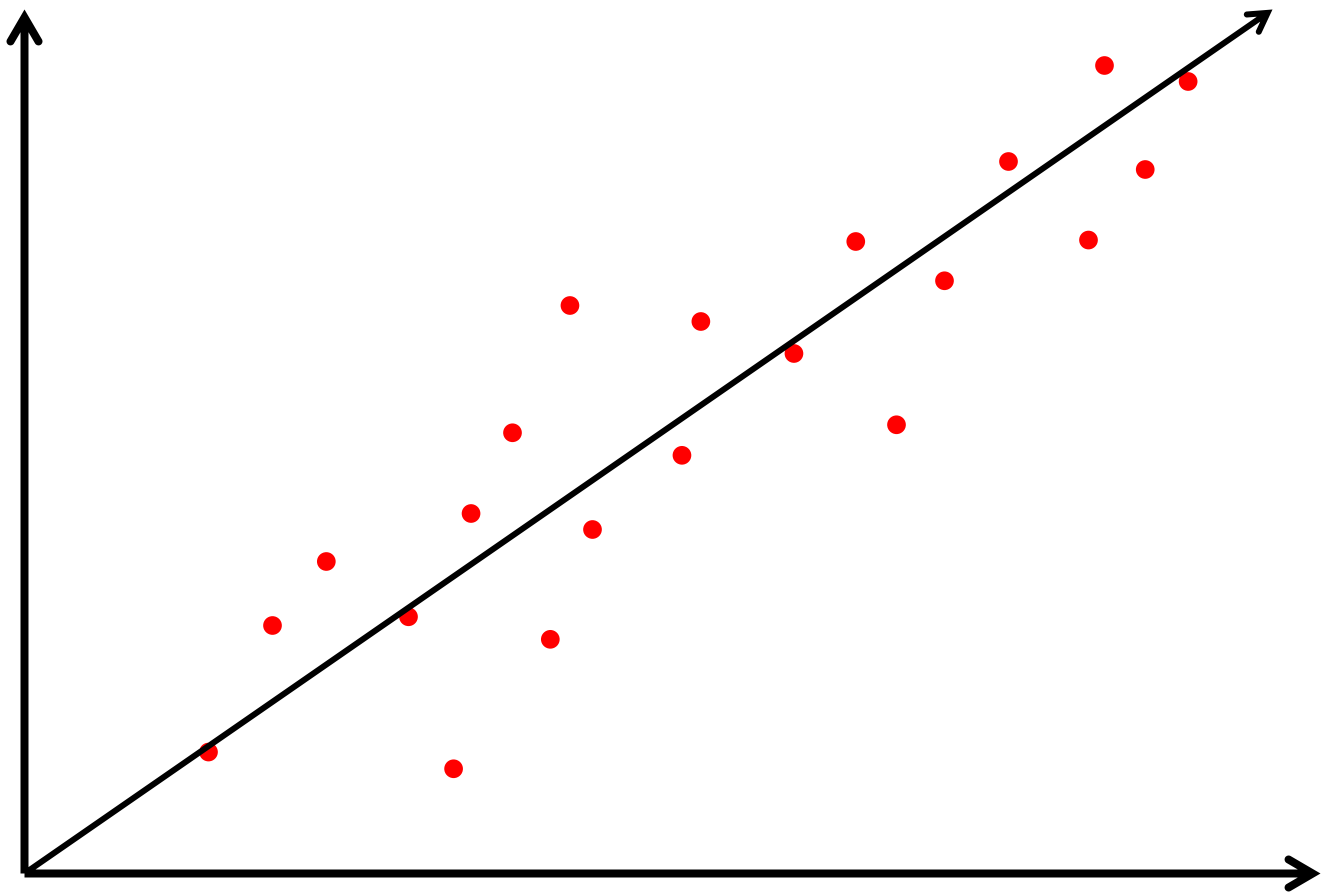


- Rewrite the coordinates in a more efficient way!
  - Old basis vectors: [1 0 0], [0 1 0], [0 0 1]
  - New basis vectors: [1 2 1], [-2 -3 1]

Intuition of Dimensionality Reduction

- Approximate an N-dimensional dataset using fewer dimensions
- By first rotating the axes into a new space
- In which the highest order dimension captures the most variance in the original dataset
- And the next dimension captures the next most variance, etc.

