Optimal Strategies for Communication and Remote Estimation With an Energy Harvesting Sensor

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***Abstract*—We consider a remote estimation problem with an energy harvesting sensor and a remote estimator. The sensor observes the state of a discrete-time source which may be a finite state Markov chain or a multidimensional linear Gaussian system. It harvests energy from its environment (say, for example, through a solar cell) and uses this energy for the purpose of communicating with the estimator. Due to randomness of the energy available for communication, the sensor may not be able to communicate all of the time. The sensor may also want to save its energy for future communications. The estimator relies on messages communicated by the sensor to produce real-time estimates of the source state. We consider the problem of finding a communication scheduling strategy for the sensor and an estimation strategy for the estimator that jointly minimizes the expected sum of communication and distortion costs over a finite time horizon. Our goal of joint optimization leads to a decentralized decision-making problem. By viewing the problem from the estimator's perspective, we obtain a dynamic programming characterization for the decentralized decision-making problem that involves optimization over functions. Under some symmetry assumptions on the source statistics and the distortion metric, we show that an optimal communication strategy is described by easily computable thresholds and that the optimal estimate is a simple function of the most recently received sensor observation.**

***Index Terms*—Decentralized decision-making, energy harvesting, Markov decision processes, remote estimation.**

I. Introduction

MANY SYSTEMS for information collection, such as sensor networks and environment monitoring networks, consist of several network nodes that can observe their environment and communicate with other nodes in the network. Such nodes are typically capable of making *decisions*, that is, they can use the information they have collected from the environment or from other nodes to decide when to make the next observation or when to communicate or how to estimate some state variable of the environment. These decisions are usually made in a *decentralized* way, that is, different nodes make decisions based on different information. Further, such decisions must be made under resource constraints. For example, a wireless node in the network must decide when to communicate under the constraint that it has a limited battery life. In this paper, we study one such decentralized decision-making problem under energy constraints.

We consider a setup where one sensor is observing an environmental process of interest which must be communicated to a remote estimator. The estimator needs to produce estimates of the state of the environmental process in real time. We assume that communication from the sensor to the estimator is energy consuming. The sensor is assumed to be harvesting energy from the environment (for example, by using a solar cell). Thus, the amount of energy available at the sensor is a random process. Given the limited and random availability of energy, the sensor has to decide when to communicate with the estimator. Since the sensor may not communicate at all times, the estimator has to decide how to estimate the state of the environmental process. Our goal is to study the effects of randomness of energy supply on the nature of optimal communication scheduling and estimation strategies.

Communication problems with energy harvesting transmitters have been studied recently (see [1], [2] and references therein). In these problems, the goal is to vary the transmission rate/power according to the availability of energy in order to maximize throughput and/or to minimize transmission time. In our problem, on the other hand, the goal is to jointly optimize the communication scheduling and the estimation strategies in order to minimize an accumulated communication and estimation cost. Problems of communication scheduling and estimation with a fixed bound on the number of transmissions, independent identically distributed (i.i.d.) sources, and without energy harvesting have been studied in [3] and [4], where scheduling strategies are restricted to be threshold based. A continuous time version of the problem with the Markov state process and a fixed number of transmissions is studied in [5]. In [6], the authors find an optimal communication schedule assuming a Kalman-like estimator. Remote estimation of a scalar linear Gaussian source with communication costs has been studied in [7], where the authors proved that a threshold-based communication schedule and a Kalman-like estimator are jointly optimal. Our analytical approach borrows extensively from the arguments in [7] and [8]. The latter considered a problem of paging and registration in a cellular network which can be viewed as a remote estimation problem.

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Problems where the estimator decides when to query a sensor or which sensor to query have been studied in [9]–[15]. In these problems, the decision making is centralized. Our problem differs from these setups because the decision to communicate is made by the sensor that has more information than the estimator, and this leads to a decentralized decision-making problem.

In order to appreciate the difficulty of joint optimization of communication and estimation strategies, it is important to recognize the role of signaling in estimation. When the sensor makes a decision on whether to communicate or not based on its observations of the source, then a decision of *not to communicate conveys information to the estimator*. For example, if the estimator knows that the sensor always communicates if the source state is outside an interval $[a, b]$, then not receiving any communication from the sensor reveals to the estimator that the state must have been inside the interval $[a, b]$. Thus, even if the source is Markov, the estimator's estimate may not simply be a function of the most recently received source state since each successive "no communication" has conveyed some information. It is this aspect of the problem that makes the derivation of jointly optimal communication and estimation strategies a difficult problem.

A. Contributions of this Paper

The main contributions of the paper are as follows.

1) We formulate the optimal communication and remote estimation of a discrete Markov source with an energy harvesting sensor as a decentralized decision-making problem, with the sensor and the estimator as the decision makers.
2) We show, under some symmetry conditions, that the globally optimal estimation strategy is to use the most recently received source state as the current estimate. Also, the optimal communication strategy for the sensor is an energy-dependent threshold-based strategy. These thresholds can be obtained by solving a simple finite-state dynamic program with two actions. Our results considerably simplify the offline computation of optimal strategies as well as their online implementation.
3) In Section V, we consider the communication and estimation problem with a multidimensional Gaussian source. Under a suitable symmetry condition, we characterize optimal strategies for this case as well. In particular, we show that for any time instant at which no message is received by the estimator, the optimal estimate is a simple linear update of the previous estimate. Moreover, the optimal strategy for the sensor is an energy-dependent threshold-based strategy. While the result in [7] is only for a scalar Gaussian source without energy harvesting, our approach addresses a multidimensional source and an energy harvesting sensor.
4) Finally, in Section VI, we show that our results apply to several special cases which include the important remote estimation problems where the sensor can afford only a fixed number of transmissions or where the sensor only has a communication cost and no constraint on the number of transmissions. To the best of our knowledge, ours is the first result that identifies globally optimal communication and estimation strategies for a multidimensional Gaussian source in the aforementioned special cases.

Notation

Random variables are denoted by uppercase letters (X, Γ, Π, Θ) and their realizations by the corresponding lowercase letters (x, γ, π, θ). The notation $X_{a:b}$ denotes the vector $(X_a, X_{a+1}, \ldots, X_b)$. Bold capital letters $\mathbf{X}$ represent random vectors, while bold small letters $\mathbf{x}$ represent their realizations. $\mathbb{P}(\cdot)$ is the probability of an event, and $\mathbb{E}(\cdot)$ is the expectation of a random variable. $\mathbb{1}_A(\cdot)$ is the indicator function of a set A. $\mathbb{Z}$ denotes the set of integers, $\mathbb{Z}_+$ denotes the set of positive integers, $\mathbb{R}$ is the set of real numbers, and $\mathbb{R}^n$ is the n- dimensional Euclidean space. $\mathbf{I}$ denotes the identity matrix. For two random variables (or random vectors) X and Y taking values in $\mathcal{X}$ and $\mathcal{Y}$, $\mathbb{P}(X = x|Y)$ denotes the conditional probability of the event $\{X = x\}$ given Y, and $\mathbb{P}(X|Y)$ denotes the conditional probability mass function (PMF) or conditional probability density of X given Y. These conditional probabilities are random variables whose realizations depend on realizations of Y.

B. Organization

In Section II, we formulate our problem for a discrete source. We present a dynamic program for our problem in Section III. This dynamic program involves optimization over a function space. In Section IV, we find optimal strategies under some symmetry assumptions on the source and the distortion function. We consider the multidimensional Gaussian source in Section V. We present some important special cases in Section VI. We conclude in Section VII. We provide some auxiliary results and proofs of key lemmas in Appendices A–E. This work is an extended version of [16].

II. Problem Formulation

A. System Model

Consider a remote estimation problem with a sensor and a remote estimator, as shown in Fig. 1. The sensor observes a discrete-time Markov process $X_t, t = 1, 2, \ldots$. The state space of this source process is a finite interval $\mathcal{X}$ of the set of integers $\mathbb{Z}$. The estimator relies on messages communicated by the sensor to produce its estimates of the process X_t. The sensor harvests energy from its environment (say, for example, through a solar cell) and uses this energy for communicating with the estimator. Let E_t be the energy level at the sensor at the beginning of time t. We assume that the energy level is discrete and takes values in the set $\mathcal{E} = \{0, 1, \ldots, B\}$, where $B \in \mathbb{Z}_+$. In the time period t, the sensor harvests a random amount N_t of energy from its environment, where N_t is a random variable taking values in the set $\mathcal{N} \subset \mathbb{Z}_+$. The sequence $N_t, t = 1, 2, \ldots$, is an i.i.d. process which is independent of the source process $X_t, t = 1, 2, \ldots$.

We assume that a successful transmission from the sensor to the estimator consumes 1 unit of energy. Also, we assume that the sensor consumes no energy if it just observes the source but does not transmit anything to the estimator. At the beginning of the time period t, the sensor makes a decision about whether to transmit its current observation and its current energy level

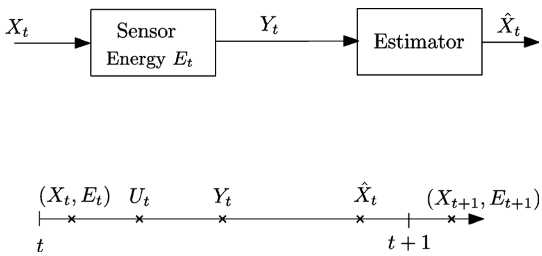

Fig. 1. System model and the time ordering of relevant variables.

to the estimator or not. We denote by $U_t \in \{0, 1\}$ the sensor's decision at time t, where $U_t = 0$ means no transmission and $U_t = 1$ means a decision to transmit. Since the sensor needs at least 1 unit of energy for transmission, we have the constraint that $U_t \leq E_t$. Thus, if $E_t = 0$, then U_t is necessarily 0. The energy level of the sensor at the beginning of the next time step can be written as

$$E_{t+1} = \min\{E_t + N_t - U_t, B\} \tag{1}$$

where B is the maximum number of units of energy that the sensor can store. The estimator receives a message Y_t from the sensor where

$$Y_t = \begin{cases} (X_t, E_t), & \text{if } U_t = 1 \\ \epsilon, & \text{if } U_t = 0 \end{cases} \tag{2}$$

where ϵ denotes that no message was transmitted. The estimator produces an estimate $\hat{X}_t$ at time t depending on the sequence of messages it received so far. The system operates for a finite time horizon T.

B. Decision Strategies

The sensor's decision at time t is chosen as a function of its observation history, the history of energy levels, and the sequence of past messages. We allow randomized strategies for the sensor (see Remark 1). Thus, at time t, the sensor makes the decision $U_t = 1$ with probability p_t where

$$p_t = f_t(X_{1:t}, E_{1:t}, Y_{1:t-1}). \tag{3}$$

The constraint $U_t \leq E_t$ implies that we have the constraint that $p_t = 0$ if $E_t = 0$. The function f_t is called the decision rule of the sensor at time t and the collection of functions $\mathbf{f} = \{f_1, f_2, \ldots, f_T\}$ is called the decision strategy of the sensor.

The estimator produces its estimate as a function of the messages

$$\hat{X}_t = g_t(Y_{1:t}). \tag{4}$$

The function g_t is called the decision rule of the estimator at time t and the collection of functions $\mathbf{g} = \{g_1, g_2, \ldots, g_T\}$ is called the decision strategy of the estimator.

C. Optimization Problem

We have the following optimization problem.

1) Problem 1: For the model described before, given the statistics of the Markov source and the initial energy level E_1, the statistics of amounts of energy harvested at each time, the sensor's energy storage limit B and the time horizon T find decision strategies $\mathbf{f}, \mathbf{g}$ for the sensor and the estimator, respectively, that minimize the following expected cost:

$$J(\mathbf{f}, \mathbf{g}) = \mathbb{E}\{\sum_{t=1}^{T} cU_t + \rho(X_t, \hat{X}_t)\} \tag{5}$$

where $c \geq 0$ is a communication cost and $\rho : \mathcal{X} \times \mathcal{X} \mapsto \mathbb{R}$ is a distortion function.

