Perception of the structure of the physical world using unknown multimodal sensors and effectors

EECS598 Presentation

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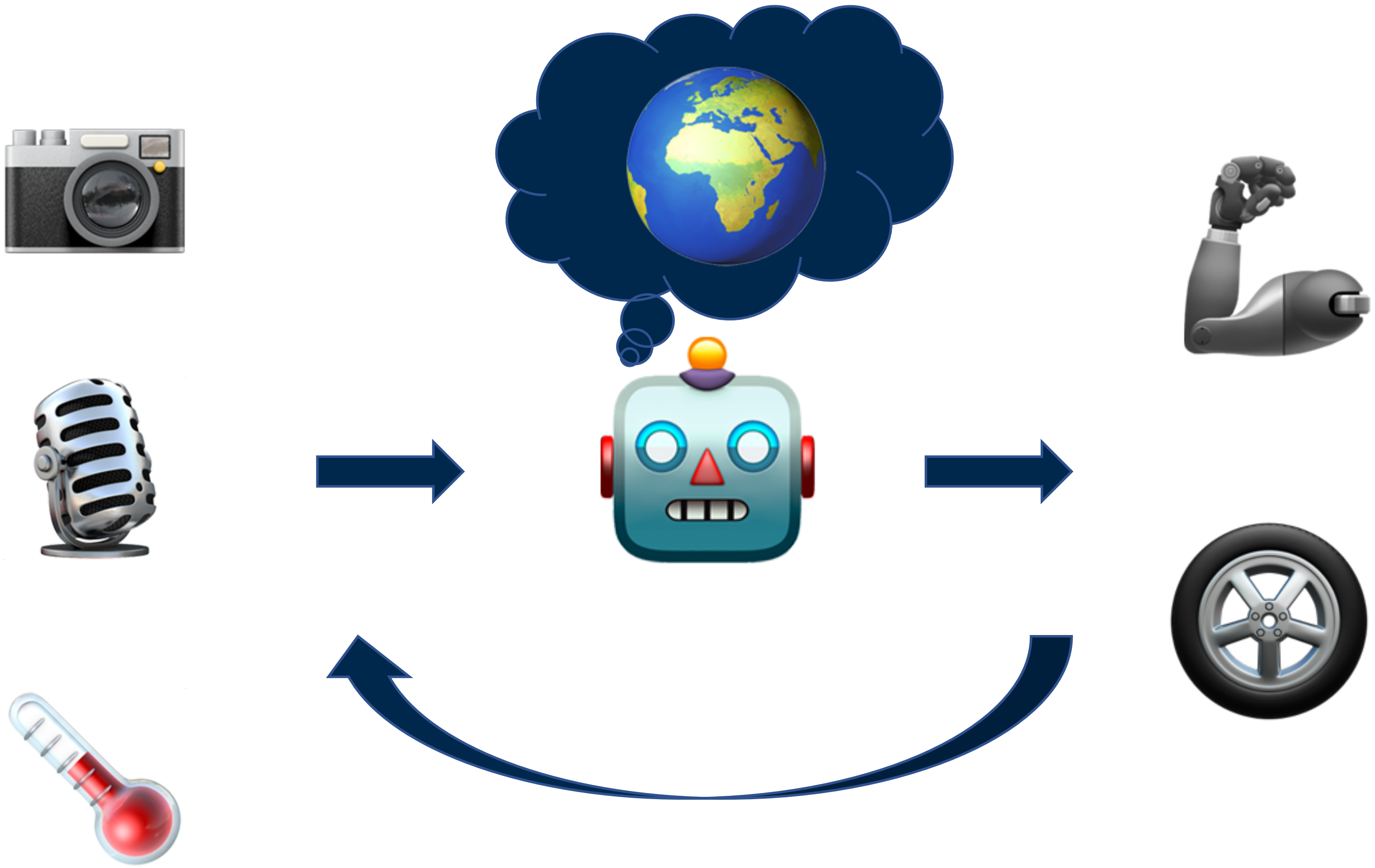
To Know the World Structure



Overview

- Mathematical Formulation
- Simulation
- Results
- Discussion

Mathematical Formulation

$$P = \varphi_a(M) \quad \text{and} \quad S = \varphi_b(P, E)$$

↓

$$\varphi(M, E) \stackrel{def}{=} \varphi_b(\varphi_a(M), E)$$

1. Isotropy Group of the Sensorimotor Law

sensorimotor laws :

$$\varphi(\cdot,\mathcal{E}) \stackrel{def}{=} \{M \mapsto \varphi(M,E), E \in \mathcal{E}\}$$

