Agenda

- Questions on anything
- Review: Graph terminology
- Shannon Switching Game
- Graph problems
- Branch & Bound
Questions?

- Any questions in general?
Graph Terminology

- What is a graph (formally)?
- What kinds of graphs can we have?
Graph Terminology

- Order of a graph
- Order of a node
- Complete graph
  - How many edges does $K_n$ have (contrary to yesterday's lecture)?
  - Examples
- Spanning subgraph
- Induced subgraph
- Example of spanning subgraph, which is not induced?
- Example of induced subgraph, which is not spanning?
- Example of spanning, induced "subgraph"?
Graph Terminology

- Parity of a node
- Isomorphism (examples)
  - Are A and B isomorphic?
- Degree sequence
- If two graphs have the same degree sequence, are they isomorphic?
- If two graphs are isomorphic, do they have the same degree sequence?
  - Examples?
Graph Terminology

- Closed and open walks
  - $x_0-x_1-x_2-...-x_m$
- Eulerian trail
  - Konigsberg bridge problem
  - Let's try it out
  - Can we find any generalizations? When does such a trail exist?
Graph Terminology

- Eularian trail (continued)
  - Closed: all even degree vertices
  - Open: exactly two even degree vertices
- Hamilton cycle (with nodes this time)
Graph Terminology

• Bipartite graph
  - When is a graph bipartite?
    • Why?
Shannon Switching Game

- Yeah, he's that guy outside the building (the statue)
- Rules
  - Two players, positive and negative
  - Let $G=(V, E)$
  - $U$ and $V$ have been distinguished
  - Negative player goal:
    - Destroy paths
  - Positive player goal:
    - Connect $U$ and $V$
- Never ends in draw. Why?
- Let's try playing for a bit
Shannon Switching Game

- Positive game, neutral game, negative game
- What's an easy way to convert a neutral game to a positive game?
- When does G produce a positive game?
Shannon Switching Game

- Positive game, neutral game, negative game
- What's an easy way to convert a neutral game to a positive game?
- When does G produce a positive game?
  - Solution: “Iff there is a subset U containing u and v of the vertex set V such that the induced multisubgraph X has two spanning trees T1 and T2 with no common edges”
  - What does this mean?
Graph Questions

- Draw a connected graph whose degree sequence equals $(5, 4, 3, 3, 3, 3, 3, 2, 2)$
- Does a graph of order $n$ with at least $(n-1)(n-2)/2 + 1$ edges have to be connected? Is it possible to have a disconnected graph of order $n$ with one fewer edge? Why?
Branch and Bound

- How is BFS better than DFS?
- How is DFS better than BFS?
- Sometimes called Best-first search
Branch and Bound

• Basic idea
  - Start off with infinite cost and null solution
  - Try to search “most promising” configurations first
  - Compare with current best
  - Update if necessary
  - Backtrack if necessary
• A* search... How does that work?
• What's the purpose of Branch and Bound and A*?
• Implications for PA3....
Questions?

- That's all for today
- Any questions?