Remark 1: It can be argued that in the problem just shown, sensor strategies can be assumed to be deterministic (instead of randomized) without compromising optimality. However, our argument for characterizing optimal strategies makes use of the possibility of randomizations by the sensor and, therefore, we allow for randomized strategies for the sensor.

2) Discussion on Our Approach: Our approach for Problem 1 makes extensive use of majorization theory-based arguments used in [7] and [8]. As in [8], we first construct a dynamic program for Problem 1 by reformulating the problem from the estimator's perspective. This dynamic program involves minimization over a function space. Unlike the approach in [8], however, we use majorization theory to argue that the value functions of this dynamic program, under some symmetry conditions, have a special property that is similar to (but not the same as) Schur-concavity [17]. We then use this property to characterize the solution of the dynamic program. This characterization then enables us to find optimal strategies. In Section V, we consider the problem with a multidimensional Gaussian source. We extend our approach for the discrete case to this problem and, under a suitable symmetry condition, we provide optimal strategies for this case as well. While the result in [7] is only for a scalar Gaussian source without energy harvesting, our approach addresses a multidimensional source and an energy harvesting sensor.

III. Preliminary Results

Lemma 1: There is no loss of performance if the sensor is restricted to decision strategies of the form

$$p_t = f_t(X_t, E_t, Y_{1:t-1}). \tag{6}$$

Proof: Fix the estimator's strategy $\mathbf{g}$ to any arbitrary choice. We will argue that for the fixed choice of $\mathbf{g}$, there is an optimal sensor strategy of the form in the lemma. To do so, we can show that with a fixed $\mathbf{g}$, the sensor's optimization problem is a Markov decision problem with $X_t, E_t, Y_{1:t-1}$ as the state of the Markov process. It is straightforward to establish that conditioned on $X_t, E_t, Y_{1:t-1}$ and p_t, the next state $(X_{t+1}, E_{t+1}, Y_{1:t})$ is independent of past source states and energy levels and past choices of transmission probabilities. Further, the expected cost at time t is a function of the state and p_t. Thus, the sensor's optimization problem is a Markov decision problem with $X_t, E_t, Y_{1:t-1}$ as the state of the Markov process. Therefore, using standard results from Markov decision theory [18, Ch. 6], it follows that an optimal sensor strategy is of the form in the lemma. Since the structure of the sensor's optimal strategy is true for an arbitrary choice of $\mathbf{g}$, it is also true for the globally optimal choice of $\mathbf{g}$. This establishes the lemma. ■

In the following analysis, we will consider only the sensor's strategies of the form in Lemma 1. Thus, at the beginning of a time instant t (before the transmission at time t occurs), the sensor only needs to know X_t, E_t, and $Y_{1:t-1}$, whereas the estimator knows $Y_{1:t-1}$. Problem 1—even with the sensor's strategy restricted to the form in Lemma 1—is a decision problem with *nonclassical information structure* [19]. One approach for addressing such problems is to view them from the perspective of a decision maker who only knows the common information among the decision makers [20]. In Problem 1, at the beginning of time t, the information at the sensor is $(X_t, E_t, Y_{1:t-1})$, while the information at the estimator is $Y_{1:t-1}$. Thus, the estimator knows the common information $(Y_{1:t-1})$ between the sensor and the estimator. We will now formulate a decision problem from the estimator's point of view and show that it is equivalent to Problem 1.

A. Equivalent Problem

We formulate a new problem in this section. Consider the model of Section II. At the end of time $t-1$, using the information $Y_{1:t-1}$, the estimator decides an estimate

$$\hat{X}_{t-1} = g_t(Y_{1:t-1}).$$

In addition, at the beginning of time t, the *estimator* decides a *function* $\Gamma_t : \mathcal{X} \times \mathcal{E} \mapsto [0,1]$, using the information $Y_{1:t-1}$. That is

$$\Gamma_t = \ell_t(Y_{1:t-1}). \tag{7}$$

Then, at time t, the sensor evaluates its transmission probability as $p_t = \Gamma_t(X_t, E_t)$. We refer to Γ_t as the *prescription* to the sensor. The sensor simply uses the prescription to evaluate its transmission probability. The estimator can select a prescription from the set $\mathcal{G}$, which is the set of all functions γ from $\mathcal{X} \times \mathcal{E}$ to $[0,1]$ such that $\gamma(x,0) = 0, \forall x \in \mathcal{X}$. It is clear that any prescription in the set $\mathcal{G}$ satisfies the energy constraint of the sensor; that is, it will result in $p_t = 0$ if $E_t = 0$. We call $\boldsymbol{\ell} := \ell_1, \ell_2, \ldots, \ell_T$ the *prescription strategy* of the estimator. Thus, in this formulation, the estimator is the only decision maker. This idea of viewing the communication and estimation problem only from the estimator's perspective has been used in [8] and [21]. A more general treatment of this approach of viewing problems with multiple decision makers from the viewpoint of an agent who knows only the common information can be found in [20]. We can now formulate the following optimization problem for the estimator.

1) Problem 2: For the model described before, given the statistics of the Markov source and the initial energy level E_1, the statistics of amounts of energy harvested at each time, the sensor's energy storage limit B, and the time horizon T, find an estimation strategy $\mathbf{g}$, and a prescription strategy $\boldsymbol{\ell}$ for the estimator that minimizes the following expected cost:

$$\hat{J}(\boldsymbol{\ell}, \mathbf{g}) = \mathbb{E}\{\sum_{t=1}^{T} cU_t + \rho(X_t, \hat{X}_t)\}. \tag{8}$$

Problems 1 and 2 are equivalent in the following sense: Consider any choice of strategies $\mathbf{f}, \mathbf{g}$ in Problem 1, and define a prescription strategy in Problem 2 as

$$\ell_t(Y_{1:t-1}) = f_t(\cdot, \cdot, Y_{1:t-1}).$$

Then, the strategies $\boldsymbol{\ell}, \mathbf{g}$ achieve the same value of the total expected cost in Problem 2 as the strategies $\mathbf{f}, \mathbf{g}$ in Problem 1. Conversely, for any choice of strategies $\boldsymbol{\ell}, \mathbf{g}$ in Problem 2, define a sensor's strategy in Problem 1 as

$$f_t(\cdot, \cdot, Y_{1:t-1}) = \ell_t(Y_{1:t-1}).$$

Then, the strategies $\mathbf{f}, \mathbf{g}$ achieve the same value of the total expected cost in Problem 1 as the strategies $\boldsymbol{\ell}, \mathbf{g}$ in Problem 2.

Because of the above equivalence, we will now focus on the estimator's problem of selecting its optimal estimate and the optimal prescriptions (Problem 2). We will then use the solution of Problem 2 to find optimal strategies in Problem 1.

Recall that E_t is the sensor's energy level at the beginning of time t. For ease of exposition, we define a post-transmission energy level at time t as $E'_t = E_t - U_t$. The estimator's optimization problem can now be described as a partially observable Markov decision problem (POMDP) as follows.

1) State processes: (X_t, E_t) is the pre-transmission state and (X_t, E'_t) is the post-transmission state.
2) Action processes: Γ_t is the pre-transmission action and $\hat{X}_t$ is the post-transmission action.
3) Controlled Markovian evolution of states: The state evolves from (X_t, E_t) to (X_t, E'_t) depending on the realizations of X_t, E_t and the choice of pre-transmission action Γ_t. The post-transmission state is $(X_t, E_t - 1)$ with probability $\Gamma_t(X_t, E_t)$ and (X_t, E_t) with probability $1 - \Gamma_t(X_t, E_t)$. The state then evolves in a Markovian manner from (X_t, E'_t) to (X_{t+1}, E_{t+1}) according to known statistics that depend on the transition probabilities of the Markov source and the statistics of the energy harvested at each time.
4) Observation process: Y_t. The observation is a function of the pre-transmission state and the pre-transmission action. The observation is (X_t, E_t) with probability $\Gamma_t(X_t, E_t)$ and ϵ with probability $1 - \Gamma_t(X_t, E_t)$.
5) Instantaneous costs: The communication cost at each time is a function of the pre-transmission state and the pre-transmission action. The communication cost is c with probability $\Gamma(X_t, E_t)$ and 0 with probability $1 - \Gamma(X_t, E_t)$. The distortion cost at each time step $\rho(X_t, \hat{X}_t)$ is a function of the post-transmission state and the post-transmission action.

The aforementioned equivalence with POMDPs suggests that the estimator's posterior beliefs on the states are its information states [18]. We, therefore, define the following probability mass functions (PMFs):

Definition 1:

1) We define the pre-transmission belief at time t as $\Pi_t := \mathbb{P}(X_t, E_t | Y_{1:t-1})$. Thus, for $(x, e) \in \mathcal{X} \times \mathcal{E}$, we have

$$\Pi_t(x, e) := \mathbb{P}(X_t = x, E_t = e | Y_{1:t-1}).$$

2) We define the post-transmission belief at time t as $\Theta_t := \mathbb{P}(X_t, E'_t|Y_{1:t})$. Thus, for $(x,e) \in \mathcal{X} \times \mathcal{E}$, we have

$$\Theta_t(x,e) := \mathbb{P}(X_t = x, E'_t = e|Y_{1:t}).$$

The following lemma describes the evolution of the beliefs Π_t and Θ_t in time.

Lemma 2: The estimator's beliefs evolve according to the following fixed transformations:

1)

$$\Pi_{t+1}(x,e) = \sum_{\substack{x' \in \mathcal{X},\\ e' \in \mathcal{E}}} \big[\mathbb{P}(X_{t+1} = x|X_t = x') \mathbb{P}(E_{t+1} = e|E'_t = e')\Theta_t(x', e')\big]. \tag{9}$$

We denote this transformation by $\Pi_{t+1} = Q^1_{t+1}(\Theta_t)$.

2)

$$\Theta_t(x,e) = \begin{cases} \delta_{\{x', e'-1\}} & \text{if } Y_t = (x', e') \\ \frac{(1-\Gamma_t(x,e))\Pi_t(x,e)}{\sum\limits_{x',e'} (1-\Gamma_t(x',e'))\Pi_t(x',e')} & \text{if } Y_t = \epsilon \end{cases}, \tag{10}$$

where $\delta_{\{x', e'-1\}}$ is a degenerate distribution at $(x', e'-1)$.

We denote this transformation by $\Theta_t = Q^2_t(\Pi_t, \Gamma_t, Y_t)$. We can now describe the optimal strategies for the estimator.

Theorem 1: Let π, θ be any PMF defined on $\mathcal{X} \times \mathcal{E}$. Define recursively the following functions:

$$\begin{aligned} W_{T+1}(\pi) &:= 0 \\ V_t(\theta) &:= \min_{a \in \mathcal{X}} \mathbb{E}[\rho(X_t, a) + W_{t+1}(\Pi_{t+1})|\Theta_t = \theta] \end{aligned} \tag{11}$$

where $\Pi_{t+1} = Q^1_{t+1}(\Theta_t)$ (see Lemma 2), and

$$W_t(\pi) := \min_{\tilde{\gamma} \in \mathcal{G}} \mathbb{E}[c\mathbb{1}_{\{U_t = 1\}} + V_t(\Theta_t)|\Pi_t = \pi, \Gamma_t = \tilde{\gamma}] \tag{12}$$

where $\Theta_t = Q^2_t(\Pi_t, \Gamma_t, Y_t)$ (see Lemma 2).

For each realization of the post-transmission belief at time t, the minimizer in (11) exists and gives the optimal estimate at time t; for each realization of the pre-transmission belief, the minimizer in (12) exists and gives the optimal prescription at time t.

Proof: The minimizer in (11) exists because $\mathcal{X}$ is finite; the minimizer in (12) exists because the conditional expectation on the right-hand side of (12) is a continuous function of $\tilde{\gamma}$, and $\mathcal{G}$ is a compact set. The optimality of the minimizers follows from standard dynamic programming arguments for POMDPs. ■

The result of Theorem 1 implies that we can solve the estimator's problem of finding optimal estimates and prescriptions by finding the minimizers in (11) and (12) in a backward inductive manner. Recall that the minimization in (12) is over the space of functions in $\mathcal{G}$. This is a difficult minimization problem. In Section IV, we consider a special class of sources and distortion functions that satisfy certain symmetry conditions. We do not solve the dynamic program but instead use it to *characterize optimal strategies of the sensor and the estimator.* Such a characterization provides us with an alternative way of finding optimal strategies of the sensor and the estimator.

