Computational Models of Perceptual Organization

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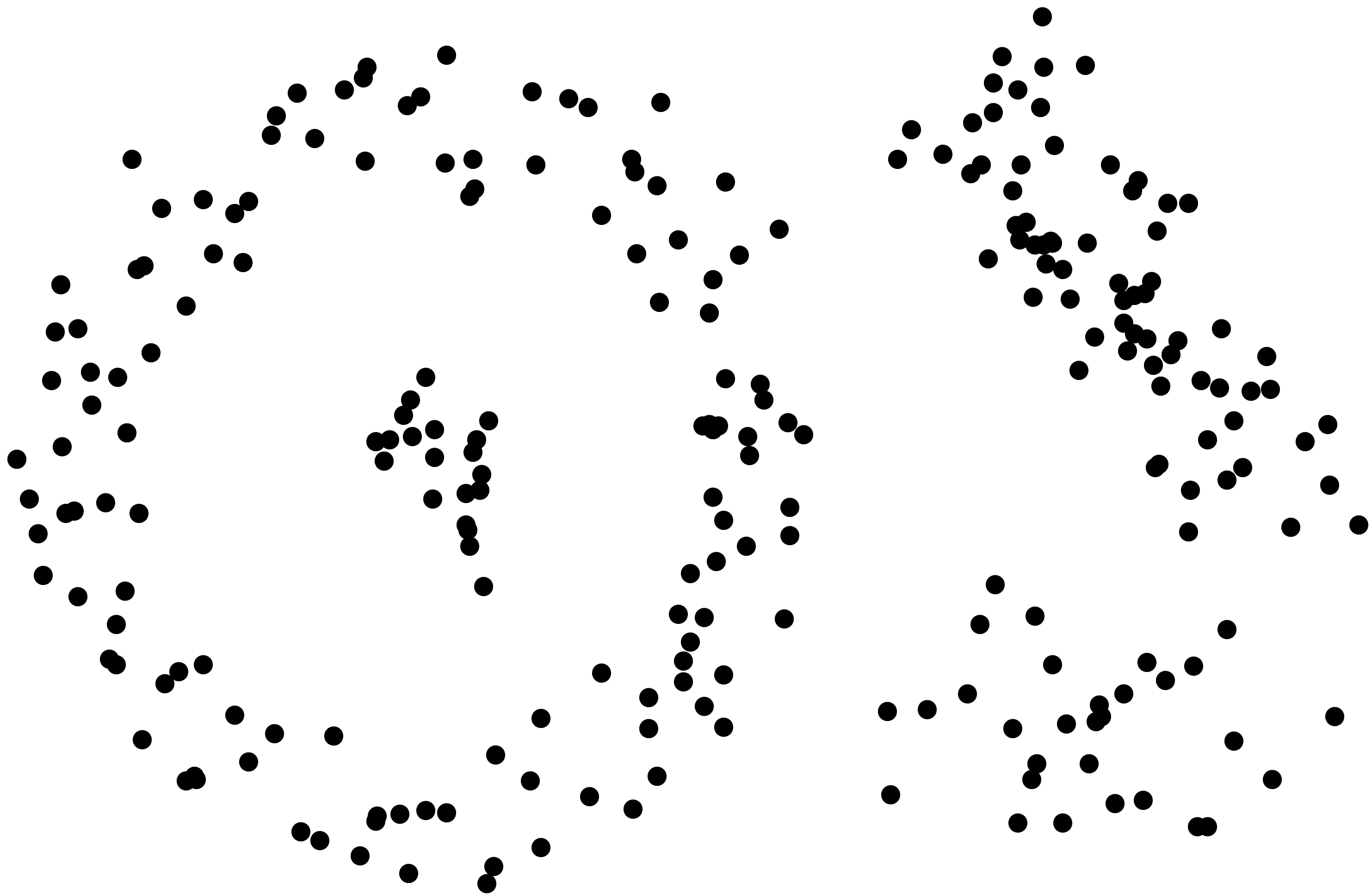
What Is Perceptual Organization



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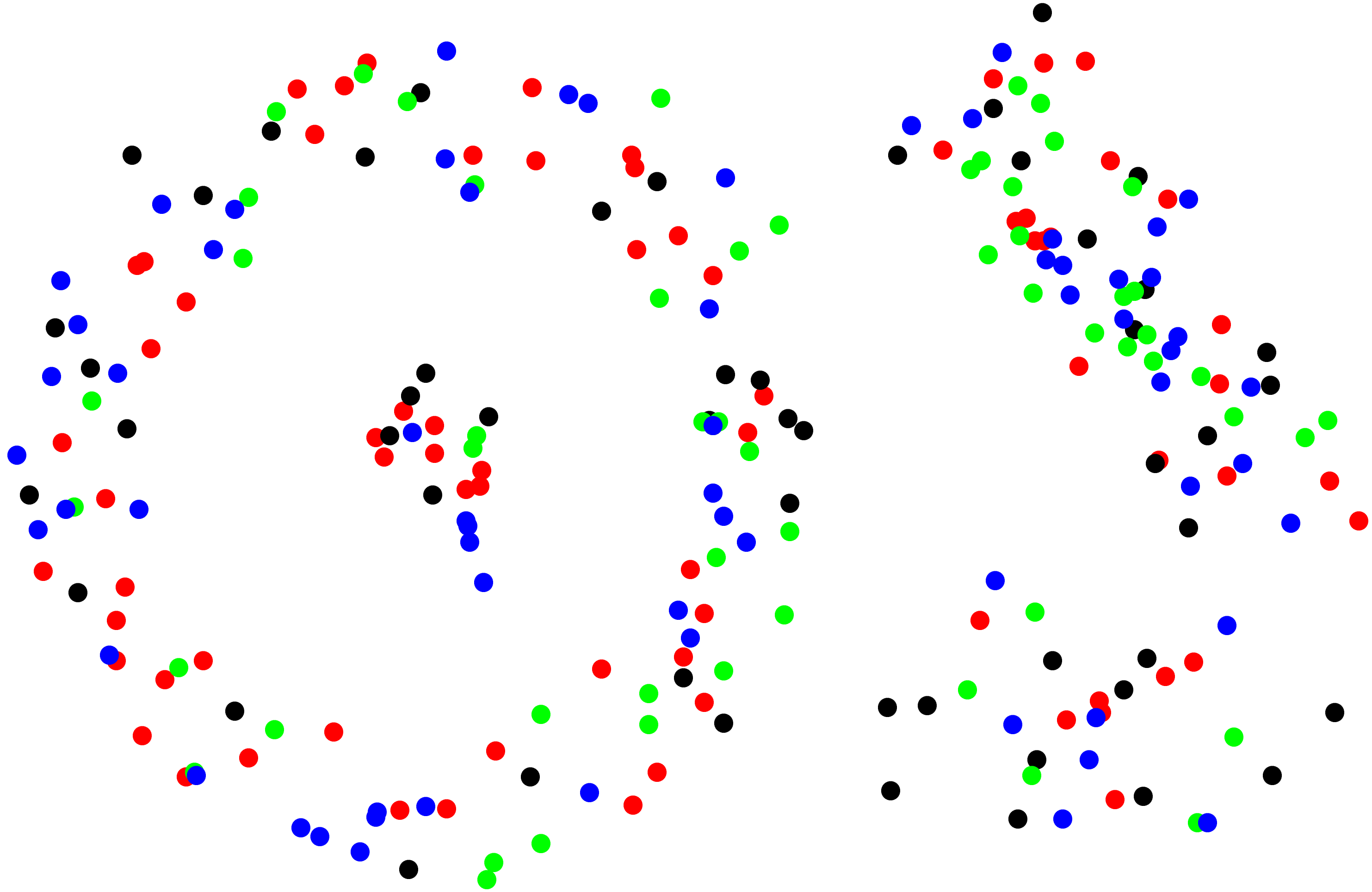

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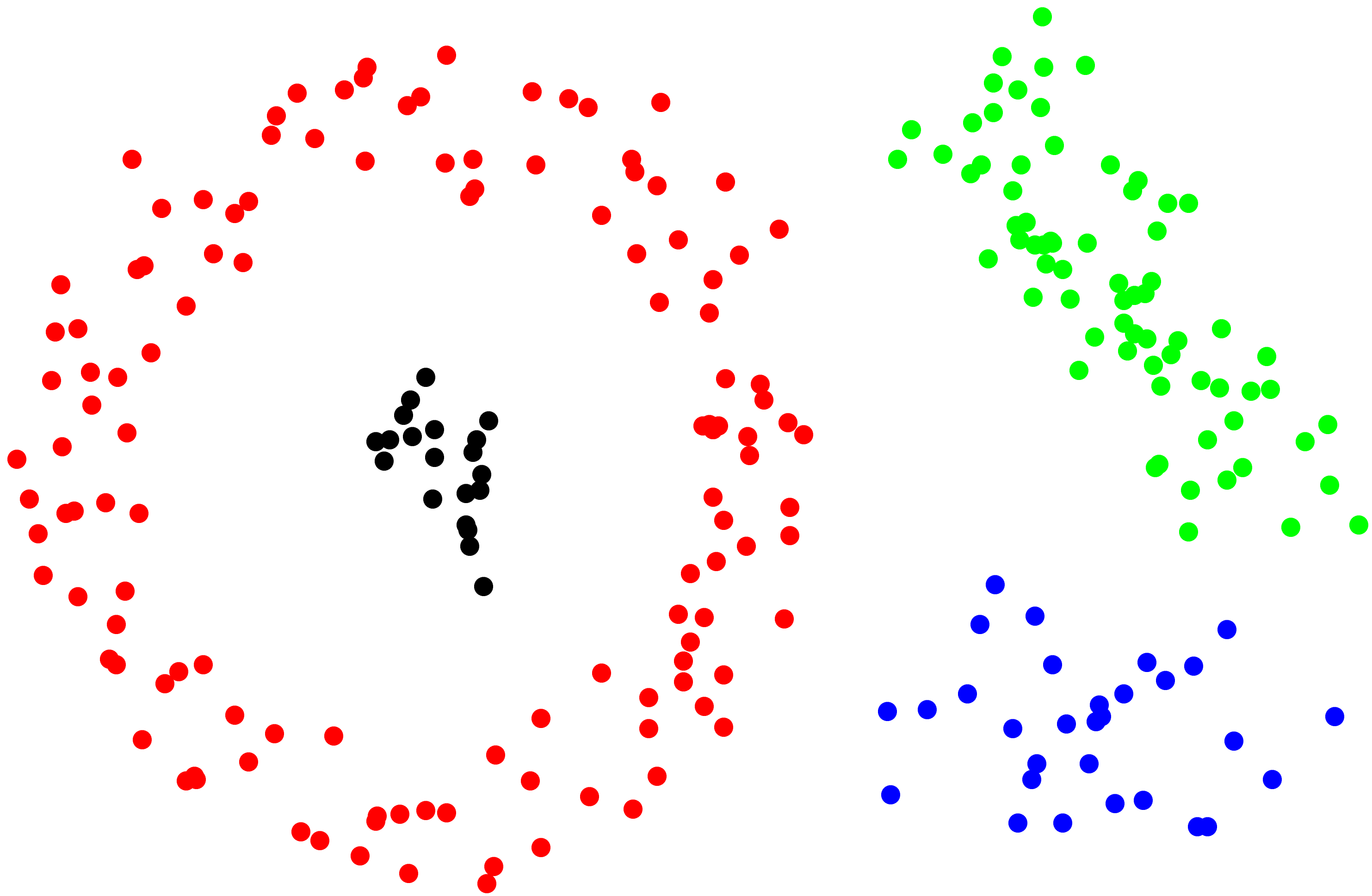
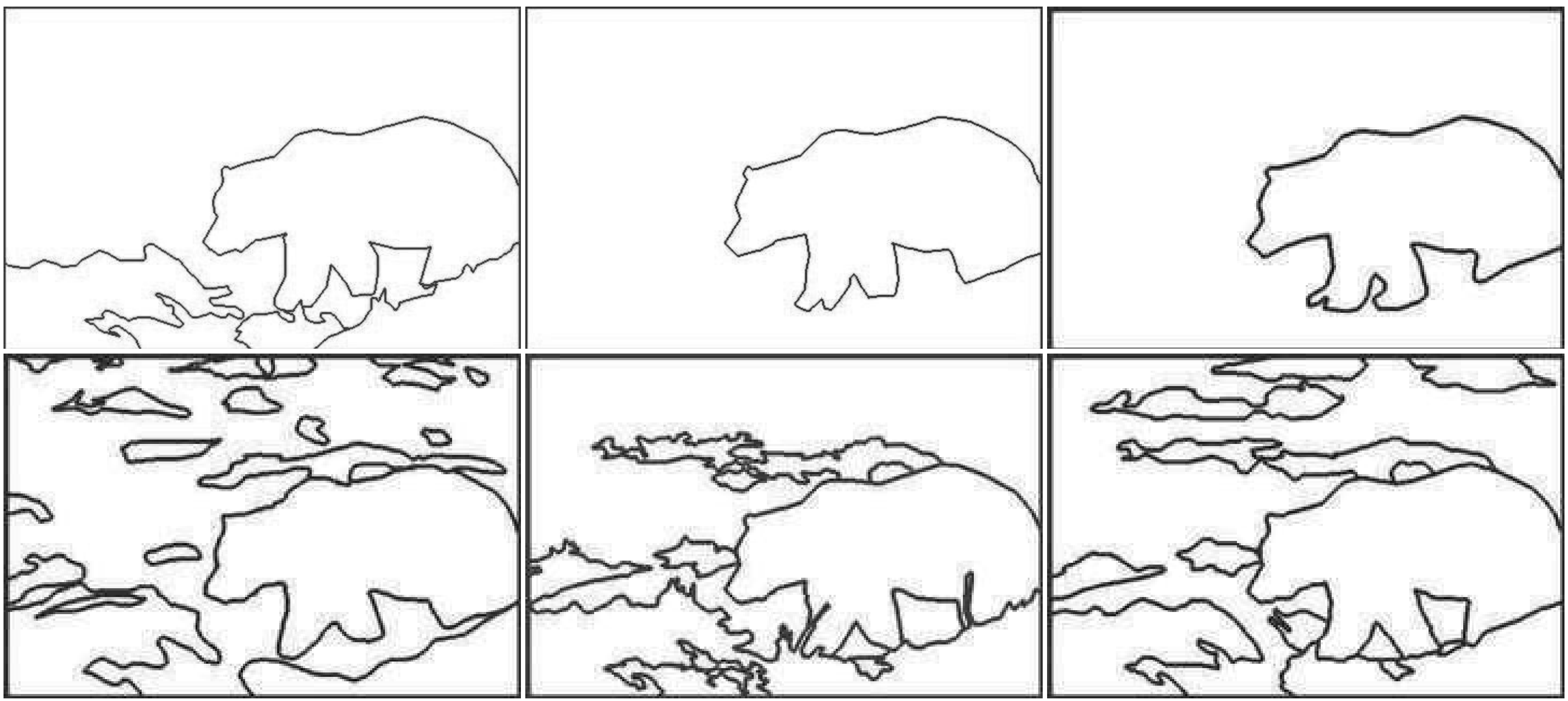


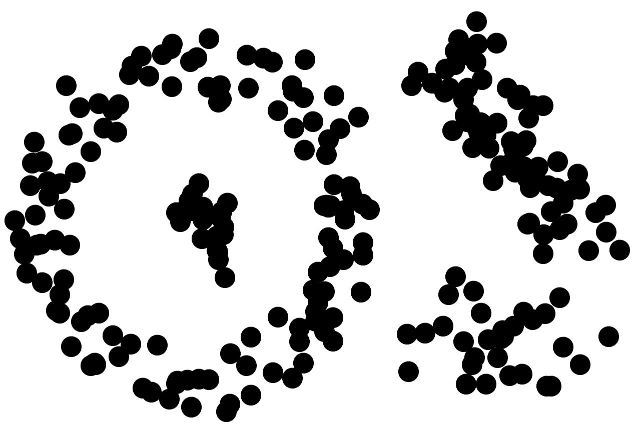
What Is Perceptual Organization

(Martin et al)



What Is Perceptual Organization

- multiple choices
- a variety of features
- content-dependent



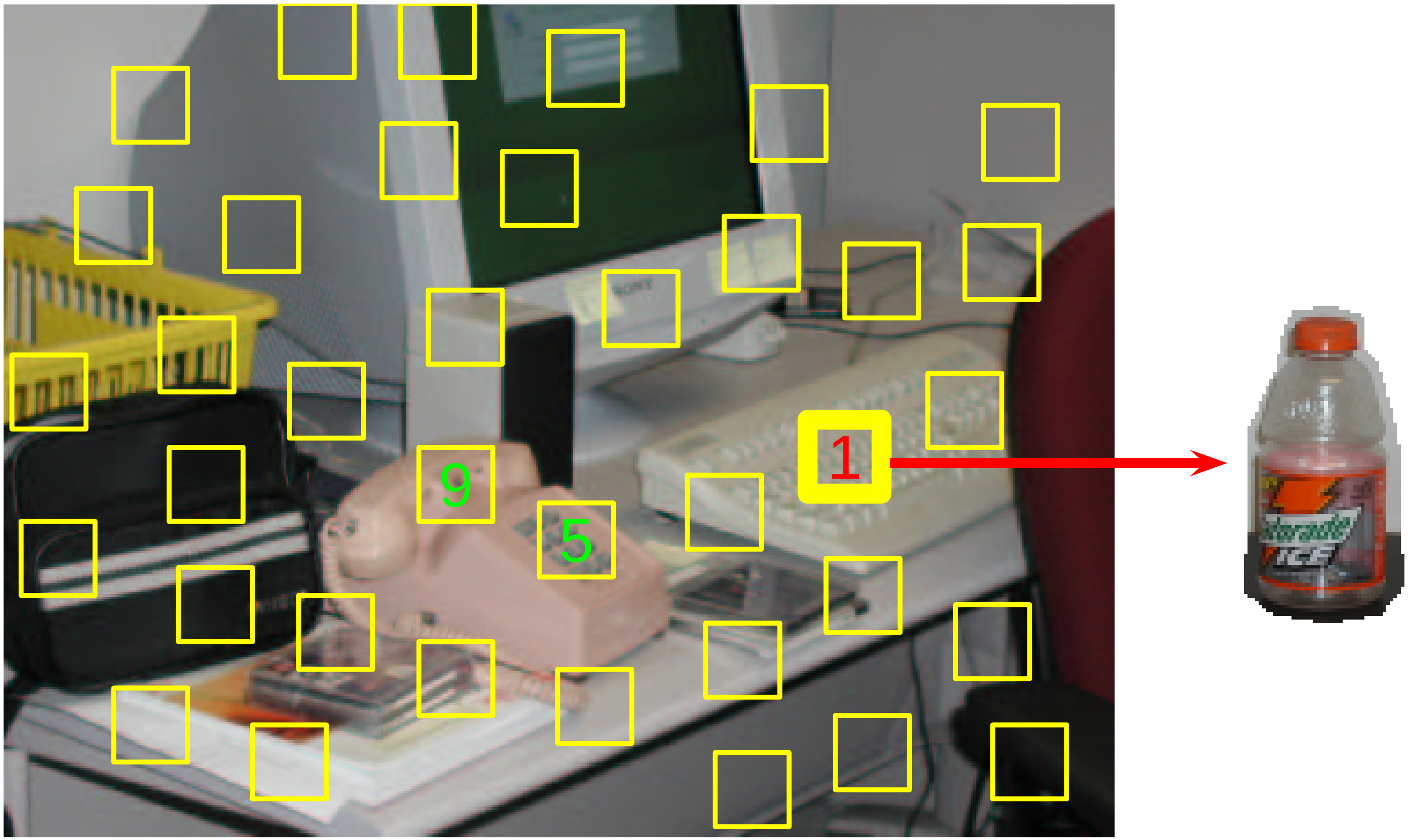
- one choice
- single feature
- content-free

Why Perceptual Organization

recognition

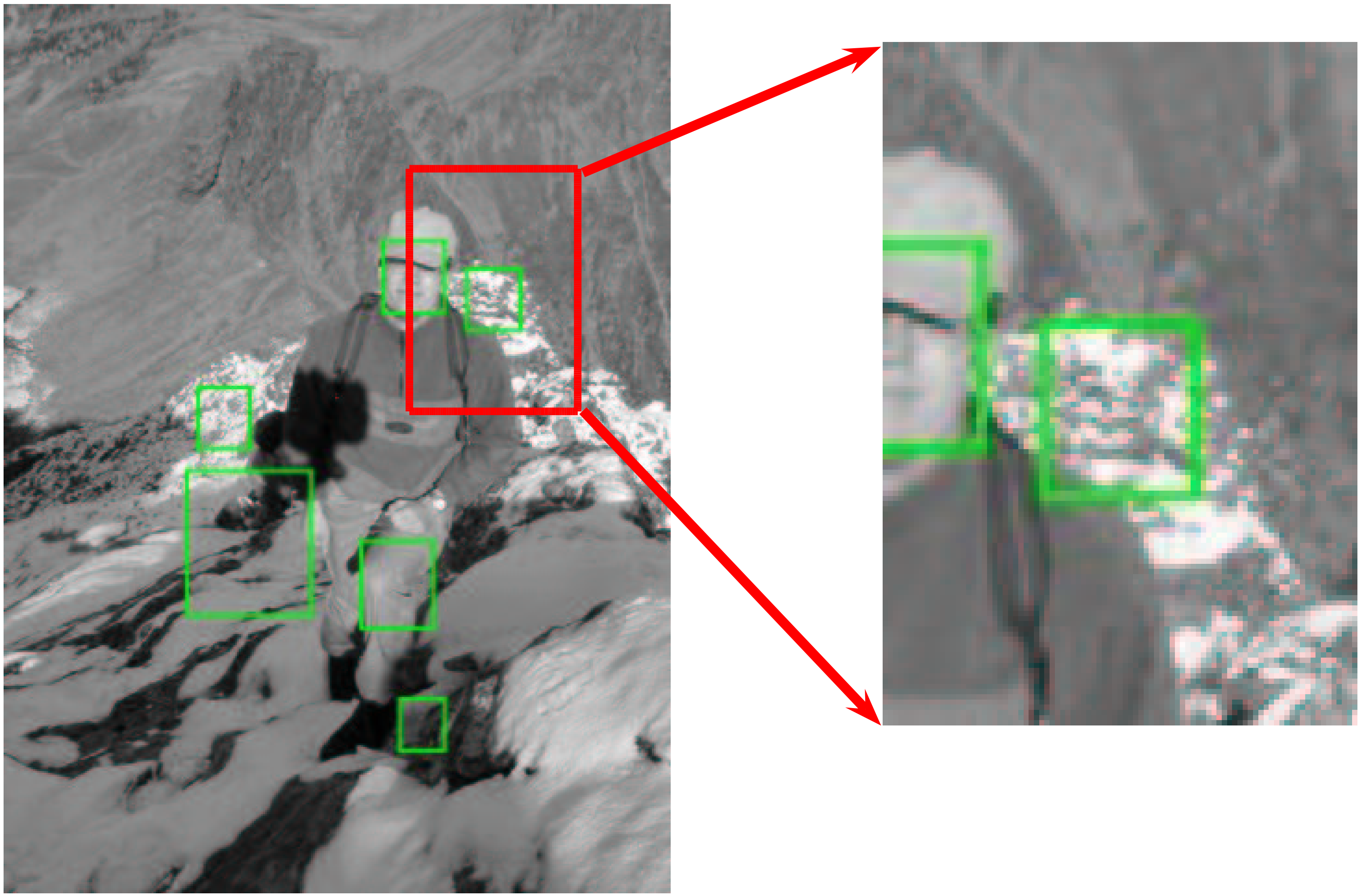
image

Why Perceptual Organization



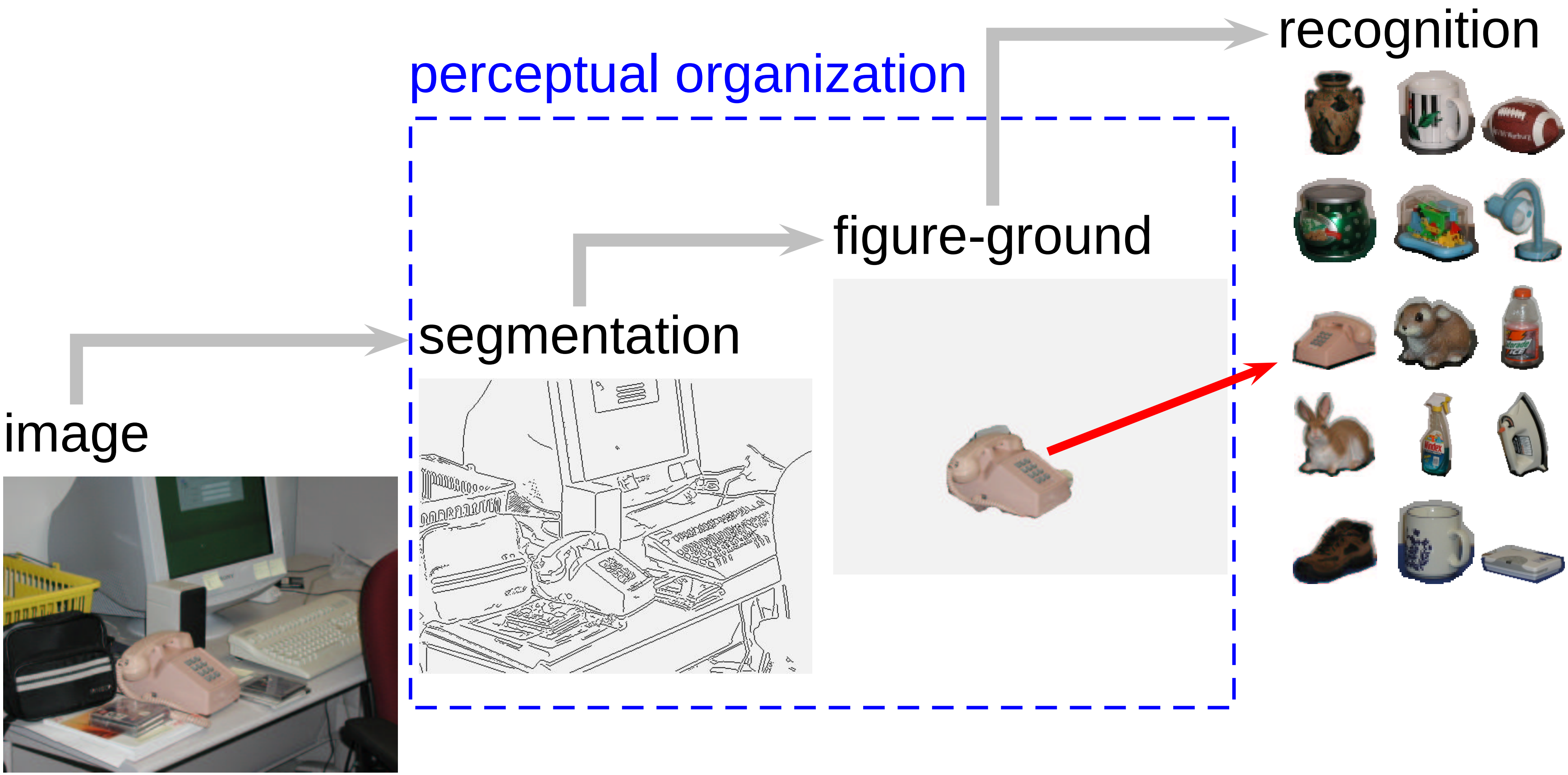
Mahamud multi-object detector

Why Perceptual Organization



Schneiderman face detector

Traditional Use of Perceptual Organization



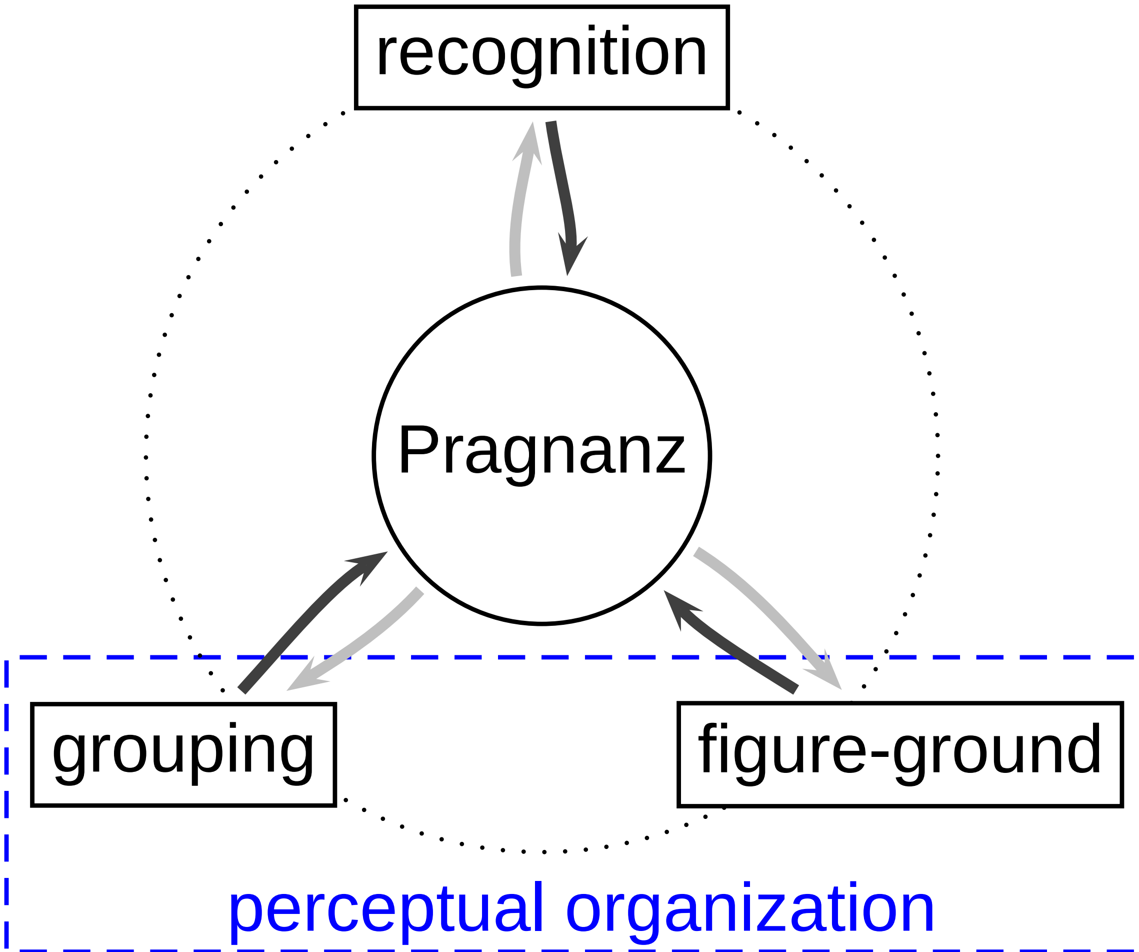
sequential processing (Marr, Lowe, Witkin, Tenenbaum, ...)