Goal:

Extract something same for all $h \circ \varphi(\cdot,\mathcal{E})$

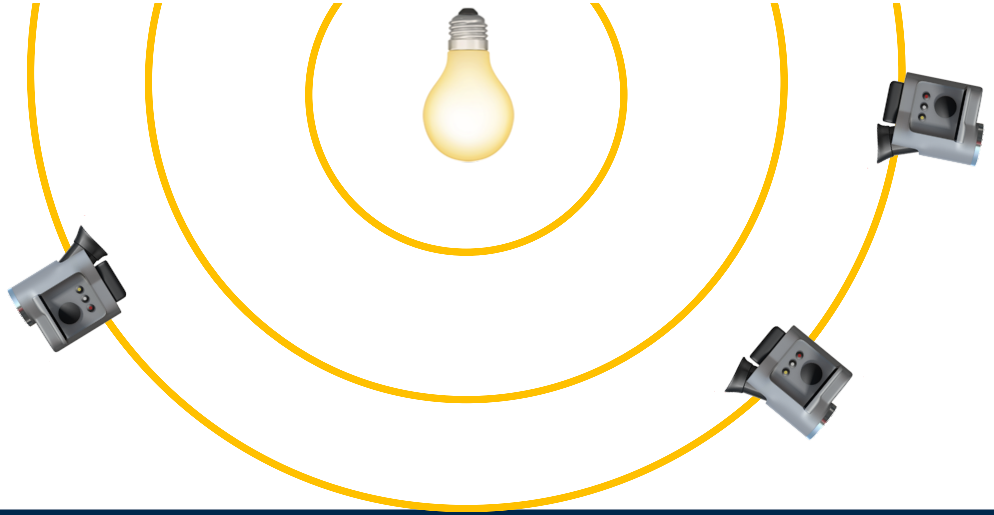
1. Isotropy Group of the Sensorimotor Law

If we note $Sym(X) \overset{def}{=} \{f : X \to X,\ f \text{ one to one mapping}\}$, and consider :

$$\Gamma(\varphi) = \{f \in Sym(\mathcal{M} \times \mathcal{E}) \text{ such that } \varphi \circ f = \varphi\}$$

then

Property 1 $\Gamma(\varphi_1) = \Gamma(\varphi_2) \Leftrightarrow \exists f \in Sym(\mathcal{S})$ *such that* $\varphi_1 = f \circ \varphi_2$



1. Isotropy Group of the Sensorimotor Law

Challenges:

1. Manifolds $\mathcal{E}$ unknow to the algorithm

2. f is *invisible* since $\varphi \circ f = \varphi$

2. Fundamental Vector Fields over Sensory Inputs

Assume:

Enough Information / Univocal observation

Condition 1 *There exists* $\mathcal{U} \times \mathcal{V} \subset \mathcal{M} \times \mathcal{E}$ *such that* $\varphi(M, \cdot)$ *is an injective immersion from* $\mathcal{V}$ *to* $\mathcal{S}$ *for any* $M \in \mathcal{U}$

2. Fundamental Vector Fields over Sensory Inputs

Under this condition, $\varphi(M, \mathcal{V})$ is a manifold for any $P \in \mathcal{U}$ and $\varphi(M, \cdot)$ is a diffeomorphism from $\mathcal{V}$ to $\varphi(M, \mathcal{V})$. We shall write $\varphi^{-1}(M, \cdot)$ its inverse. Choosing $M_0 \in \mathcal{U}$, it is thus possible to define an action ϕ^{M_0} of G over the manifold $\varphi(M_0, \mathcal{V})$:

$$\phi^{M_0}(g, S) \stackrel{def}{=} \varphi(M_0, g_2(\varphi^{-1}(M_0, S))) \quad \forall S \in \varphi(M_0, \mathcal{V})$$

As a consequence (see for instance [2]), for any left invariant vector field X on G there is an associated fundamental vector field $X^{\mathcal{S}}$ on $\varphi(M_0, \mathcal{V})$[1] :

$$X^{\mathcal{S}}(S) \stackrel{def}{=} \frac{d}{dt} \phi^{M_0}(e^{-tX}, S)_{|t=0} \quad \forall S \in \varphi(M_0, \mathcal{V})$$

[1]To avoid heavy notations we have written $X^{\mathcal{S}}$ instead of $X^{\varphi(M_0, \mathcal{V})}$.