IV. Characterizing Optimal Strategies

A. Definitions

Definition 2: A probability distribution μ on $\mathbb{Z}$ is said to be almost symmetric and unimodal (a.s.u.) about a point $a \in \mathbb{Z}$, if for any $k = 0, 1, 2, \ldots,$

$$\mu(a+k) \geq \mu(a-k) \geq \mu(a+k+1). \tag{13}$$

If a distribution μ is a.s.u. about 0 and $\mu(x) = \mu(-x)$, then μ is said to be a.s.u. and even. Similar definitions hold if μ is a sequence, that is, $\mu : \mathbb{Z} \mapsto \mathbb{R}$.

Definition 3: We call a source neat if the following assumptions hold:

1) The *apriori* probability of the initial state of the source $\mathbb{P}(X_1)$ is a.s.u. and even, and has finite support.
2) The time evolution of the source is given as

$$X_{t+1} = X_t + Z_t \tag{14}$$

where $Z_t, t = 1, 2, \ldots, T-1$ are i.i.d random variables with a finite support, a.s.u. and even distribution μ.

Remark 2: Note that the finite support of the distributions of X_1 and Z_t and the finiteness of the time horizon T imply that the state of a neat source always lies within a finite interval in $\mathbb{Z}$. This finite interval is the state space $\mathcal{X}$.

We borrow the following notation and definition from the theory of majorization.

Definition 4: Given $\mu \in \mathbb{R}^n$, let $\mu_\downarrow = (\mu_{[1]}, \mu_{[2]}, \ldots, \mu_{[n]})$ denote the nonincreasing rearrangement of μ with $\mu_{[1]} \geq \mu_{[2]} \geq \cdots \geq \mu_{[n]}$. Given two vectors μ and ν from $\mathbb{R}^n$, we say that ν majorizes μ, denoted by $\mu \prec \nu$ if the following conditions hold:

$$\begin{aligned} \sum_{i=1}^k \mu_{[i]} &\leq \sum_{i=1}^k \nu_{[i]}, \qquad \text{for } 1 \leq k \leq n-1 \\ \sum_{i=1}^n \mu_{[i]} &= \sum_{i=1}^n \nu_{[i]}. \end{aligned}$$

We now define a relation $\mathbf{R}$ among possible information states and a property $\mathbf{R}$ of real-valued functions of information states.

Definition 5 (Binary Relation $\mathbf{R}$): Let θ and $\tilde{\theta}$ be two distributions on $\mathcal{X} \times \mathcal{E}$. We say $\theta \mathbf{R} \tilde{\theta}$ if, and only if (iff):

1) For each $e \in \mathcal{E}$, $\theta(\cdot, e) \prec \tilde{\theta}(\cdot, e)$
2) For all $e \in \mathcal{E}$, $\tilde{\theta}(\cdot, e)$ is a.s.u. about the same point $x \in \mathcal{X}$.

From the definition before, it is straightforward to see that if $\theta \mathbf{R} \tilde{\theta}$, then θ and $\tilde{\theta}$ have the same marginal distribution on $\mathcal{E}$ (since $\theta(\cdot, e) \prec \tilde{\theta}(\cdot, e)$ implies that $\sum_{x \in \mathcal{X}} \theta(x,e) = \sum_{x \in \mathcal{X}} \tilde{\theta}(x,e)$). Thus, θ and $\tilde{\theta}$ imply the same distribution on the energy of the sensor. Moreover, for each e, $\tilde{\theta}$ is more "symmetrically concentrated" about the same point $x \in \mathcal{X}$ which means that the marginal of $\tilde{\theta}$ on $\mathcal{X}$ is concentrated around x. Thus, intuitively, $\tilde{\theta}$ should be a better distribution for estimating the source state than θ.

Definition 6 (Property $\mathbf{R}$*):* Let V be a function that maps distributions on $\mathcal{X} \times \mathcal{E}$ to the set of real numbers $\mathbb{R}$. We say that V satisfies Property $\mathbf{R}$ iff for any two distributions θ and $\tilde{\theta}$

$$\theta \mathbf{R} \tilde{\theta} \Longrightarrow V(\theta) \geq V(\tilde{\theta}).$$

B. Analysis

In this section, we will consider Problem 1 under the assumptions that:

A1) the source is neat (See Definition 3);

A2) the distortion function $\rho(x,a)$ is either $\rho(x,a) = \mathbb{1}_{\{x \neq a\}}$ or $\rho(x,a) = |x-a|^k$, for some $k > 0$.

Our goal is to show that Assumptions A1 and A2, combined with the result of Theorem 1, allow for a much simpler characterization of optimal strategies for the sensor and the estimator than the one provided by the dynamic program of Theorem 1. We defer the discussion of practical justifications of these assumptions to Section IV-E.

Lemma 3: Let θ be a distribution on $\mathcal{X} \times \mathcal{E}$ such that for all $e \in \mathcal{E}$, $\theta(\cdot, e)$ is a.s.u. about the same point $x' \in \mathcal{X}$. Then, the minimum in (11) is achieved at x'.

Proof: Using Lemma 2, the expression in (11) can be written as

$$V_t(\theta) := W_{t+1}(Q^1_{t+1}(\theta)) + \min_{a \in \mathcal{X}} \mathbb{E}[\rho(X_t, a) | \Theta_t = \theta].$$

Thus, the minimum is achieved at the point that minimizes the expected distortion function $\rho(X_t, a)$ given that X_t has the distribution θ. The a.s.u. assumption of all $\theta(\cdot, e)$ about x', and the nature of distortion functions given in Assumption A2 imply that x' is the minimizer. ■

We now want to characterize the minimizing $\tilde{\gamma}$ in (12). Toward that end, we start with the following claim.

1) Claim 1: The value functions $W_t, t = 1, 2, \ldots T+1$, and $V_t, t = 1, 2, \ldots, T$, satisfy Property $\mathbf{R}$.

Proof: See Appendix C. ■

Recall that (12) in the dynamic program for the estimator defines W_t as

$$W_t(\pi) := \min_{\tilde{\gamma}} \mathbb{E}[c\mathbb{1}_{\{U_t=1\}} + V_t(\Theta_t) | \Pi_t = \pi, \gamma_t = \tilde{\gamma}]. \quad (15)$$

The following lemma is a consequence of Claim 1.

Lemma 4: Let π be a distribution on $\mathcal{X} \times \mathcal{E}$ such that $\pi(\cdot, e)$ is a.s.u. about the same point $a \in \mathcal{X}$ for all $e \in \mathcal{E}$. Then, the minimum in the definition of $W_t(\pi)$ is achieved by a prescription $\tilde{\gamma} : \mathcal{X} \times \mathcal{E} \mapsto [0,1]$ of the form

$$\tilde{\gamma}(x,e) = \begin{cases} 1, & \text{if } |x-a| > n(e,\pi) \\ 0, & \text{if } |x-a| < n(e,\pi) \\ \alpha(e,\pi), & \text{if } x = a + n(e,\pi) \\ \beta(e,\pi), & \text{if } x = a - n(e,\pi) \end{cases} \quad (16)$$

where for each $e \in \mathcal{E}$, $\alpha(e,\pi), \beta(e,\pi) \in [0,1], \alpha(e,\pi) \leq \beta(e,\pi)$ and $n(e,\pi)$ is a non-negative integer.

Proof: See Appendix D. ■

Lemmas 3 and 4 can be used to establish a threshold structure for optimal prescriptions and a simple recursive optimal estimator for Problem 2. At time $t = 1$, by assumption A1, Π_1 is such that $\Pi_1(\cdot, e)$ is a.s.u. about 0 for all $e \in \mathcal{E}$. Hence, by Lemma 4, an optimal prescription at time $t = 1$ has the threshold structure of (16). If a transmission occurs at time $t = 1$, then the resulting post-transmission belief Θ_1 is a delta-function and consequently $\Theta_1(\cdot, e), e \in \mathcal{E}$ are a.s.u. about the same point. If a transmission does not occur at time $t = 1$, then, using Lemma 2 and the threshold nature of the prescription, it can be shown that the resulting post-transmission belief is such that $\Theta_1(\cdot, e), e \in \mathcal{E}$ are a.s.u. about 0. Thus, it follows that Θ_1 will always be such that all $\Theta_1(\cdot, e), e \in \mathcal{E}$ are a.s.u. about the same point and because of Lemma 3, this point will be the optimal estimate. Using Lemma 2 and the a.s.u. property of $\Theta_1(\cdot, e)$, it follows that the next pre-transmission belief Π_2 will always be such that $\Pi_2(\cdot, e), e \in \mathcal{E}$ are a.s.u. about the same point (by arguments similar to those in Lemma 12 in Appendix C). Hence, by Lemma 4, an optimal prescription at time $t = 2$ has the threshold structure of (16). Proceeding sequentially as before establishes the following result.

Theorem 2: In Problem 2, under Assumptions A1 and A2, there is an optimal prescription and estimation strategy such that:

1) The optimal estimate is given as

$$\hat{X}_t = \begin{cases} \hat{X}_{t-1} & \text{if } y_t = \epsilon \\ x & \text{if } y_t = (x,e) \end{cases} \quad (17)$$

where $\hat{X}_0 := 0$.

2) The pre-transmission belief at any time t, $\Pi_t(\cdot, e)$, is a.s.u. about $\hat{X}_{t-1}$, for all $e \in \mathcal{E}$.

3) The prescription at any time has the threshold structure of Lemma 4.

As argued in Section III-A, Problem 2 and Problem 1 are equivalent. Hence, the result of Theorem 2 implies the following result for Problem 1.

Theorem 3: In Problem 1 under assumptions A1 and A2, there exist optimal decision strategies $\mathbf{f}, \mathbf{g}$ for the sensor and the estimator given as

$$g^*_t(y_{1:t}) = \begin{cases} a, & \text{if } y_t = \epsilon \\ x, & \text{if } y_t = (x,e) \end{cases} \quad (18)$$

$$f^*_t(x, e, y_{1:t-1}) = \begin{cases} 1, & \text{if } |x-a| > n_t(e,\pi_t) \\ 0, & \text{if } |x-a| < n_t(e,\pi_t) \\ \alpha_t(e,\pi_t), & \text{if } x = a + n_t(e,\pi_t) \\ \beta_t(e,\pi_t), & \text{if } x = a - n_t(e,\pi_t) \end{cases} \quad (19)$$

where $a = 0$ for $t = 1$, $a = g^*_{t-1}(y_{1:t-1})$ for $t > 1$, and $\pi_t = \mathbb{P}(X_t, E_t | y_{1:t-1})$.

Theorem 3 can be interpreted as follows: it says that the optimal estimate is the most recently received value of the source (the optimal estimate is 0 if no source value has been received). Further, there is a threshold rule at the sensor. The sensor transmits with probability 1 if the difference between the current source value and the most recently transmitted value exceeds a threshold that depends on the sensor's current energy level and the estimator's pre-transmission belief; it does not transmit if the difference between the current source value and the most recently transmitted value is strictly below the threshold.

C. Optimal Thresholds

Theorem 3 gives a complete characterization of the optimal estimation strategy, but it only provides a *structural form* of the optimal strategy for the sensor. Our goal now is to find the exact characterization of the thresholds and the randomization probabilities in the structure of the optimal strategy of the sensor. We denote the optimal estimation strategy of Theorem 3 by $\mathbf{g}^*$ and the class of sensor strategies that satisfy the threshold structure of Theorem 3 as $\mathcal{F}$. We know that the global minimum of the expected cost is $J(\mathbf{f}, \mathbf{g}^*)$, for some $\mathbf{f} \in \mathcal{F}$. Any sensor strategy $\mathbf{f}'$ that achieves a cost $J(\mathbf{f}', \mathbf{g}^*) \leq J(\mathbf{f}, \mathbf{g}^*)$, for all $\mathbf{f} \in \mathcal{F}$ must be a globally optimum sensor strategy.

