Collisions between (2+1)D rotating propeller solitons

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We study theoretically the collisions between (2 + 1)D rotating-dipole-type bimodal solitons and find that such interactions exhibit many interesting exchanges of angular momentum.
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Solitons are self-trapped wave packets that behave as particles: A single soliton propagates without loss of energy and momentum. This analogy holds also for collisions between solitons, which can be elastic or inelastic, depending on whether the system is integrable. Soliton collisions have been thoroughly investigated over the years.[1] In the (1 + 1)D integrable case, collisions are elastic: The number of solitons is conserved, each soliton conserves its power and linear momentum, and no energy is transferred from bound (soliton) states to radiation states. But, in such a system, (2 + 1)D solitons are unstable, so soliton interactions are restricted to a single plane. In saturable nonlinearities collisions are much richer, because in such media (2 + 1)D solitons are stable. Thus collisions can occur between (2 + 1)D solitons with trajectories in different planes. Also, in this system collisions at shallow angles are inelastic.[1] Thus far, various types of interaction, e.g., fusion, fission, and spiraling of solitons, have been identified in saturable nonlinearities.[1] The ability of saturable nonlinearities to support stable (2 + 1)D self-trapping can be exploited to launch (2 + 1)D composite (or multimode) solitons. These are (2 + 1)D solitons that comprise multiple components, each populating a different mode of their jointly induced waveguide. Multimode solitons were first suggested in the form of temporal solitons[2–4] and later on in the (1 + 1)D spatial domain,[5] where they were first observed in 1998.[6] In (2 + 1)D, composite bright solitons were suggested first in a vortex-type form[7–11] and later in a dipole-type form,[12] for which the first mode is an elliptical beam and the second mode is a two-dimensional dipole. Experimentally, such solitons were observed in a dipole-type[13,14] and in a multipole[15,16] form. Recently we demonstrated, theoretically and experimentally, so-called rotating propeller solitons.[17] These solitons consist of a bell-shaped mode jointly trapped with a dipole mode, rotating in unison. We call them propeller solitons because their planes of equal phase are shaped as propeller blades. Such solitons have their entire intensity structures rotating during propagation, making it easy to observe manifestations of their angular momentum. Here, we study numerically collisions between propeller solitons and show that they exhibit interesting features that result from exchanges of angular momentum between the field constituents of the interacting solitons.

The system is described by the normalized equations[17,18]

$$i\,\frac{\partial \psi_j}{\partial z} + \frac{1}{2}\,\nabla_\perp{}^2 \psi_j - \frac{\psi_j}{1+I} = 0\,, \qquad (1)$$

where $j = 1, 2$ are the two modes in the composite soliton, ψ_j is the slowly varying envelope, and $I = |\psi_1|^2 + |\psi_2|^2$. The nonlinearity is saturable, as can be found in electronic transitions in a homogeneously broadened two-level system or (with some approximation) in photorefractives. We find the wave functions of ψ_1 and ψ_2 by using the weakly nonlinear procedure of Ref. 17, which yields $\psi_1(r, z) = u_1(r)\exp(i\mu_1 z)$ and $\psi_2(r, \theta, z) = u_2(r)[R_+\exp(i\theta + i\Omega_+ z) + R_-\exp(-i\theta + i\Omega_- z)]\exp(i\mu_2 z)$, where R_+ and R_- are parameters that determine the rotation rate, μ_1 and μ_2 are propagation constants, and Ω_+ and Ω_- depend on R_+ and R_-. The radial structures u_1 and u_2 are found by use of a relaxation code. Both R_+ and R_- must be nonzero (otherwise ψ_2 is a vortex), and $R_+ \neq R_-$ (otherwise ψ_2 is a stationary dipole). The angular momentum at $z = 0$ is due to the topological charge of the counterrotating vortices. We numerically find (by launching the solution and observing its evolution) that a propeller soliton is stable for more than 80 diffraction lengths, the total power of each field is conserved, and the linear and angular momenta of all fields together are conserved.[17]

Collisions of solitons in saturable nonlinearities depend strongly on the relation between collision angle θ and critical angle θ_c.[1] When a soliton is viewed as a guided mode of its own self-induced waveguide,[19] θ_c is the (complementary) critical angle for total internal reflection in the waveguide, as is known from waveguide theory. When they collide at $\theta > \theta_c$, scalar solitons go through each other unaffected,[1] whereas for $\theta < \theta_c$ solitons can couple light into each other's waveguides; hence the collisions are inelastic, leading to fusion or fission. θ_c plays a key role also for composite solitons.[7–10] The other key factor in a collision of propeller solitons is their angular momentum. The propeller can have right- or left-handed rotation, so collisions between corotating propeller solitons differ from those between counterrotating solitons. The final key factor is the orientation of ψ_2: Inasmuch as this mode has two poles (labeled + and − in the figures), one can arrange the input such that the poles of one propeller are of the same sign as those of the other, or of opposite sign. The relative arrangement determines the forces that the solitons exert on each other.[1] These factors suggest that collisions between such solitons exhibit interesting features. Here we concentrate on the generic effects. We study collisions between two such solitons of identical initial power and identical rotation rate and with trajectories in the same plane.