Sample Dimensionality Reduction



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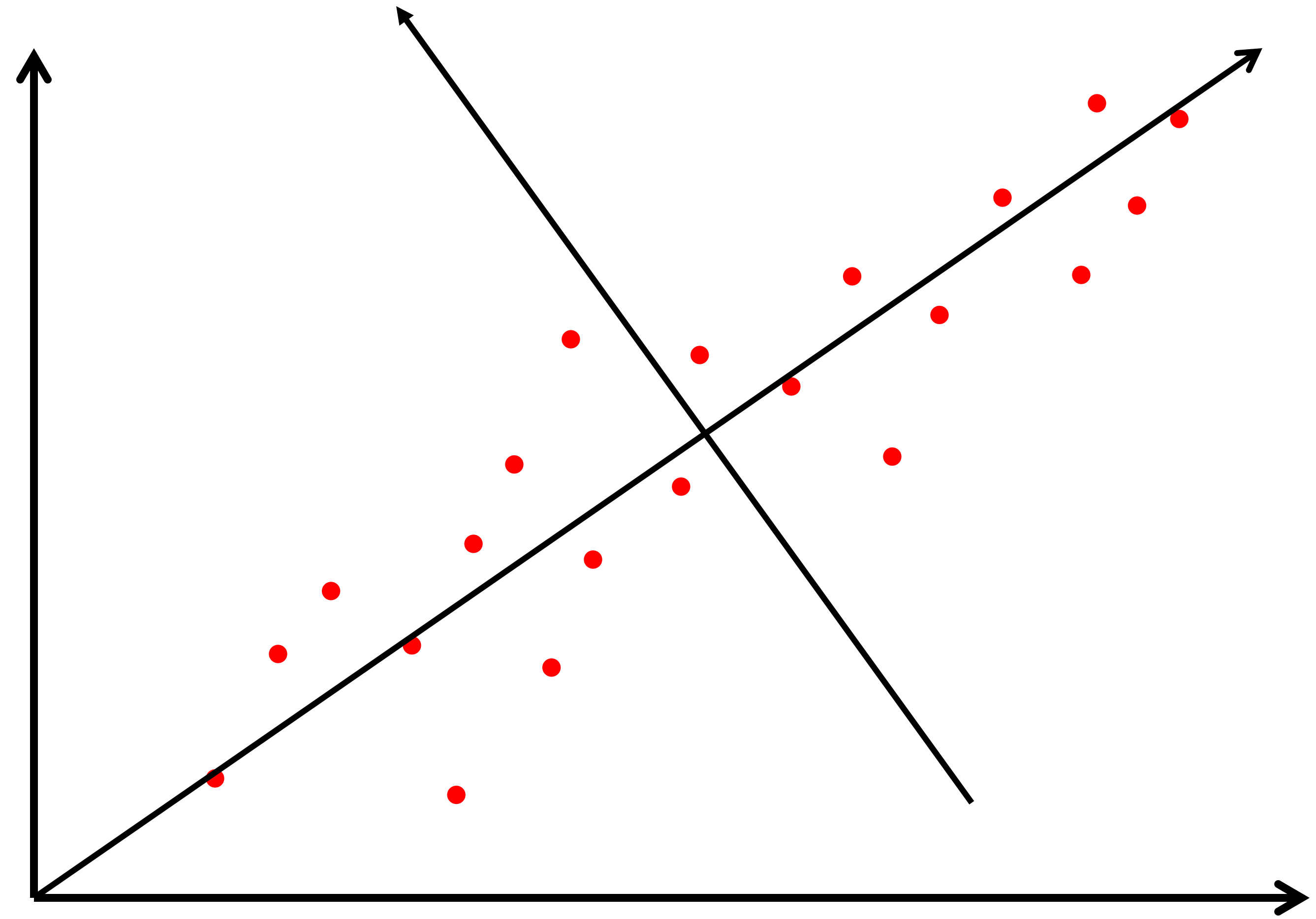