Given that the strategy for the estimator is fixed to $\mathbf{g}^*$, we will address the question of finding the best sensor strategy among all possible strategies (including those not in $\mathcal{F}$). The answer to this question can be found by a standard dynamic program (see Theorem 4 below). We denote by $\mathbf{f}^*$ the strategy specified by the dynamic program. We have that $J(\mathbf{f}, \mathbf{g}^*) \geq J(\mathbf{f}^*, \mathbf{g}^*)$, for all $\mathbf{f}$ (including those not in $\mathcal{F}$). Thus, $\mathbf{f}^*$ is a globally optimal sensor strategy. Further, $\mathbf{f}^*$ is in the set $\mathcal{F}$. Thus, the dynamic program of Theorem 4 provides a way of computing the optimal thresholds of Theorem 3.

Theorem 4: Given that the strategy for the estimator is fixed to $\mathbf{g}^*$ as defined in Theorem 3, the best sensor strategy (from the class of all possible strategies) is of the form $U_t = f_t^*(D_t, E_t)$, where $D_t := X_t - g_{t-1}^*(Y_{1:t-1})$. Further, this strategy is described by the following dynamic program:

$$J_{T+1}(\cdot, \cdot) := 0,$$

and for positive energy levels $e > 0$

$$\begin{aligned} J_t(d, e) := \min\{c + \mathbb{E}[J_{t+1}(Z_t, \min(e-1+N_t, B))], \tilde{\rho}(d) \\ + \mathbb{E}[J_{t+1}(d + Z_t, \min(e + N_t, B))]\} \end{aligned} \quad (20)$$

where $\tilde{\rho}(d)$ is $\mathbb{1}_{\{d \neq 0\}}$ if the distortion metric is $\rho(x, a) = \mathbb{1}_{\{x \neq a\}}$ and $\tilde{\rho}(d)$ is $|d|^k$ if the distortion metric is $\rho(x, a) = |x - a|^k$. For $e > 0$, the optimal action for a realization (d, e) of (D_t, E_t) is $U_t = 1$ iff $J_t(d, e)$ is equal to the first term on the right-hand side of (20). If $e = 0$, $J_t(\cdot, 0)$ is the second term on the right-hand side of (20) evaluated at $e = 0$ and the optimal action is $U_t = 0$.

Proof: Once the estimator's strategy is fixed to $\mathbf{g}^*$, the sensor's optimization problem is a standard Markov decision problem (MDP) with $D_t = X_t - g_{t-1}^*(Y_{1:t-1})$ and E_t as the two-dimensional state. The result of the lemma is the standard dynamic program for MDPs. ■

Consider the definition of $J_t(d, e)$ in (20). For a fixed $e > 0$, the first term on the right-hand side of (20) does not depend on d, while it can be easily shown that the second term is nondecreasing in d. These observations imply that for each $e > 0$, there is a threshold value of d below which $U_t = 0$ and above which $U_t = 1$ in the optimal strategy. Thus, the $\mathbf{f}^*$ of Theorem 4 satisfies the threshold structure of Theorem 3. Comparing the strategy $\mathbf{f}^*$ specified by Theorem 4 and the form of sensor strategies in Theorem 3, we see that:

1) the thresholds in $\mathbf{f}^*$ depend *only on the current energy level of the sensor* and not on the pre-transmission belief π_t, whereas the thresholds in Theorem 3 could depend on the energy level and π_t.
2) the strategy $\mathbf{f}^*$ is purely deterministic, whereas Theorem 3 allowed for possible randomizations at two points.

D. Discussion of the Results

The problem of finding globally optimal strategies for the sensor and the estimator (Problem 1) is a difficult optimization problem. It is clear that the number of possible strategy choices is prohibitively large. For example, the number of deterministic strategies for the sensor is $\prod_{t=1}^{T} 2^{|\mathcal{X}||\mathcal{E}||\mathcal{Y}|^{t-1}}$, where $\mathcal{Y}$ is the set of all possible values of Y_t; the number of randomized strategies for the sensor is clearly infinite. The result of Theorem 1 provides a backward-inductive solution of the problem. However, this solution still involves optimization over an infinite space of functions at each time step. By making use of the structure provided by Assumptions A1 and A2, the results of Theorems 3 and 4 lead to considerable simplification of the problem. Recall that Theorem 3 completely specifies the globally optimal strategy for the estimator. Moreover, the dynamic program of Theorem 4 is a simple finite-state dynamic program with a state space no larger than $2|\mathcal{X}| \times |\mathcal{E}|$ and an action space of size 2 (since there are only two terms in the minimization in (20)). Numerically solving this dynamic program requires solving, at most, $2T|\mathcal{X}||\mathcal{E}|$ optimization problems, each over a set of size 2. The threshold structure further simplifies the dynamic program because if $U_t = 1$ is optimal for some value of (d, e), then $U_t = 1$ is optimal for all (d', e) with $d' > d$. Thus, our results significantly reduce the computational cost of solving Problem 1. Moreover, the threshold nature of the optimal sensor strategy and the simple form of the optimal estimation strategy make them easily implementable.

It is interesting to observe that a reasonable (but potentially suboptimal) approach for Problem 1 would have been to *assume* that the estimator simply uses the most recently received message as its current estimate. Our result shows that this reasonable strategy is indeed globally optimal. Assuming this strategy for the estimator, the problem of finding best strategy for the sensor is a centralized decision-making problem. The result of Theorem 4 is essentially a solution for this centralized problem.

E. Role of Assumptions A1 and A2

Assumptions A1 and A2 were critical for obtaining the results of Theorems 3 and 4. These assumptions provided the necessary structure to the dynamic program of Theorem 1 that allowed us to prove the optimality of the strategies of Theorem 3. Since most commonly used distortion functions, such as absolute error and squared error, are of the form in Assumption A2, the main limitation of the aforementioned analysis is due to Assumption A1. This assumption can be justified if the source statistics are such that small changes in the source are more likely than larger changes. Sources that can be modeled as symmetric random walks where the probability of a jump decreases with jump size would fit this description. Sources where the state is perturbed by a zero-mean Gaussian noise (considered in Section V) would also satisfy Assumption A1.

Without Assumptions A1 and A2, it is unclear if a solution of the dynamic program of Theorem 1 can be easily obtained. In the absence of these assumptions, one can still use the estimator's strategy of Theorem 3 as a simple, intuitive heuristic choice for the estimator's strategy and optimize for the sensor strategy using a dynamic program similar to that in Theorem 4.

V. Multidimensional Gaussian Source

In this section, we consider a variant of Problem 1, with a multidimensional Gaussian source. The state of the source evolves according to the equation

$$\mathbf{X}_{t+1} = \lambda \mathbf{A} \mathbf{X}_t + \mathbf{Z}_t \tag{21}$$

where $\mathbf{X}_t = (X_t^1, X_t^2, \ldots, X_t^n)$, $\mathbf{Z}_t = (Z_t^1, Z_t^2, \ldots, Z_t^n)$ are random vectors taking values in $\mathbb{R}^n$, $\lambda > 0$ is a real number and $\mathbf{A}$ is an orthogonal matrix (that is, the transpose of $\mathbf{A}$ is the inverse of $\mathbf{A}$ and, more important for our purpose, $\mathbf{A}$ preserves norms). The initial state $\mathbf{X}_1$ has a zero-mean Gaussian distribution with covariance matrix $s_1\mathbf{I}$, and $\mathbf{Z}_1, \mathbf{Z}_2, \ldots, \mathbf{Z}_{T-1}$ are i.i.d. random vectors with a zero-mean Gaussian distribution and covariance matrix $s_2\mathbf{I}$. The energy dynamics for the sensor are the same as in Problem 1.

At the beginning of the time period t, the sensor makes a decision about whether to transmit its current observation vector and its current energy level to the estimator or not. The estimator receives a message $\mathbf{Y}_t$ from the sensor where $\mathbf{Y}_t = (\mathbf{X}_t, E_t)$, if $U_t = 1$ and $\mathbf{Y}_t = \epsilon$ otherwise. The estimator produces an estimate $\hat{\mathbf{X}}_t = (\hat{X}_t^1, \ldots, \hat{X}_t^n)$ at time t depending on the sequence of messages it received so far. The system operates for a finite time horizon T.

The sensor and estimator make their decisions according to deterministic strategies $\mathbf{f}$ and $\mathbf{g}$ of the form $U_t = f_t(\mathbf{X}_t, E_t, \mathbf{Y}_{1:t-1})$ and $\hat{\mathbf{X}}_t = g_t(\mathbf{Y}_{1:t})$. We assume that for any time and any realization of past messages, the set of source and energy states for which transmission occurs is an open or a closed subset of $\mathbb{R}^n \times \mathcal{E}$. We have the following optimization problem.

1) Problem 3: For the model described before, given the statistics of the Markov source and the initial energy level E_1, the statistics of amounts of energy harvested at each time, the sensor's energy storage limit B and the time horizon T find decision strategies $\mathbf{f}, \mathbf{g}$ for the sensor and the estimator that minimize the following expected cost:

$$J(\mathbf{f}, \mathbf{g}) = \mathbb{E}\{\sum_{t=1}^{T} cU_t + \left\|\mathbf{X}_t - \hat{\mathbf{X}}_t\right\|^2\} \tag{22}$$

where $c \geq 0$ is a communication cost and $\|\cdot\|$ is the Euclidean norm.

Remark 3: Note that we have assumed here that the sensor is using a deterministic strategy that employs only the current source and energy state and the past transmissions to make the decision at time t. Using arguments analogous to those used in proving Lemma 1, it can be shown that this restriction leads to no loss of optimality. While randomization was used in our proofs for the problem with discrete source (Problem 1), it is not needed when the source state space is continuous.

Following the arguments of Sections III and IV, we can view the problem from the estimator's perspective, who, at each time t, selects a prescription for the sensor before the transmission and then an estimate on the source after the transmission. Since we have deterministic policies, the prescriptions are binary-valued functions. We can define at each time t, the estimator's pre-transmission (post-transmission) beliefs as conditional probability densities on $\mathbb{R}^n \times \mathcal{E}$ given the transmissions $\mathbf{Y}_{1:t-1}(\mathbf{Y}_{1:t})$.

Lemma 5: The estimator's beliefs evolve according to the following fixed transformations:

1) $\Pi_{t+1}(\mathbf{x}, e) = \lambda^{-n} \int_{\mathbf{x}' \in \mathbb{R}^n} \sum_{e' \in \mathcal{E}} [\mathbb{P}(E_{t+1} = e | E'_t = e') \mu(\mathbf{x} - \mathbf{x}') \Theta_t(\lambda^{-1}\mathbf{A}^{-1}\mathbf{x}', e')]$ where μ is the probability density function of $\mathbf{Z}_t$; we denote this transformation by $\Pi_{t+1} = Q_{t+1}^1(\Theta_t)$;

2)

$$\Theta_t(\mathbf{x}, e) = \begin{cases} \delta_{\{\mathbf{x}', e'-1\}}, & \text{if } \mathbf{Y}_t = (\mathbf{x}', e') \\ \frac{(1-\Gamma_t(\mathbf{x}, e))\mathbf{\Pi_t}(\mathbf{x}, e)}{\int_{\mathbf{x}'} \sum_{e'} (1-\Gamma_t(\mathbf{x}', e'))\Pi_t(\mathbf{x}', e')}, & \text{if } \mathbf{Y}_t = \epsilon \end{cases} \tag{23}$$

where $\delta_{\{\mathbf{x}', e'-1\}}$ is a degenerate distribution at $(\mathbf{x}', e'-1)$; we denote this transformation by $\Theta_t = Q_t^2(\Pi_t, \Gamma_t, \mathbf{Y}_t)$.

Further, we can establish the following analogue of Theorem 1 by using dynamic programming arguments [22, Ch. 8 and 10].