Perceptual Organization without Object Knowledge

difficult and brittle

(Canny, Geman & Geman, Shah & Mumford, Witkin, Jacobs, ...)

Our Overall Approach



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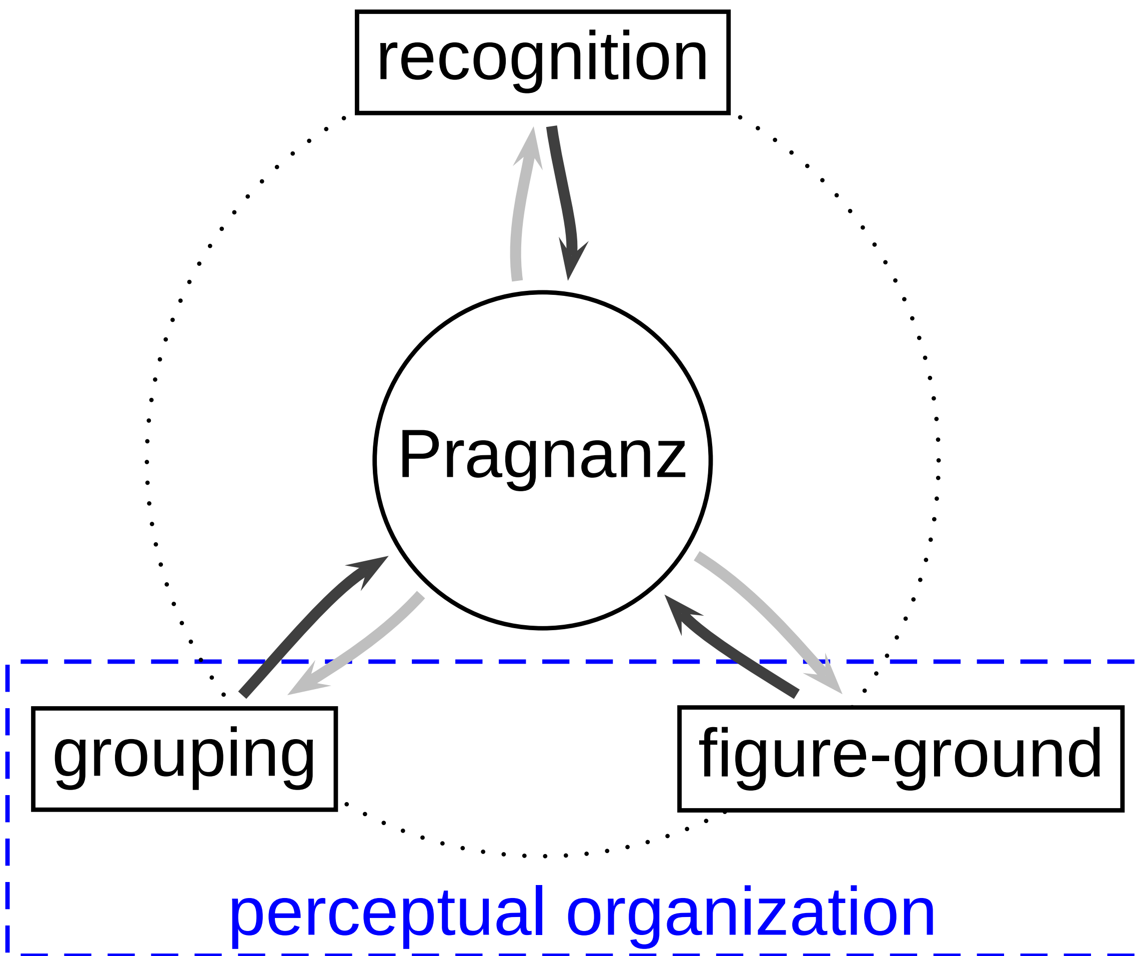
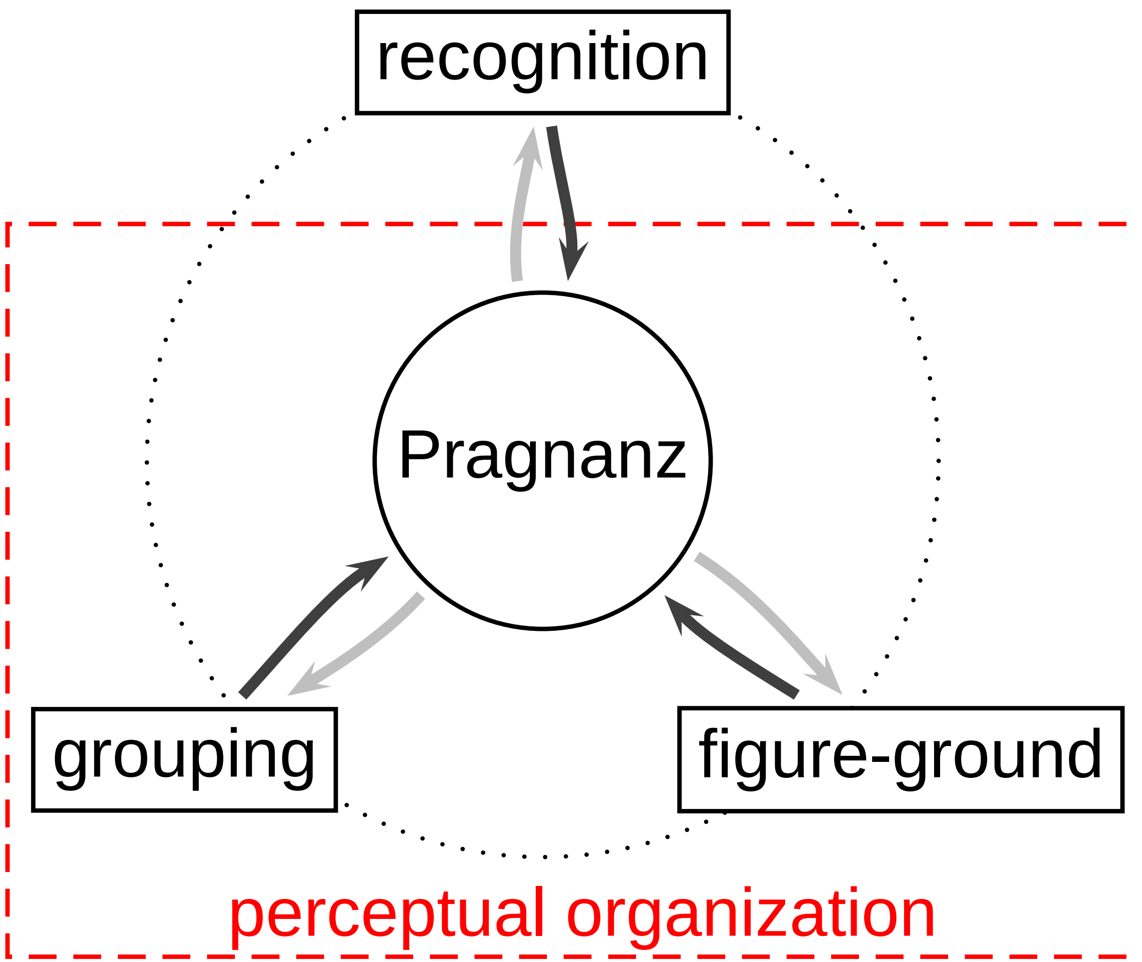


interactive processing (Grossberg, McClelland,Grenandar,Mumford,Lee,...)

Our Overall Approach



A criterion

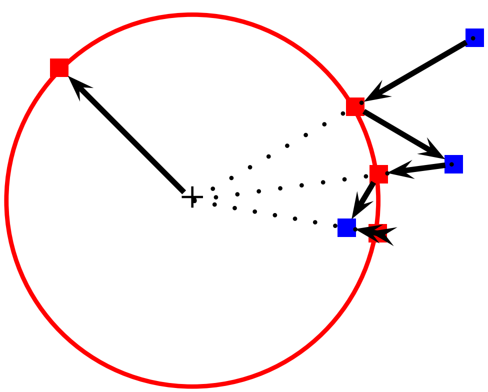
A fast solution

A wide range of images

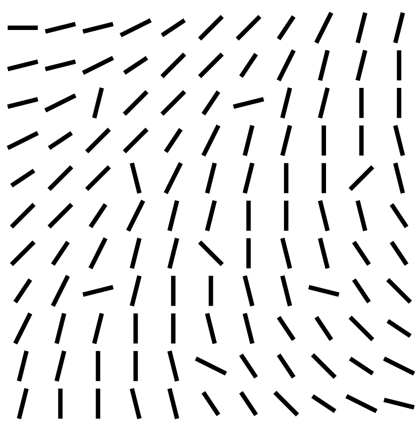
interactive processing (Grossberg, McClelland,Grenandar,Mumford,Lee,...)

Outline

1. Computational framework: spectral clustering
2. Expand the repertoire of grouping cues: dissimilarity
3. Guide grouping with partial cues
4. Guide grouping with object knowledge
5. Summary and future work



1



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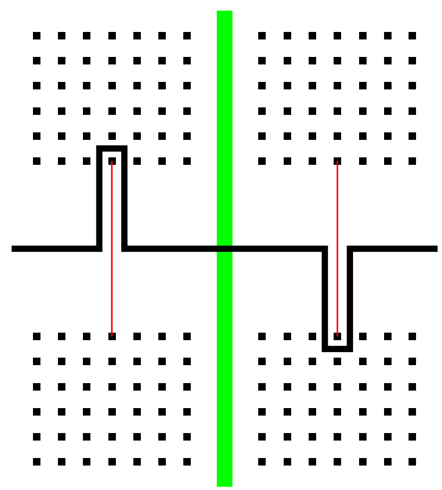

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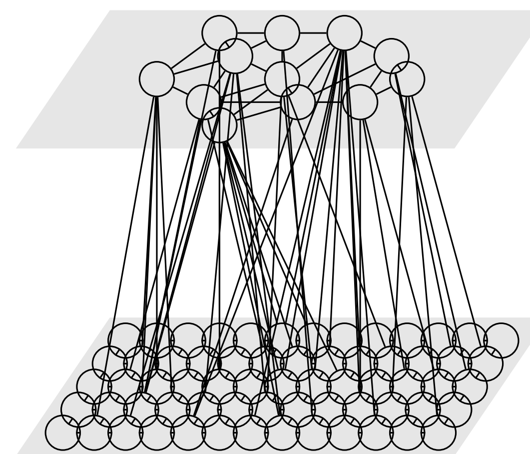
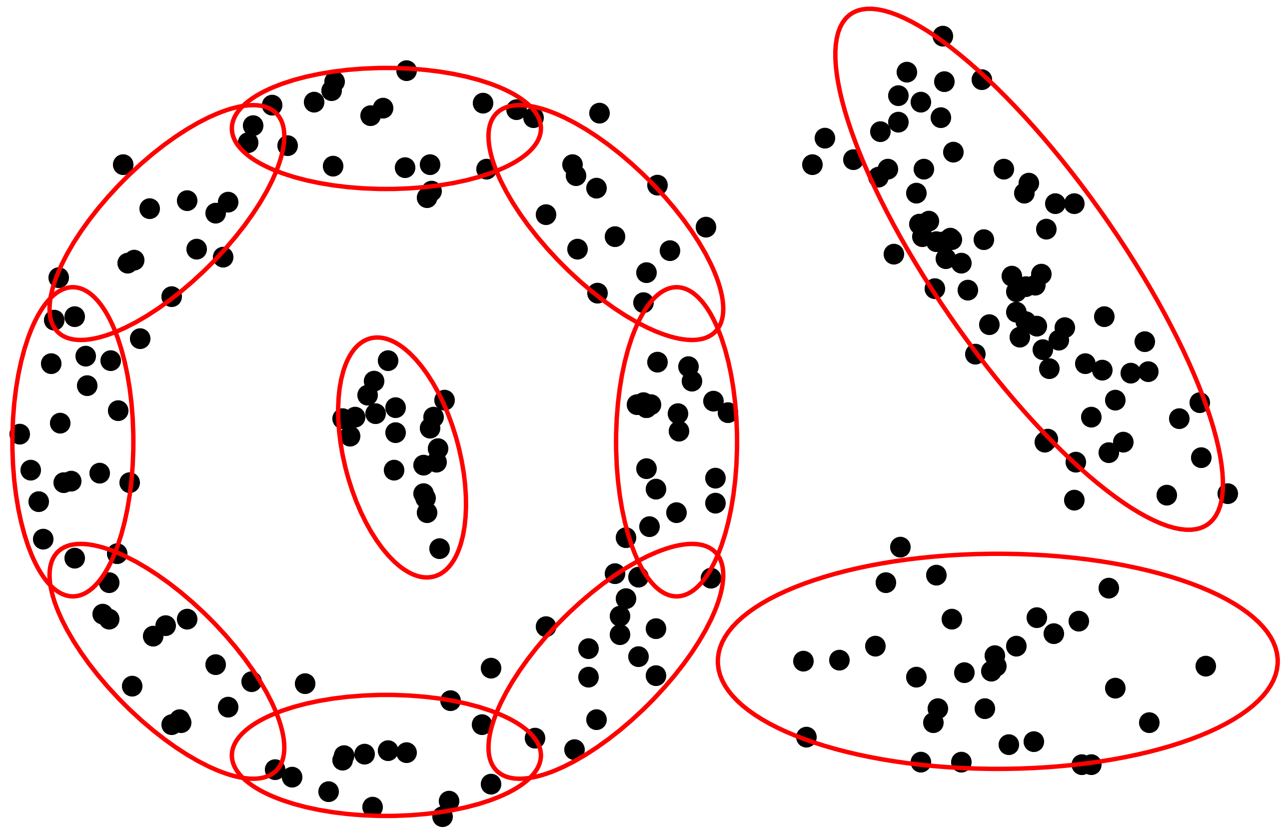


4

Generative Approach for Data Clustering

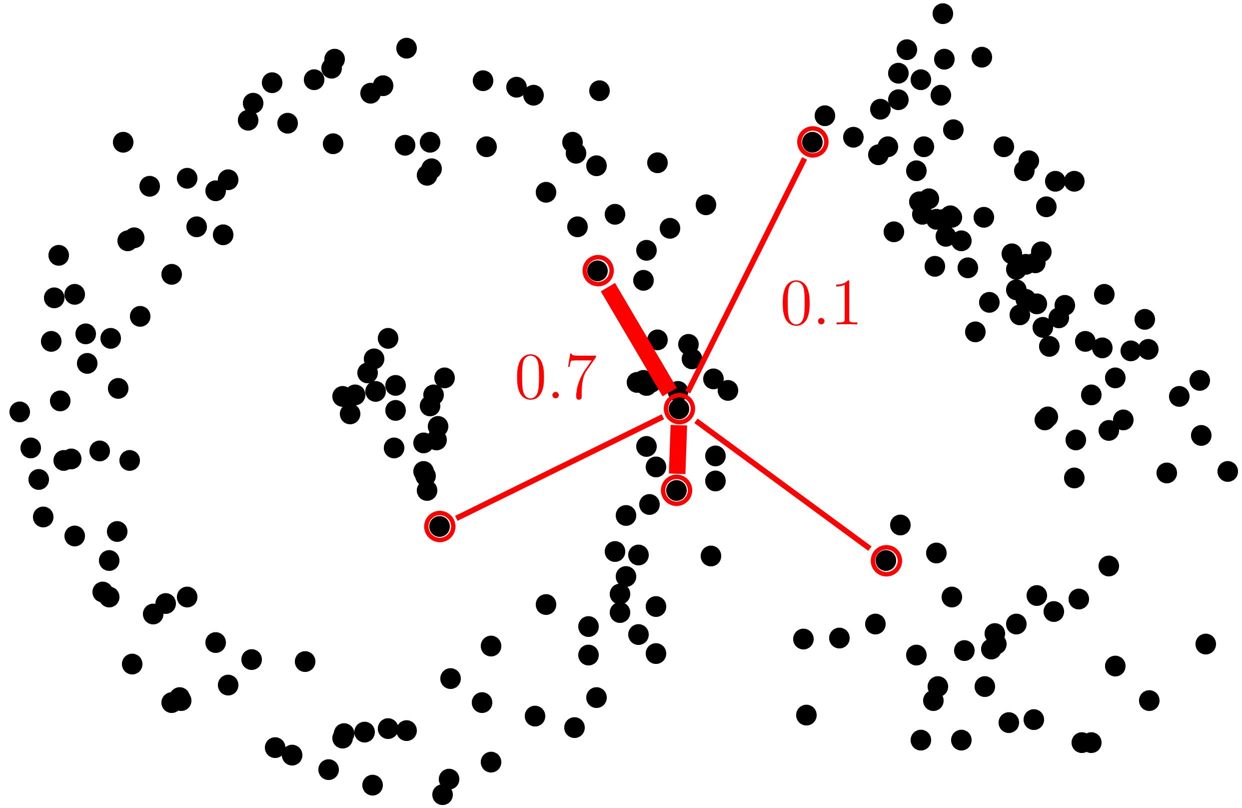


Key: Assumptions on the global structure of the data

Pros: Intuitive interpretation; analysis = synthesis

Cons: Model inadequacy and computational intractability

Discriminative Approach for Clustering

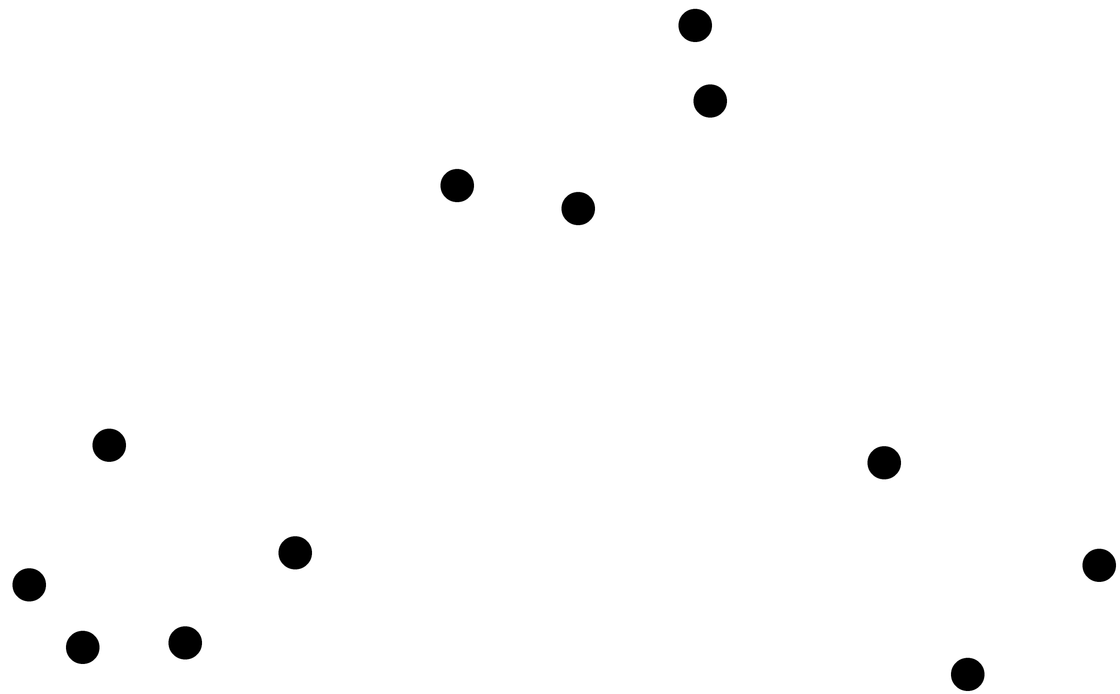


Key: Same group or not

Pros: Adaptable to all data structures; tractable computation

Cons: No interpretation of the groups

Grouping in a Graph-Theoretic Framework



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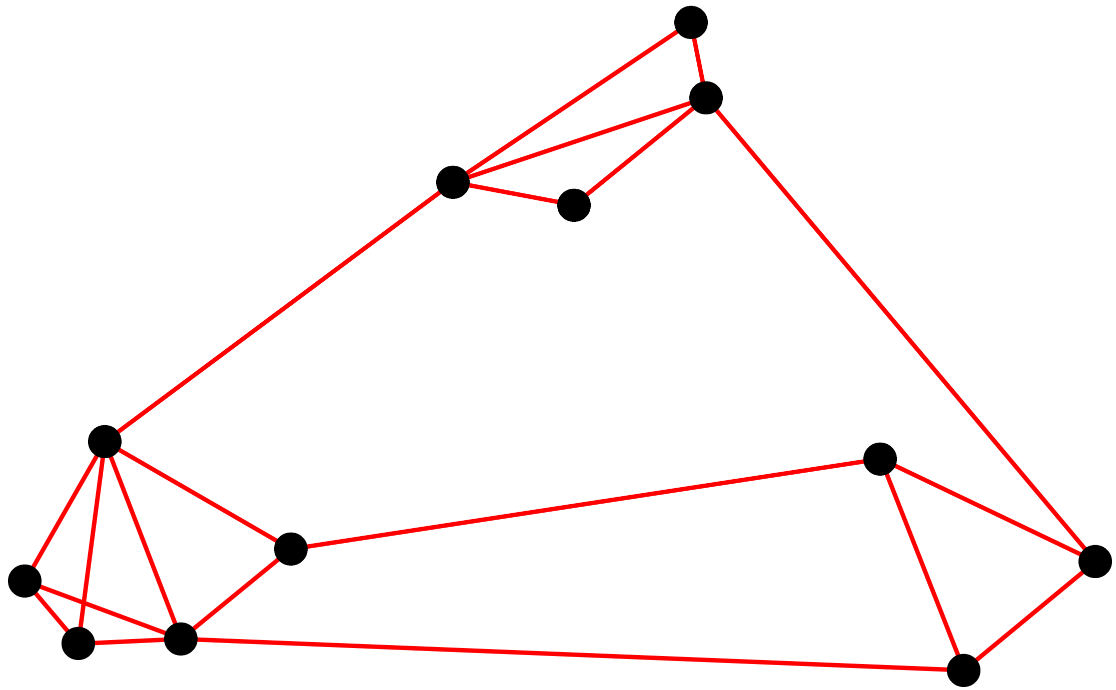

Representation: $\mathbb{G} = \{\mathbb{V}, \mathbb{E}, W\} = \{$ nodes, edges, weights $\}$

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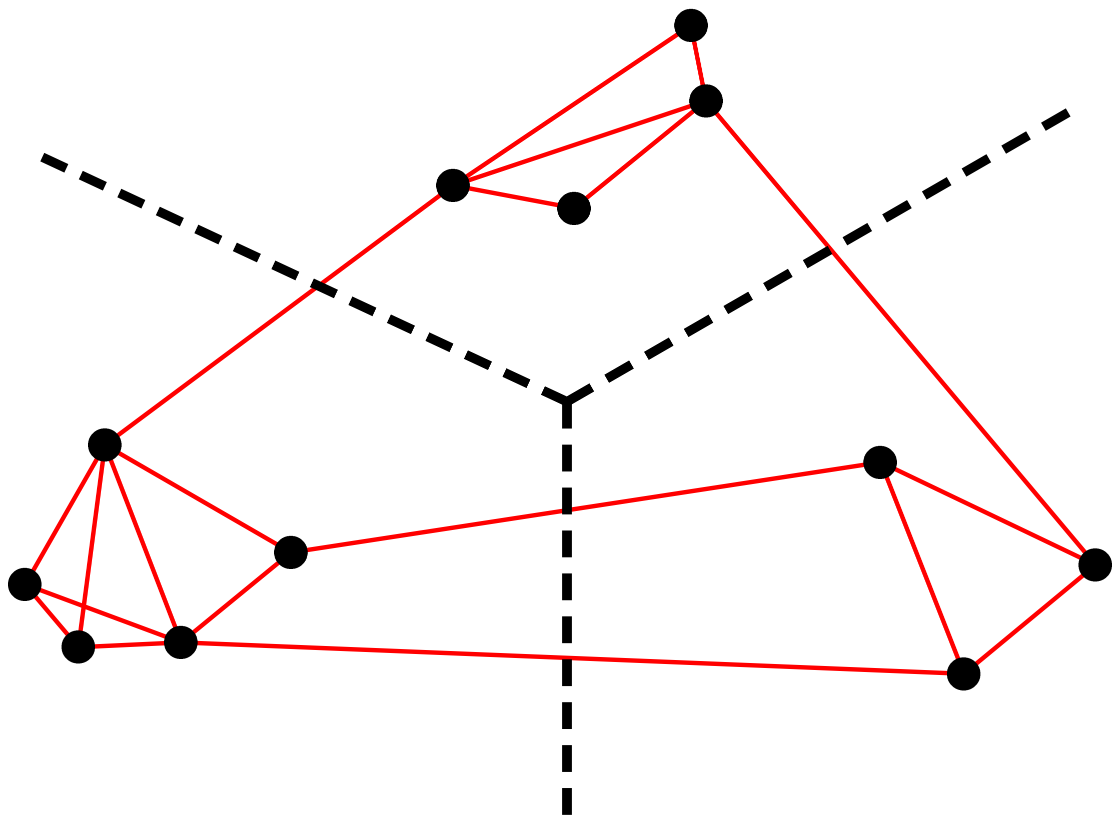
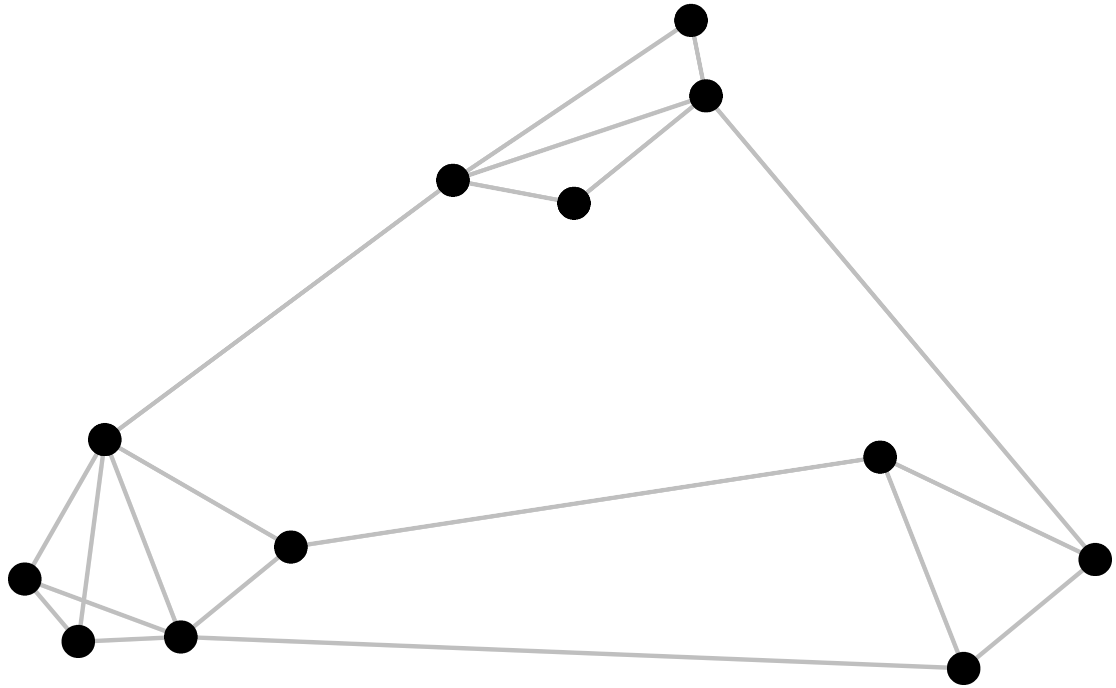


Representation: $\mathbb{G} = \{\mathbb{V}, \mathbb{E}, W\} = \{$ nodes, edges, weights $\}$

Clustering: $\Gamma_{\mathbb{V}}^{K} = \{\mathbb{V}_1, \ldots, \mathbb{V}_K\} = K$-way node partitioning

(Shi & Malik, Zabih, Boykov, Veksler, Kolmogorov,...)

