Holographic solitons

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We propose a new kind of an optical spatial soliton: the holographic soliton. This soliton consists of two mutually coherent field components that interfere, induce a periodic change in the refractive index, and simultaneously are Bragg diffracted from the grating. Holographic solitons are formed when the broadening tendency of diffraction is balanced by phase modulation that is due to Bragg diffraction from the induced grating. Holographic solitons are solely supported by cross-phase modulation arising from the induced grating, not involving self-phase modulation at all. © 2002 Optical Society of America

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Optical spatial solitons are supported by many different nonlinear mechanisms,[1] which generally can be classified into two generic types. The first type is the self-phase-modulation self-focusing mechanism through which an optical beam modifies the refractive index and induces an effective positive lens; i.e., the refractive index in the center of the beam is larger than at the beam's margins. The refractive-index structure of the medium then resembles that of a graded-index waveguide. When the optical beam that has induced the waveguide is also a guided mode of the waveguide that it has induced, the beam's propagation becomes stationary.[2] The vast majority of the physical phenomena that support optical solitons belong to this self-focusing type, which has been implemented for observation of solitons in optical Kerr media,[3,4] atom vapor,[5,6] photorefractive[7,8] and photovoltaic[9] crystals, thermal nonlinearities,[10] liquid crystals,[11] and more. The second mechanism that can support spatial solitons arises from the nonlinear phase coupling that results from symmetric energy exchange between two or more mutually coherent beams. Each beam continuously loses some of its energy and regains the same amount, such that the net power in each beam is conserved. In this interaction, the field that constitutes the acquired energy (to each of the beams) is phase retarded relative to the primary field of each beam. Thus, as the acquired field is added to the primary field, it effectively slows the phase velocity of the beam. Hence, if the interaction occurs in such a way that the effect is more intense at the center of the beam (or for the lowest spatial frequencies of the beam), then it reduces or eliminates the broadening effects of diffraction. Currently, the only known soliton that is supported solely by such a mechanism is the quadratic soliton, which consists of multifrequency beams.[12,13] However, phase coupling between two mutually coherent beams can also be established through a grating in the refractive index that is induced (in real time) by interference between the beams. In this case the beams are coupled through Bragg reflections. Each beam is Bragg reflected and coherently added into the other beam. This can lead to focusing of narrow beams when the reflected beams are $\pi/2$ phase retarded with respect to the primary beams and the index change that gives rise to the grating is an increasing function of the optical intensity. Inasmuch as the focusing effect results from the induced index grating, we termed the effect holographic focusing.[14]

Holographic focusing was most probably present in the pioneering first experimental demonstration of spatial Kerr solitons,[3] which utilized interference between two beams to arrest transverse instability. Effects related to holographic focusing have been reported by Vaupel *et al.*,[15,16] who demonstrated mutual focusing/defocusing of two beams with slightly different frequencies in a photorefractive crystal. Holographic focusing was also recently predicted in plasma.[17] Recently a vector soliton composed of counterpropagating coherent fields that were partly supported by holographic focusing was experimentally demonstrated.[18] Here we propose a soliton for which the focusing mechanism is solely holographic focusing, that is, a holographic soliton that does not rely on self-phase-modulation self-focusing but exists only by virtue of the induced periodic modulation of the refractive index.

A prerequisite for obtaining holographic focusing is that the induced index grating be in phase with the intensity interference grating. When the index grating is π shifted from the interference grating then the grating leads to holographic defocusing, which one can use to obtain dark holographic solitons. But, if the index grating is $\pm\pi/2$ phase shifted with respect to the intensity grating, the interaction will yield an asymmetric (i.e., unidirectional) energy exchange between the beams,[19] so it fundamentally cannot support solitons. Thus the $\pi/2$ phase-shifted component of the grating must be minimized; otherwise, if the rate of asymmetric energy exchange is significant for the propagation distance in the medium, this component will destroy the solitons.

Consider the schematic shown in Fig. 1. Holographic solitons necessitate that the nonlinearity be anisotropic, such that self-focusing of each beam separately is eliminated by use of vectorial (anisotropic) effects. For example, assume that the medium is a

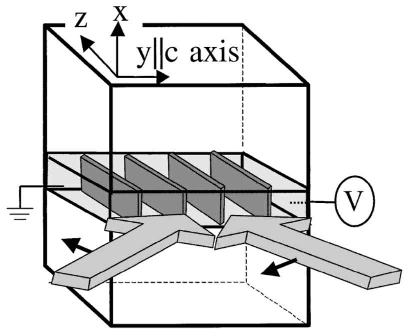

Fig. 1. Schematic of a holographic soliton in a photorefractive medium.