Theorem 5: Let π, θ be any pair pre-transmission and post-transmission beliefs. Define recursively the following functions:

$$W_{T+1}(\pi) := 0$$
$$V_t(\theta) := \inf_{\mathbf{a} \in \mathbb{R}^n} \mathbb{E}[\|\mathbf{X}_t - \mathbf{a}\|^2 + W_{t+1}(\Pi_{t+1}) | \Theta_t = \theta] \tag{24}$$

where $\Pi_{t+1} = Q_{t+1}^1(\Theta_t)$ (see Lemma 5), and

$$W_t(\pi) := \inf_{\tilde{\gamma} \in \mathcal{G}} \mathbb{E}[c\mathbb{1}_{\{U_t=1\}} + V_t(\Theta_t) | \Pi_t = \pi, \Gamma_t = \tilde{\gamma}] \tag{25}$$

where $\mathcal{G}$ is the set of all functions γ from $\mathbb{R}^n \times \mathcal{E}$ to $\{0, 1\}$ such that $\gamma^{-1}(\{0\}) = \mathbb{R}^n \times \{0\} \cup (\cup_{e=1}^{B} \mathcal{I}_e \times \{e\})$, where $\mathcal{I}_e$ is an open or closed subset of $\mathbb{R}^n$. Then, $V_1(\pi_1)$, where π_1 is the density of $\mathbf{X}_1$, is a lower bound on the cost of any strategy. A strategy that at each time and for each realization of pre-transmission and post-transmission beliefs selects a prescription, and an estimate that achieves the infima in (24) and (25) is optimal. Further, even if the infimum is not always achieved, it is possible to find a strategy with performance arbitrarily close to the lower bound.

As in Theorem 1, the dynamic program of Theorem 5 involves optimization over functions. Intuitively, the source and distortion in Problem 3 have the same structures as required by Assumptions A1 and A2 in Section IV. However, because the source is now continuous valued, the analysis of Section IV needs to be modified in order to completely characterize the solution of the dynamic program in Theorem 5. The following theorem is the analogue of Theorem 3 for Problem 3.

Theorem 6: In Problem 3, it is without loss of optimality[1] to restrict to strategies $\mathbf{f}^*, \mathbf{g}^*$ that are given as

$$g_t^*(\mathbf{y}_{1:t}) = \begin{cases} \lambda\mathbf{A}\mathbf{a}, & \text{if } \mathbf{y}_t = \epsilon \\ \mathbf{x}, & \text{if } \mathbf{y}_t = (\mathbf{x}, e) \end{cases} \quad (26)$$

$$f_t^*(\mathbf{x}, e, \mathbf{y}_{1:t-1}) = \begin{cases} 1, & \text{if } \|\mathbf{x} - \lambda\mathbf{A}\mathbf{a}\| \geq r_t(e, \pi_t) \\ 0, & \text{if } \|\mathbf{x} - \lambda\mathbf{A}\mathbf{a}\| < r_t(e, \pi_t) \end{cases} \quad (27)$$

where $\mathbf{a} = \mathbf{0}$ for $t = 1$, $\mathbf{a} = g_{t-1}^*(\mathbf{y}_{1:t-1})$ for $t > 1$, $\pi_t = \mathbb{P}(X_t, E_t|\mathbf{y}_{1:t-1})$, and $r_t(e, \pi_t) \geq 0$.

Proof: See Appendix E. ■

Further, the optimal values of thresholds can be obtained by the following dynamic program which is similar to the dynamic program in Theorem 4.

Theorem 7: Given that the strategy for the estimator is fixed to $\mathbf{g}^*$, the best sensor strategy (from the class of all possible strategies) is of the form $U_t = f_t^*(\mathbf{D}_t, E_t)$, where $\mathbf{D}_t := \mathbf{X}_t - \lambda\mathbf{A}g_{t-1}^*(Y_{1:t-1})$. Further, this strategy is described by the following dynamic program:

$$J_{T+1}(\cdot,\cdot) := 0$$

and for positive energy levels $e > 0$

$$J_t(\mathbf{d}, e) := \min\{c + \mathbb{E}[J_{t+1}(\mathbf{Z}_t, \min(e - 1 + N_t, B))], \|\mathbf{d}\|^2 + \mathbb{E}[J_{t+1}(\lambda\mathbf{A}\mathbf{d} + \mathbf{Z}_t, \min(e + N_t, B))]\}. \quad (28)$$

For $e > 0$, the optimal action for a realization $(\mathbf{d}, e)$ of $(\mathbf{D}_t, E_t)$ is $U_t = 1$ iff $J_t(\mathbf{d}, e)$ is equal to the first term on the right-hand side of (28). If $e = 0$, $J_t(\cdot, 0)$ is the second term on the right-hand side of (28) evaluated at $e = 0$ and the optimal action is $U_t = 0$.

VI. Special Cases

By making suitable assumptions on the source, the energy storage limit B of the sensor, and the statistics of initial energy level, and the energy harvested at each time, we can derive the following special cases of Problem 1 in Section II and Problem 3 in Section V.

1) Fixed Number of Transmissions: Assume that the initial energy level is $E_1 = K$ $(K \leq B)$ with probability 1 and that the energy harvested at any time is $N_t = 0$ with probability 1. Under these assumptions, Problem 1 can be interpreted as capturing the scenario when the sensor can afford, at most, K transmissions during the time horizon with no possibility of energy harvesting. This is similar to the model in [3].

2) No Energy Constraint: Assume that the storage limit is $B = 1$, and that the initial energy level and the energy harvested at each time are 1 with probability 1. Then, it follows that at any time t, $E_t = 1$ with probability 1. Thus, the sensor is *always guaranteed to have energy to communicate*. Under these assumptions, Problem 1 can be interpreted as capturing the scenario when the sensor has no energy constraints (it still has energy costs because of the term cU_t in the objective). This is similar to the model in [7].

3) I.I.D. Source: The analyses of Sections IV and V can be used if the source evolution is given as $X_{t+1} = Z_t$, where Z_t, $t = 1, 2, \ldots, T-1$, are the i.i.d. noise variables. For i.i.d. sources, the optimal estimate is the mean value of the source in case of no transmission. Also, the dynamic program of Theorem 4 can be used for finite-valued i.i.d. sources by replacing D_t with X_t and changing (20) to $J_t(d, e) := \min\{c + \mathbb{E}[J_{t+1}(X_{t+1}, \min(e - 1 + N_t, B))], \tilde{\rho}(d) + \mathbb{E}[J_{t+1}(X_{t+1}, \min(e + N_t, B))]\}$. A similar dynamic program can be written for the Gaussian source.

[1]That is, there is a strategy of the form in the theorem whose performance is arbitrarily close to the lower bound $V_1(\pi_1)$

VII. Conclusion

We considered the problem of finding globally optimal communication scheduling and estimation strategies in a remote estimation problem with an energy harvesting sensor and a finite-valued or a multidimensional Gaussian source. We established the global optimality of a simple energy-dependent threshold-based communication strategy and a simple estimation strategy. Our results considerably simplify the offline computation of optimal strategies as well as their online implementation.

Our approach started with providing a POMDP-based dynamic program for the decentralized decision-making problem. Dynamic programming solutions often rely on finding a key property of value functions (such as concavity or quadraticness) and exploiting this property to characterize the solution. In dynamic programs that arise from decentralized problems, however, value functions involve minimization over functions [20] and, hence, the usual properties of value functions are either not applicable or not useful. In such problems, there is a need to find the right property of value functions that can be used to characterize optimal solutions. We believe that this work demonstrates that in some problems where majorization-based properties related to Schur concavity may be the right value function property to exploit.

Appendix A
Lemmas From [8], Section VI

A) For the Discrete Source:

Lemma 6: If μ is a.s.u. and even and ξ is a.s.u. about a, then the convolution $\xi * \mu$ is a.s.u about a.

Lemma 7: If μ is a.s.u. and even, $\tilde{\xi}$ is a.s.u. and $\xi \prec \tilde{\xi}$, then $\xi * \mu \prec \tilde{\xi} * \mu$.

B) For the Multidimensional Gaussian Source:

Lemma 8: If μ and ν are two non-negative integrable functions on $\mathbb{R}^n$ and $\mu \prec \nu$, then $\int_{\mathbb{R}^n} \mu^\sigma(\mathbf{x})h(\mathbf{x})d\mathbf{x} \leq \int_{\mathbb{R}^n} \nu^\sigma(\mathbf{x})h(\mathbf{x})d\mathbf{x}$ for any symmetric unimodal function h.

Lemma 9: If μ and ν are two non-negative integrable functions on $\mathbb{R}^n$, then $\int_{\mathbb{R}^n} \mu(\mathbf{x})\nu(\mathbf{x})d\mathbf{x} \leq \int_{\mathbb{R}^n} \mu^\sigma(\mathbf{x})\nu^\sigma(\mathbf{x})d\mathbf{x}$. (This lemma is known as the Hardy Littlewood Inequality [23].)

Lemma 10: If μ is symmetric unimodal about $\mathbf{0}$, $\tilde{\xi}$ is symmetric unimodal, and $\xi \prec \tilde{\xi}$, then $\xi * \mu \prec \tilde{\xi} * \mu$.

Appendix B
Other Preliminary Lemmas

Lemma 11: Let h_1 be a non-negative, integrable function from $\mathbb{R}^n$ to $\mathbb{R}$ such that h_1 is symmetric unimodal about a point $\mathbf{a}$. Let h_2 be a probability density function (pdf) on $\mathbb{R}^n$ which is symmetric unimodal about $\mathbf{0}$. Then, $h_1 * h_2$ is symmetric unimodal about $\mathbf{a}$.

Proof: For ease of exposition, we will assume that h_1 and h_2 are symmetric unimodal about $\mathbf{0}$. If h_1 is symmetric unimodal about a nonzero point, then to obtain $h_1 * h_2$, we can first do a translation of h_1 so that it is symmetric unimodal about $\mathbf{0}$ to carry out the convolution and translate the result back.

Consider two points $\mathbf{x} \neq \mathbf{y}$ such that $\|\mathbf{x}\| = \|\mathbf{y}\|$. Then, we can always find an orthogonal matrix such that $\mathbf{y} = Q\mathbf{x}$. Then

$$(h_1 * h_2)(\mathbf{y}) = (h_1 * h_2)(Q\mathbf{x}) = \int h_1(\mathbf{z}) h_2(Q\mathbf{x} - \mathbf{z}) d\mathbf{z}. \quad (29)$$

Carrying out a change of variables so that $\mathbf{z} = Q\mathbf{z}'$, the above integral becomes

$$\begin{aligned}&\int h_1(Q\mathbf{z}') h_2(Q\mathbf{x} - Q\mathbf{z}') d\mathbf{z}' \\ &= \int h_1(Q\mathbf{z}') h_2(Q(\mathbf{x} - \mathbf{z}')) d\mathbf{z}' \\ &= \int h_1(\mathbf{z}') h_2(\mathbf{x} - \mathbf{z}') d\mathbf{z}' = (h_1 * h_2)(\mathbf{x}) \quad (30)\end{aligned}$$

where we used the symmetric nature of h_1 and h_2 and the fact that the orthogonal matrix preserves norm. Thus, any two points with the same norm have the same value of $h_1 * h_2$. This establishes the symmetry of $h_1 * h_2$. Next, we look at unimodality. We follow an argument similar to the one used in [24]. Because of symmetry, it suffices to show that $(h_1 * h_2)(x_1, 0, 0, \ldots, 0)$ is nonincreasing for $x_1 \in [0, \infty)$. (Here, $(x_1, 0, \ldots, 0)$ is the n dimensional vector with all but the first coordinates as 0.) Note that

$$\begin{aligned}&(h_1 * h_2)((x_1, 0, \ldots, 0)) \\ &= \int h_2(\mathbf{z}) h_1((x_1, 0, \ldots, 0) - \mathbf{z}) d\mathbf{z} \\ &= \mathbb{E}[h_1((x_1, 0, \ldots, 0) - \mathbf{Z})] \quad (31)\end{aligned}$$

where $\mathbf{Z}$ is a random vector with pdf h_2. Define a new random variable $Y_{x_1} := h_1((x_1, 0, \ldots, 0) - \mathbf{Z})$. Then

$$\mathbb{E}[h_1((x_1, 0, \ldots, 0) - \mathbf{Z})] = \mathbb{E}[Y_{x_1}] = \int_0^\infty \mathbb{P}(Y_{x_1} > t) dt. \quad (32)$$

We now prove that for any given $t \geq 0$, $\mathbb{P}(Y_{x_1} > t)$ is nonincreasing in x_1. This would imply that the integral in (32) and, hence, $(h_1 * h_2)((x_1, 0, \ldots, 0))$ is nonincreasing in x_1.