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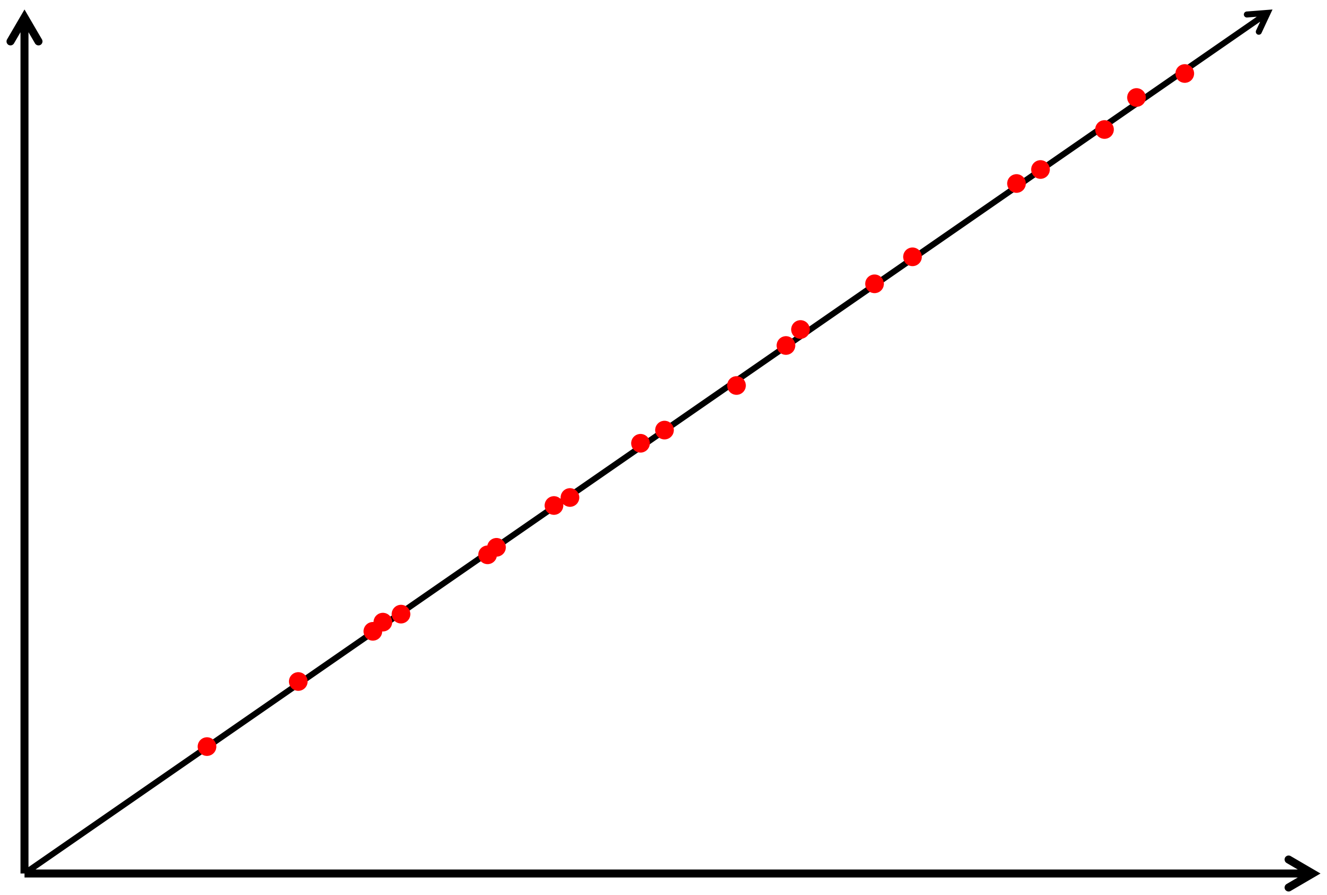
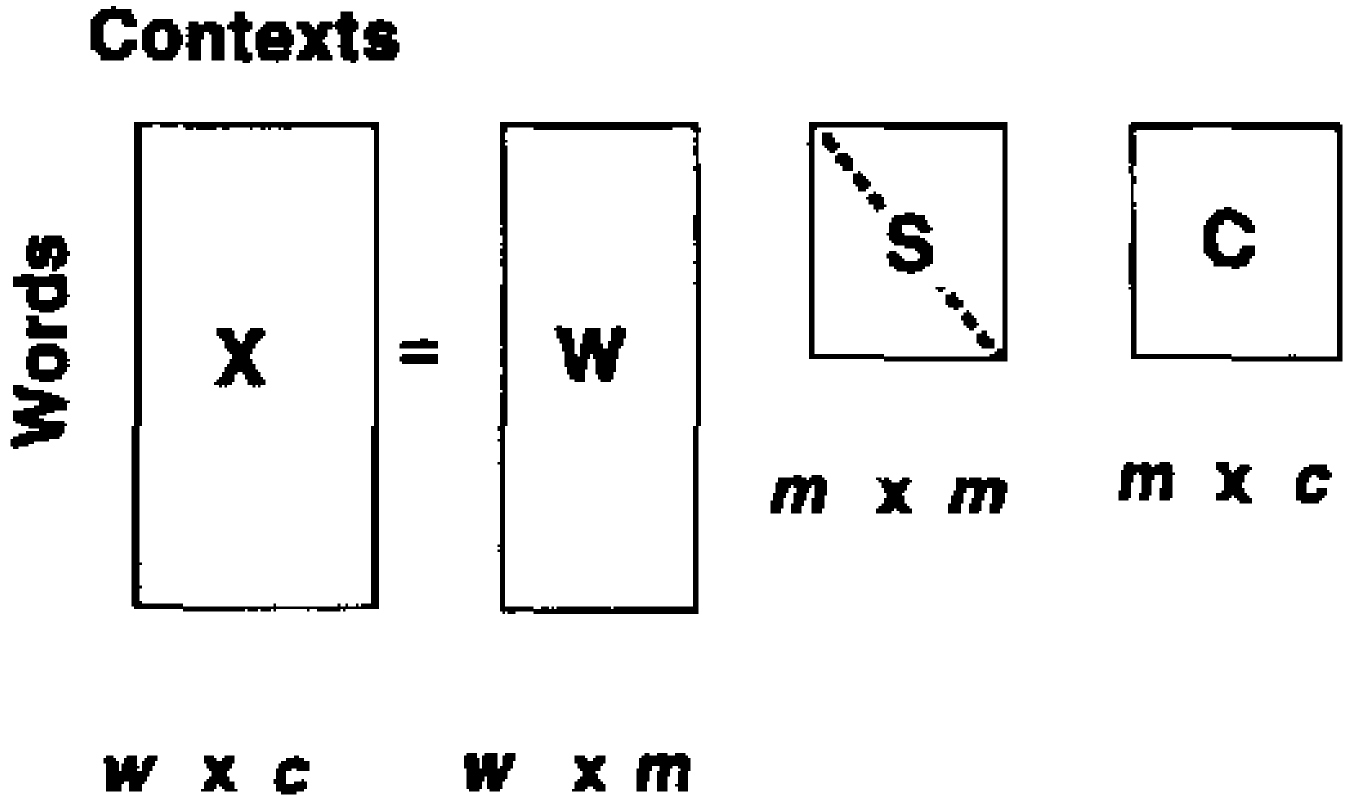