photorefractive crystal of $4mm$ point group symmetry (e.g., SBN) with its crystalline c axis along y, which is also the direction of the applied field. Two mutually coherent beams with their polarization in the yz plane (taking advantage of the large electro-optic coefficient r_{33}) are focused at the input face of the medium. The intensity of each beam is uniform in y. The input beams, however, are narrow in x, and in the absence of nonlinearity they are free to diffract in x. In this configuration, with the electric field applied normal to the narrow direction of the beam, the photorefractive screening nonlinearity does not induce any waveguiding structure in x. Thus, in this configuration, there are no focusing effects in x induced by the photorefractive screening nonlinearity. However, the two beams interact (through the grating that they form), focusing together, and form a holographic soliton. Let the optical field be written as $E = A \exp\{i[k \cos(\theta)z + k \sin(\theta)y - \omega t]\}[\hat{y} \cos(\theta) + \hat{z} \sin(\theta)] + B \exp\{i[k \cos(\theta)z - k \sin(\theta)y - \omega t]\}[\hat{y} \cos(\theta) + \hat{z} \sin(\theta)] +$ c.c., where A and B are the complex envelope amplitudes of the beams, the wave vector is $k = \omega n_0/c$, ω is the temporal frequency, c is the speed of light in vacuum, n_0 is the linear index of refraction, and θ is the angle between the k vector of each beam and the z axis ($\theta \ll 1$). To within a proportionality factor, the intensity is $I \propto |E|^2 = |A|^2 + |B|^2 + \{A^*B \exp[-2ik \sin(\theta)y] + B^*A \exp[2ik \sin(\theta)y]\}\cos(2\theta)$. The refractive-index change in this configuration is given by

$$\Delta n = \Delta n_0 \frac{AB^* \exp[2ik \sin(\theta)y] + \text{c.c.}}{|A|^2 + |B|^2 + I_B} \cos(2\theta)\,, \quad (1)$$

where I_B is the uniformly illuminated background intensity (or background irradiance) and Δn_0 is a complex factor that depends on the specific material parameters and the applied voltage.[20] In what follows, we assume that Δn_0 is real, that is, that the index grating is 0 or π phase shifted relative to the intensity interference grating.[21] To study the time-independent propagation of the beams, we substitute E into the Helmholtz equation. Assuming that $|\Delta n| \ll n_0$ [such that $n^2 = (n_0 + \Delta n)^2 \cong n_0{}^2 + 2n_0\Delta n_0$] and that $\theta \ll 1$ (the paraxial approximation), selecting synchronous terms and seeking solutions that do not depend on y lead to

$$\frac{\partial^2 A}{\partial x^2} + 2ik\cos(\theta)\frac{\partial A}{\partial z} + \frac{2k^2\Delta n_0 \cos(2\theta)}{n_0} \times \frac{|B|^2}{|A|^2 + |B|^2 + I_B} A = 0\,,$$

$$\frac{\partial^2 B}{\partial x^2} + 2ik\cos(\theta)\frac{\partial B}{\partial z} + \frac{2k^2\Delta n_0 \cos(2\theta)}{n_0} \times \frac{|A|^2}{|A|^2 + |B|^2 + I_B} B = 0\,. \quad (2)$$

We seek stationary symmetric intensity profiles for which only the phase is allowed to evolve with propagation. Thus we substitute into Eqs. (2) solutions of the form $A = B = u(\xi)\exp(i\beta\zeta)\sqrt{I_B}$, where $\zeta = k|\Delta n_0|\cos(2\theta)z/[n_0 \cos(\theta)]$ and $\xi = [2k^2|\Delta n_0|\cos(2\theta)/n_0]^{1/2}x$ are dimensionless independent variables, and obtain

$$u'' - \beta u \pm \frac{|u|^2 u}{2|u|^2 + 1} = 0\,, \quad (3)$$

where the plus (minus) means that $\Delta n_0 > 0$ (<0) and the prime stands for the derivative with respect to variable ξ. Using quadrature, we obtain the first integral of Eq. (3):

$$u''(\xi)^2 - u''(0)^2 - \beta[u(\xi)^2 - u(0)^2] \pm [u(\xi)^2 - u(0)^2]/2 \mp \ln\{[1 + 2u(\xi)^2]/[1 + 2u(0)^2]\}/4 = 0\,. \quad (4)$$

We are now in a position to solve Eq. (3) subject to the boundary conditions of bright and dark solitons.