The symmetric unimodal nature of h_1 implies that $Y_{x_1} > t$ if and only if $\|(x_1, 0, \ldots, 0) - \mathbf{Z}\| < r$ (or $\|(x_1, 0, \ldots, 0) - \mathbf{Z}\| \leq r$) for some constant r whose value varies with t. Thus

$$\begin{aligned}\mathbb{P}(Y_{x_1} > t) &= \mathbb{P}(\|(x_1, 0, \ldots, 0) - \mathbf{Z}\| < r) \\ &= \int_{\mathbb{S}(x_1, r)} h_2(\mathbf{z}) d\mathbf{z} \quad (33)\end{aligned}$$

where $\mathbb{S}(x_1, r)$ is the n-dimensional (open) sphere centered at $(x_1, 0, \ldots, 0)$ with radius r. It can be easily verified that the symmetric unimodal nature of h_2 implies that as the center of the sphere $\mathbb{S}(x_1, r)$ is shifted away from the origin (keeping the radius fixed), the integral in (33) cannot increase. This concludes the proof. ■

APPENDIX C
PROOF OF CLAIM 1

Since $W_{T+1}(\pi) := 0$ for any choice of π, it trivially satisfies Property $\mathbf{R}$. We will now proceed in a backward-inductive manner.

Step 1: If W_{t+1} satisfies Property $\mathbf{R}$, we will show that V_t satisfies Property $\mathbf{R}$ too.

Using Lemma 2, the expression in (11) can be written as

$$V_t(\theta) := W_{t+1}(Q^1_{t+1}(\theta)) + \min_{a \in \mathcal{X}} \mathbb{E}[\rho(X_t, a) | \Theta_t = \theta]. \quad (34)$$

We will look at the two terms in the above expression separately and show that each term satisfies Property $\mathbf{R}$. To do so, we will use the following lemmas.

Lemma 12: $\theta \mathbf{R} \tilde{\theta} \Longrightarrow Q^1_{t+1}(\theta) \mathbf{R} Q^1_{t+1}(\tilde{\theta})$.

Proof: Let $\pi = Q^1_{t+1}(\theta)$ and $\tilde{\pi} = Q^1_{t+1}(\tilde{\theta})$. Then, from Lemma 2

$$\begin{aligned}&\pi(x, e) \\ &= \sum_{\substack{x' \in \mathbb{Z},\\ e' \in \mathcal{E}}} [\mathbb{P}(X_{t+1} = x | X_t = x') \mathbb{P}(E_{t+1} = e | E'_t = e') \theta_t(x', e')] \\ &= \sum_{e' \in \mathcal{E}} \Big[\mathbb{P}(E_{t+1} = e | E'_t = e') \\ &\quad \times \sum_{x' \in \mathbb{Z}} [\mathbb{P}(X_{t+1} = x | X_t = x') \theta(x', e')] \Big] \\ &= \sum_{e' \in \mathcal{E}} \Big[\mathbb{P}(E_{t+1} = e | E'_t = e') \sum_{x' \in \mathbb{Z}} [\mathbb{P}(Z_t = x - x') \theta(x', e')] \Big] \\ &= \sum_{e' \in \mathcal{E}} \mathbb{P}(E_{t+1} = e | E'_t = e') \zeta(x, e'), \quad (35)\end{aligned}$$

where $\zeta(x, e') = \sum_{x' \in \mathbb{Z}} [\mathbb{P}(Z_t = x - x') \theta(x', e')]$. Similarly

$$\tilde{\pi}(x, e) = \sum_{e' \in \mathcal{E}} \mathbb{P}(E_{t+1} = e | E'_t = e') \tilde{\zeta}(x, e') \quad (36)$$

where $\tilde{\zeta}(x, e') = \sum_{x' \in \mathbb{Z}} [\mathbb{P}(Z_t = x - x') \tilde{\theta}(x', e')]$. In order to show that $\pi(\cdot, e) \prec \tilde{\pi}(\cdot, e)$, it suffices to show that $\zeta(\cdot, e') \prec \tilde{\zeta}(\cdot, e')$ and that $\tilde{\zeta}(\cdot, e')$ is a.s.u about the same point for all $e' \in \mathcal{E}$. It is clear that

$$\zeta(\cdot, e') = \mu * \theta(\cdot, e'), \quad \tilde{\zeta}(\cdot, e') = \mu * \tilde{\theta}(\cdot, e')$$

where μ is the distribution of Z_t, and $*$ denotes convolution. We now use the result in Lemmas 6 and 7 from Appendix A to conclude that $\mu * \theta(\cdot, e') \prec \mu * \tilde{\theta}(\cdot, e')$ and that $\mu * \tilde{\theta}(\cdot, e')$ is a.s.u. about the same point as $\tilde{\theta}(\cdot, e')$. Thus, we have established that for all $e \in \mathcal{E}$, $\pi(\cdot, e) \prec \tilde{\pi}(\cdot, e)$. Similarly, we can argue that $\tilde{\pi}(\cdot, e)$, $e \in \mathcal{E}$, are a.s.u. about the same point since $\tilde{\zeta}(\cdot, e')$, $e' \in \mathcal{E}$, are all a.s.u about the same point. Thus

$$\theta \mathbf{R} \tilde{\theta} \Longrightarrow Q^1_{t+1}(\theta) \mathbf{R} Q^1_{t+1}(\tilde{\theta}).$$

■

The above relation, combined with the assumption that W_{t+1} satisfies Property $\mathbf{R}$, implies that the first term in (34) satisfies Property $\mathbf{R}$. The following lemma addresses the second term in (34).

Lemma 13: Define $L(\theta) := \min_{a \in \mathcal{X}} \mathbb{E}[\rho(X_t, a) | \Theta_t = \theta]$. $L(\cdot)$ satisfies Property $\mathbf{R}$.

Proof: For any $a \in \mathcal{X}$, the conditional expectation in the definition of $L(\theta)$ can be written as

$$\sum_{x\in\mathbb{Z}} \rho(x,a)\Big\{\sum_{e\in\mathcal{E}} \theta(x,e)\Big\} = \sum_{x\in\mathbb{Z}} \rho(x,a) m_X\theta(x) \tag{37}$$

where $m_X\theta(x) = \sum_{e\in\mathcal{E}} \theta(x,e)$ is the marginal distribution of θ. Recall that the distortion function $\rho(x,a)$ is a nondecreasing function of $|x-a|$. Let d_i be the value of the distortion when $|x-a| = i$. Let $\mathcal{D} := \{0, d_1, d_1, d_2, d_2, d_3, d_3, \ldots, d_M, d_M\}$, where M is the cardinality of $\mathcal{X}$. It is clear that the expression in (37) is an inner product of some permutation of $\mathcal{D}$ with $m_X\theta$. For any choice of a, such an inner product is lowerbounded as

$$\sum_{x\in\mathcal{X}} \rho(x,a) m_X\theta(x) \geq \langle \mathcal{D}_\uparrow, m_X\theta_\downarrow\rangle \tag{38}$$

which implies that

$$L(\theta) \geq \langle \mathcal{D}_\uparrow, m_X\theta_\downarrow\rangle \tag{39}$$

where $\langle\cdot,\cdot\rangle$ represents the inner product, $\mathcal{D}_\uparrow$ is the nondecreasing rearrangement of $\mathcal{D}$, and $m_X\theta_\downarrow$ is the nonincreasing rearrangement of $m_X\theta$. If $\theta\mathbf{R}\tilde{\theta}$, then it follows that $m_X\theta \prec m_X\tilde{\theta}$ and $m_X\tilde{\theta}$ is a.s.u. about some point $b \in \mathcal{X}$. It can be easily established that $m_X\theta \prec m_X\tilde{\theta}$ implies that

$$\langle \mathcal{D}_\uparrow, m_X\theta_\downarrow\rangle \geq \langle \mathcal{D}_\uparrow, m_X\tilde{\theta}_\downarrow\rangle. \tag{40}$$

Further, since $m_X\tilde{\theta}$ is a.s.u. about b, $\sum_{x\in\mathcal{X}} \rho(x,b) m_X\tilde{\theta}(x) = \langle \mathcal{D}_\uparrow, m_X\tilde{\theta}_\downarrow\rangle$. Thus

$$L(\tilde{\theta}) = \langle \mathcal{D}_\uparrow, m_X\tilde{\theta}_\downarrow\rangle. \tag{41}$$

Combining (39)–(41) proves the lemma. ■

Thus, both terms in (34) satisfy Property $\mathbf{R}$ and, hence, V_t satisfies Property $\mathbf{R}$.

Step 2: If V_t satisfies Property $\mathbf{R}$, we will show that W_t satisfies Property $\mathbf{R}$ too.

Consider two distributions π and $\tilde{\pi}$ such that $\pi\mathbf{R}\tilde{\pi}$. Recall that (12) defined $W_t(\pi)$ as

$$\begin{aligned} W_t(\pi) &= \min_{\hat{\gamma}} \mathbb{E}[c\mathbb{1}_{\{U_t=1\}} + V_t(\Theta_t)|\Pi_t = \pi, \gamma_t = \hat{\gamma}] \\ &=: \min_{\hat{\gamma}} \mathbb{W}(\pi, \hat{\gamma}) \end{aligned} \tag{42}$$

where $\mathbb{W}(\pi,\hat{\gamma})$ denotes the conditional expectation in (42). Suppose that the minimum in the definition of $W_t(\pi)$ is achieved by some prescription γ, that is, $W_t(\pi) = \mathbb{W}(\pi,\gamma)$. Using γ, we will construct another prescription $\tilde{\gamma}$ such that $\mathbb{W}(\tilde{\pi},\tilde{\gamma}) \leq \mathbb{W}(\pi,\gamma)$. This will imply that $W_t(\tilde{\pi}) \leq W_t(\pi)$, thus establishing the statement of step 2. We start with

$$\begin{aligned} \mathbb{W}(\pi,\gamma) &= \mathbb{E}[c\mathbb{1}_{\{U_t=1\}} + V_t(\Theta_t)|\Pi_t = \pi, \gamma_t = \gamma] \\ &= c\mathbb{P}(U_t = 1|\Pi_t = \pi, \gamma_t = \gamma) \\ &\quad + \mathbb{E}[V_t(\Theta_t)|\Pi_t = \pi, \gamma_t = \gamma] \\ &= c\sum_{x,e} \pi(x,e)\gamma(x,e) \\ &\quad + \mathbb{E}[V_t(Q_t^2(\pi, Y_t, \gamma))|\Pi_t = \pi, \gamma_t = \gamma]. \end{aligned} \tag{43}$$

The second term in (43) can be further written as

$$\begin{aligned} &\mathbb{P}(Y_t = \epsilon|\Pi_t = \pi, \gamma_t = \gamma) \\ &\quad\times [V_t(Q_t^2(\pi, Y_t = \epsilon, \gamma))|\Pi_t = \pi, \gamma_t = \gamma] \\ &\quad + \sum_{x,e}\Big[\mathbb{P}(Y_t = (x,e)|\Pi_t = \pi, \gamma_t = \gamma) \\ &\qquad\qquad \times [V_t(Q_t^2(\pi, Y_t = (x,e), \gamma))|\Pi_t = \pi, \gamma_t = \gamma]\Big] \\ &= \sum_{x',e'} \pi(x',e')(1-\gamma(x',e')) \times V_t(\theta^\gamma) \\ &\quad + \sum_{x,e} \pi(x,e)\gamma(x,e)V_t(\delta(x,e-1)) \end{aligned} \tag{44}$$

where θ^γ is the distribution resulting from π and γ when $Y_t = \epsilon$ (see Lemma 2). Substituting (44) in (43) gives the minimum value to be

$$\begin{aligned} &c\sum_{x,e} \pi(x,e)\gamma(x,e) + \sum_{x,e} \pi(x,e)\gamma(x,e)V_t(\delta_{(x,e-1)}) \\ &\qquad + \sum_{x',e'} \pi(x',e')(1-\gamma(x',e')) \times V_t(\theta^\gamma). \end{aligned} \tag{45}$$

We will now use the fact that V_t satisfies Property $\mathbf{R}$ to conclude that $V_t(\delta_{(x,e-1)})$ does not depend on x. That is, $V_t(\delta_{(x,e-1)}) = K(e-1), \forall x \in \mathcal{X}$, where $K(e-1)$ is a number that depends only on $e-1$. Consider $\delta_{(x,e-1)}$ and $\delta_{(x',e-1)}$. It is easy to see that $\delta_{(x,e-1)}\mathbf{R}\delta_{(x',e-1)}$ and $\delta_{(x',e-1)}\mathbf{R}\delta_{(x,e-1)}$. Since V_t satisfies Property $\mathbf{R}$, it implies that $V_t(\delta_{(x',e-1)}) \leq V_t(\delta_{(x,e-1)})$ and $V_t(\delta_{(x,e-1)}) \leq V_t(\delta_{(x',e-1)})$. Thus, $V_t(\delta_{(x,e-1)}) = V_t(\delta_{(x',e-1)}) = K(e-1)$. The expression in (45) now becomes

$$\begin{aligned} &c\sum_{x,e} \pi(x,e)\gamma(x,e) + \sum_{x,e} \pi(x,e)\gamma(x,e)K(e-1) \\ &\qquad + \sum_{x',e'} \pi(x',e')(1-\gamma(x',e')) \times V_t(\theta^\gamma). \end{aligned} \tag{46}$$

We define $\lambda(e) := \sum_{x\in\mathcal{X}} \pi(x,e)(1-\gamma(x,e))$.