Singular Value Decomposition



(assuming the matrix has rank m)

Landuaer and Dumais 1997

Singular Value Decomposition

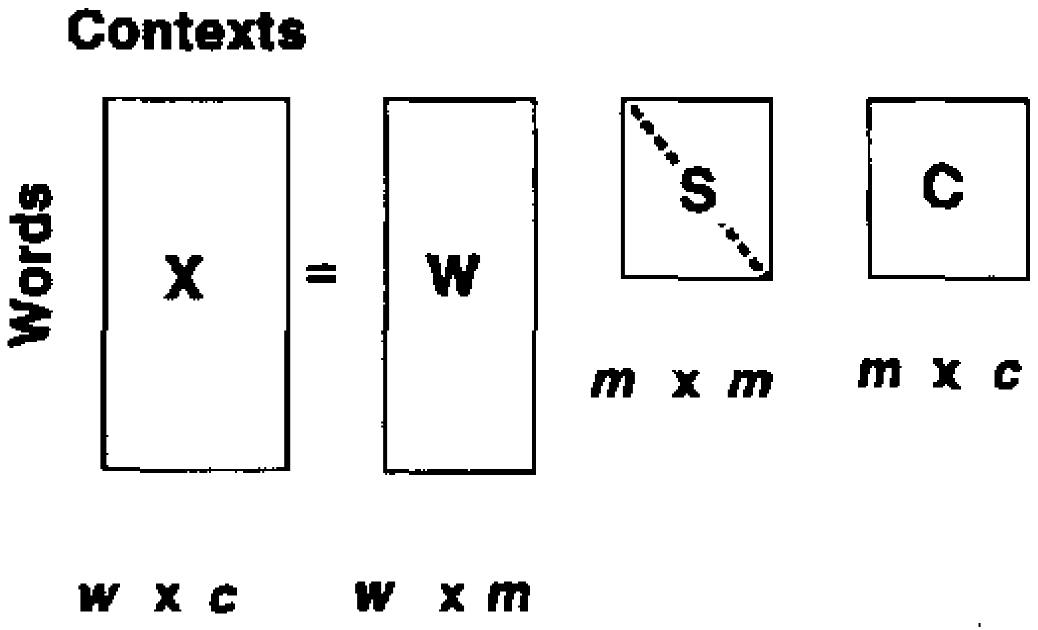
Any rectangular w x c matrix ***X*** *equals the product of 3 matrices:*

W: rows corresponding to original but m columns represents a dimension in a new latent space, such that

- m column vectors are orthogonal to each other
- Columns are ordered by the amount of variance in the dataset each new dimension accounts for

S: diagonal *m* x *m* matrix of **singular values** expressing the importance of each dimension.

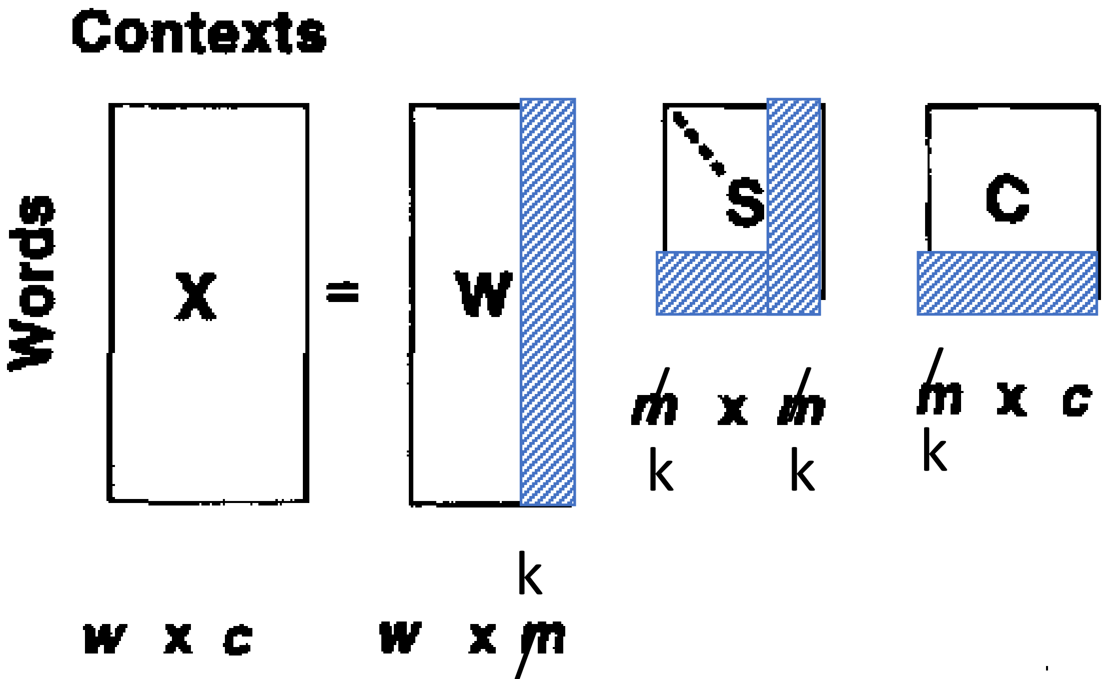
C: columns corresponding to original but m rows corresponding to singular values