2. Fundamental Vector Fields over Sensory Inputs

Measuring Rod:

$$\frac{d}{dt}\,\phi_1(e^{-tX}, M_0)_{|t=0} \in T\mathcal{M}_{|M_0}$$

Motor Command $M_X(t)$ **s.t. :**

$$M_X(0) = M_0 \quad \text{and} \quad \dot{M}_X(0) = -\frac{d}{dt}\phi_1(e^{-tX}, M_0)$$

Fundamental Field:

Property 2 $X^S(S) = \frac{d}{dt}\varphi(M_X(t), \varphi^{-1}(M_0, S))_{|t=0} \quad \forall\, S \in \varphi(M_0, \mathcal{V})$

3. Discovery of the Measuring Rods

If φ Not Singular:

Property 3 $\frac{\partial \varphi}{\partial M}(M_0, E_0)\left[\dot{M} - \dot{M}_X\right] = 0 \Rightarrow \frac{d}{dt}\varphi(M(t), \cdot)_{|t=0} = X^{\mathcal{S}}(\varphi(M_0, \cdot))$

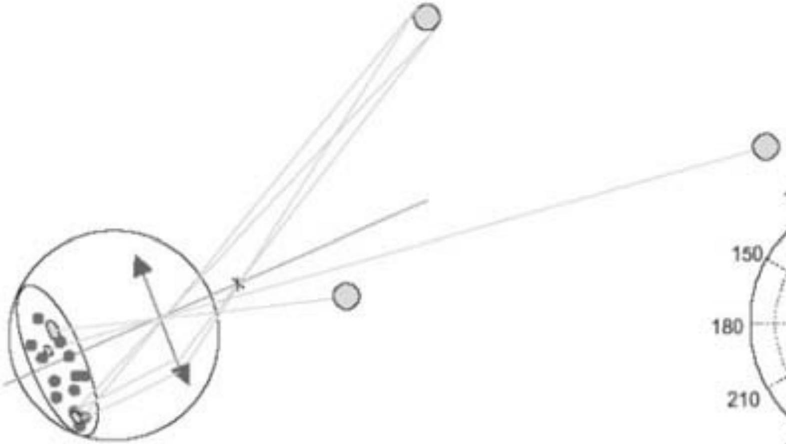
(Particular choice of one vector of $T\mathcal{M}_{|M_0}$ is not important)

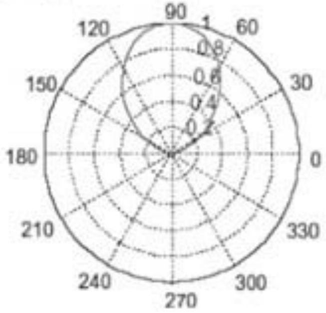
⬇

(Search for Rods → Search for Sensory Images)

$$\forall X \; \frac{\partial \varphi}{\partial M}(M_0, E_0)\frac{d}{dt}\phi_1(e^{-tX}, M_0)_{|t=0} \in T\varphi(M_0, \mathcal{V})_{|S_0} \bigcap T\varphi(\mathcal{U}, E_0)_{|S_0}$$

Simulation





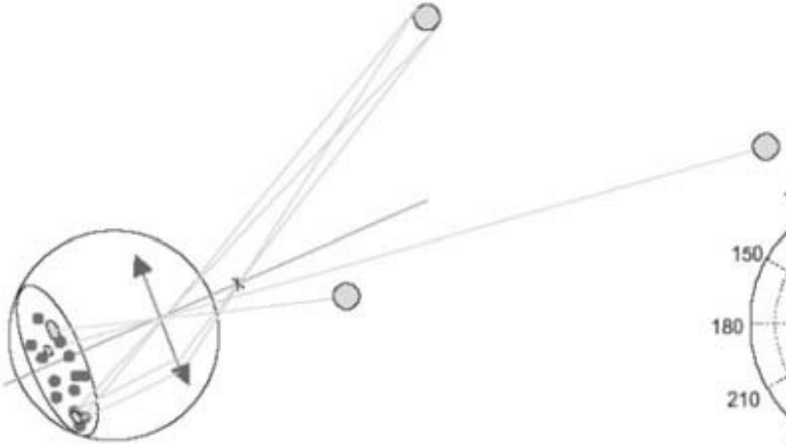
Sensory Inputs:

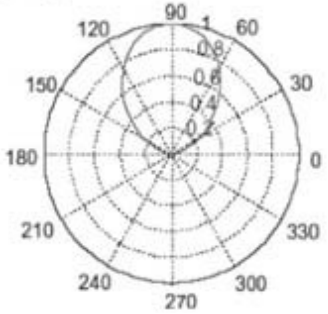
2 x 40(photo sensors) + 2(auditory sensors) + 8(tactile sensors) = 90

Motor outputs:

6(head configuration) + 2 x 3(eye orientation) + 2x2(aperture) =16

Simulation





Sensory Encoding:

$$W_S \in \mathcal{M}(s, 90)$$

Motor Encoding:

$$W_M \in \mathcal{M}(16, m)$$

s.t. $s = m = 300$

Simulation

Algorithm:

Stage 1:

Fix environment and extracts sensory input combinations that are meaningful as regards its own mobility.

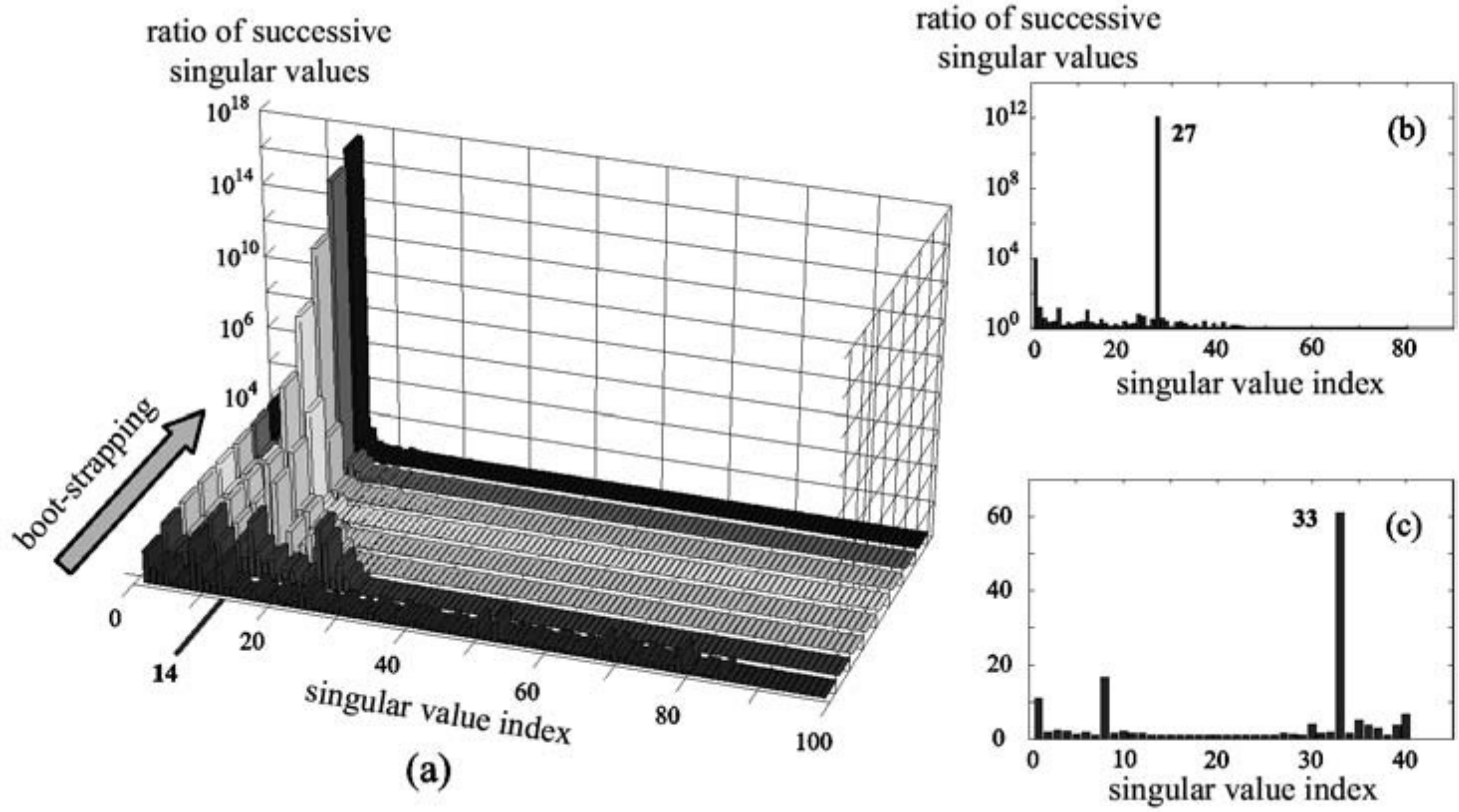
Stage 2:

Fix motor output and move the environment, estimate the tangent space to sensory inputs.

State 3:

Calculate the intersection of the 2 tangent spaces.

Results

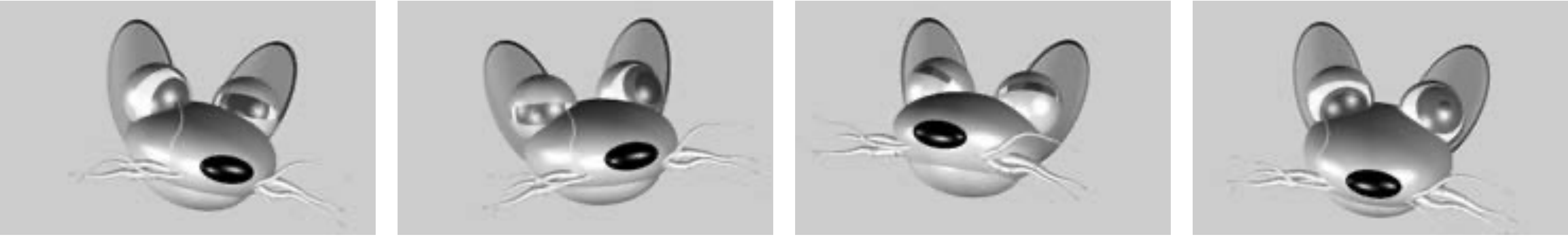


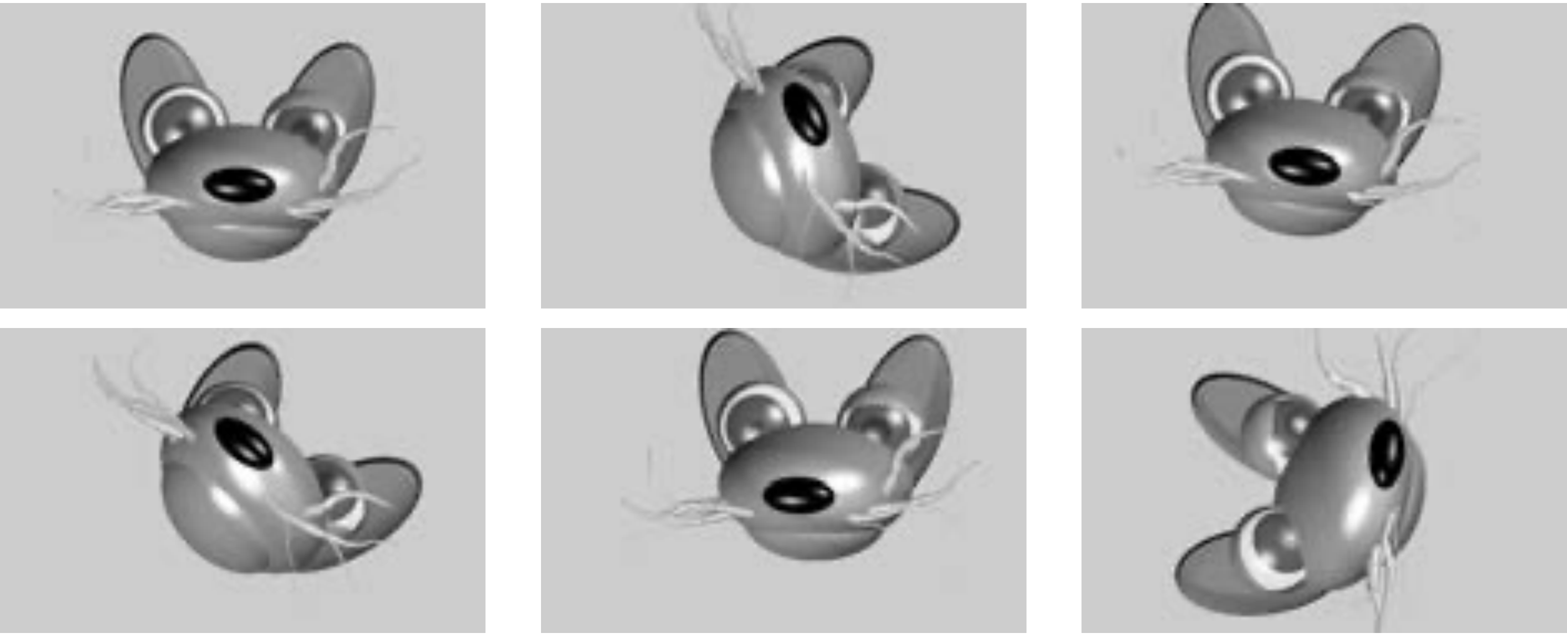
(a)
16-2 = 14
effective parameters of control of the body with respect to the sensory inputs