Links in Graph Cuts



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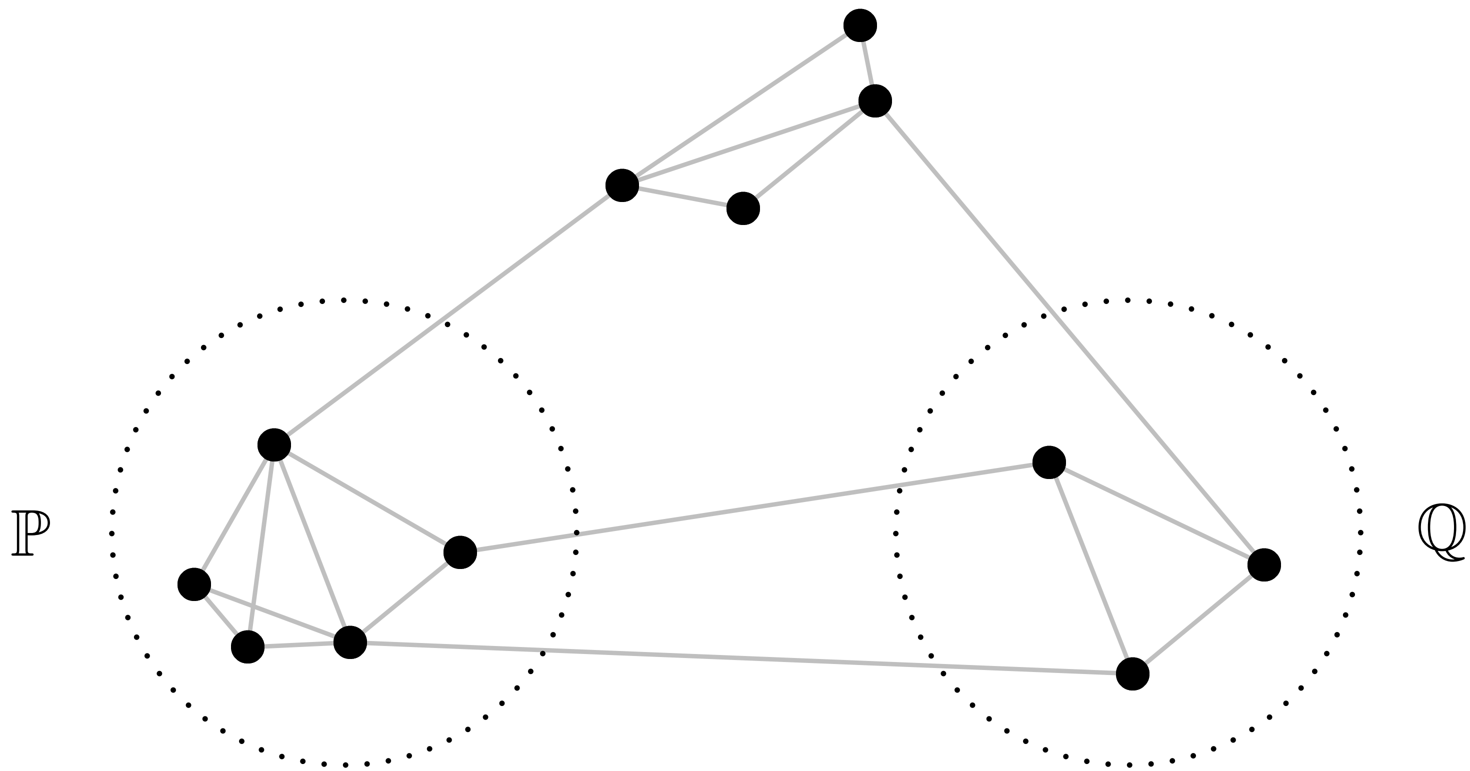

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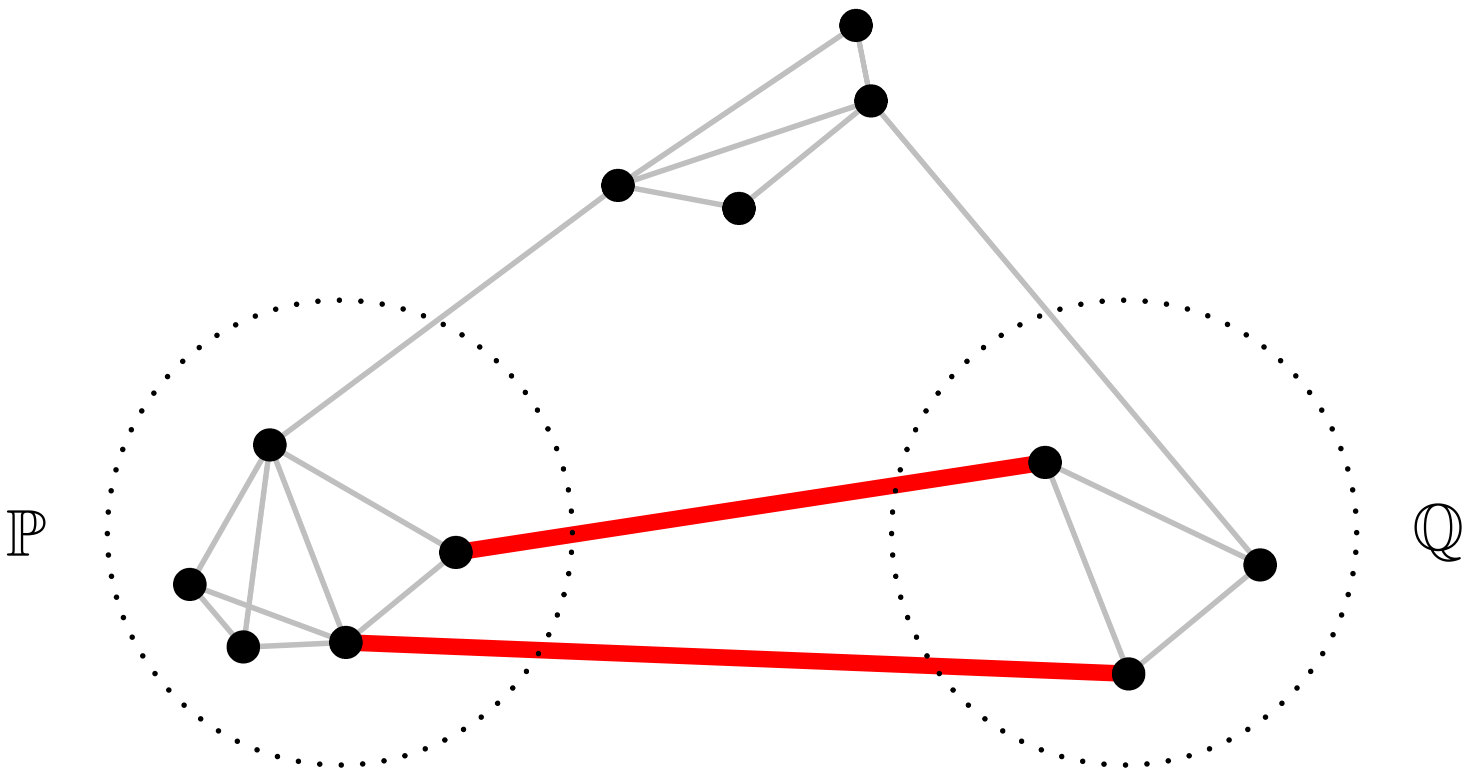
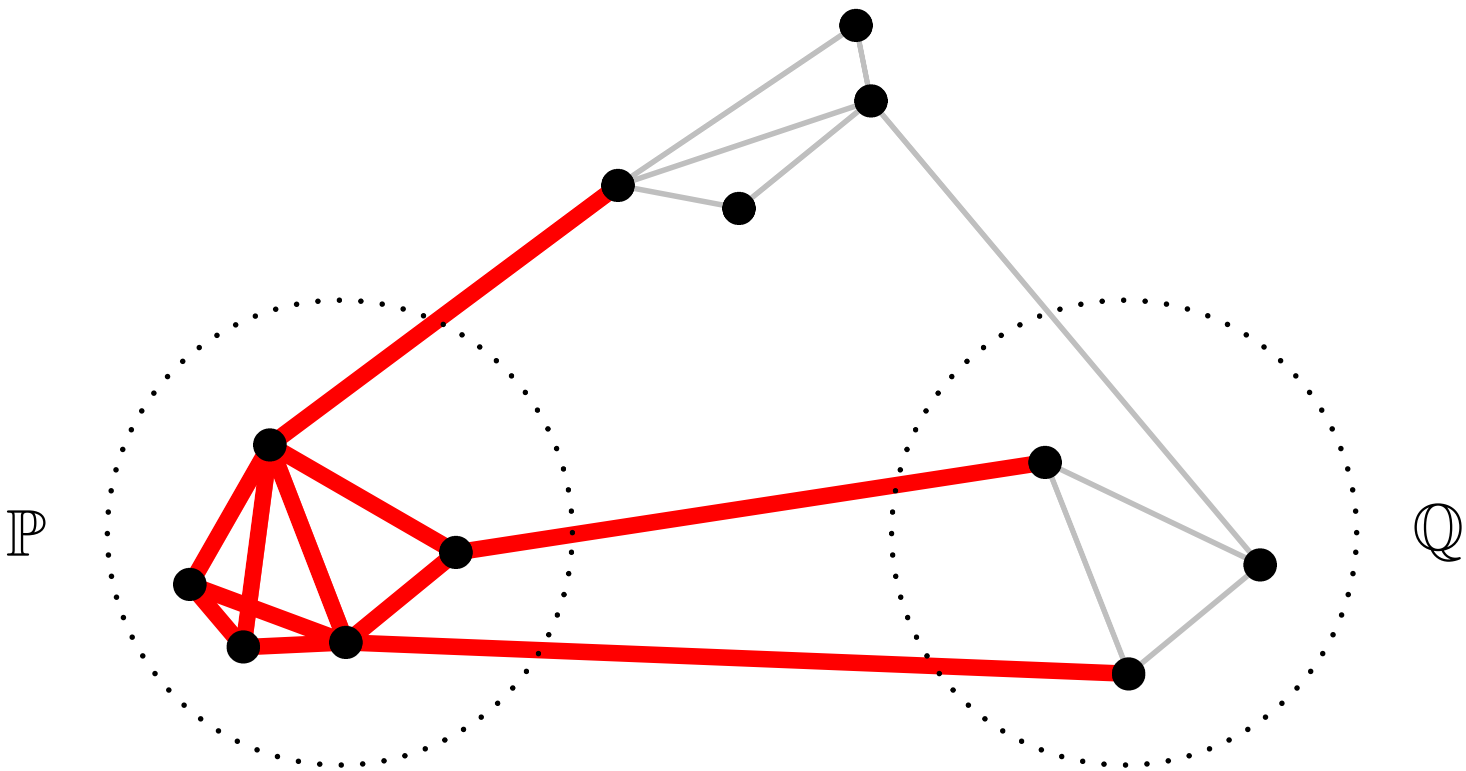


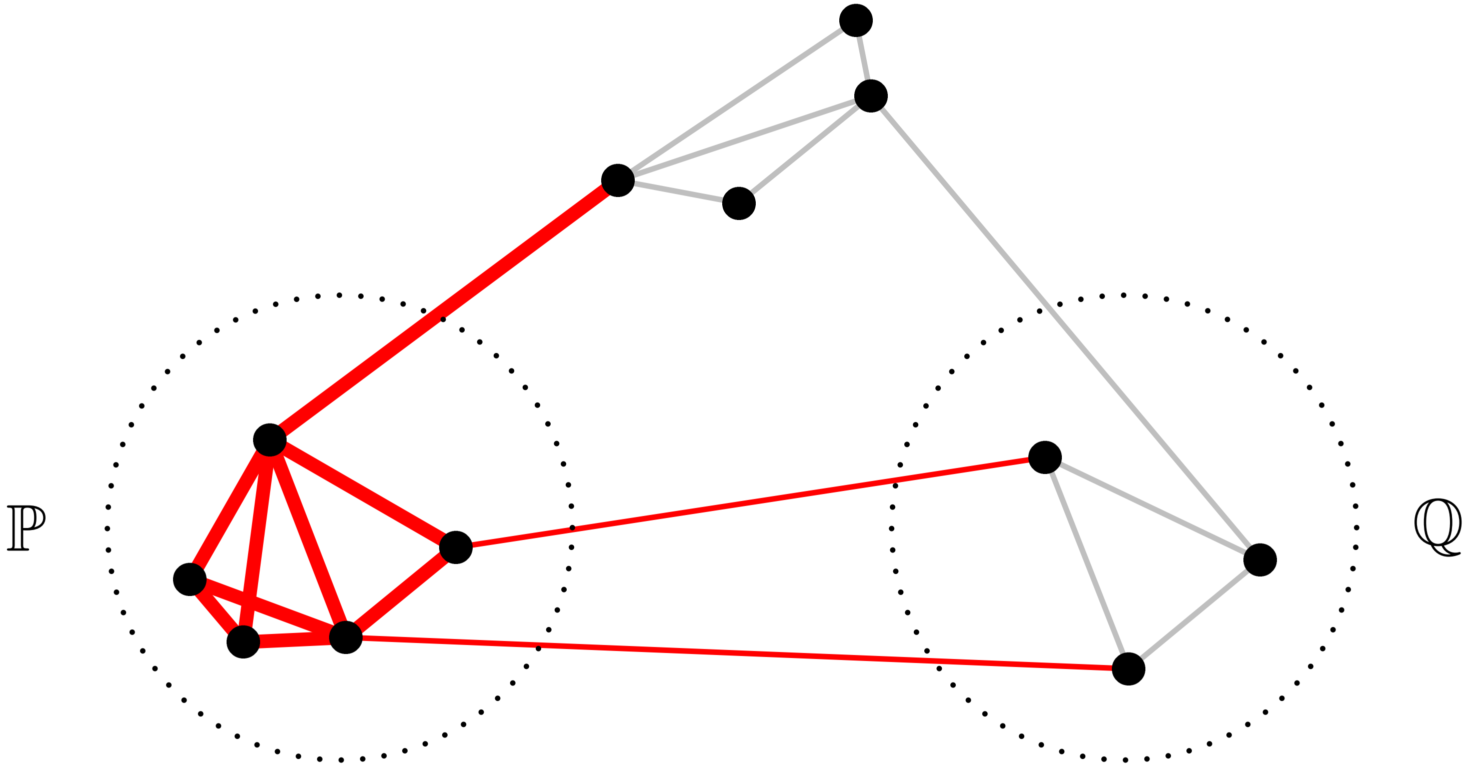
$$\text{links}(\mathbb{P}, \mathbb{Q}) = \sum_{p \in \mathbb{P},\, q \in \mathbb{Q}} W(p, q)$$

Degree in Graph Cuts



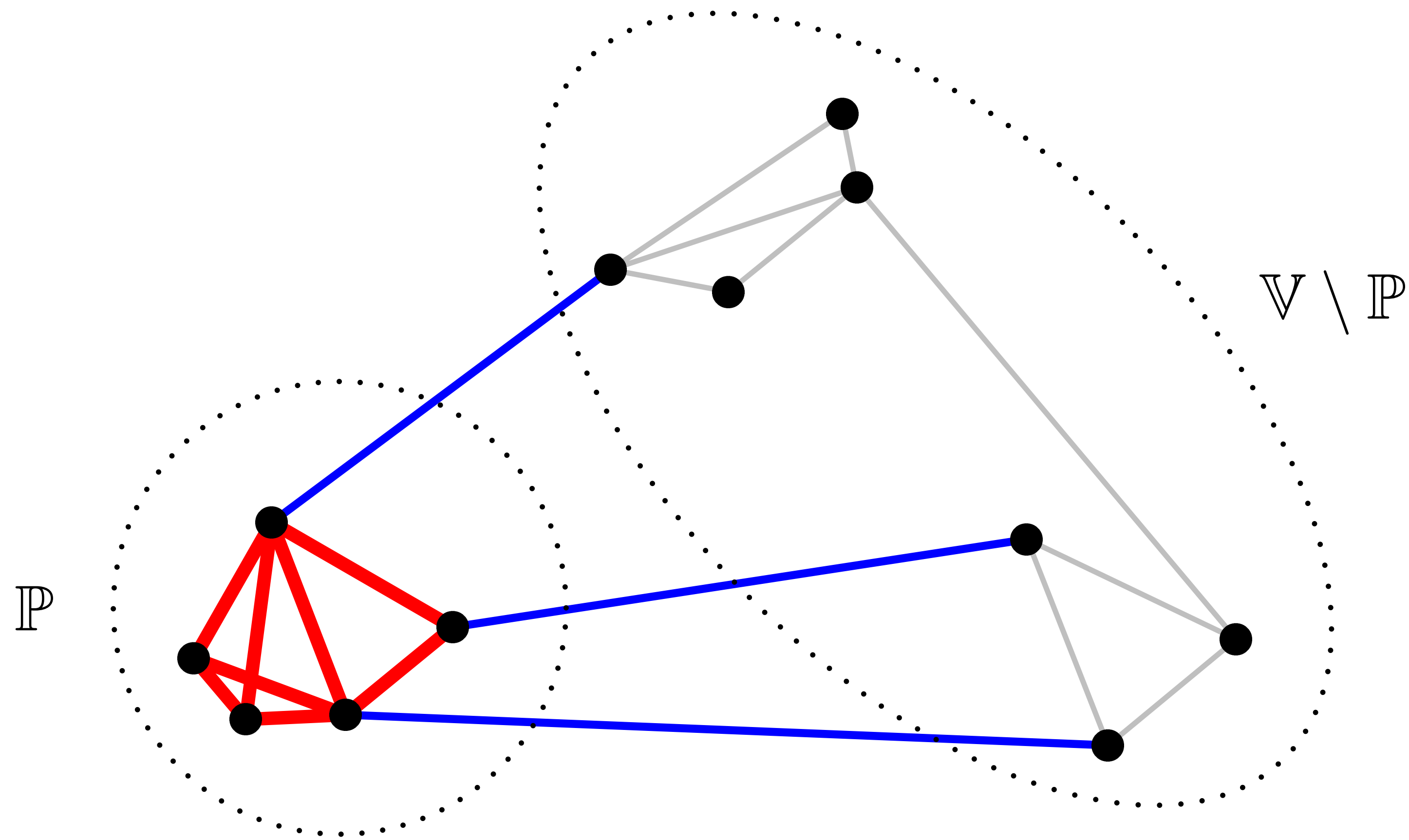
$$\text{degree}(\mathbb{P}) = \text{links}(\mathbb{P}, \mathbb{V})$$

Linkratio in Graph Cuts



$$\text{linkratio}(\mathbb{P}, \mathbb{Q}) = \frac{\text{links}(\mathbb{P}, \mathbb{Q})}{\text{degree}(\mathbb{P})}$$

Goodness of Grouping in Graph Cuts

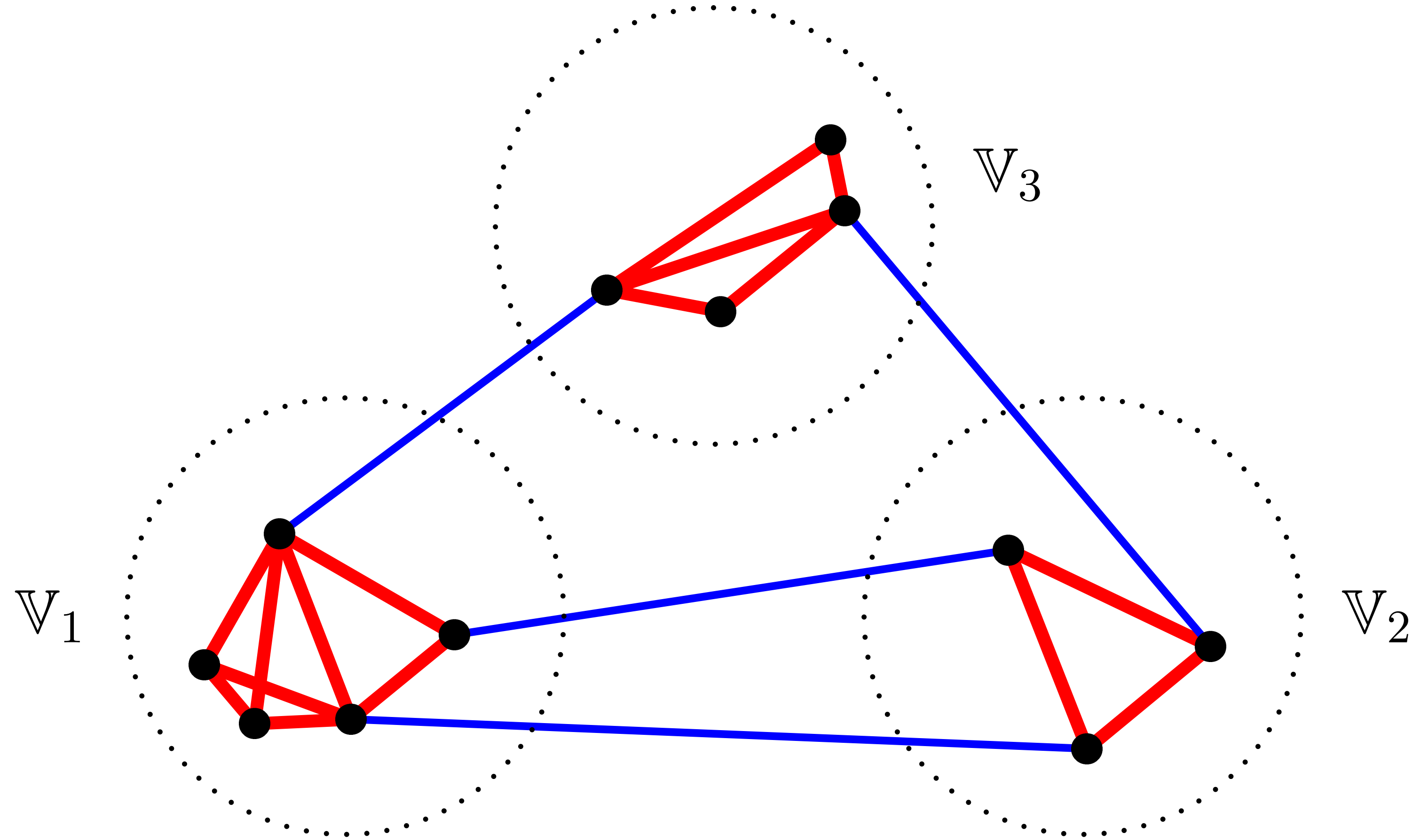


Maximize within-group connections: $\mathrm{linkratio}(\mathbb{P}, \mathbb{P})$

Minimize between-group connections: $\mathrm{linkratio}(\mathbb{P}, \mathbb{V} \setminus \mathbb{P})$

Equivalent: $\mathrm{linkratio}(\mathbb{P}, \mathbb{P}) + \mathrm{linkratio}(\mathbb{P}, \mathbb{V} \setminus \mathbb{P}) = 1$

K-Way Normalized Cuts



$$\max \quad \text{knassoc}(\Gamma_{\mathbb{V}}^{K}) = \frac{1}{K}\sum_{l=1}^{K} \text{linkratio}(\mathbb{V}_l, \mathbb{V}_l)$$

$$\min \quad \text{kncuts}(\Gamma_{\mathbb{V}}^{K}) = \frac{1}{K}\sum_{l=1}^{K} \text{linkratio}(\mathbb{V}_l, \mathbb{V} \setminus \mathbb{V}_l)$$

A Principled Solution to Normalized Cuts

$$\max \quad \mathrm{knassoc}(\Gamma_{\mathbb{V}}^{K}) = \frac{1}{K}\sum_{l=1}^{K} \mathrm{linkratio}(\mathbb{V}_l, \mathbb{V}_l)$$

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NP complete even for $K = 2$ and planar graphs

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Fast solution to find near-global optima:

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Fast solution to find near-global optima:

1. Find global optima in the relaxed continuous domain

   optima = eigenvectors of $(W, D) \times$ rotations

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NP complete even for $K = 2$ and planar graphs

Fast solution to find near-global optima:

1. Find global optima in the relaxed continuous domain
   optima = eigenvectors of $(W, D) \times$ rotations
2. Find a discrete solution closest to continuous optima
   closeness = measured in L_2 norm between solutions