First we launched two propellers at $\theta > \theta_c$. The solitons went through each other almost unaffected, with only a tiny perturbation, irrespective of whether they corotated or counterrotated and of their initial arrangement. When we launched the propeller solitons at $\theta < \theta_c$ (including $\theta = 0$), the solitons interacted in a fascinating fashion. Consider the first collision [Fig. 1(a)], for which we launched two corotating propellers at $\theta = 0$, arranged symmetrically (dipoles in parallel). The collision outcome was three fission products [Fig. 1(c)]. In the center, a new propeller soliton emerged, rotating in the same direction yet a little slower than each of the input solitons. On the sides, two self-trapped beams, in which both field constituents populated the lowest mode, emerged. The side beams propagated in a plane that was different from the original (x, z) plane of the input trajectories. The equal-phase planes of the incoming propellers resembled propeller blades, which means that each pole of the dipole had a Poynting vector tilted with respect to the (x, z) plane. The incoming propellers were destroyed during the collision process because their inner poles bound and formed a rotating dipole. The surviving outer poles propagated in the direction determined by their individual Poynting vectors, at the point where they broke free from the inner poles [$z = 10$; Fig. 1(a)]. Examining the central soliton revealed that, after the soliton had stabilized (at $z = 90$), the power of each component was conserved (to within $0.03\%/z$, which part escaped to radiation). Furthermore, the total angular momentum L_z of the central composite soliton (the sum of the angular momenta of its constituents, calculated with respect to the center of mass of the whole system, i.e., parallel to the z axis in the center between the input solitons) was conserved (to within our numerical accuracy). In the side beams, the power in each mode decreased at $0.13{-}0.21\%/z$, and L_z decreased at $0.04\%/z$; i.e., L_z decreased considerably more slowly than the power, indicating that the small amount of power escaping to radiation did not carry much angular momentum. A striking feature is the cyclic exchange of angular momentum between ψ_1 and ψ_2 of the outer beams [Fig. 1(b)], while their total L_z is conserved (with slow decay). This effect reflects a tiny back-and-forth wobbling of ψ_2 and ψ_1 about their common center of mass in the outer beam. The central (new) propeller soliton also exhibited a cyclic exchange of L_z between its modes but on a much smaller scale. This seems a general feature of collisions between propeller solitons: The modal constituents of emerging daughter solitons exchange angular momentum cyclically. The fact that collisions between propeller solitons result in new solitons, one of which is a new propeller soliton for which power and angular momentum are almost ideally conserved, is a striking indication that propeller solitons are robust.

The second collision [Fig. 2(a)] was at $\theta < \theta_c$ but with counterrotating propellers. Three new solitons emerged from the collision, but now the central soliton was no longer a dipole-type composite soliton. Instead, both of its constituents were bell shaped (both populating the fundamental mode), with different amplitudes [Fig. 2(b)]. This central composite soliton propagated with a trajectory in a plane inclined with respect to the collision plane. In contradistinction to the collision of corotating propeller solitons (of Fig. 1), the outer beams were counterrotating propellers. Interestingly, the leftmost outgoing beam had right-handed rotation, i.e., counter to the incoming rightmost propeller, which had left-handed rotation (the opposite occurred for the rightmost beam). The poles in the dipole of the outer propeller solitons exchanged energy similarly to the perturbed stationary dipole,[12] but here, in addition, the dipole rotated. The central soliton lost the propeller structure because of the symmetric counterrotating initial arrangement of the poles, and the two outer emerging solitons had opposite angular momenta like the input beams that initiated them. When we repeated the simulation of Fig. 2(a) (collision of counterrotating propellers) but with opposite polarity of one of the dipoles, again three daughter solitons emerged. However, the central