Bright solitons: For lowest-order bright solitons one requires as boundary conditions that (i) $u(\infty) = u'(\infty) = u''(\infty) = 0$, (ii) $u'(0) = 0$, and (iii) $u''(0)/u(0) < 0$. Condition (i) ensures the decay of the field and all its derivatives far from $\xi = 0$, and conditions (ii) and (iii) ensure a local maximum at $\xi = 0$. From condition (iii) we find that bright solitons exist only for $\Delta n_0 > 0$. Taking the limit $\xi \to \infty$ in Eq. (4) and applying the boundary conditions lead to $\beta = 1/2 - \ln(1 + 2u_0{}^2)/4u_0{}^2$, where $u_0 \equiv u(0)$. We numerically integrate Eq. (3) for various values of u_0 and obtain the waveforms shown in Fig. 2a.

Dark solitons: For dark solitons the boundary conditions are that (i) $u(\infty) = u_\infty$, (ii) $u'(\infty) = u''(\infty) = 0$, (iii) $u(0) = 0$, and (iv) there be a real (nonzero) $u'(0)$. The first two conditions ensure a constant value of the field far from $\xi = 0$; the last condition eliminates solutions that are periodic in ξ. From condition (iii) we find that dark solitons exist only for $\Delta n_0 < 0$. Taking the limit $\xi \to \infty$ in Eq. (4) and applying the boundary conditions lead to $\beta = (-u_\infty{}^2/2u_\infty{}^2 + 1)$, which is substituted into Eq. (3) to yield

$$u'(0)^2 = u_\infty{}^2[u_\infty{}^2/(2u_\infty{}^2 + 1) - 1/2] + \ln(2u_\infty{}^2 + 1)/4\,.$$

We numerically integrate Eq. (3) for various values of u_∞ and obtain the waveforms shown in Fig. 2b.

Figure 2c shows the existence curves of bright and dark holographic solitons and exemplifies the relationship among the peak amplitudes of these solitons, the nonlinearity required for their support, and their

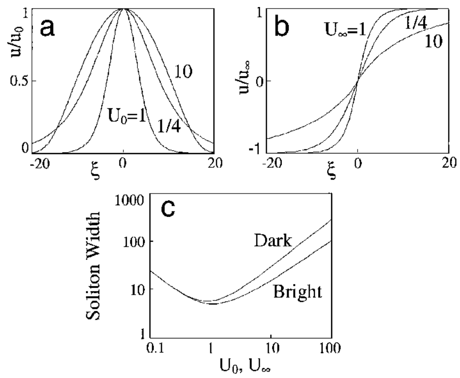

Fig. 2. a, Normalized wave functions of the bright holographic soliton versus transverse coordinate ξ for several values of the peak amplitude (u_0). b, Normalized wave functions of the dark holographic soliton versus ξ for several values of the amplitude at infinity (u_∞). c, FWHM of the intensity of bright and dark holographic soliton versus u_0 and u_∞, respectively.

width. Notice that, given the peak amplitude and width, there is only one value of Δn_0 that gives rise to solitons. This situation is similar to that of almost all other known types of soliton in any medium[1]; based on past experience with spatial solitons, this is not a limiting factor because the solitons dynamically adjust their widths and peak amplitudes to fit this value.

As we have just explained, holographic solitons do not rely on self-focusing but on Bragg scattering from the grating induced by its field components. There are, however, several other distinct differences between holographic solitons and spatial solitons that result from self-phase-modulation. First, the self-phase-modulation self-focusing is insensitive to ω (or is weakly dependent on it), whereas holographic focusing and solitons occur only for those beams that induced the grating. A second difference between the two focusing mechanisms has to do with their response times. The nonlinear response time, τ, characterizes the time that it takes Δn to respond to intensity variations, and it can range from $\sim$100 fs in semiconductors (the dephasing time) to many seconds in photorefractive, thermal, and other nonlinearities associated with transport (of charge, temperature, etc.). The constant τ is also the response time of the conventional focusing mechanism. Holographic focusing has two different characteristic response times. The first is the formation time of the grating, which is τ. The second is the switching time: the response time for holographic focusing of one of the beams due to blocking of the second beam, once the Δn grating is already set. This response is always (irrespective of τ) extremely fast ($\approx$the dephasing time), because the holographic focusing on the first beam results from the phase-delayed Bragg-reflected portion of the second beam. Thus, once Δn is set, one can instantaneously turn off (or on) the holographic focusing effect on the forward beam by blocking (or unblocking) the backward beam. This means that holographic spatial solitons can be switched on and off extremely fast, even in slow nonlinear media. Finally, we note that holographic solitons can be generated in any nonlinear medium in which conventional self-focusing can be eliminated but grating effects can be large. Photorefractives are natural candidates to observe such solitons, but holographic solitons can exist in liquid crystals, polymers, and other media as well.

In conclusion, we have proposed spatial solitons that rely solely on holographic focusing and have suggested an experimental configuration with which to obtain them. The holographic soliton requires Bragg matching, although it is possible that a phase mismatch will also lead to soliton phenomena, as it does for quadratic solitons.

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