Step 1: Find Continuous Global Optima

$$\overset{+}{O}$$

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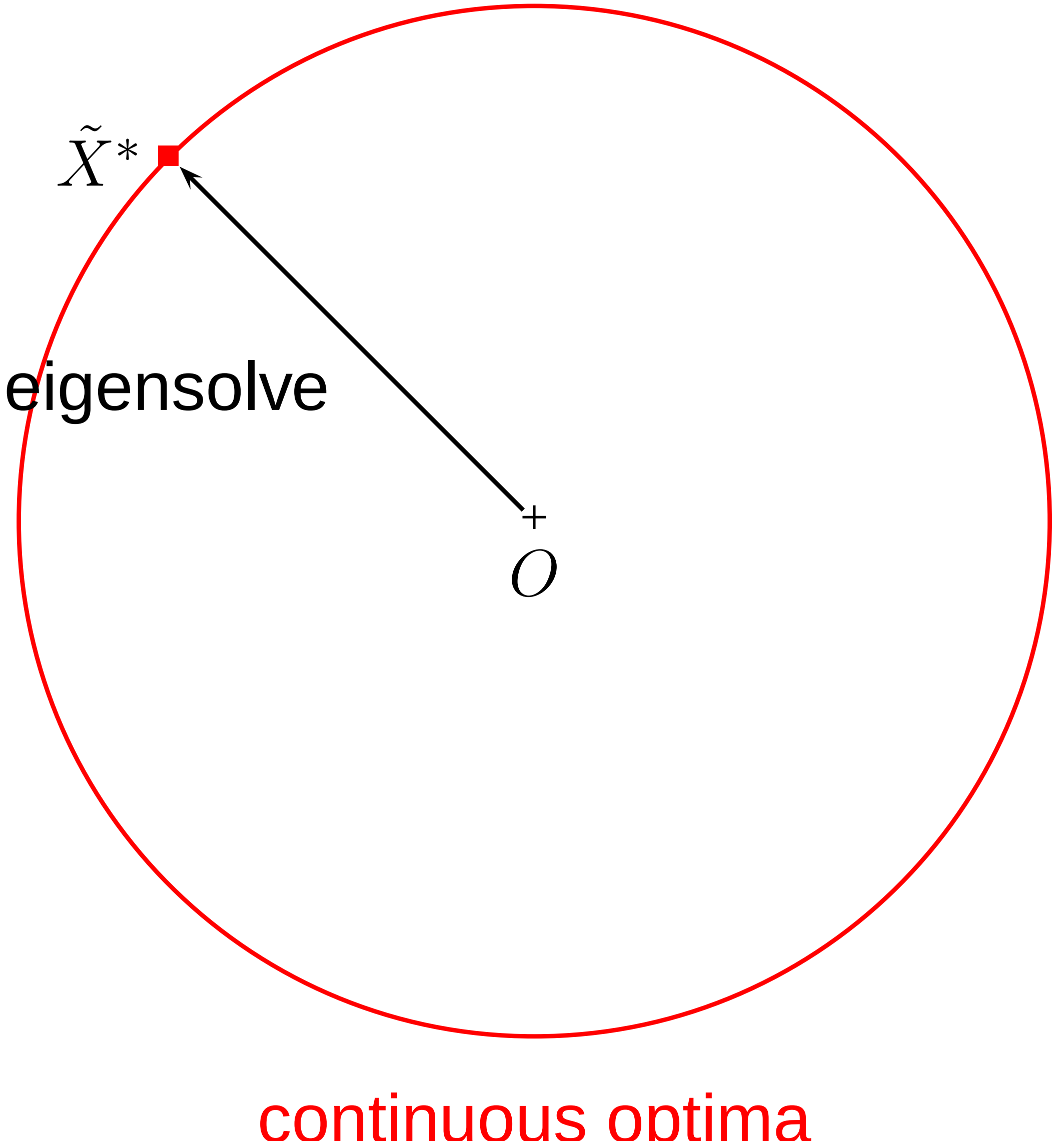
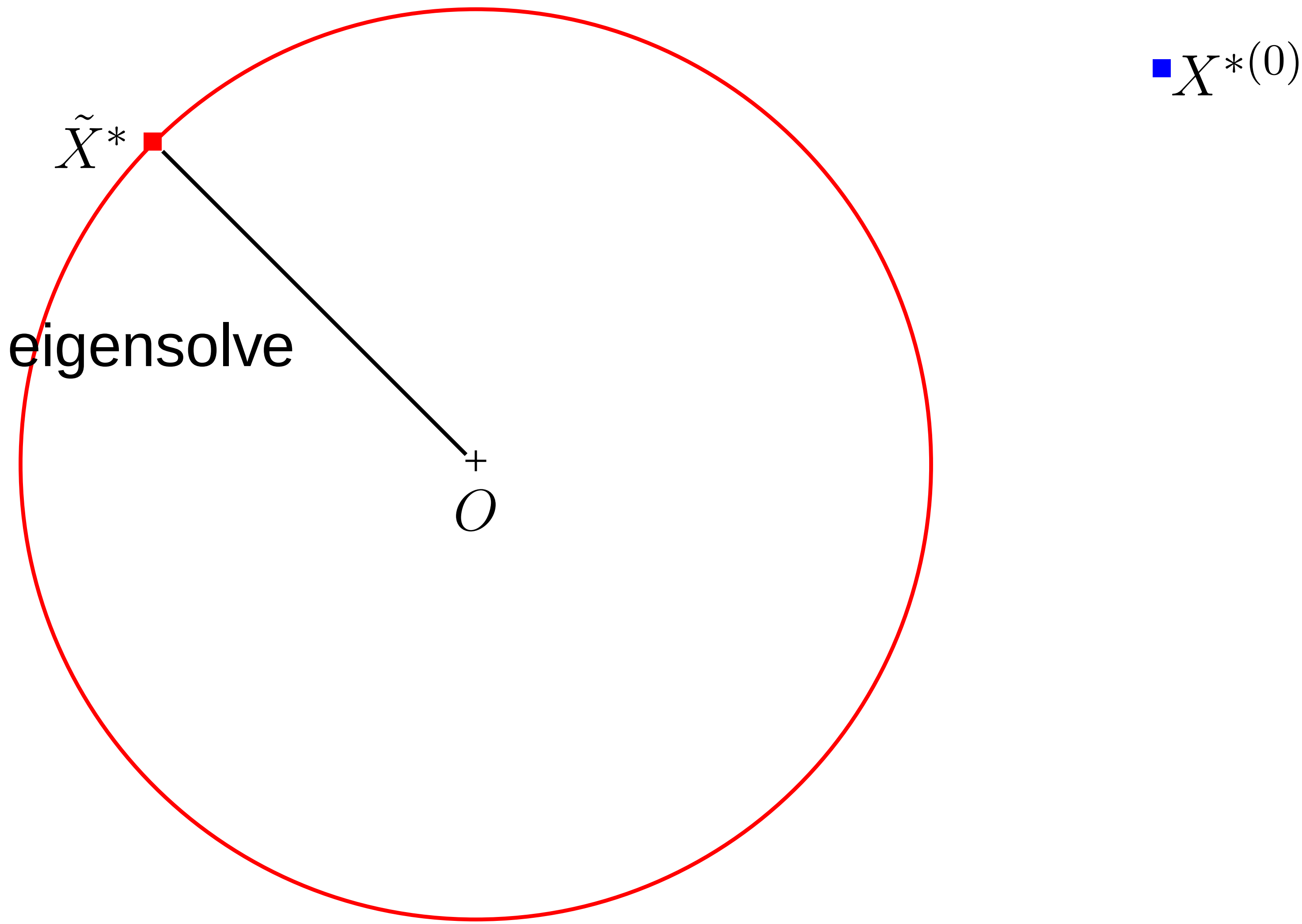


Step 2: Discretize Continuous Optima



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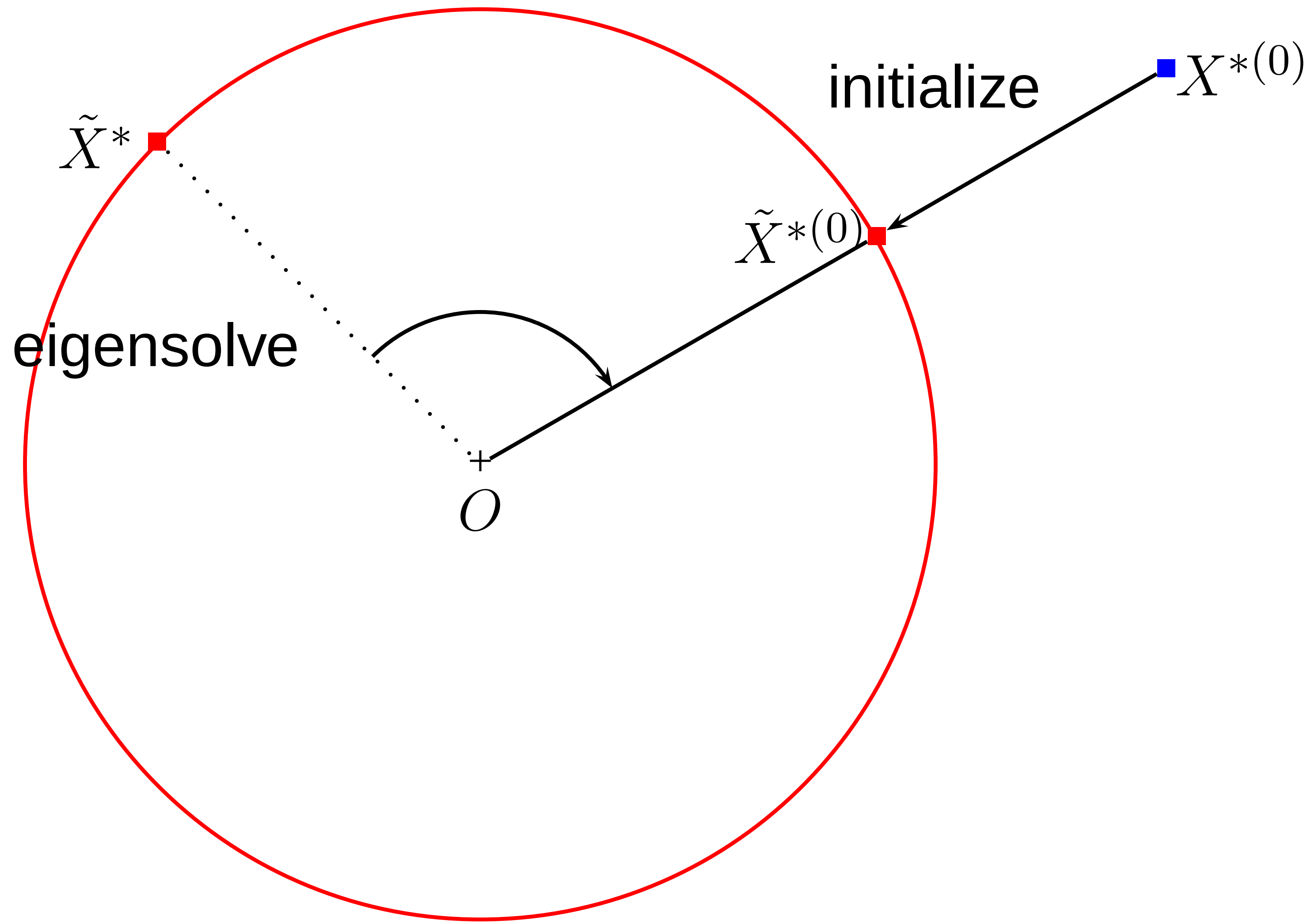

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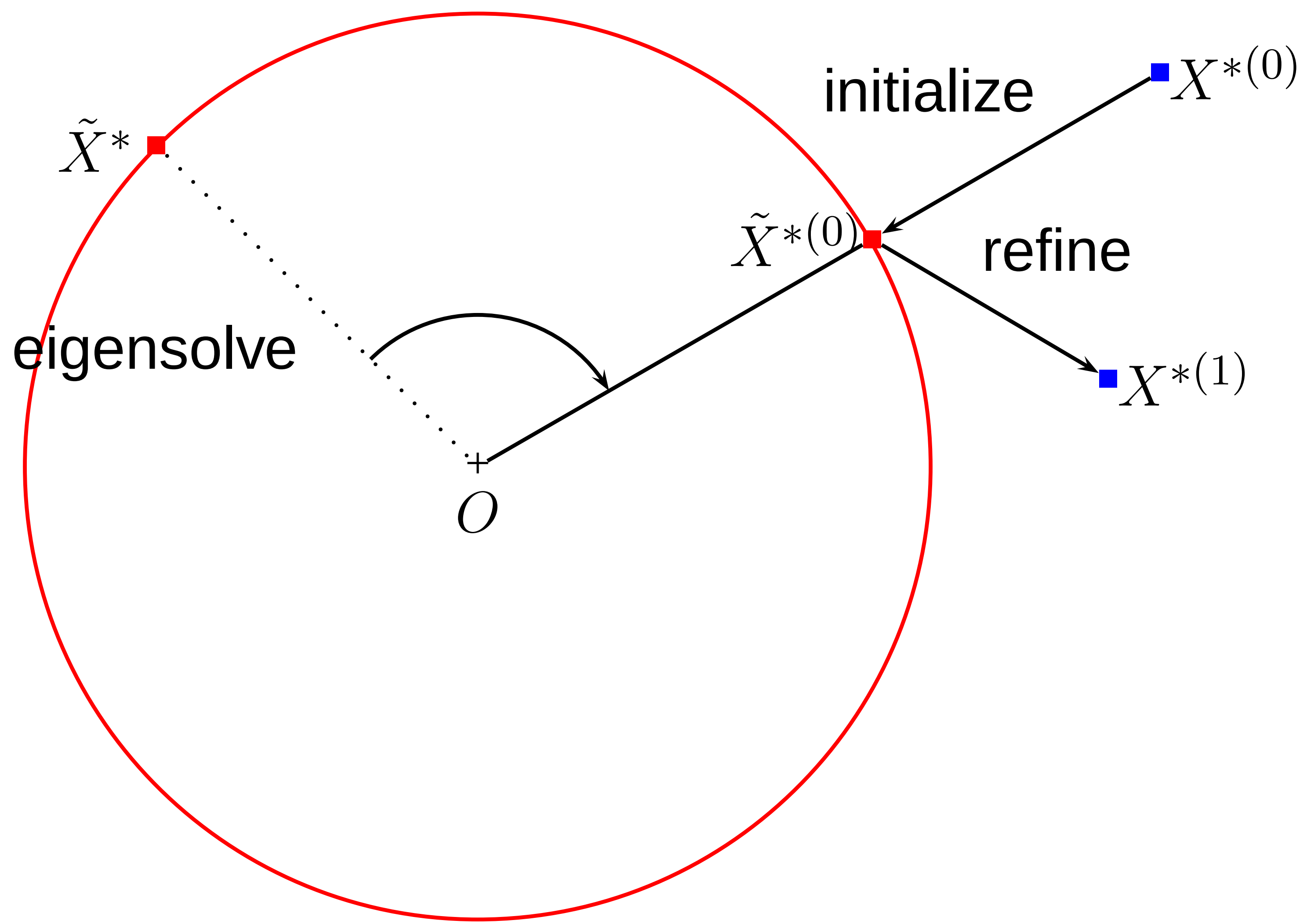

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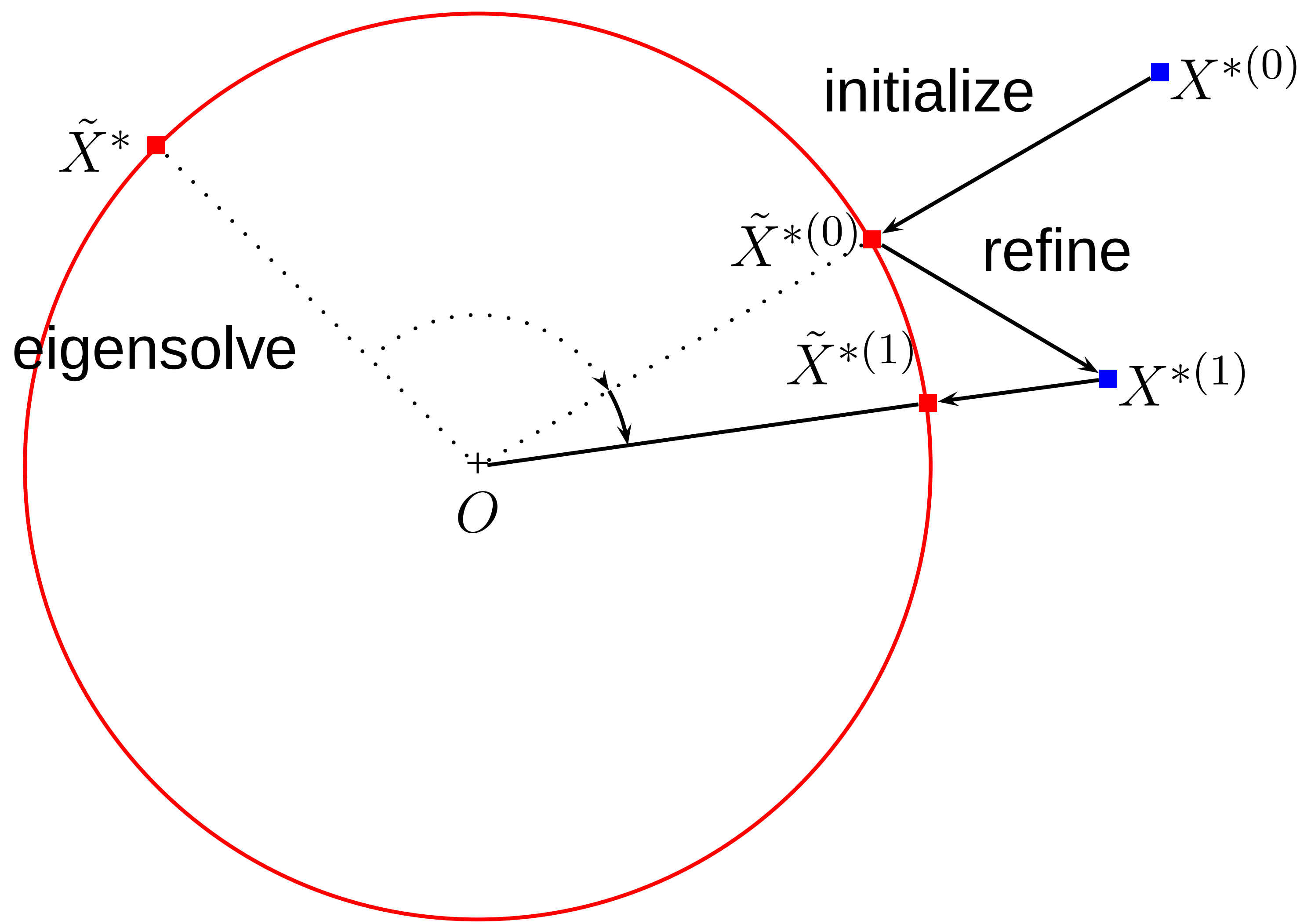

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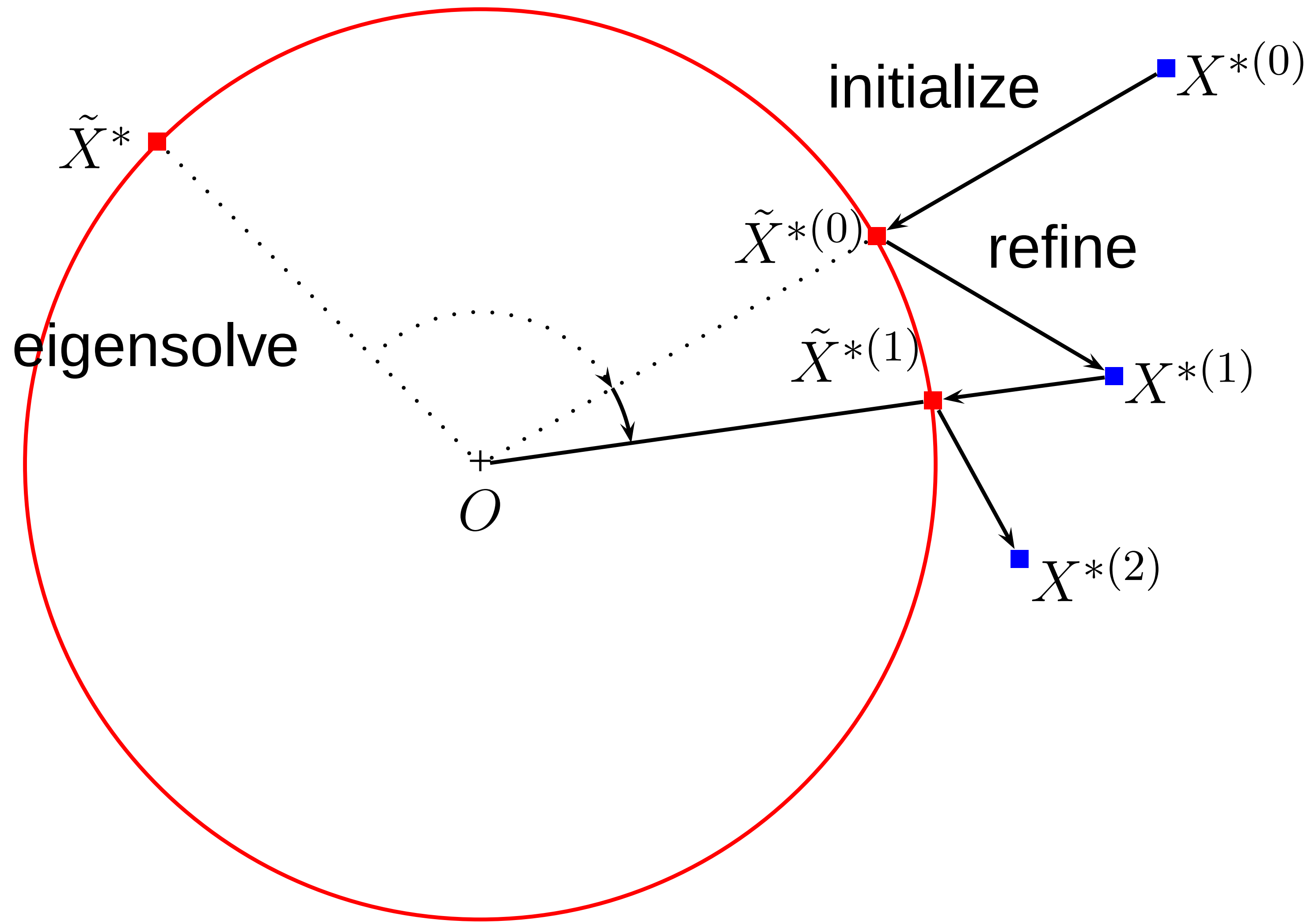

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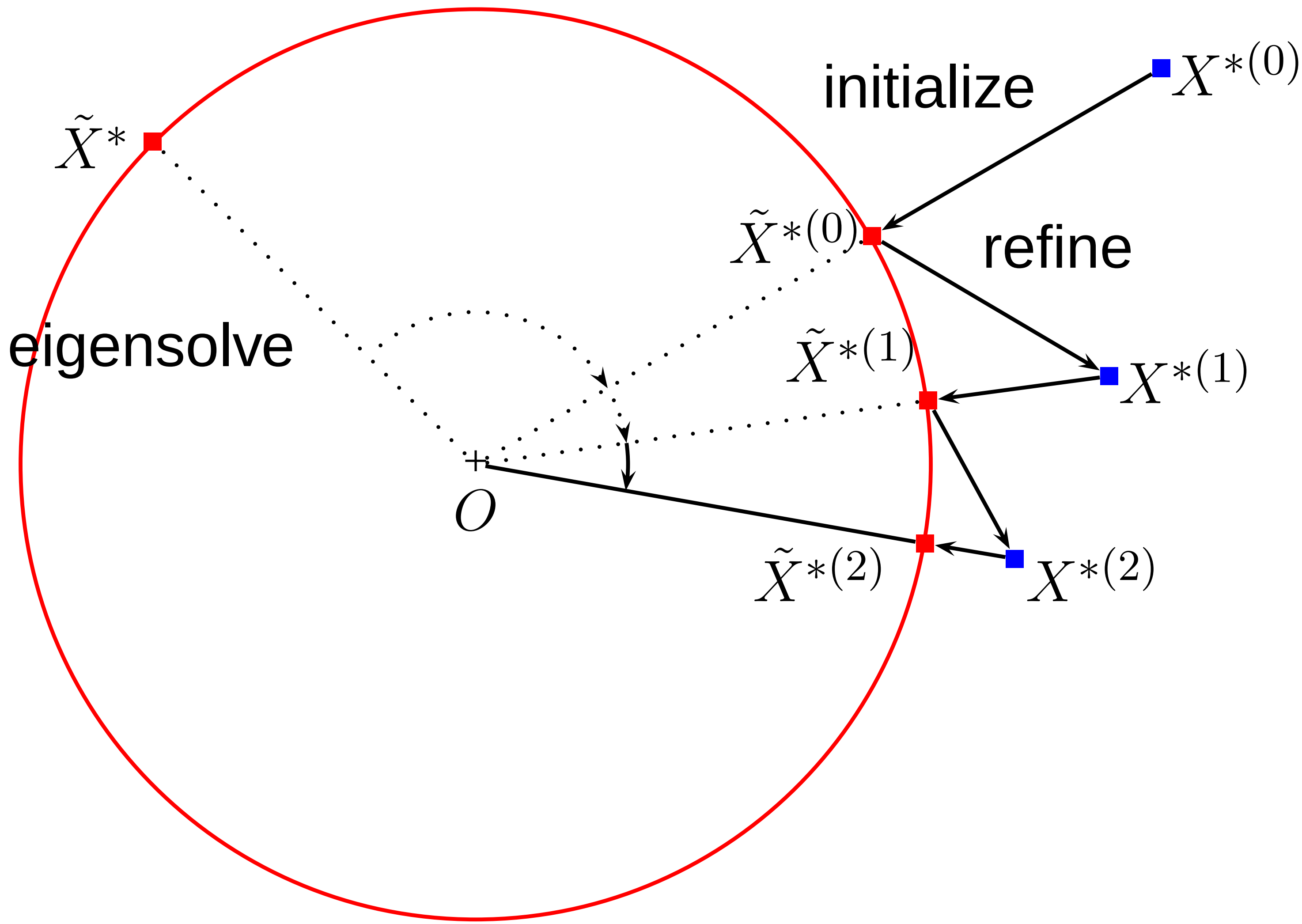

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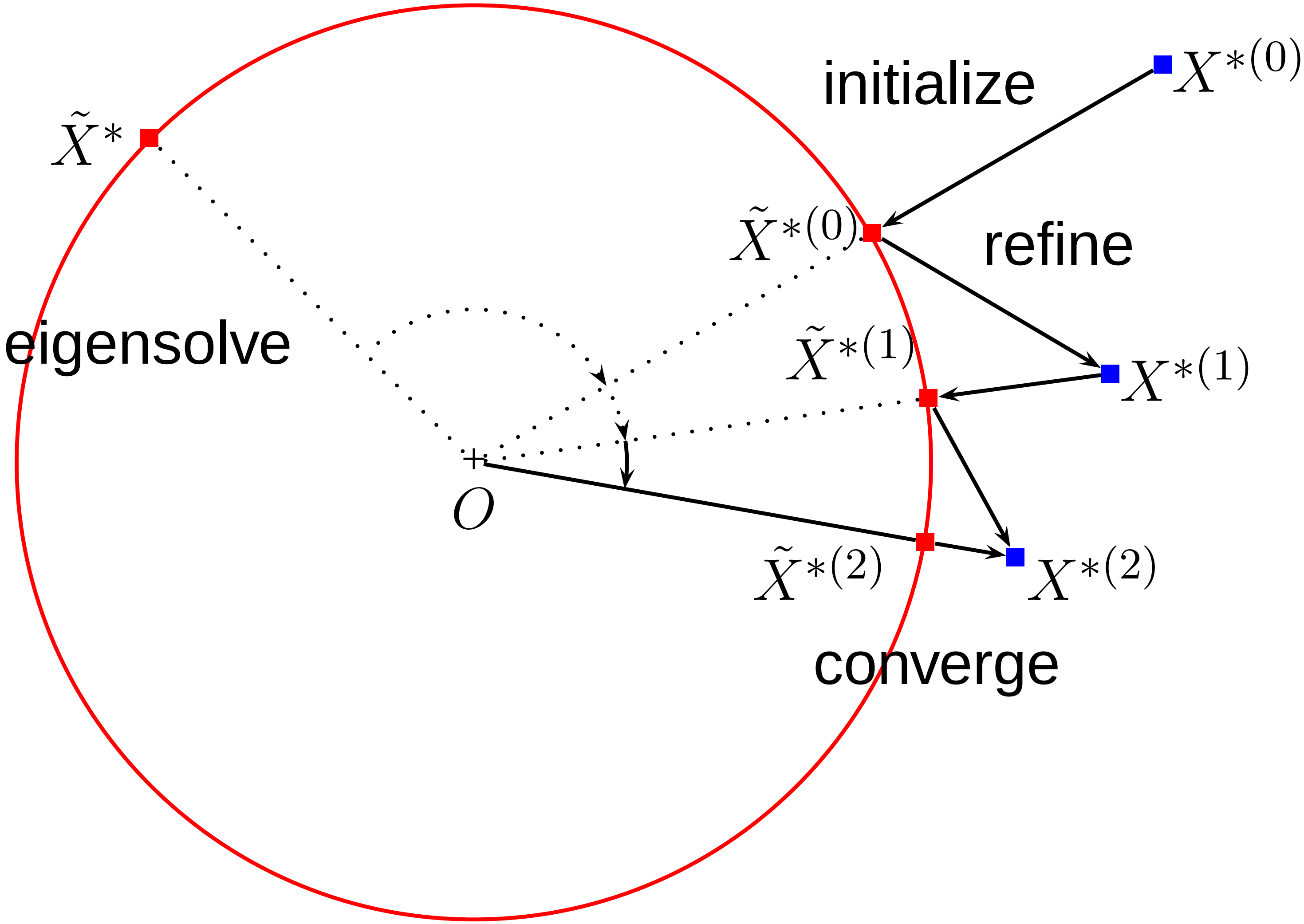