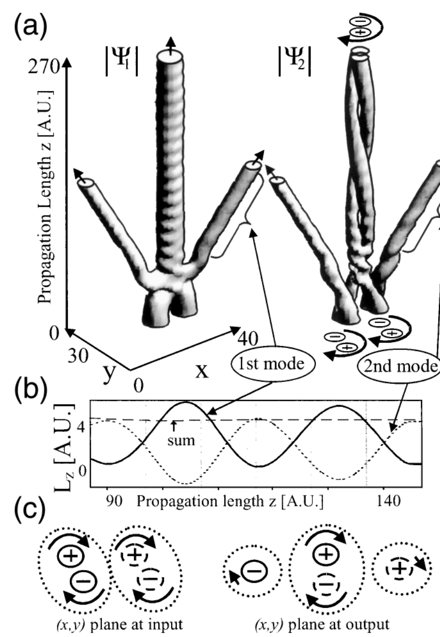

Fig. 1. (a) Collision of two corotating, symmetrically arranged propeller solitons at $\theta = 0$. The propagation length of 100 z units is 120 diffraction lengths. (b) Evolution of the angular momentum of ψ_1 (solid curve), of ψ_2 (dotted curve), and total L_z (dashed curve) in the rightmost outgoing beam. Each component has oscillating angular momentum, but their sum is constant. The leftmost outgoing beam has identical behavior. (c) Input–output configurations of the dipole mode. The initial conditions (in normalized units) are as follows: For ψ_1, FWHM, 2.9; peak intensity, 4. For ψ_2 peak-to-peak separation, 3.6; peak intensity, 0.49; distance between the input propellers, 8. The rotation rate of the envelope is $\approx 9°/z$. For $\lambda = 0.488\ \mu$m and $\Delta n_0/n_0 = 10^{-4}$, the FWHM of ψ_1 is 9.7 μm, the peak-to-peak separation of ψ_2 is 11.7 μm, and the distance between the input propellers is 26.2 μm.

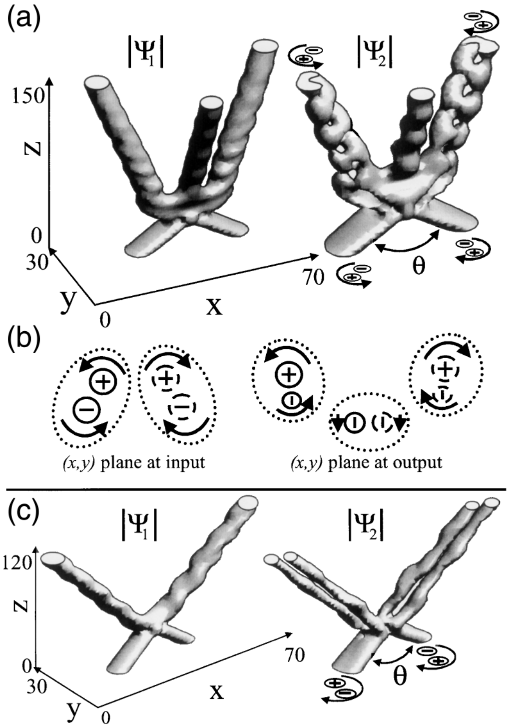

Fig. 2. (a) Collision of two counterrotating, symmetrically arranged propeller solitons at $\theta < \theta_c$. The collision angle in dimensional units is $\theta \times (2\Delta n_0/n_0)^{1/2}$; e.g., for $\Delta n_0/n_0 = 10^{-4}$ the collision angle is 0.57°. (b) Illustration of the input–output configurations of the dipole mode. The negative poles collide, and each breaks into two nonequal portions. The central parts fuse and together form a nonrotating (−) pole. The outer parts are captured by the original positive poles, and each forms a nonsymmetric rotating dipole. (c) Collision of two corotating, antisymmetrically arranged propeller solitons for the same θ as in (a). The distance between the input propellers is 20 [normalized units] = 65.5 μm (for the parameters of Fig. 1) in both cases. Other initial conditions are as in Fig. 1.

soliton was then a stationary, nonrotating, dipole-type composite soliton. Thus, from a collision between counterrotating oppositely arranged propeller solitons, a stationary dipole composite soliton emerged.

In the third collision, shown in Fig. 2(c), we launched two corotating propellers in an antisymmetric arrangement, at $\theta < \theta_c$. Then there was no fission: Two composite solitons emerged from the collision. The propellers went through each other, coming out slightly perturbed, rotating in the initial direction but with trajectories in tilted planes. Each outgoing beam conserved its L_z, but its modes exchanged angular momentum [similarly to Fig. 1(b)]. This passing of solitons through each other is unique in saturable systems: In all other collisions of solitons at $\theta < \theta_c$ in such media, the solitons strongly interact. It seems that here the interaction effects were canceled out.

We studied several collision cases. Collisions of high symmetry (e.g., all solitons of the same P and L_z and launched in a symmetric or antisymmetric scheme) can be classified into one of the above cases. Collisions of lower symmetry offer new properties. At zero angles, in some cases we observed chaotic-looking behavior, and there was no clear outcome for a long propagation distance. To conclude, we have studied collisions between dipole-type composite solitons that carry angular momentum. The angular momentum gives rise to fascinating features, which arise from transmission of angular momentum from the input solitons to the fission products, which are solitons themselves. We have found that propeller solitons are robust, as they can survive collisions even under the critical angle, much like their stationary cousins. In fact, in all the collisions between rotating propeller solitons for which the outcome is self-trapped beams, at least one of the emerging beams is a (stationary or rotating) dipole-type composite soliton.

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