(b)
9 × 3 = 27
for the 9 light sources moving in a three dimensional space

(c)
41 − 35 = 6
dimension 6 reflecting the Lie group of Euclidian transformations SE(3)

Results





Random Motor Output

Motor Output using Measuring Rods

Discussion Questions

1.

Is there a way of extracting *objects* in the environment from the sensorimotor law, even though nothing is known about the sensors and effectors?

Can human do this? Or are human doing this?

Discussion Questions

2.

Can we apply this idea to current deep learning methods to explicitly show how the model understands the data? By constructing some form of measuring laws?

Discussion Questions

3.

Can the algorithm know anything about the world structure with only sensor inputs or the motor outputs, but not both? Like unsupervised learning can learn the data structure without associated labels?

Discussion Summary #1

Post:

The author is exploring a way for an algorithm to detect objects in the environment with only sensorimotor data. When a baby is born, he knows nothing about the environment, the objects in the environment, and the sensor he possesses. However, the baby is able to learn about the environment and his body using sensorimotor information, by finding a law that maps sensory information to motor information. And the author tries to simulate that using algorithm. He successfully estimated the dimension of motor inputs in his environment. However, is it true we born without knowledge of the environment and our sensors?

--Haoyuan Ma

Response:

I would assume that we are born without any knowledge about the environment or how to use our sensory systems. As we grow, we keep acquiring new knowledge about both.

On the other hand, I think it's very difficult to prove this true, or false, as it would be difficult to correctly measure how much babies know about what. We can deduce and infer from how they act or react, but the concept is rather unclear. Nonetheless, it would be interesting to see if we can prove that humans (or more generally organisms) are born with some knowledge to start out with.

-- Katsumi Ibaraki

Discussion Summary #2

Post:

The authors in this paper mention that this builds upon their previous work, where the same authors showed an algorithm can determine the dimensionality of its environment by only considering the relation between sensory inputs and motor effects. They also mention in the conclusion that this work is just a step towards their final goal of "extracting objects" using only the sensorimotor law. I wonder how far the authors were able to go with this approach and if they achieved that final goal, I did not see anything that directly related from a brief literature search.

--Alan Van Omen

Response:

I also think final goal wasn't achieved. I agree with Nathaniel that the results were the computed measurements of similarity which could be extrapolated to movements of the parts of the rat (head, eyes, etc) but I don't think that it provided a compelling argument that they were able to extract objects using this process.

-- **Maxwell Li**

Discussion Summary #3

Post:

The author didn't explain the ultimate goal: to show that there is a way of extracting objects in the environment from the sensorimotor law, even though nothing is known about the sensors and effectors. I'm confused about how to apply sensorimotor law without getting access to the sensors and effectors.

--**Ruifeng Xu**

Response:

The idea behind this paper is to simulate the process by which a newborn can learn about the environment and the body using only sensorimotor information. The authors try to find a law that maps sensory information to motor information, and by using this law, the algorithm is able to estimate the dimension of motor inputs in the environment. However, it is important to note that this is a simplification of the complex process of human perception and development, and there is still much to be learned about how babies and humans in general learn about their environment and themselves.

-- **Jemuel Stanley Premkumar**