We will now construct another prescription $\tilde{\gamma}$. To that end, we first define the sequence $\mathcal{S} = \{0, 1, -1, 2, -2, 3, -3, \ldots\ldots\}$ and let $s(n)$ denote the n^{th} element of this sequence. Recall that $\tilde{\pi}(\cdot, e)$ is a.s.u. about the same point $a \in \mathcal{X}$ for all $e \in \mathcal{E}$. For each $e \in \mathcal{E}$, define

$$n^*(e) := \min\{n : \sum_{k=1}^{n} \tilde{\pi}(a+s(k), e) \geq \lambda(e)\}$$

$$\text{and}$$

$$\alpha(e) := \frac{\lambda(e) - \sum\limits_{k=1}^{n^*(e)-1} \tilde{\pi}(a+s(k), e)}{\tilde{\pi}(a+s(n^*(e)), e)}.$$

Define $\tilde{\gamma}(\cdot,\cdot)$ as

$$\tilde{\gamma}(a+s(k), e) = \begin{cases} 0, & \text{if } k < n^*(e) \\ (1-\alpha(e)), & \text{if } k = n^*(e) \\ 1, & \text{if } k > n^*(e). \end{cases} \tag{47}$$

We can show that with the above choice of $\tilde{\gamma}$

$$\sum_x \pi(x,e)(1-\gamma(x,e)) = \sum_x \tilde{\pi}(x,e)(1-\tilde{\gamma}(x,e)) \tag{48}$$

and

$$\sum_x \pi(x,e)\gamma(x,e) = \sum_x \tilde{\pi}(x,e)\tilde{\gamma}(x,e). \tag{49}$$

Using the same analysis used to obtain (46), we can now evaluate the expression

$$\mathbb{E}[c\mathbb{1}_{\{U_t=1\}} + V_t(\Theta_t)|\Pi_t = \tilde{\pi}, \gamma_t = \tilde{\gamma}]$$

to be

$$c\sum_{x,e}\tilde{\pi}(x,e)\tilde{\gamma}(x,e) + \sum_{x,e}\tilde{\pi}(x,e)\tilde{\gamma}(x,e)K(e-1) + \sum_{x',e'}\tilde{\pi}(x',e')(1-\tilde{\gamma}(x',e'))\times V_t(\tilde{\theta}^{\tilde{\gamma}}) \tag{50}$$

where $\tilde{\theta}^{\tilde{\gamma}}$ is the distribution resulting from $\tilde{\pi}$ and $\tilde{\gamma}$ when $Y_t = \epsilon$ (see Lemma 2). Using (49) in (50), we obtain the expression

$$c\sum_{x,e}\pi(x,e)\gamma(x,e) + \sum_{x,e}\pi(x,e)\gamma(x,e)K(e-1) + \sum_{x',e'}\pi(x',e')(1-\gamma(x',e'))\times V_t(\tilde{\theta}^{\tilde{\gamma}}). \tag{51}$$

Comparing (46) and (51), we observe that all terms in the two expressions are identical except for the last term $V_t(\cdot)$. Using the expressions for θ^{γ} and $\tilde{\theta}^{\tilde{\gamma}}$ from Lemma 2, and the fact that $\pi\mathbf{R}\tilde{\pi}$, it can be shown that $\theta^{\gamma}\mathbf{R}\tilde{\theta}^{\tilde{\gamma}}$. Thus, $V_t(\tilde{\theta}^{\tilde{\gamma}}) \leq V_t(\theta^{\gamma})$. This implies that the expression in (51) is no more than the expression in (46). This establishes the statement of Step 2.

APPENDIX D
PROOF OF LEMMA 4

Suppose that the minimum in the definition of $W_t(\pi)$ is achieved by some prescription γ. Using γ, we will construct another prescription $\tilde{\gamma}$ of the form in (16) which also achieves the minimum. The construction of $\tilde{\gamma}$ is identical to the construction of $\tilde{\gamma}$ in Step 2 of the proof of Claim 1 (using π instead of $\tilde{\pi}$ to define $n^*(e), \alpha(e)$). The a.s.u. assumption of π and the nature of constructed $\tilde{\gamma}$ imply that $\tilde{\gamma}$ is of the form required in the Lemma.

APPENDIX E
PROOF OF THEOREM 6

We need the following definitions for the proof.

Definition 7: A function $\nu : \mathbb{R}^n \mapsto \mathbb{R}$ is said to be symmetric and unimodal about a point $\mathbf{a} \in \mathbb{R}^n$, if $\|\mathbf{x}-\mathbf{a}\| \leq \|\mathbf{y}-\mathbf{a}\|$ implies that $\nu(\mathbf{x}) \geq \nu(\mathbf{y})$. Further, we use the convention that a Dirac-delta function at $\mathbf{a}$ is also symmetric unimodal about $\mathbf{a}$.

For a Borel set A in $\mathbb{R}^n$, we denote by $\mathcal{L}(A)$ the Lebesgue measure of A.

Definition 8: For a Borel set A in $\mathbb{R}^n$, we denote by A^{σ} the symmetric rearrangement of A. That is, A^{σ} is an open ball centered at $\mathbf{0}$, whose volume is $\mathcal{L}(A)$. Given an integrable, nonnegative function $h : \mathbb{R}^n \mapsto \mathbb{R}$, we denote by h^{σ} its symmetric nondecreasing rearrangement. That is

$$h^{\sigma}(\mathbf{x}) = \int_0^{\infty} \mathbb{1}_{\{\mathbf{a}\in\mathbb{R}^n | h(\mathbf{a})>t\}^{\sigma}}(\mathbf{x})dt.$$

Definition 9: Given two integrable, non-negative functions h_1 and h_2 from $\mathbb{R}^n$ to $\mathbb{R}$, we say that h_1 majorizes h_2, denoted by $h_2 \prec h_1$, if the following holds:

$$\int_{\|\mathbf{x}\|\leq t} h_2^{\sigma}(\mathbf{x})d\mathbf{x} \leq \int_{\|\mathbf{x}\|\leq t} h_1^{\sigma}(\mathbf{x})d\mathbf{x} \quad \forall\, t > 0 \tag{52}$$

and

$$\int_{\mathbb{R}^n} h_2^{\sigma}(\mathbf{x})d\mathbf{x} = \int_{\mathbb{R}^n} h_1^{\sigma}(\mathbf{x})d\mathbf{x}.$$

The condition in (52) is equivalent to saying that for every Borel set $\mathbb{B} \subset \mathbb{R}^n$, there exists another Borel set $\mathbb{B}' \subset \mathbb{R}^n$ such that $\mathcal{L}(\mathbb{B}) = \mathcal{L}(\mathbb{B}')$ and $\int_{\mathbb{B}} h_2(\mathbf{x})d\mathbf{x} \leq \int_{\mathbb{B}'} h_1(\mathbf{x})d\mathbf{x}$.

Definition 10 (Binary Relation $\mathbf{R}^n$): Let θ and $\tilde{\theta}$ be two post-transmission beliefs. We say $\theta\mathbf{R}^n\tilde{\theta}$ iff:

1) for each $e \in \mathcal{E}, \theta(\cdot,e) \prec \tilde{\theta}(\cdot,e)$;
2) for all $e \in \mathcal{E}, \tilde{\theta}(\cdot,e)$ is symmetric and unimodal about the same point $x \in \mathcal{X}$.

A similar relation is defined for pre-transmission beliefs.

Definition 11 (Property $\mathbf{R}^n$): Let V be a function that maps probability measures on $\mathbb{R}^n \times \mathcal{E}$ to the set of real numbers $\mathbb{R}$. We say that V satisfies Property $\mathbf{R}^n$ iff for any two distributions θ and $\tilde{\theta}$

$$\theta\mathbf{R}^n\tilde{\theta} \Longrightarrow V(\theta) \geq V(\tilde{\theta}).$$

We can now state the analogue of Claim 1.

Claim 2: The value functions in Theorem 5 W_t $t = 1,2,\ldots T+1$ and $V_t, t = 1,2,\ldots,T$ satisfy Property $\mathbf{R}^n$.

Proof: See Appendix F ■

Because of Claim 2, we can follow arguments similar to those in Section IV to conclude the following: At time $t = 1$, because $\pi_1(\cdot,e)$ is symmetric unimodal about $\mathbf{0}$ for all e, it is sufficient to consider symmetric threshold-based prescriptions of the form

$$\gamma(\mathbf{x},e) = \begin{cases} 1, & \text{if } \|\mathbf{x}\| \geq r_t(e,\pi_1) \\ 0, & \text{if } \|\mathbf{x}\| < r_t(e,\pi_1) \end{cases} \tag{53}$$

on the right-hand side of (25) for time $t = 1$. Using such prescriptions implies that $\theta_1(\cdot,e)$ is always symmetric unimodal about some point $\mathbf{a}$, which is the optimal estimate in (24) at time $t = 1$. Further, $\pi_2(\cdot,e)$ will also be symmetric unimodal about $\lambda\mathbf{A}\mathbf{a}$ and, therefore, it is sufficient to restrict to symmetric threshold-based prescriptions in (25) at time $t = 2$. Proceeding sequentially until time T allows us to conclude that at each time, we only need to consider pre- and post-transmission beliefs that are symmetric unimodal, prescriptions that are symmetric threshold based, and estimates that are equal to the point about which belief is symmetric. This allows us to conclude the result of Theorem 6.

APPENDIX F
PROOF OF CLAIM 2

The proof follows a backward-inductive argument similar to the proof of Claim 1.

Step 1: If W_{t+1} satisfies Property $\mathbf{R}^n$, we will show that V_t satisfies Property $\mathbf{R}^n$ too.

Using Lemma 5, the expression in (24) can be written as

$$V_t(\theta) := W_{t+1}(Q^1_{t+1}(\theta)) + \inf_{\mathbf{a}\in\mathbb{R}^n} \mathbb{E}[\rho(\mathbf{X}_t, \mathbf{a})|\Theta_t = \theta]. \quad (54)$$

We will look at the two terms on the right-hand side of (54) separately and show that each term satisfies Property $\mathbf{R}^n$.

Lemma 14: $\theta\mathbf{R}^n\tilde{\theta} \Longrightarrow Q^1_{t+1}(\theta)\mathbf{R}^n Q^1_{t+1}(\tilde{\theta})$.