SVD applied to term-document matrix: Latent Semantic Analysis

Deerwester et al (1988)

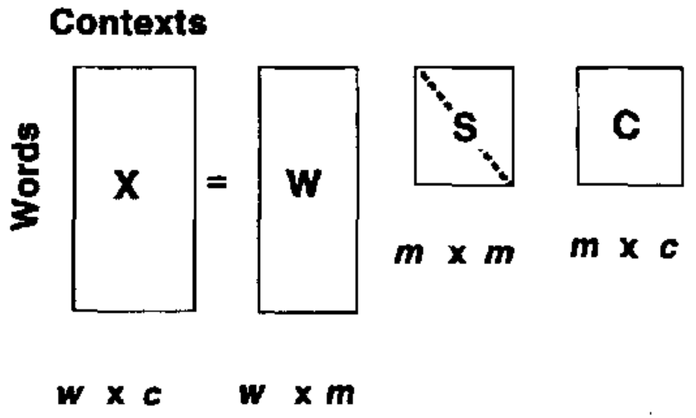
- If instead of keeping all m dimensions, we just keep the top k singular values. Let's say 300.
- The result is a least-squares approximation to the original X
- But instead of multiplying, we'll just make use of W.
- Each row of W:
  - A k-dimensional vector
  - Representing word W



SVD on Term-Document Matrix: Example

- The matrix X

	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1



Matrix **W**

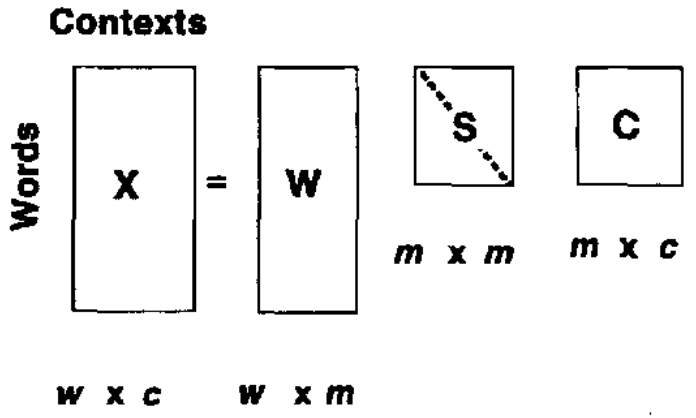
	1	2	3	4	5
ship	−0.44	−0.30	0.57	0.58	0.25
boat	−0.13	−0.33	−0.59	0.00	0.73
ocean	−0.48	−0.51	−0.37	0.00	−0.61
wood	−0.70	0.35	0.15	−0.58	0.16
tree	−0.26	0.65	−0.41	0.58	−0.09

Matrix **S**

	1	2	3	4	5
1	2.16	0.00	0.00	0.00	0.00
2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	1.28	0.00	0.00
4	0.00	0.00	0.00	1.00	0.00
5	0.00	0.00	0.00	0.00	0.39

Matrix **C**

	d_1	d_2	d_3	d_4	d_5	d_6
1	−0.75	−0.28	−0.20	−0.45	−0.33	−0.12
2	−0.29	−0.53	−0.19	0.63	0.22	0.41
3	0.28	−0.75	0.45	−0.20	0.12	−0.33
4	0.00	0.00	0.58	0.00	−0.58	0.58
5	−0.53	0.29	0.63	0.19	0.41	−0.22



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Similarity between ***ship*** and ***boat*** ***vs*** ***ship*** and ***wood*** ?