Final solution: $(X^{*(2)}, \tilde{X}^{*(2)})$

Pixel Similarity based on Intensity Edges

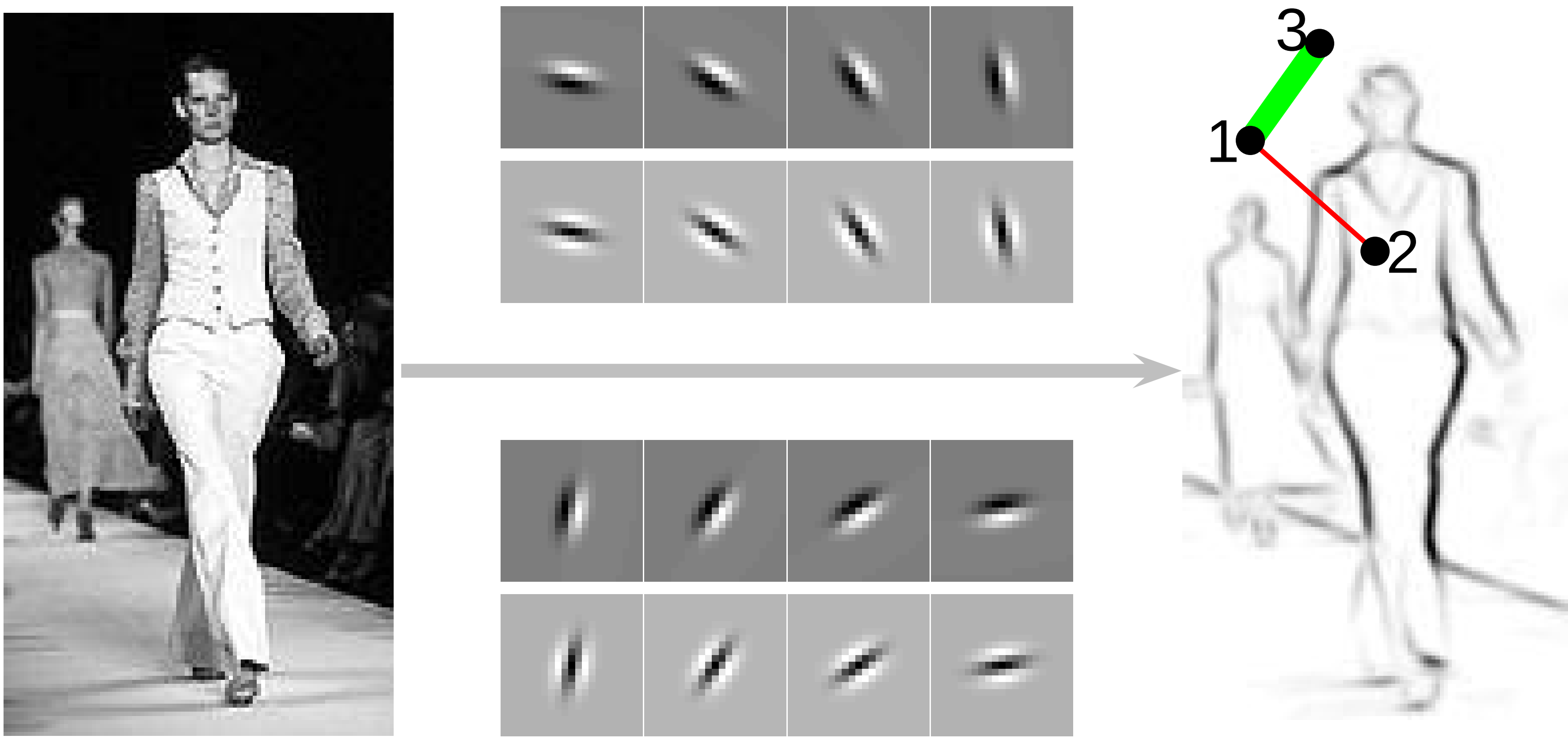


image oriented filter pairs edge magnitudes

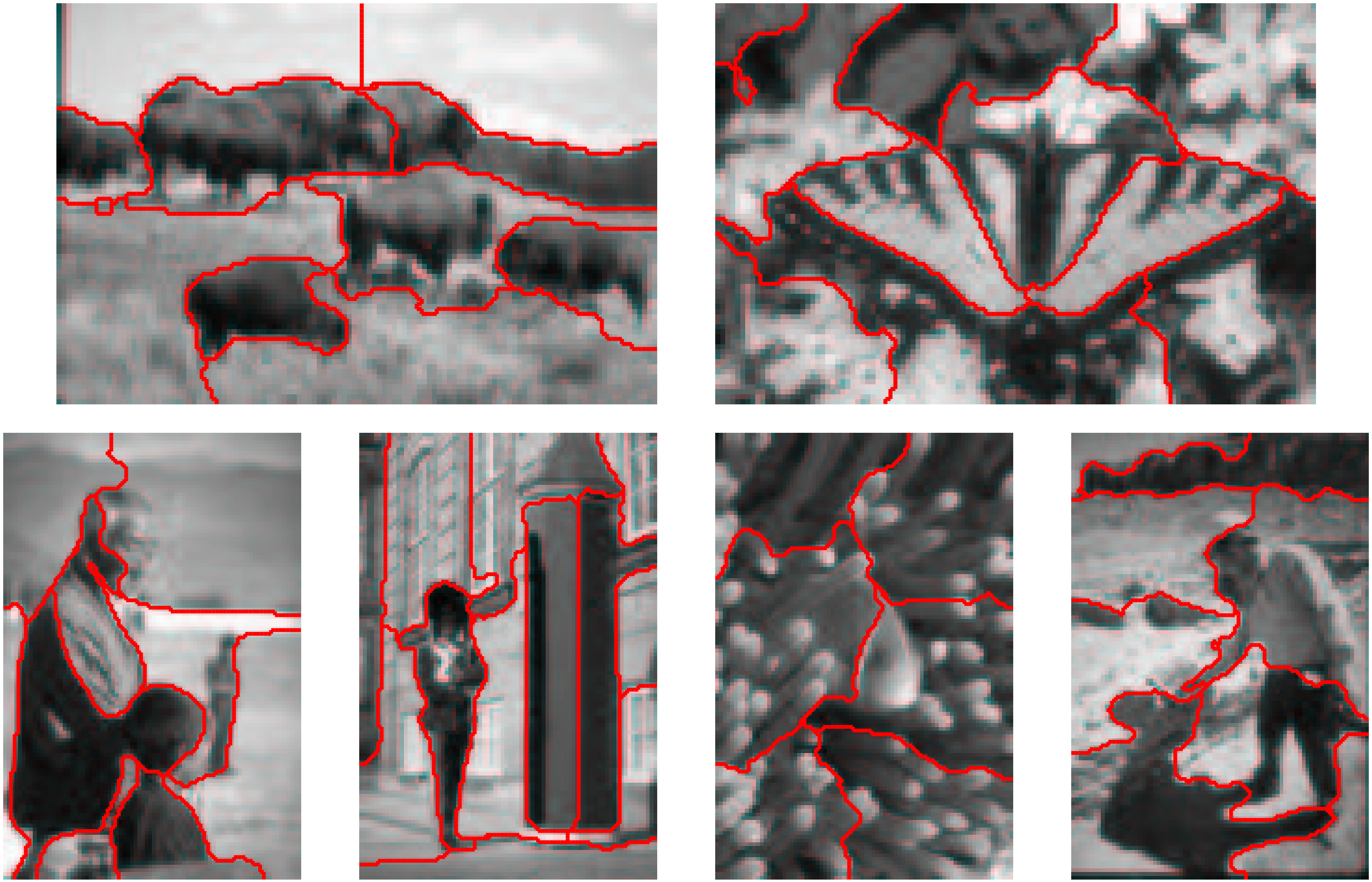
Discrete Optima Generated by Eigenvectors

Discrete Optima Generated by Eigenvectors

$K = 4:\quad 0.9901 \quad 0.9899 \quad 0.9881$

Not many local discrete optima, all good quality

Multiclass Real Image Segmentation



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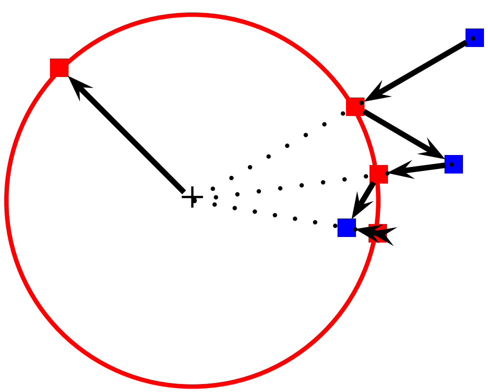
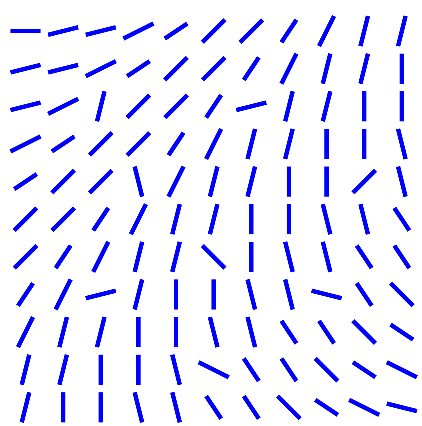


1



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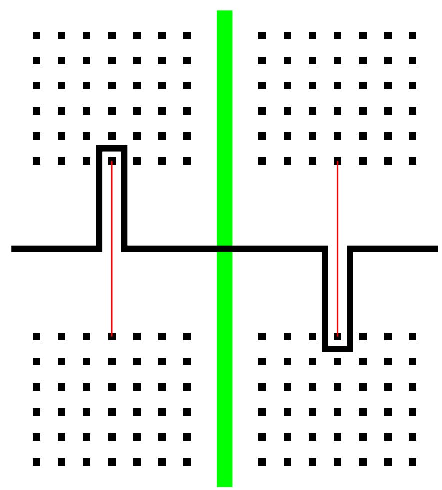

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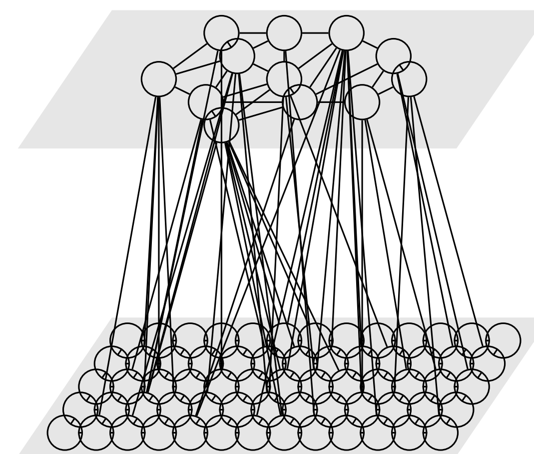

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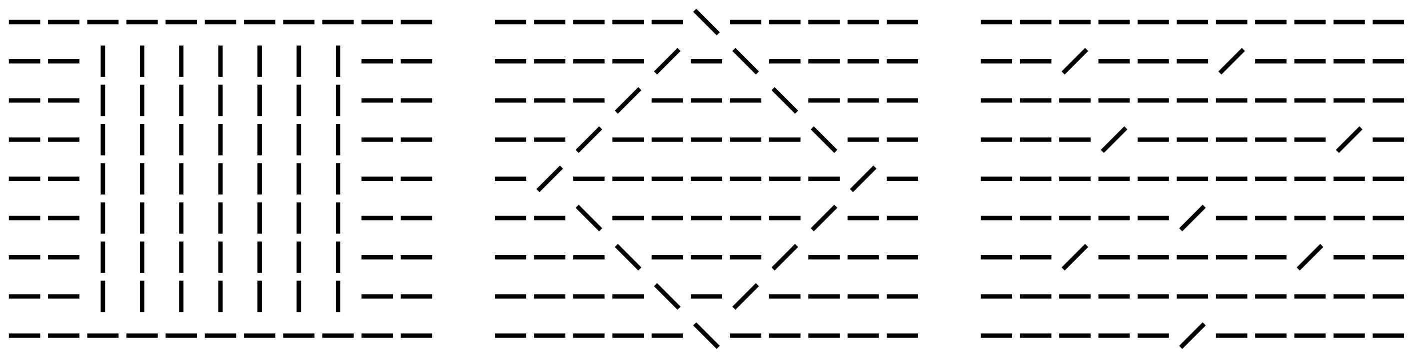
Perceptual Popout



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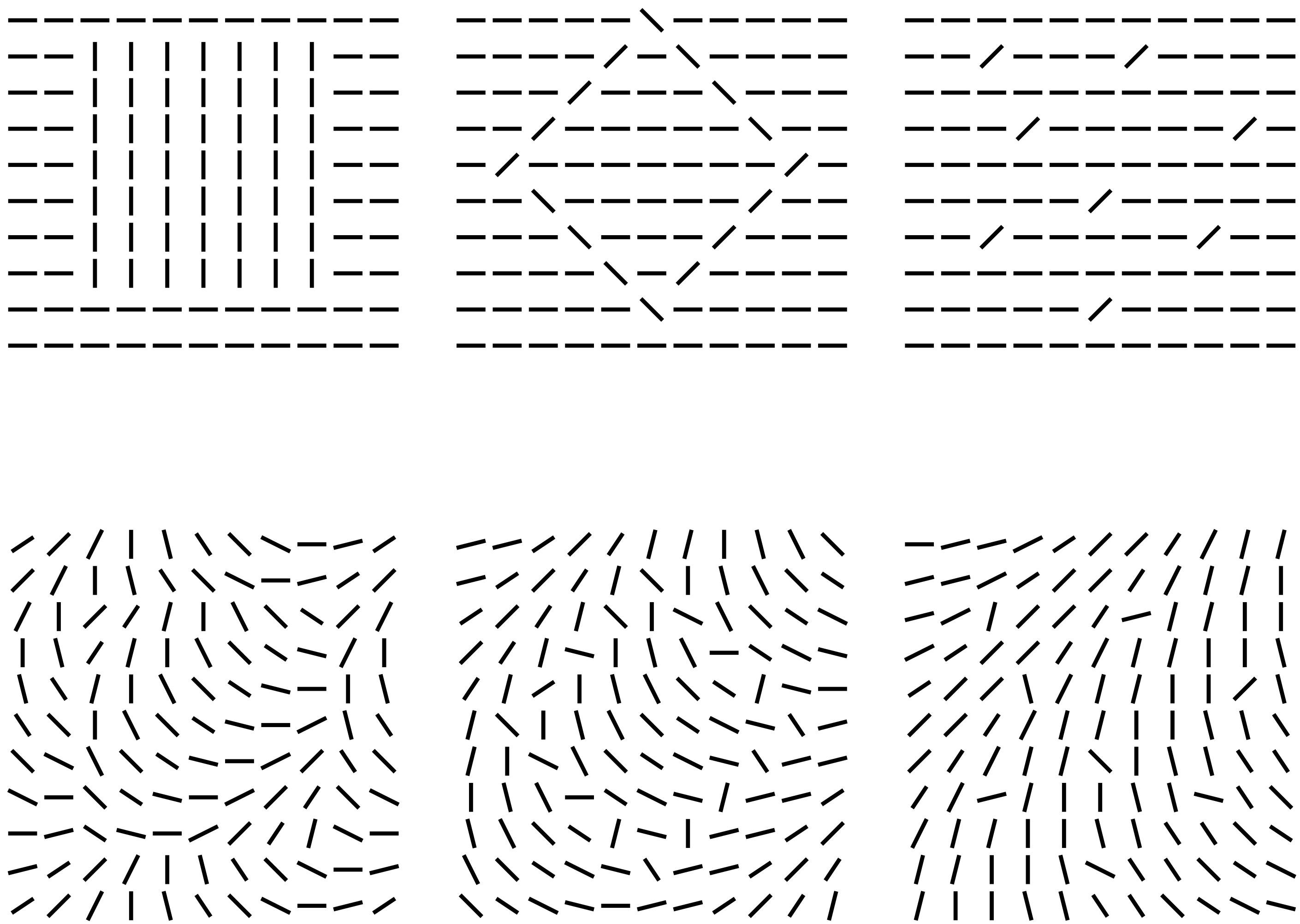
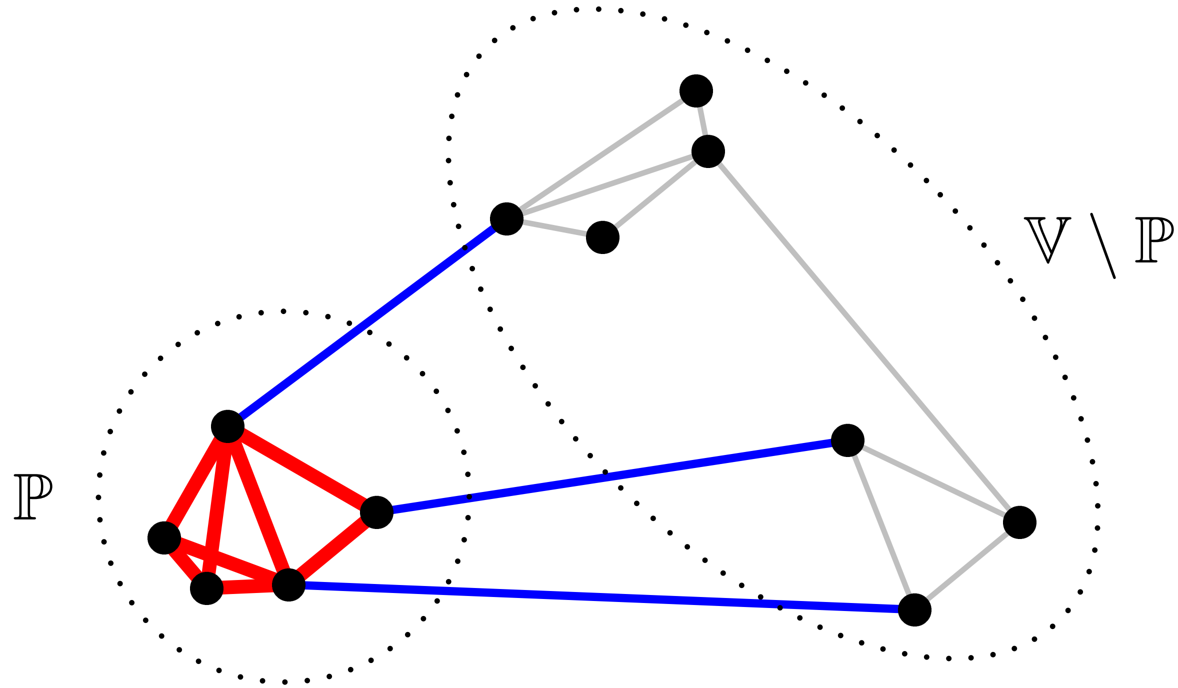


Goodness of Grouping: Attraction and Repulsion



Maximize within-group attraction: $\text{linkratio}(\mathbb{P}, \mathbb{P}; A)$

Minimize between-group attraction: $\text{linkratio}(\mathbb{P}, \mathbb{V} \setminus \mathbb{P}; A)$

Equivalent: $\text{linkratio}(\mathbb{P}, \mathbb{P}; A) + \text{linkratio}(\mathbb{P}, \mathbb{V} \setminus \mathbb{P}; A) = 1$

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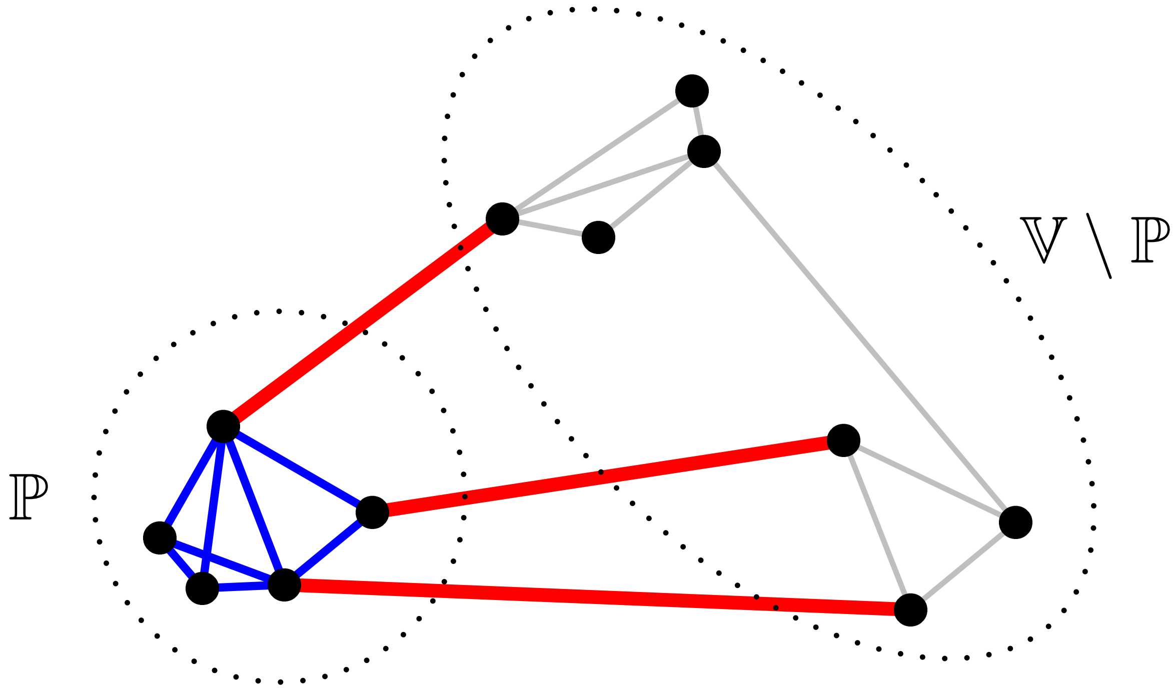

Maximize between-group repulsion: $\mathrm{linkratio}(\mathbb{P}, \mathbb{V} \setminus \mathbb{P}; R)$

Minimize within-group repulsion: $\mathrm{linkratio}(\mathbb{P}, \mathbb{P}; R)$

Equivalent: $\mathrm{linkratio}(\mathbb{P}, \mathbb{P}; R) + \mathrm{linkratio}(\mathbb{P}, \mathbb{V} \setminus \mathbb{P}; R) = 1$

Normalized Cuts with Attraction and Repulsion

- Criteria

$$\mathrm{knassoc}(\Gamma_{\mathbb{V}}^{K}) = \frac{1}{K}\sum_{l=1}^{K}\frac{\mathrm{links}(\mathbb{V}_l, \mathbb{V}_l; A) + \mathrm{links}(\mathbb{V}_l, \mathbb{V}\setminus\mathbb{V}_l; R)}{\mathrm{degree}(\mathbb{V}_l; A) + \mathrm{degree}(\mathbb{V}_l; R)}$$

$$\mathrm{kncuts}(\Gamma_{\mathbb{V}}^{K}) = \frac{1}{K}\sum_{l=1}^{K}\frac{\mathrm{links}(\mathbb{V}_l, \mathbb{V}\setminus\mathbb{V}_l; A) + \mathrm{links}(\mathbb{V}_l, \mathbb{V}_l; R)}{\mathrm{degree}(\mathbb{V}_l; A) + \mathrm{degree}(\mathbb{V}_l; R)}$$

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- Equivalent weight matrix and degree matrix

$$\begin{aligned}\hat{W} &= A - R + D_R\\ \hat{D} &= D_A + D_R\end{aligned}$$

Negative Weights and Regularization

- Negative weights:

$$
\begin{aligned}
W &= A - R \\
&= \text{(positive entries + offset)} - \text{(negative entries + offset)}
\end{aligned}
$$

Negative Weights and Regularization

- Negative weights:

$$W = A - R$$
$$= \text{(positive entries + offset)} - \text{(negative entries + offset)}$$

- Equivalent matrices: $(\hat{W} + D_{\text{offset}},\ \hat{D} + 2D_{\text{offset}})$

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- Negative weights:

  $$W = A - R$$

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- Regularization: increase the degrees of nodes without changing the sizes of weights between two nodes.

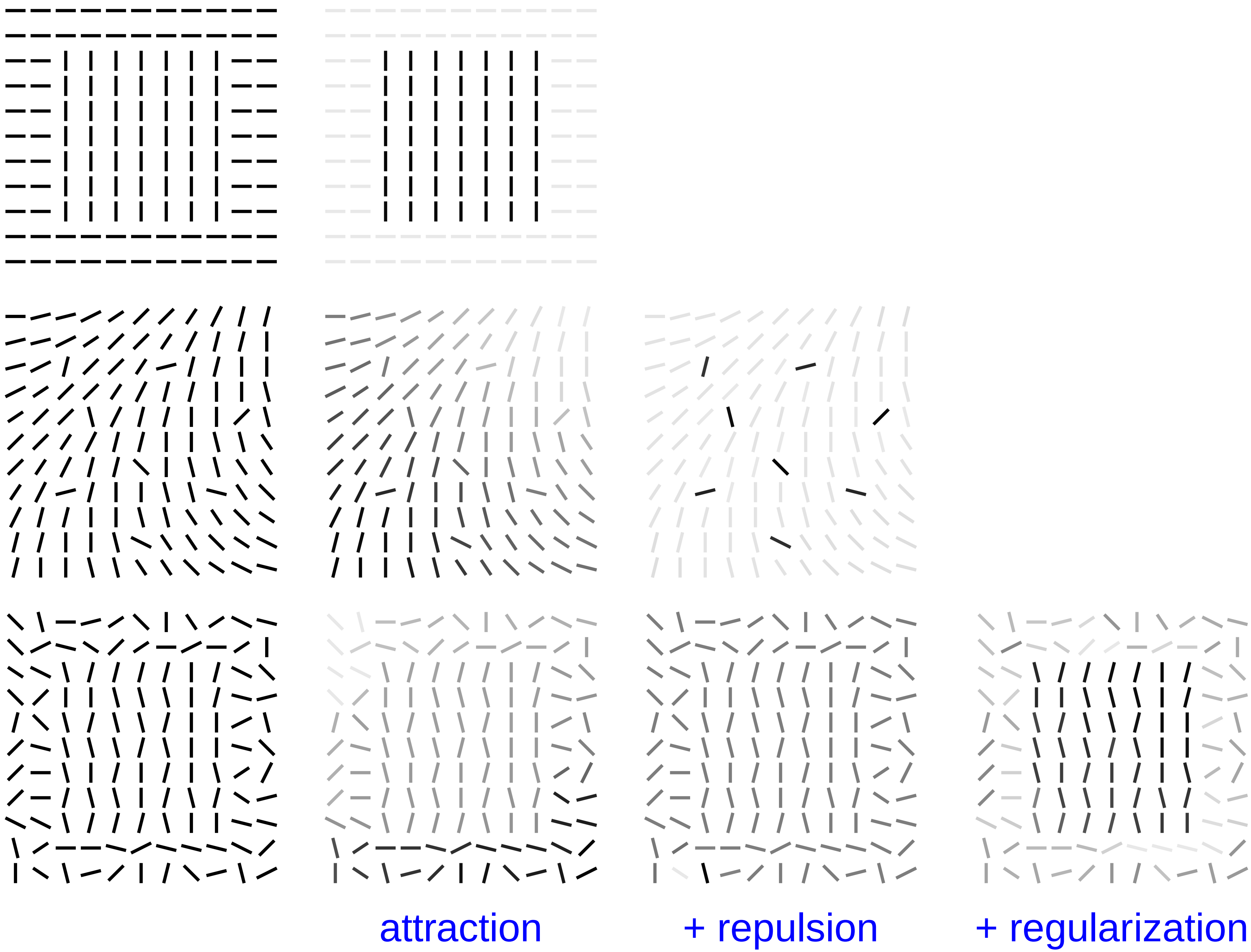
Negative Weights and Regularization

- Negative weights:

  $$W = A - R$$
  $$= \text{(positive entries + offset)} - \text{(negative entries + offset)}$$

- Equivalent matrices: $(\hat{W} + D_{\text{offset}},\ \hat{D} + 2D_{\text{offset}})$
- Regularization: increase the degrees of nodes without changing the sizes of weights between two nodes.
- Decrease the sensitivity of $\mathrm{linkratio}$ for nodes with little connections.