Proof: Let $\pi = Q^1_{t+1}(\theta)$ and $\tilde{\pi} = Q^1_{t+1}(\tilde{\theta})$. Then, following steps similar to those in the proof of Claim 1

$$\pi(\mathbf{x}, e) = \sum_{e'\in\mathcal{E}} \mathbb{P}(E_{t+1} = e|E'_t = e')\zeta(\mathbf{x}, e') \quad (55)$$

where $\zeta(\mathbf{x}, e') = \lambda^{-n}\int_{\mathbf{x}'\in\mathbb{R}^n}[\mu(\mathbf{x}-\mathbf{x}')\theta(\lambda^{-1}\mathbf{A}^{-1}\mathbf{x}', e')]d\mathbf{x}'$. Similarly

$$\tilde{\pi}(\mathbf{x}, e) = \sum_{e'\in\mathcal{E}} \mathbb{P}(E_{t+1} = e|E'_t = e')\tilde{\zeta}(\mathbf{x}, e') \quad (56)$$

where $\tilde{\zeta}(\mathbf{x}, e') = \lambda^{-n}\int_{\mathbf{x}'\in\mathbb{R}^n}[\mu(\mathbf{x}-\mathbf{x}')\tilde{\theta}(\lambda^{-1}\mathbf{A}^{-1}\mathbf{x}', e')]d\mathbf{x}'$. In order to show that $\pi(\cdot, e) \prec \tilde{\pi}(\cdot, e)$, it suffices to show that $\lambda^n\zeta(\cdot, e') \prec \lambda^n\tilde{\zeta}(\cdot, e')$ and that $\tilde{\zeta}(\cdot, e')$ are symmetric unimodal about the same point for all $e' \in \mathcal{E}$. It is clear that

$$\lambda^n\zeta(\cdot, e') = \mu * \eta(\cdot, e'), \quad \lambda^n\tilde{\zeta}(\cdot, e') = \mu * \tilde{\eta}(\cdot, e')$$

where $\eta(\mathbf{x}, e') = \theta(\lambda^{-1}\mathbf{A}^{-1}\mathbf{x}, e')$ and $\tilde{\eta}(\mathbf{x}, e') = \tilde{\theta}(\lambda^{-1}\mathbf{A}^{-1}\mathbf{x}, e')$. Recall that $\theta(\cdot, e) \prec \tilde{\theta}(\cdot, e)$ and that $\tilde{\theta}(\cdot, e)$ is symmetric unimodal about a point. It can then be easily shown, using the orthogonal nature of matrix $\mathbf{A}$, that $\eta(\cdot, e) \prec \tilde{\eta}(\cdot, e)$ and that $\tilde{\eta}(\cdot, e)$ is symmetric unimodal about a point. We now use the result in Lemmas 10 and 11 to conclude that $\mu * \eta(\cdot, e') \prec \mu * \tilde{\eta}(\cdot, e')$ and that $\mu * \tilde{\eta}(\cdot, e')$ is symmetric unimodal about the same point as $\tilde{\eta}(\cdot, e')$. Thus, we have established that for all $e \in \mathcal{E}$, $\pi(\cdot, e) \prec \tilde{\pi}(\cdot, e)$.

To prove that $\tilde{\pi}(\cdot, e)$ is symmetric and unimodal about the same point for all e, it suffices to show that $\tilde{\zeta}(\cdot, e')$ are symmetric and unimodal about the same point for all e'. Since $\tilde{\zeta}(\cdot, e')$ is a convolution of $\tilde{\eta}(\cdot, e')$ and μ, its symmetric unimodal nature follows from Lemma 11. ■

Lemma 15: Define $L(\theta) := \inf_{\mathbf{a}\in\mathbb{R}^n}\mathbb{E}[\|\mathbf{X}_t - \mathbf{a}\|^2|\Theta_t = \theta]$. $L(\cdot)$ satisfies Property $\mathbf{R}^n$.

Proof: Let $\theta\mathbf{R}^n\tilde{\theta}$ such that $\tilde{\theta}(\cdot, e)$ is symmetric unimodal about $\mathbf{b}$ for all e. For any $\mathbf{a} \in \mathbb{R}^n$, the conditional expectation in the definition of $L(\theta)$ can be written as

$$\sum_{e\in\mathcal{E}}\int_{\mathbf{x}\in\mathbb{R}^n}\|\mathbf{x}-\mathbf{a}\|^2\theta(\mathbf{x}, e)d\mathbf{x}. \quad (57)$$

Consider any e with a positive probability under θ (that is, $\int_{\mathbf{x}\in\mathbb{R}^n}\theta(\mathbf{x}, e)d\mathbf{x} > 0$). For a constant $c > 0$, consider the function $\nu_c(\mathbf{x}) = c - \min\{c, \|\mathbf{x}-\mathbf{a}\|^2\}$. Then

$$\begin{aligned}\int_{\mathbf{x}\in\mathbb{R}^n}\nu_c(\mathbf{x})\theta(\mathbf{x}, e)d\mathbf{x} &\le \int_{\mathbf{x}\in\mathbb{R}^n}\nu_c^\sigma(\mathbf{x})\theta^\sigma(\mathbf{x}, e)d\mathbf{x}\\ &= \int_{\mathbf{x}\in\mathbb{R}^n}(c - \min\{c, \|\mathbf{x}\|^2\})\theta^\sigma(\mathbf{x}, e)d\mathbf{x} \quad (58)\end{aligned}$$

where we used Lemma 9 in (58). Using the fact that $\theta(\cdot, e) \prec \tilde{\theta}(\cdot, e)$ and Lemma 8, we have

$$\begin{aligned}&\int_{\mathbf{x}\in\mathbb{R}^n}(c - \min\{c, \|\mathbf{x}\|^2\})\theta^\sigma(\mathbf{x}, e)d\mathbf{x}\\ &\le \int_{\mathbf{x}\in\mathbb{R}^n}(c - \min\{c, \|\mathbf{x}\|^2\})\tilde{\theta}^\sigma(\mathbf{x}, e)d\mathbf{x}\\ &= \int_{\mathbf{x}\in\mathbb{R}^n}(c - \min\{c, \|\mathbf{x}-\mathbf{b}\|^2\})\tilde{\theta}(\mathbf{x}, e)d\mathbf{x} \quad (59)\end{aligned}$$

where $\mathbf{b}$ is the point about which $\tilde{\theta}$ is symmetric unimodal. Therefore, for any $\mathbf{a} \in \mathbb{R}^n$

$$\begin{aligned}&\int_{\mathbf{x}\in\mathbb{R}^n}(c - \min\{c, \|\mathbf{x}-\mathbf{a}\|^2\})\theta(\mathbf{x}, e)d\mathbf{x}\\ &\le \int_{\mathbf{x}\in\mathbb{R}^n}(c - \min\{c, \|\mathbf{x}-\mathbf{b}\|^2\})\tilde{\theta}(\mathbf{x}, e)d\mathbf{x}\\ &\Longrightarrow \int_{\mathbf{x}\in\mathbb{R}^n}(\min\{c, \|\mathbf{x}-\mathbf{a}\|^2\})\theta(\mathbf{x}, e)d\mathbf{x}\\ &\ge \int_{\mathbf{x}\in\mathbb{R}^n}(\min\{c, \|\mathbf{x}-\mathbf{b}\|^2\})\tilde{\theta}(\mathbf{x}, e)d\mathbf{x}. \quad (60)\end{aligned}$$

As c goes to infinity, the above inequality implies that

$$\int_{\mathbf{x}\in\mathbb{R}^n}(\|\mathbf{x}-\mathbf{a}\|^2)\theta(\mathbf{x}, e)d\mathbf{x} \ge \int_{\mathbf{x}\in\mathbb{R}^n}(\|\mathbf{x}-\mathbf{b}\|^2)\tilde{\theta}(\mathbf{x}, e)d\mathbf{x}. \quad (61)$$

Summing up (61), for all e establishes that

$$\sum_{e\in\mathcal{E}}\int_{\mathbf{x}\in\mathbb{R}^n}\|\mathbf{x}-\mathbf{a}\|^2\theta(\mathbf{x}, e)d\mathbf{x} \ge \sum_{e\in\mathcal{E}}\int_{\mathbf{x}\in\mathbb{R}^n}\|\mathbf{x}-\mathbf{b}\|^2\tilde{\theta}(\mathbf{x}, e)d\mathbf{x}. \quad (62)$$

Taking infimum over a in the left-hand side (LHS) of the above inequality, proves the lemma. ■

Thus, both terms in (34) satisfy Property $\mathbf{R}^n$ and, hence, V_t satisfies Property $\mathbf{R}^n$.

Step 2: If V_t satisfies Property $\mathbf{R}$, we will show that W_t satisfies Property $\mathbf{R}$ too.

Consider two distributions π and $\tilde{\pi}$ such that $\pi\mathbf{R}\tilde{\pi}$ and $\tilde{\pi}$ is symmetric unimodal about $\mathbf{b}$. Recall that (25) defined $W_t(\pi)$ as

$$\begin{aligned} W_t(\pi) &= \inf_{\hat{\gamma}} \mathbb{E}[c\mathbb{1}_{\{U_t=1\}} + V_t(\Theta_t)|\Pi_t = \pi, \gamma_t = \hat{\gamma}] \\ &=: \inf_{\hat{\gamma}} \mathbb{W}(\pi, \hat{\gamma}). \end{aligned} \tag{63}$$

For any γ, we will construct another prescription $\tilde{\gamma}$ such that $\mathbb{W}(\tilde{\pi}, \tilde{\gamma}) \leq \mathbb{W}(\pi, \gamma)$. This will imply that $W_t(\tilde{\pi}) \leq W_t(\pi)$, thus establishing the statement of step 2. We start with

$$\begin{aligned} \mathbb{W}(\pi, \gamma) &= \mathbb{E}[c\mathbb{1}_{\{U_t=1\}} + V_t(\Theta_t)|\Pi_t = \pi, \gamma_t = \gamma] \\ &= c\sum_e \int \pi(\mathbf{x}, e)\gamma(\mathbf{x}, e)d\mathbf{x} \\ &\quad + \mathbb{E}[V_t(Q_t^2(\pi, Y_t, \gamma))|\Pi_t = \pi, \gamma_t = \gamma]. \end{aligned} \tag{64}$$

The second term in (64) can be further written as

$$\sum_{e'} \int \pi(\mathbf{x}', e')(1 - \gamma(\mathbf{x}', e'))d\mathbf{x}' \times V_t(\theta^\gamma) + \sum_e \int \pi(\mathbf{x}, e)\gamma(\mathbf{x}, e)V_t(\delta(\mathbf{x}, e-1))d\mathbf{x} \tag{65}$$

where θ^γ is the distribution resulting from π and γ when $Y_t = \epsilon$ (see Lemma 5). Substituting (65) in (64) and using the fact that $V_t(\delta(\mathbf{x}, e-1)) = K(e-1)$ gives

$$\begin{aligned} &c\sum_e \int \pi(\mathbf{x}, e)\gamma(\mathbf{x}, e)d\mathbf{x} + \sum_e \int \pi(\mathbf{x}, e)\gamma(\mathbf{x}, e)K(e-1)d\mathbf{x} \\ &\quad + \sum_{e'} \int \pi(\mathbf{x}', e')(1 - \gamma(\mathbf{x}', e'))d\mathbf{x}' \times V_t(\theta^\gamma). \end{aligned} \tag{66}$$

We define $\lambda(e) := \int \pi(\mathbf{x}, e)(1 - \gamma(\mathbf{x}, e))$. We construct $\tilde{\gamma}$ as follows. Define $r \geq 0$ to be the radius of an open ball centered at $\mathbf{b}$ such that $\int_{\|\mathbf{x}-\mathbf{b}\|<r} \pi(\mathbf{x}, e) = \lambda(e)$. Then, define

$$\tilde{\gamma}(\mathbf{x}, e) = \begin{cases} 0, & \text{if } \|\mathbf{x} - \mathbf{b}\| < r \\ 1, & \text{otherwise} \end{cases}. \tag{67}$$

Using the expressions for θ^γ and $\tilde{\theta}^{\tilde{\gamma}}$ from Lemma 5 and the fact that $\pi\mathbf{R}\tilde{\pi}$, it can be shown that $\theta^\gamma\mathbf{R}\tilde{\theta}^{\tilde{\gamma}}$. This establishes the result of Step 2.

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