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More details

- 300 dimensions are commonly used
- The cells are commonly weighted by a product of two weights (TF-IDF)
  - Local weight: Log term frequency
  - Global weight: either idf or an entropy measure

Let’s return to PPMI word-word matrices

- Can we apply to SVD to them?

SVD applied to term-term matrix

$$
\underset{|V| \times |V|}{\begin{bmatrix} \\ \\ & X & \\ \\ \\ \end{bmatrix}} = \underset{|V| \times |V|}{\begin{bmatrix} \\ \\ & W & \\ \\ \\ \end{bmatrix}} \underset{|V| \times |V|}{\begin{bmatrix} \sigma_1 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sigma_V \end{bmatrix}} \underset{|V| \times |V|}{\begin{bmatrix} \\ \\ & C & \\ \\ \\ \end{bmatrix}}
$$

(assuming the matrix has rank |V|, may not be true)

SVD applied to term-term matrix

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\underset{|V| \times |V|}{\begin{bmatrix} & & \\ & C & \\ & & \end{bmatrix}}
$$

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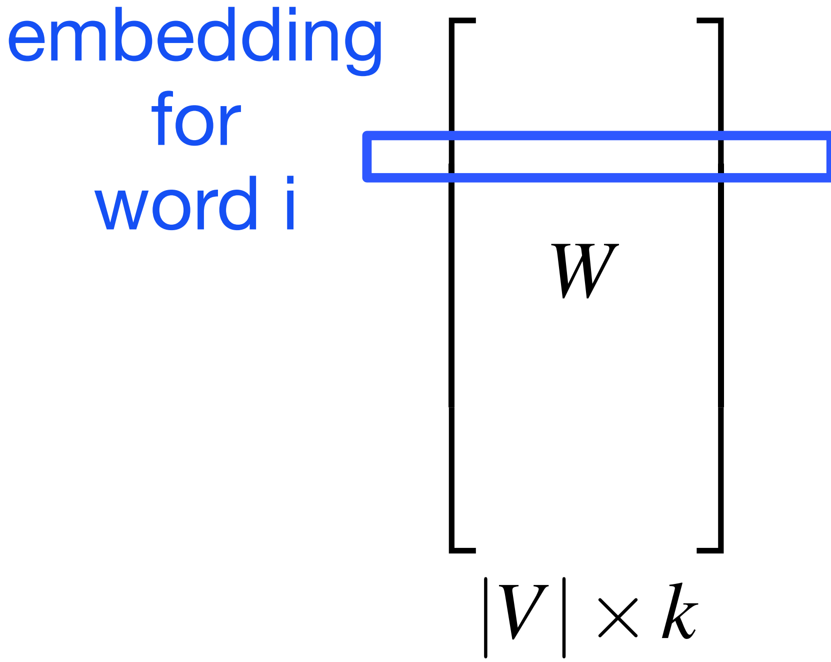
Truncated SVD on term-term matrix

$$\underset{|V| \times |V|}{\begin{bmatrix} \\ \\ X \\ \\ \\ \end{bmatrix}} = \underset{|V| \times k}{\begin{bmatrix} \\ \\ W \\ \\ \\ \end{bmatrix}} \underset{k \times k}{\begin{bmatrix} \sigma_1 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \sigma_k \end{bmatrix}} \underset{k \times |V|}{\begin{bmatrix} & C & \end{bmatrix}}$$

Truncated SVD produces embeddings

- Each row of W matrix is a k-dimensional representation of each word *w*
- K might range from 50 to 1000
- Generally we keep the top k dimensions, but some experiments suggest that getting rid of the top 1 dimension or even the top 50 dimensions is helpful (Lapesa and Evert 2014).



Embeddings versus sparse vectors

- Dense SVD embeddings sometimes work better than sparse PPMI matrices at tasks like word similarity
  - Denoising: low-order dimensions may represent unimportant information
  - Truncation may help the models generalize better to unseen data.
  - Having a smaller number of dimensions may make it easier for classifiers to properly weight the dimensions for the task.
  - Dense models may do better at capturing higher order co-occurrence.