Roles of Attraction, Repulsion, Regularization



Segmentation with Repulsion and Regularization

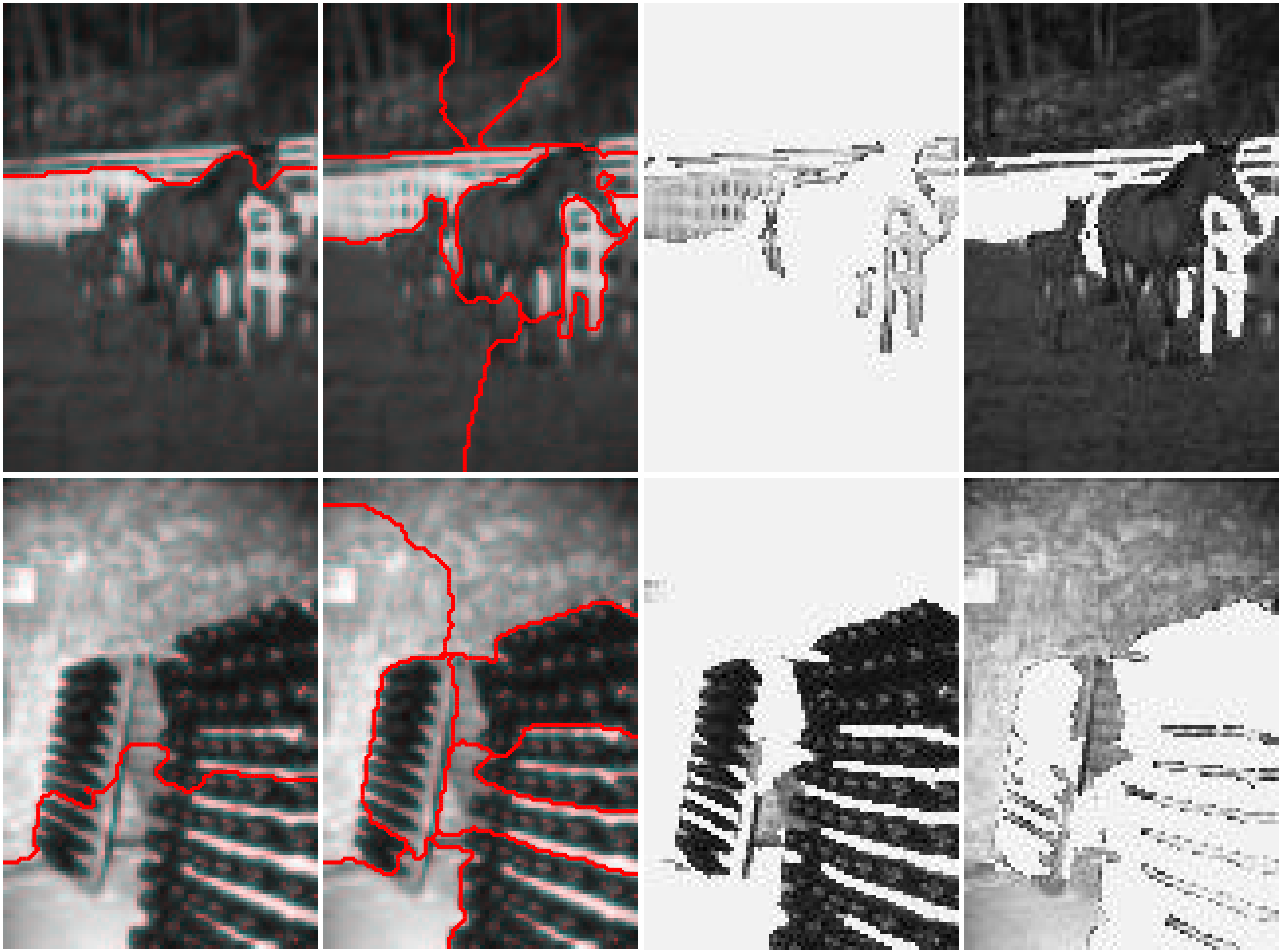
attraction

attraction, repulsion and regularization

Segmentation with Repulsion and Regularization

attraction attraction, repulsion and regularization

Segmentation with Repulsion and Regularization



attraction

attraction, repulsion and regularization

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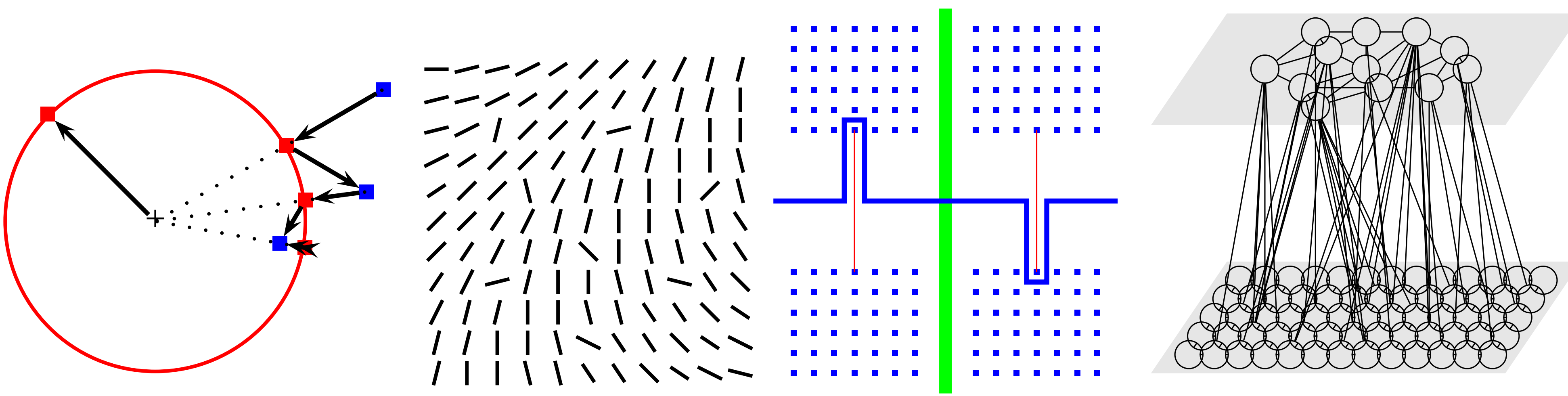
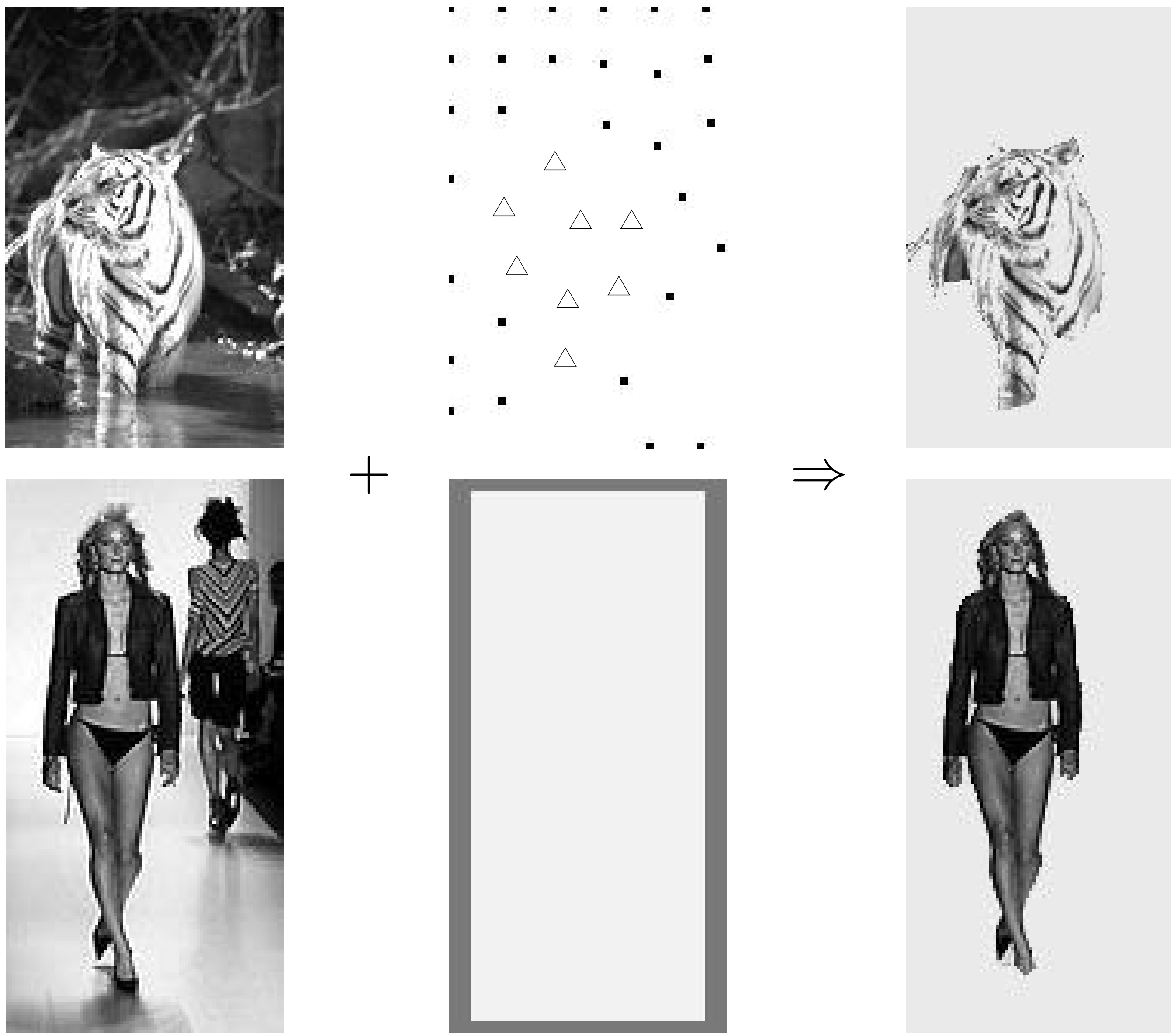
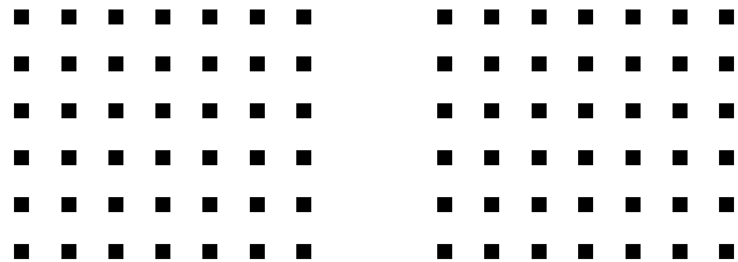


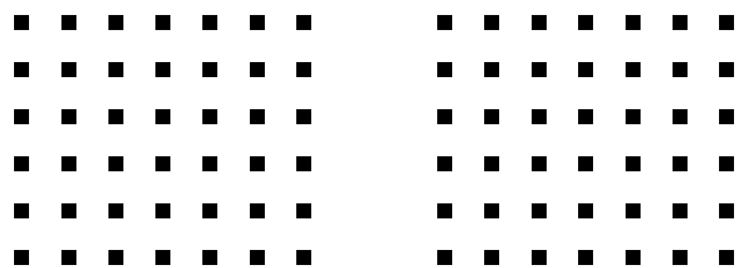
1 2 3 4

Grouping with Partial Cues



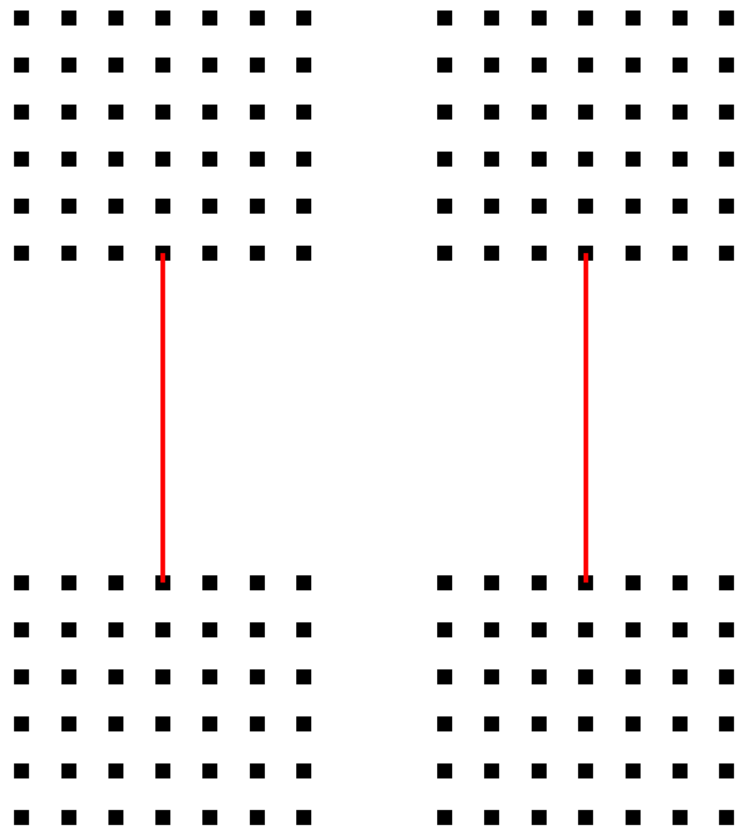
Basic Formulation: Grouping with Constraints





maximize $\varepsilon(\Gamma_{\mathbb{V}}^{K})$

Basic Formulation: Grouping with Constraints



maximize $\varepsilon(\Gamma_{\mathbb{V}}^{K})$

subject to $X(i,l) = X(j,l)$

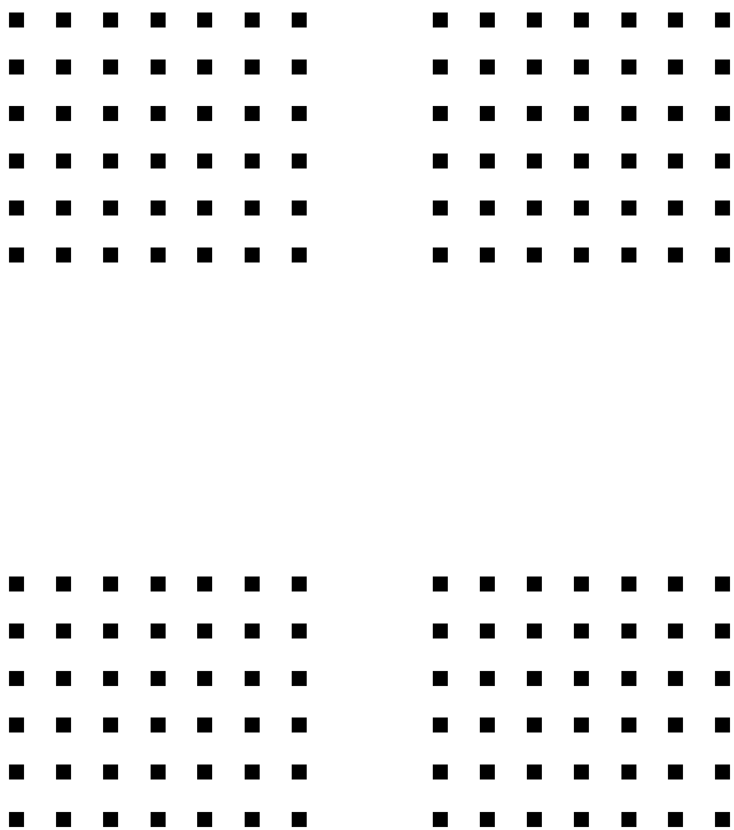
Computing Constrained Normalized Cuts

- Constrained eigenvalue problem

- Efficient solution using a projector onto the feasible space

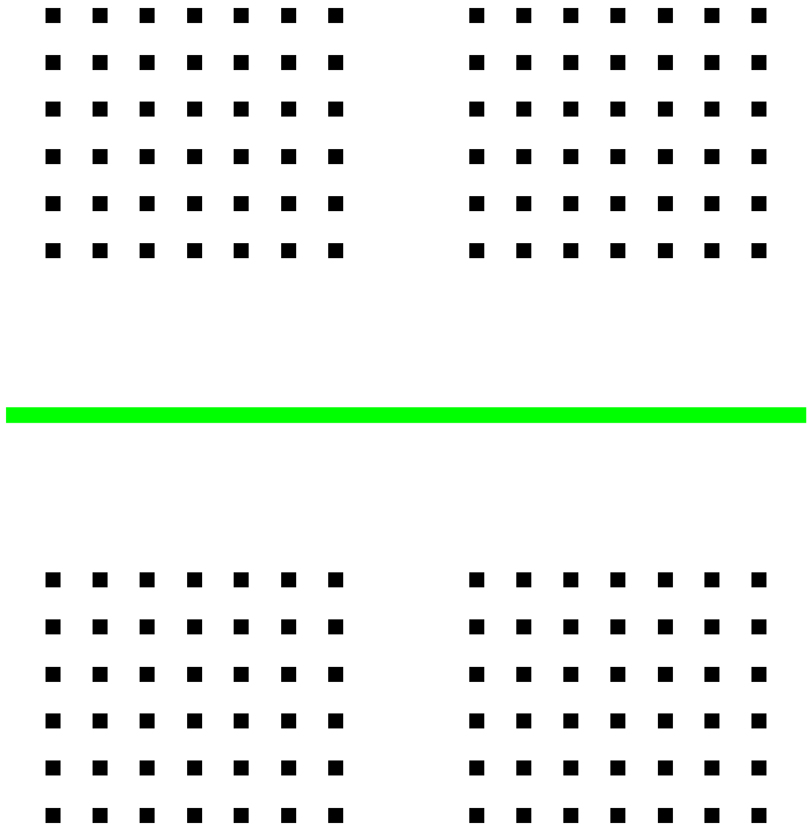
- Generalize Rayleigh-Ritz theorem to projected matrices

Why Simple Constraints Are Insufficient



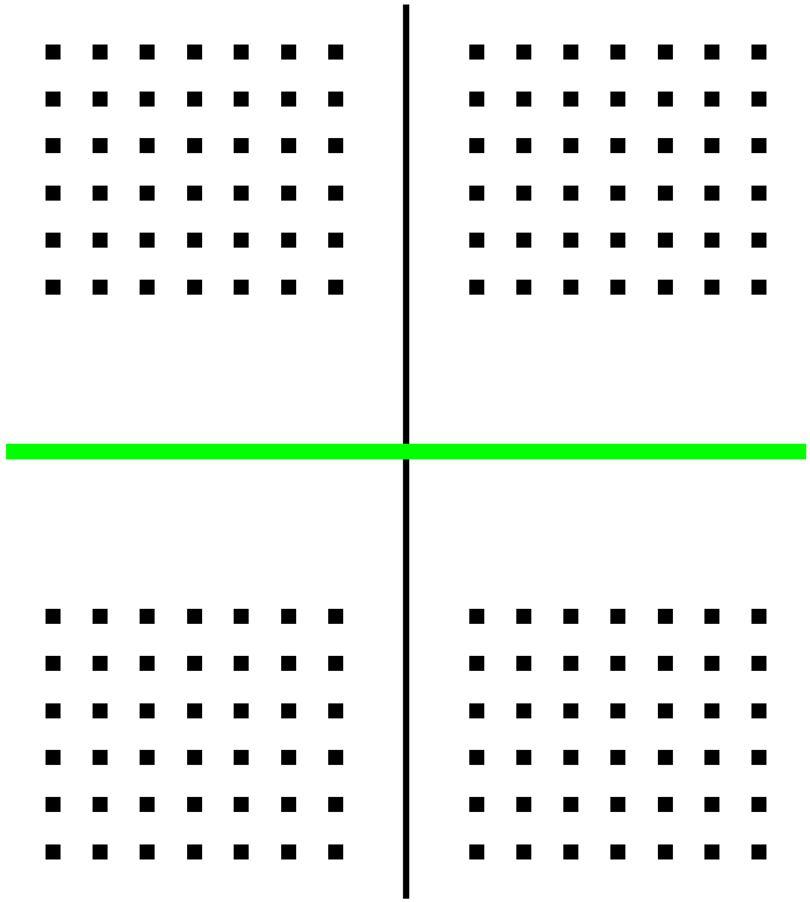
natural grouping

Why Simple Constraints Are Insufficient



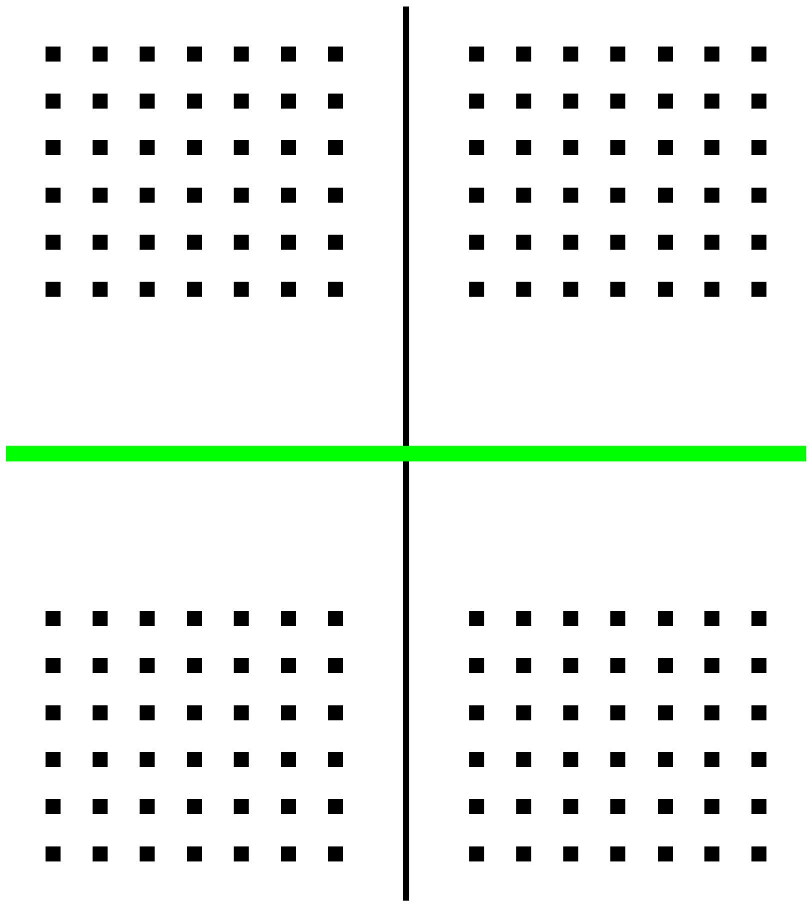
natural grouping

Why Simple Constraints Are Insufficient

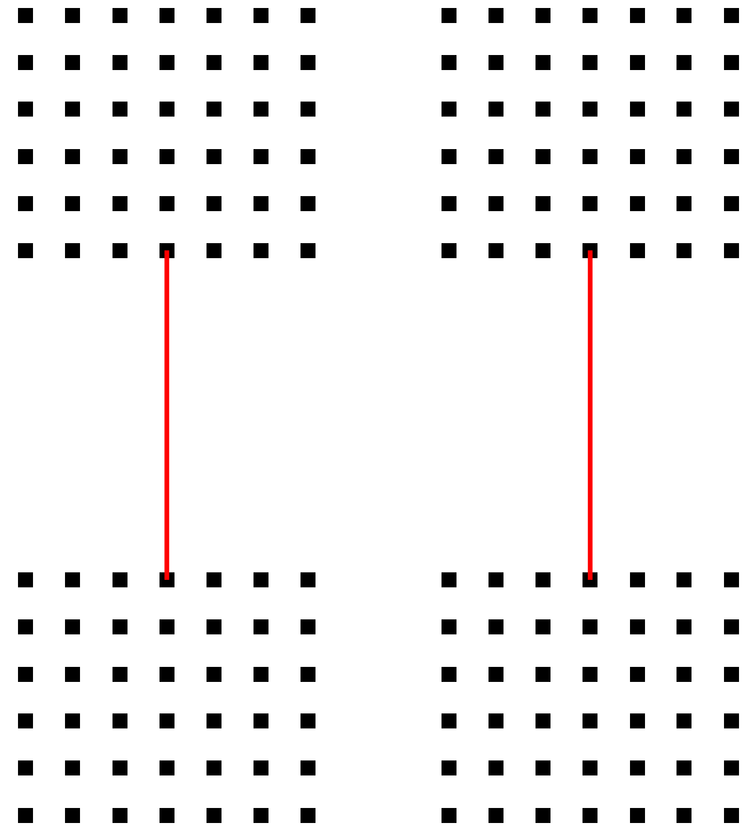


natural grouping

Why Simple Constraints Are Insufficient

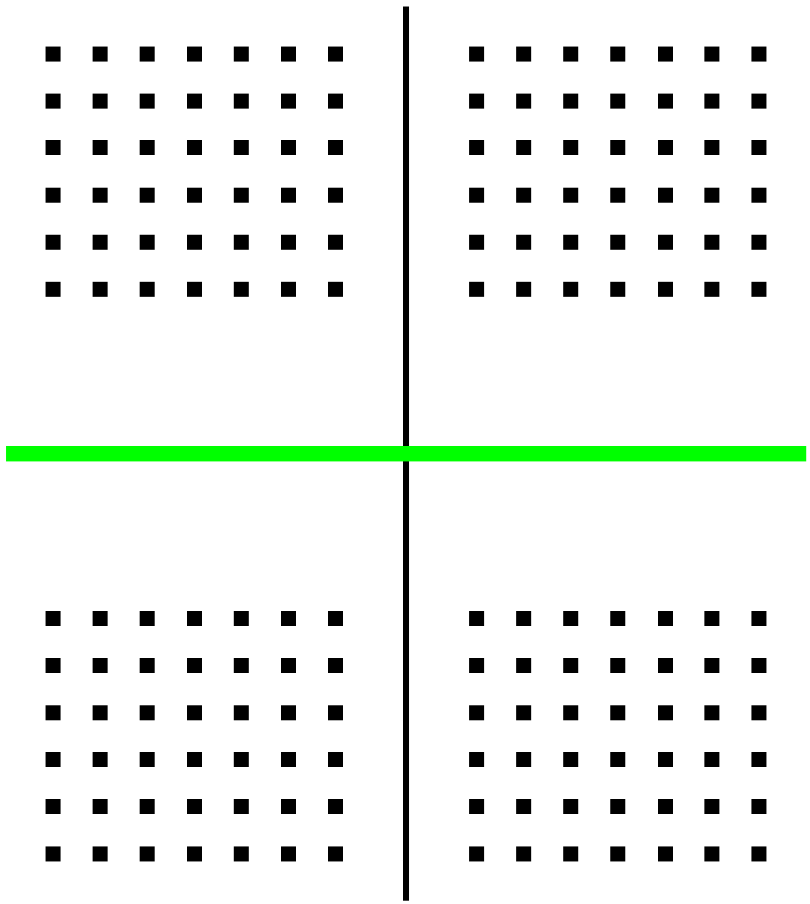


natural grouping

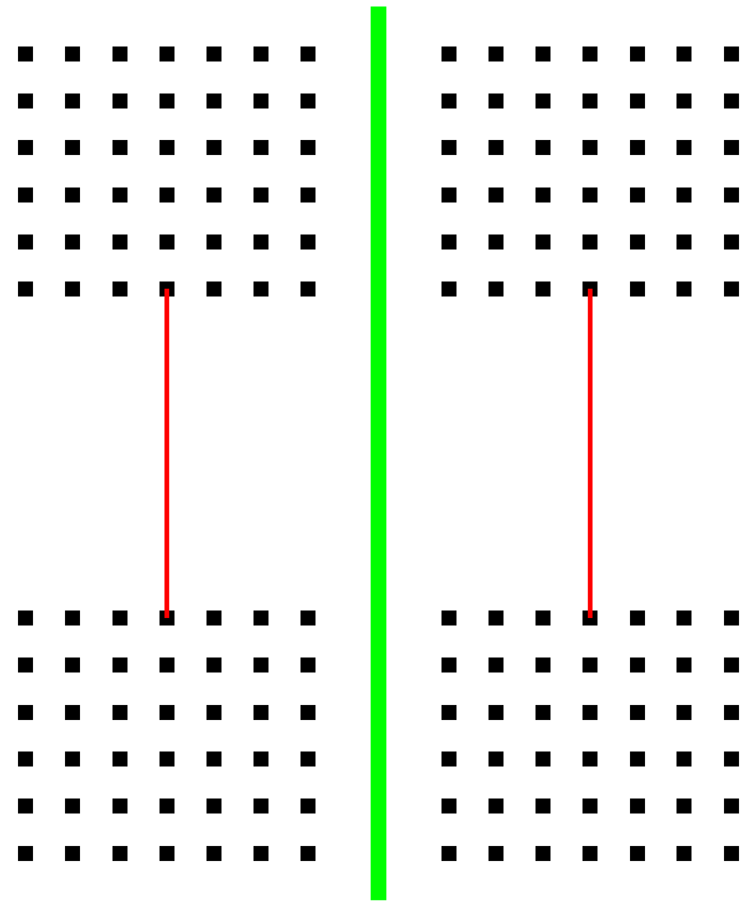


constrained grouping

Why Simple Constraints Are Insufficient

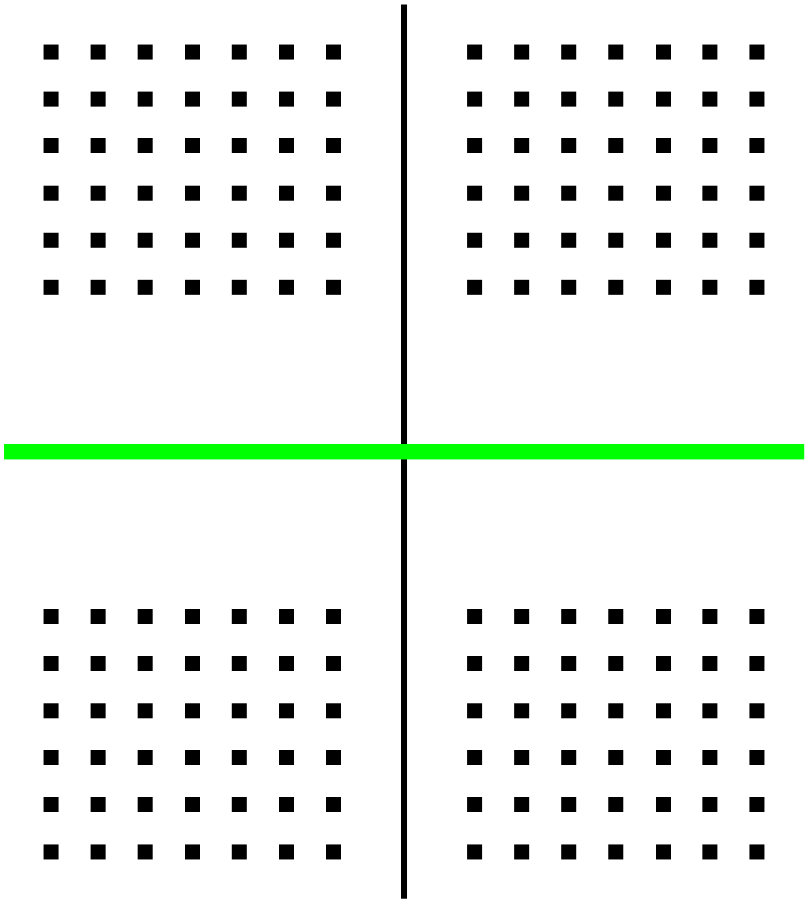


natural grouping

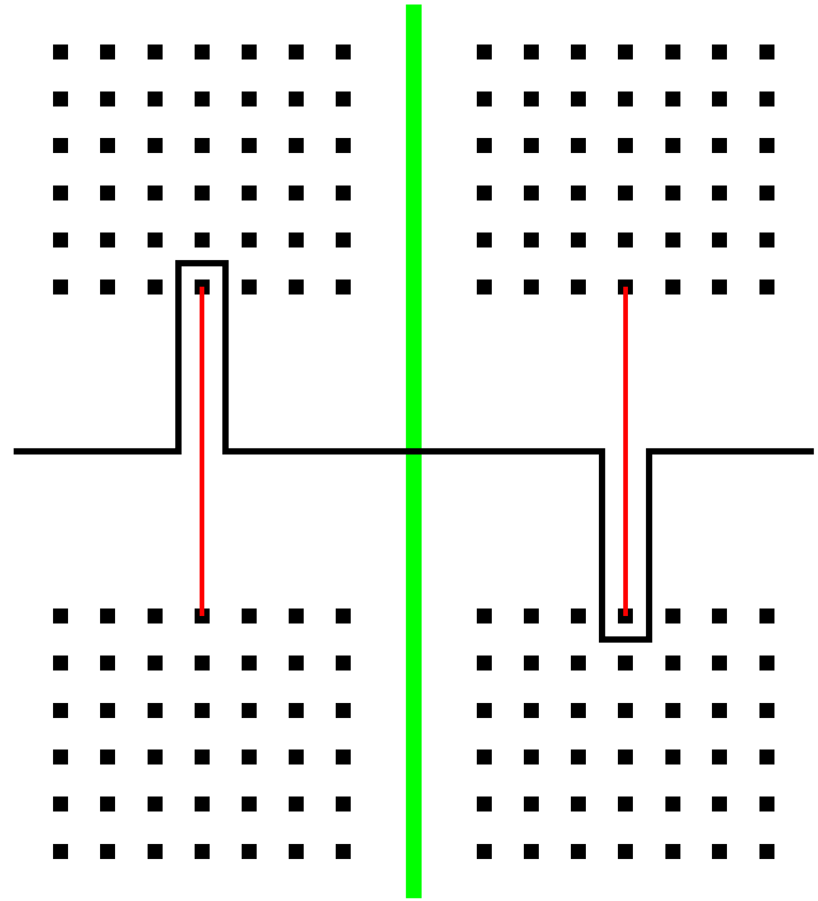


constrained grouping

Why Simple Constraints Are Insufficient



natural grouping



constrained grouping

Remedy: Propagate Constraints

- General formulation:

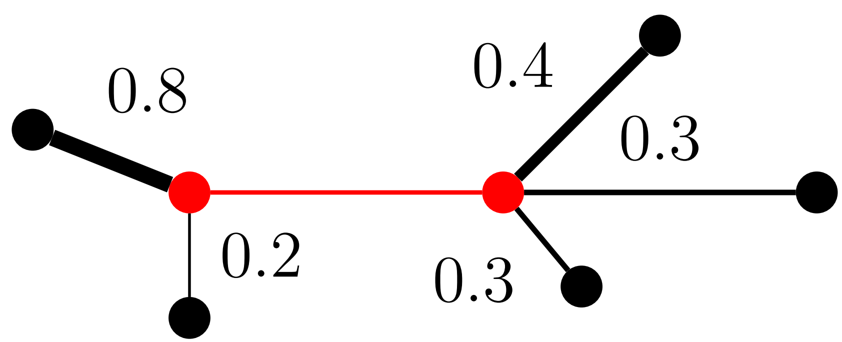
$$\begin{aligned} \text{maximize} \quad & \varepsilon(\Gamma_{\mathbb{V}}^{K}) \\ \text{subject to} \quad & \color{red}{S} \cdot X(i,l) = \color{red}{S} \cdot X(j,l) \end{aligned}$$

Remedy: Propagate Constraints

- General formulation:

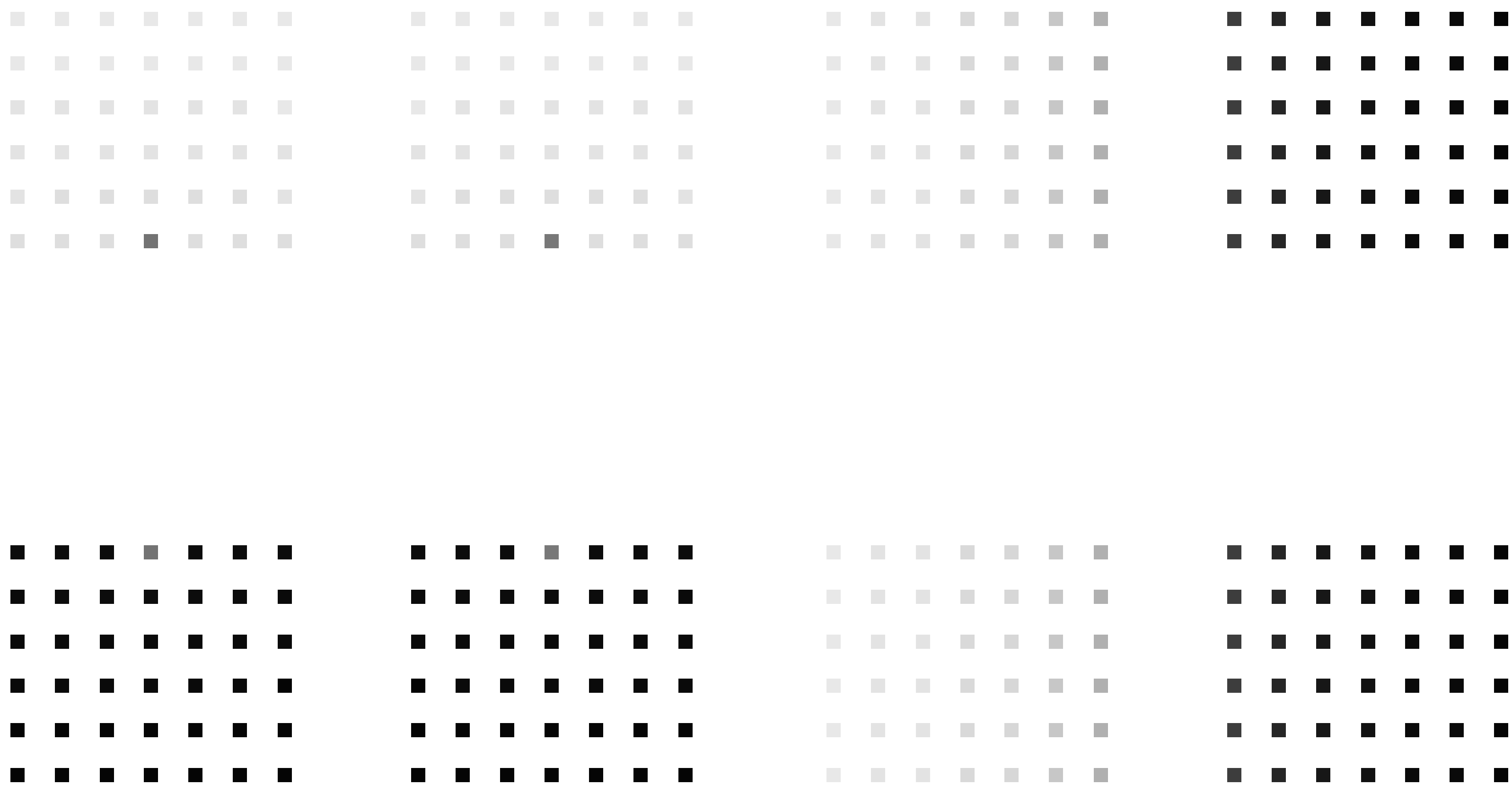
$$\begin{aligned} \text{maximize} \quad & \varepsilon(\Gamma_{\mathbb{V}}^{K}) \\ \text{subject to} \quad & S \cdot X(i,l) = S \cdot X(j,l) \end{aligned}$$

- Normalized cuts:



$$\begin{aligned} S &= P, \quad \text{or} \\ \sum_k P_{ik} X(k,l) &= \sum_k P_{jk} X(k,l), \quad \text{or} (P^T U)^T X = 0 \end{aligned}$$

Clustering Points with Sparse Grouping Cues



simple bias

smoothed bias

Image Segmentation with Biased Grouping

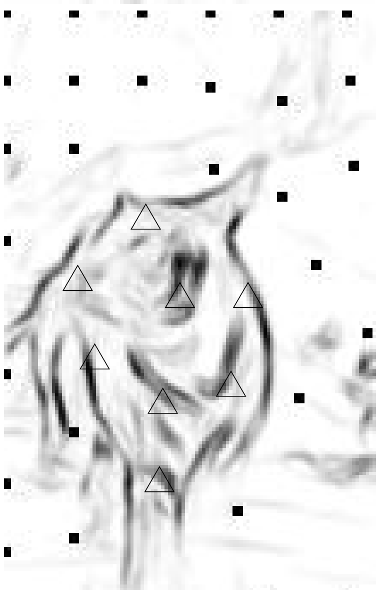
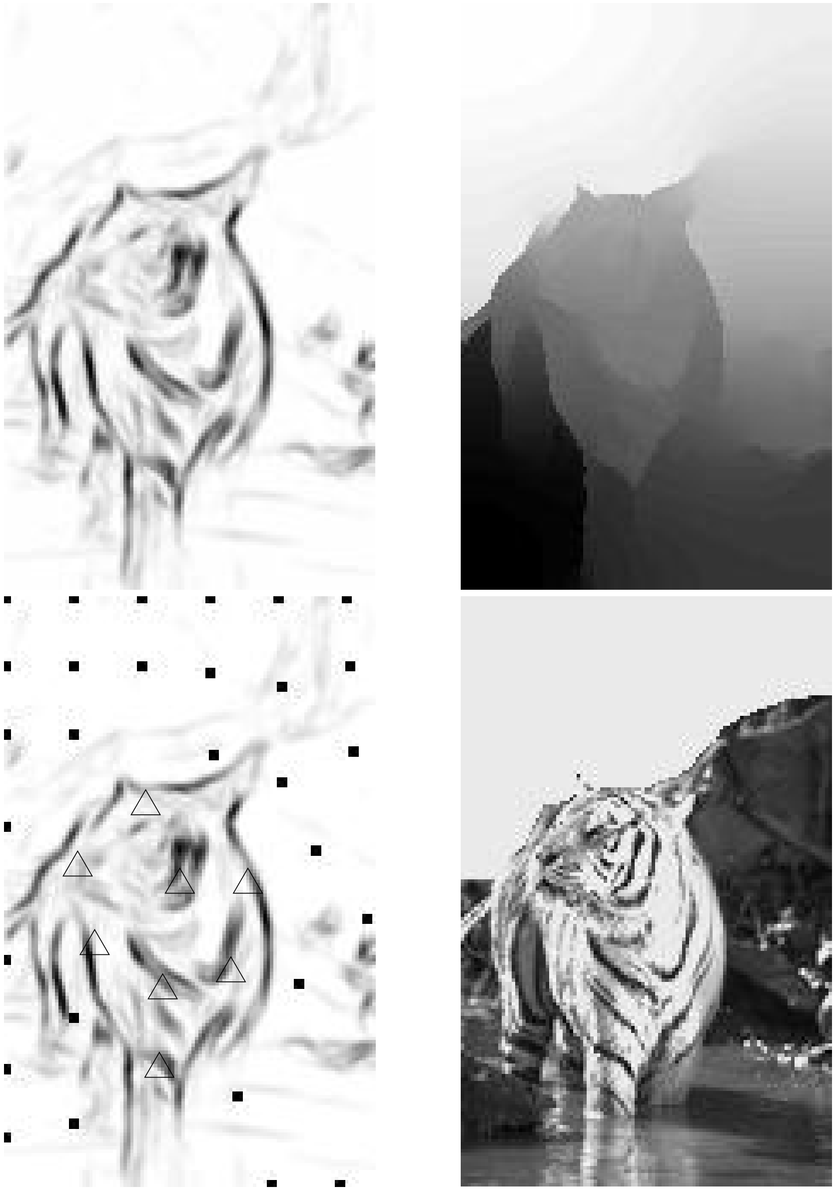
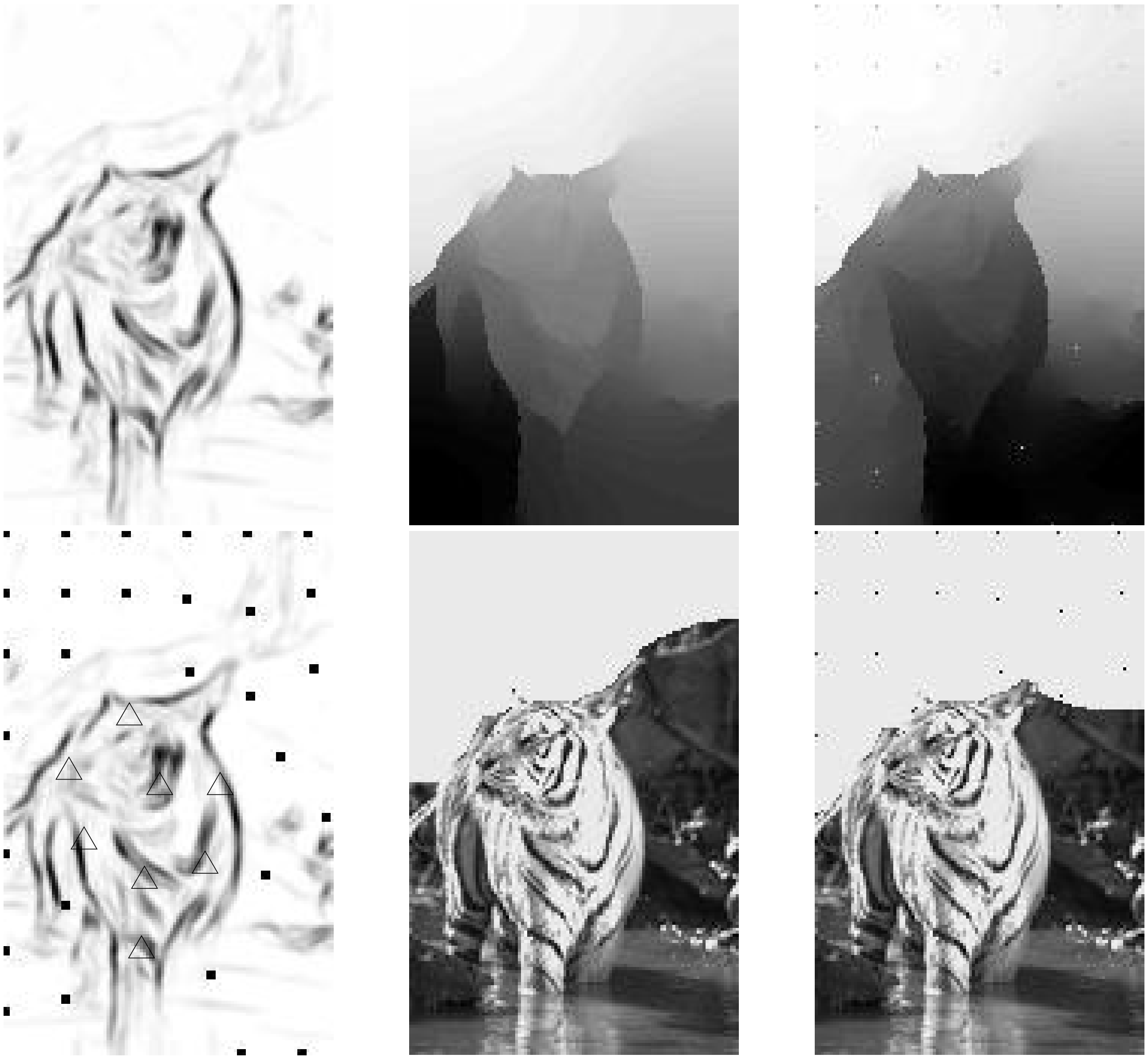


Image Segmentation with Biased Grouping



no bias

Image Segmentation with Biased Grouping



no bias simple bias

Image Segmentation with Biased Grouping

no bias   simple bias   smoothed bias

Image Segmentation with Biased Grouping

Image Segmentation with Biased Grouping

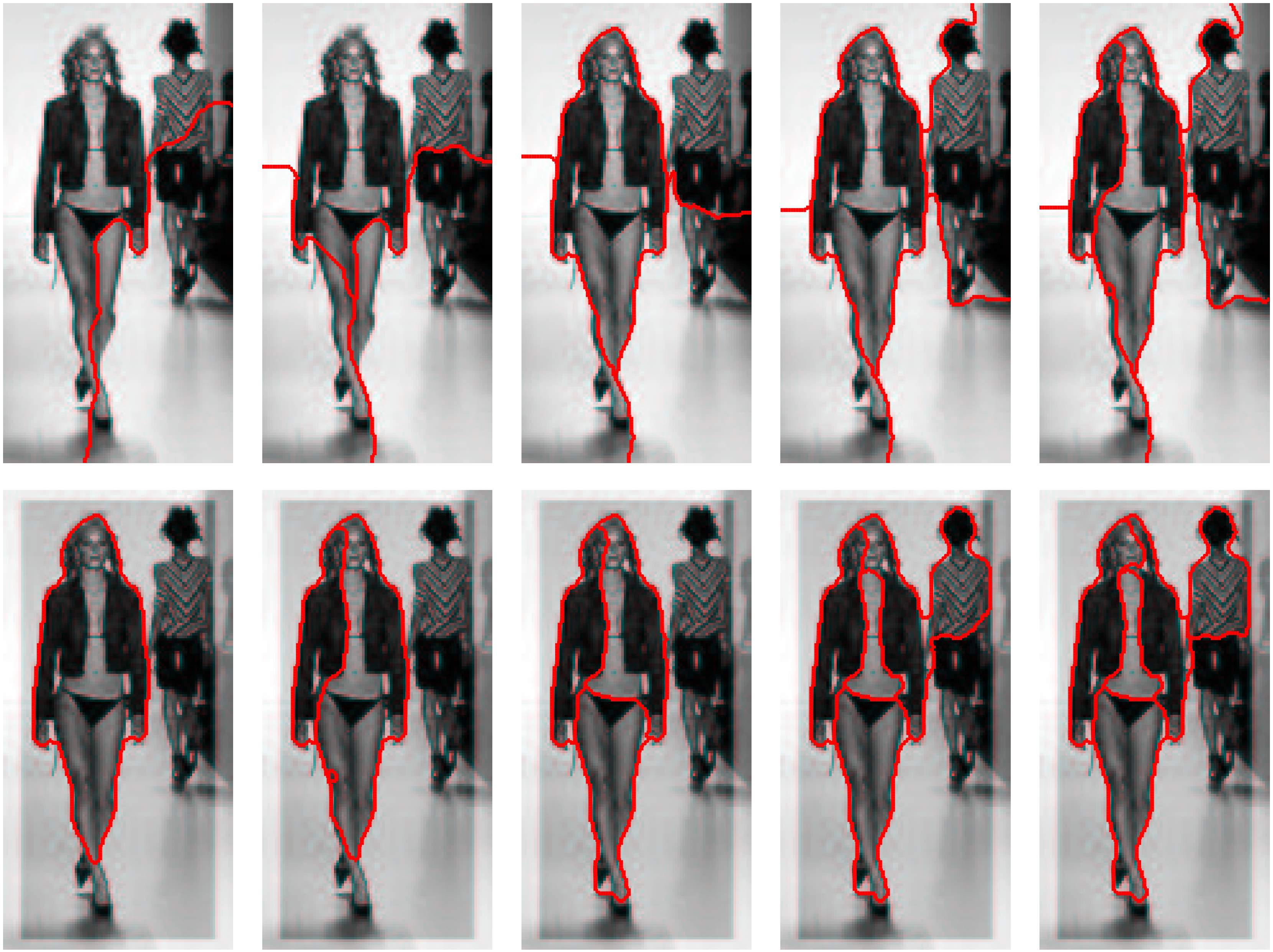
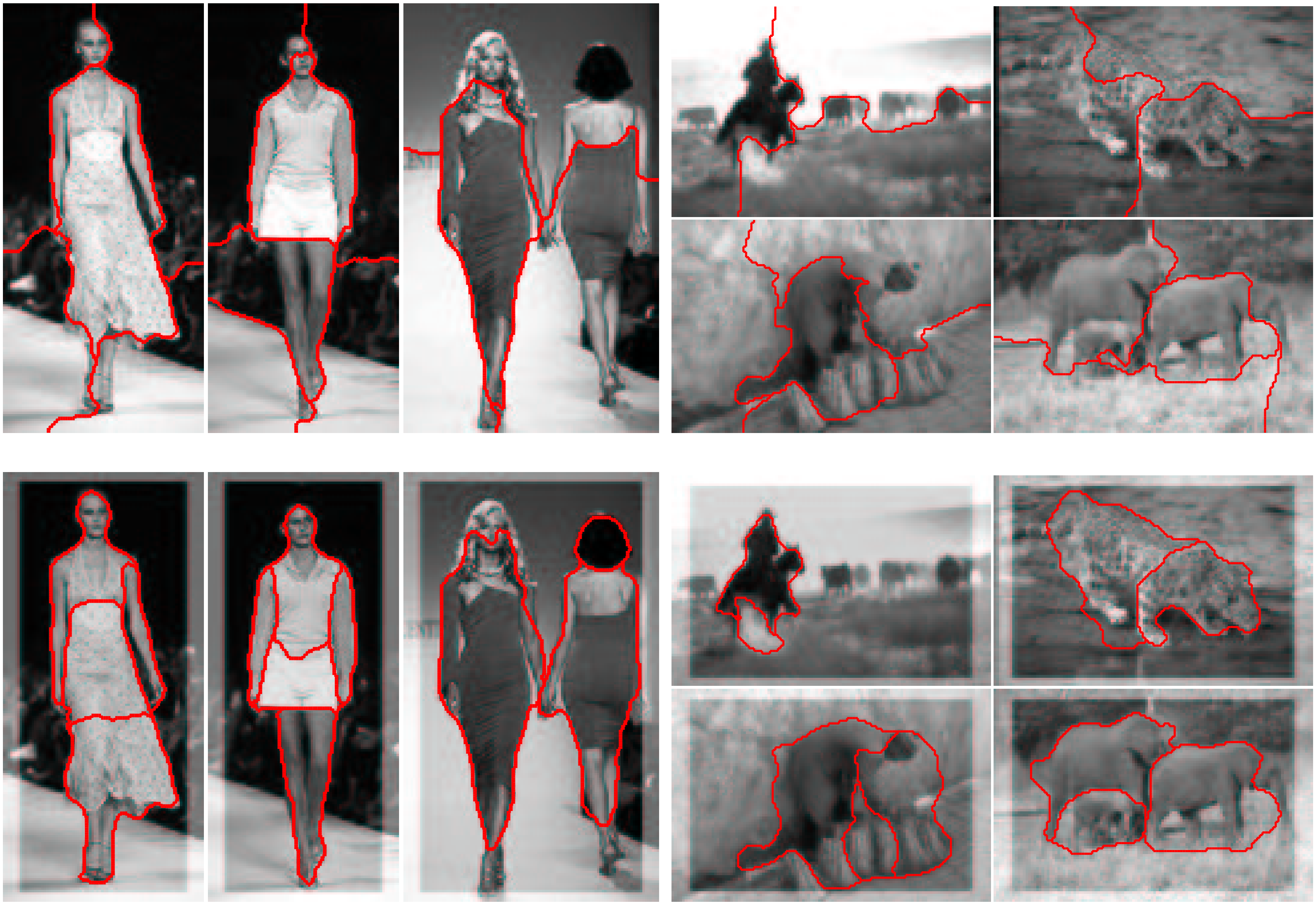


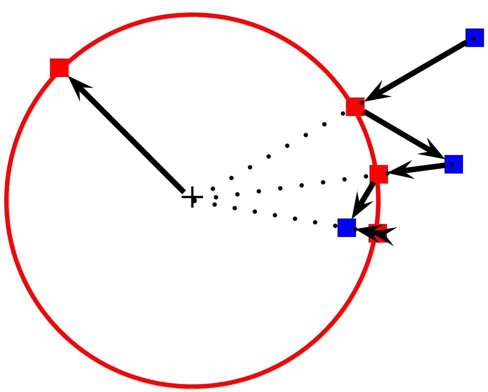
Image Segmentation with Spatial Attention

Image Segmentation with Spatial Attention



Outline

1. Computational framework: spectral clustering
2. Expand the repertoire of grouping cues: dissimilarity
3. Guide grouping with partial cues
4. Guide grouping with object knowledge
5. Summary and future work



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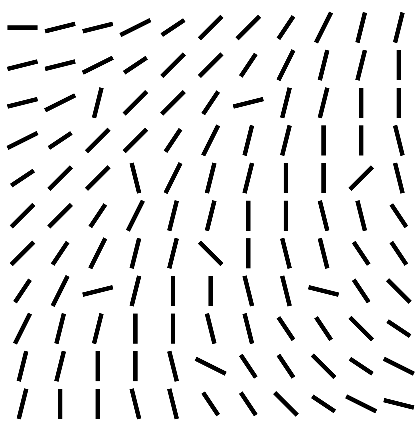

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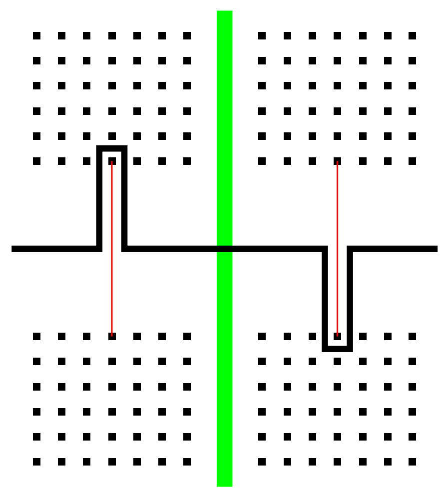

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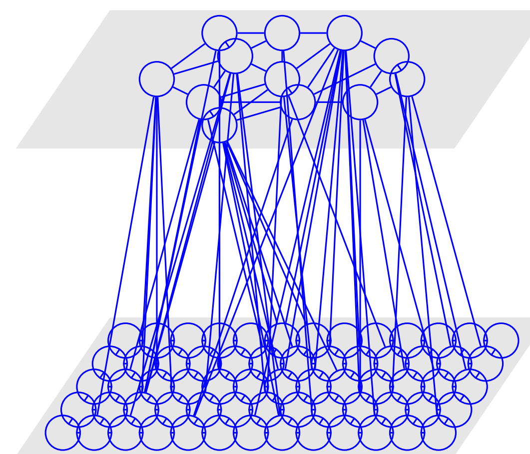
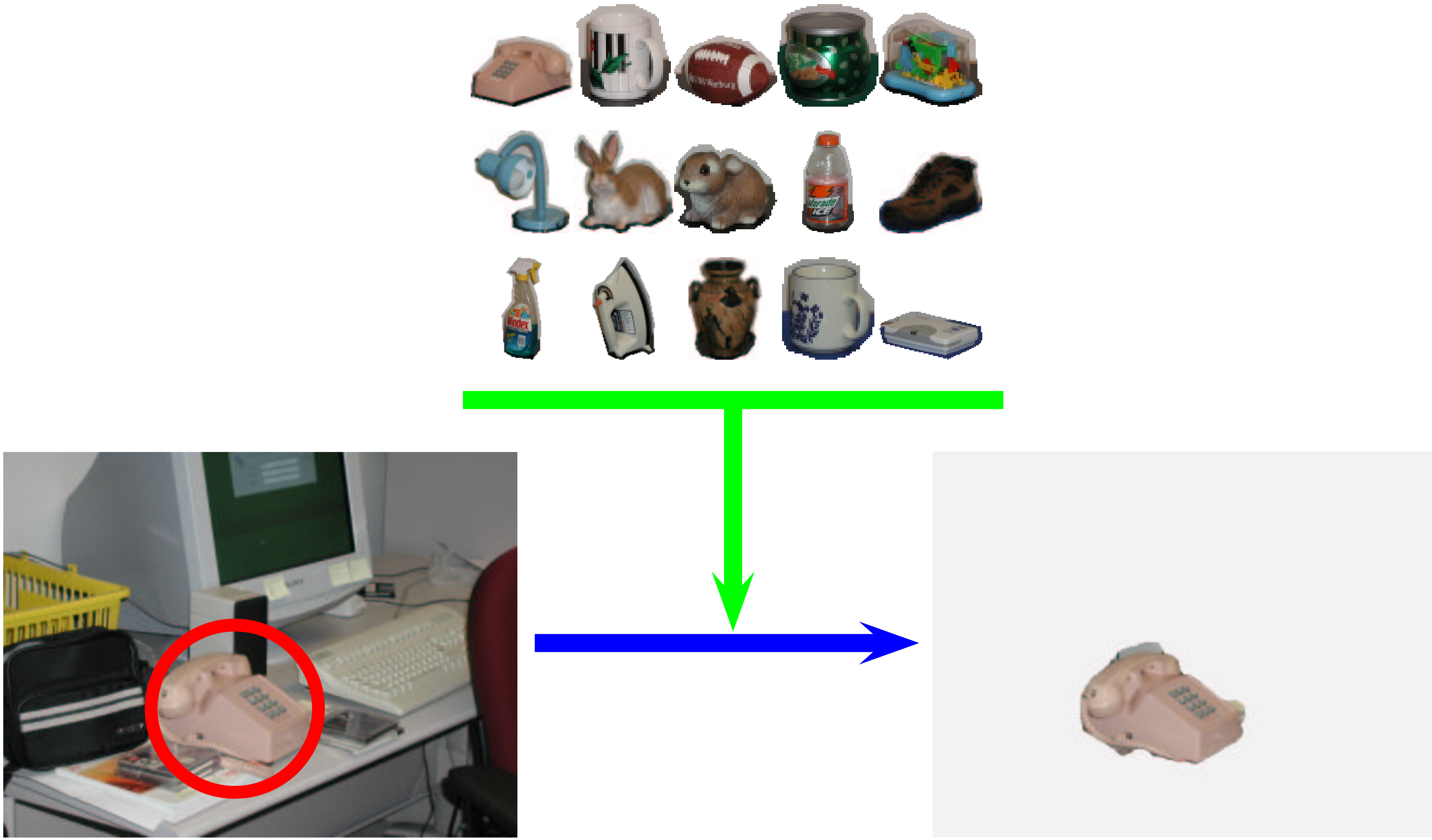


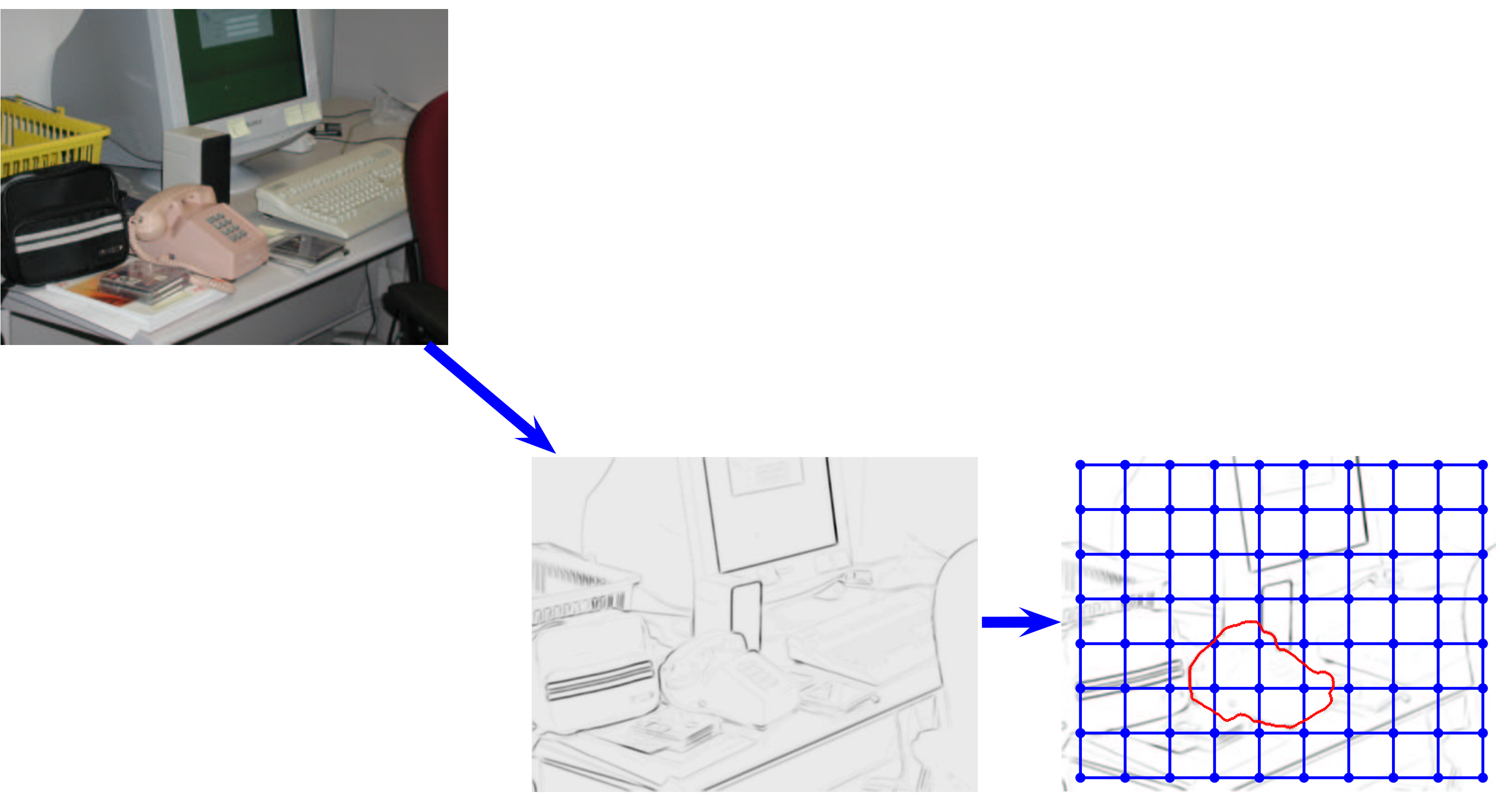
4

Object Segmentation

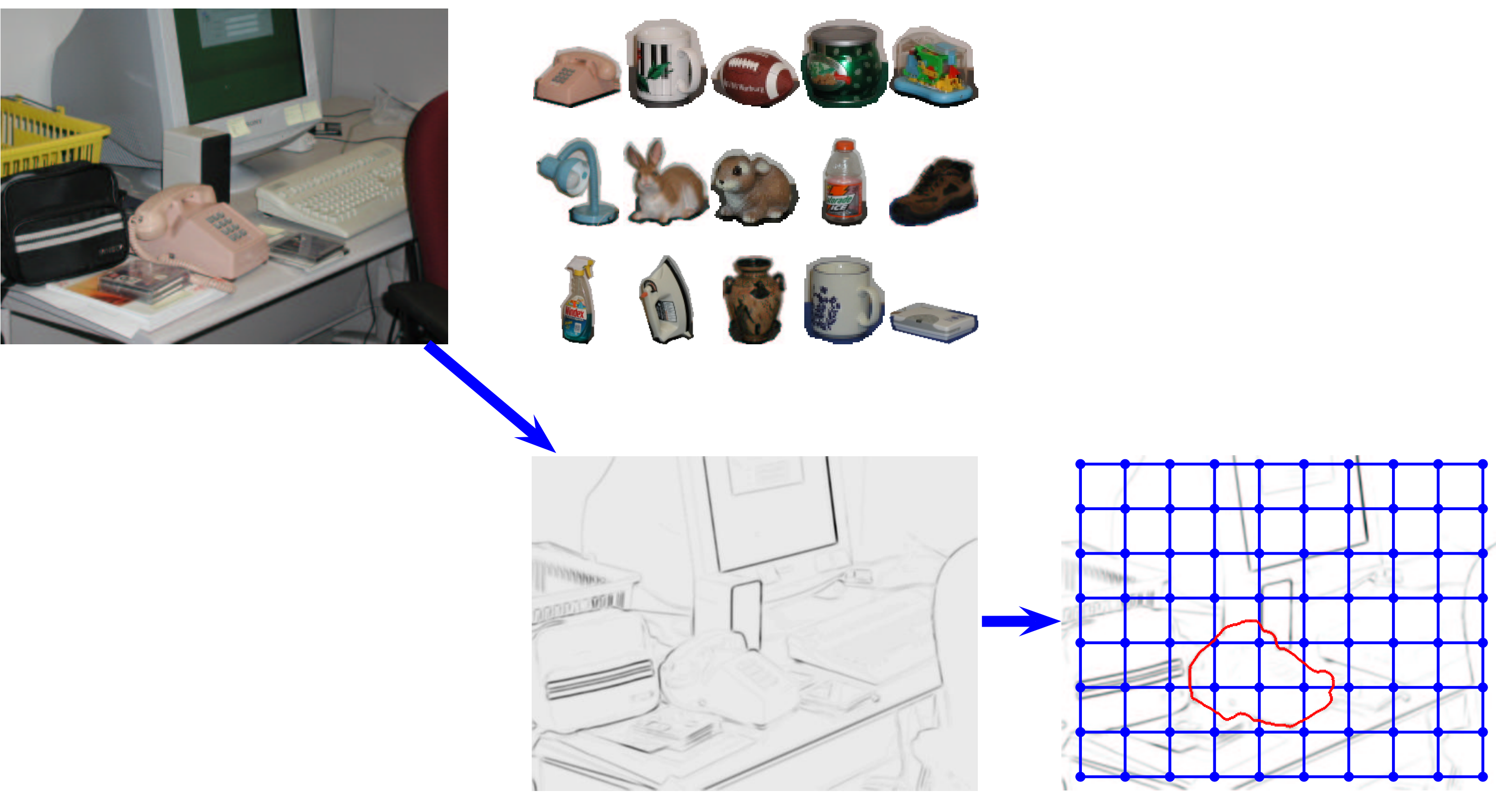


Our Approach to Object Segmentation

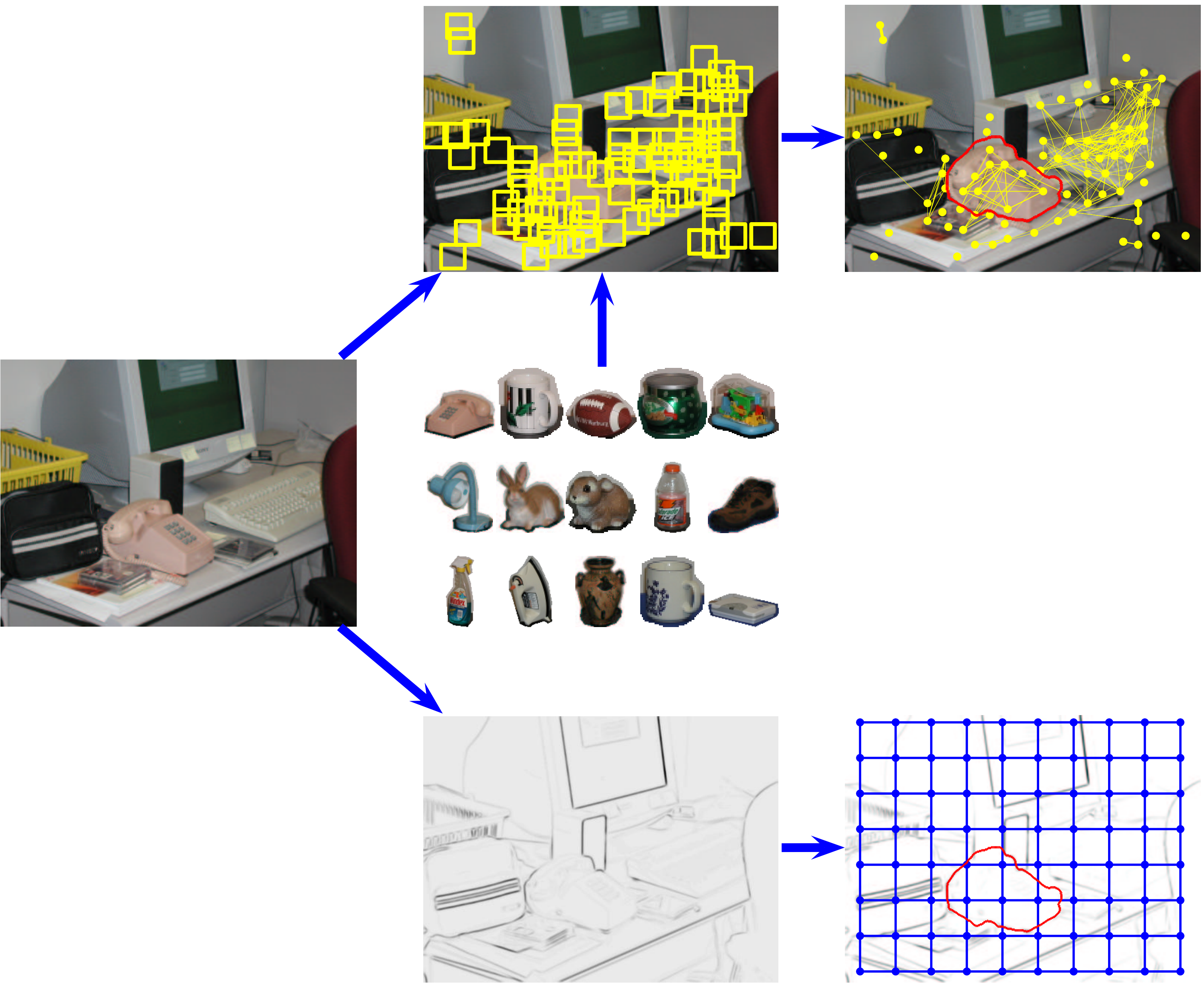
Our Approach to Object Segmentation



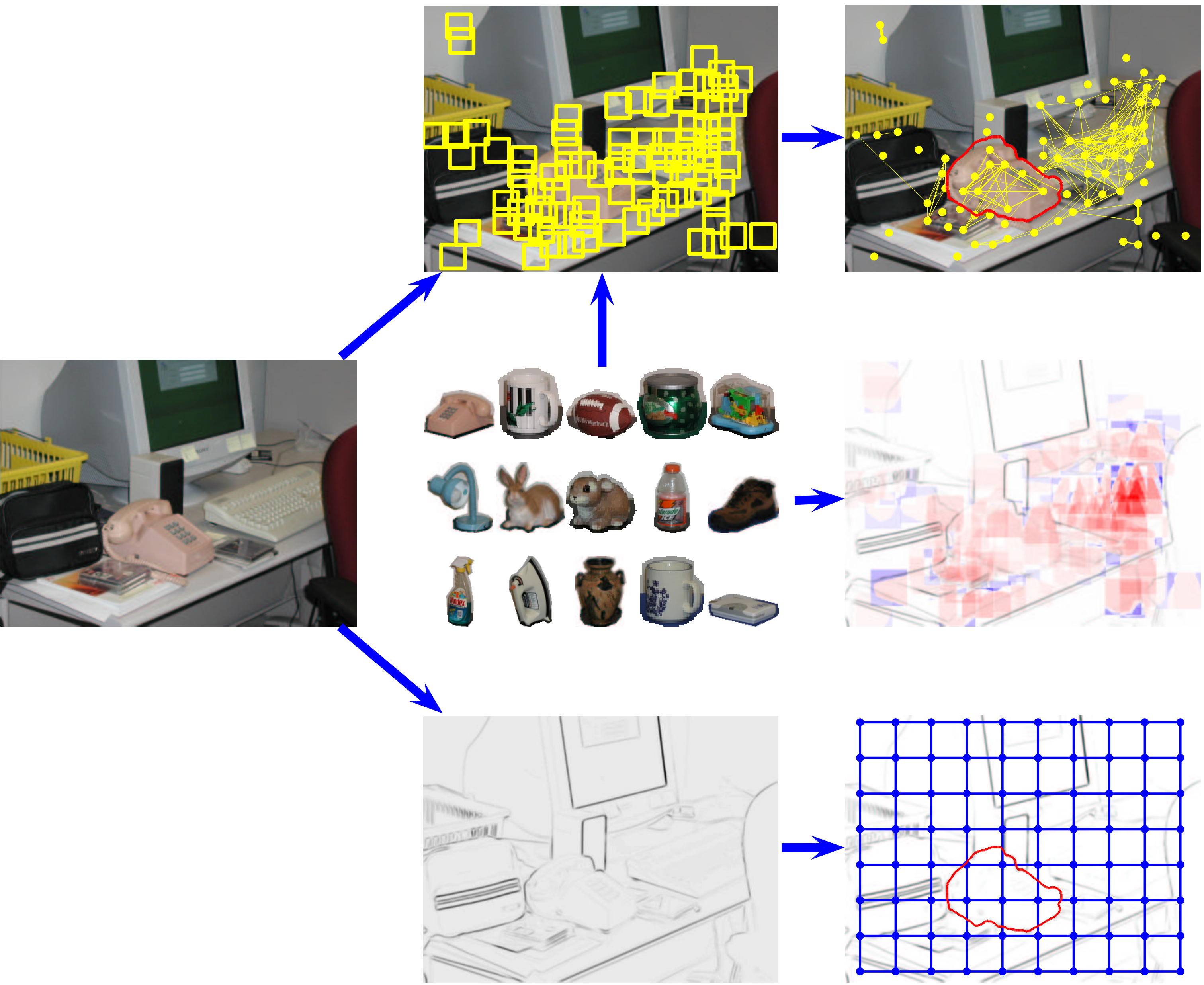
Our Approach to Object Segmentation



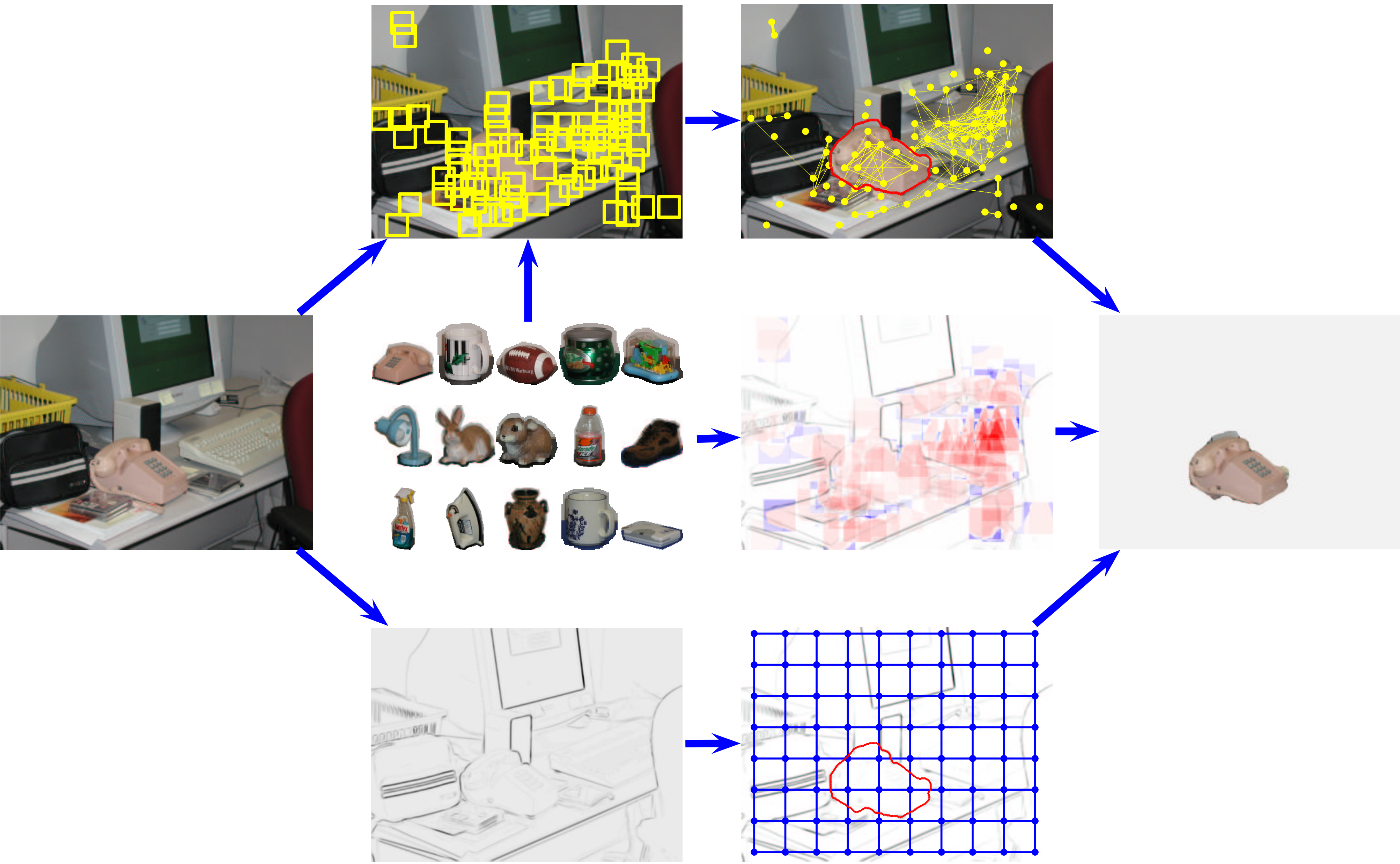
Our Approach to Object Segmentation



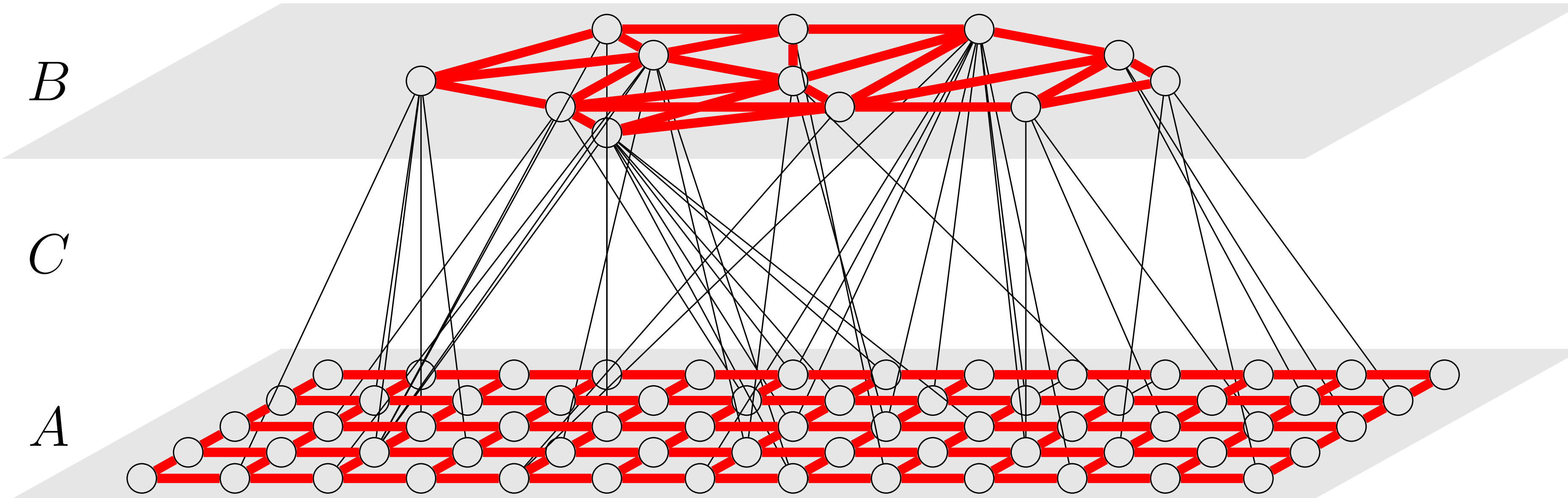
Our Approach to Object Segmentation



Our Approach to Object Segmentation



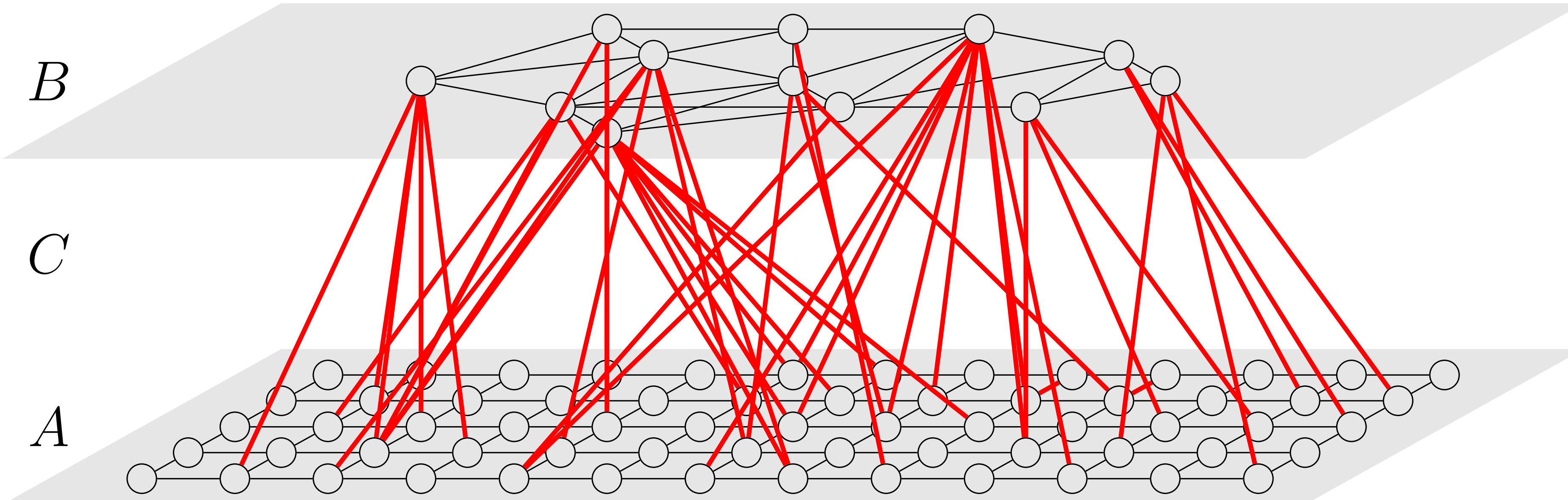
Joint Pixel-Patch Grouping: Criterion



$$\bar{\varepsilon}(\Gamma_{\mathbb{V}}^{K}, \Gamma_{\mathbb{U}}^{K}; A, B) = \frac{1}{K}\sum_{l=1}^{K} \frac{\text{linkratio}(\mathbb{U}_l, \mathbb{U}_l; B) \cdot \text{degree}(\mathbb{U}_l; B)}{\text{degree}(\mathbb{V}_l; A) + \text{degree}(\mathbb{U}_l; B)}$$

$$+ \frac{1}{K}\sum_{l=1}^{K} \frac{\text{linkratio}(\mathbb{V}_l, \mathbb{V}_l; A) \cdot \text{degree}(\mathbb{V}_l; A)}{\text{degree}(\mathbb{V}_l; A) + \text{degree}(\mathbb{U}_l; B)}$$

Joint Pixel-Patch Grouping: Consistency

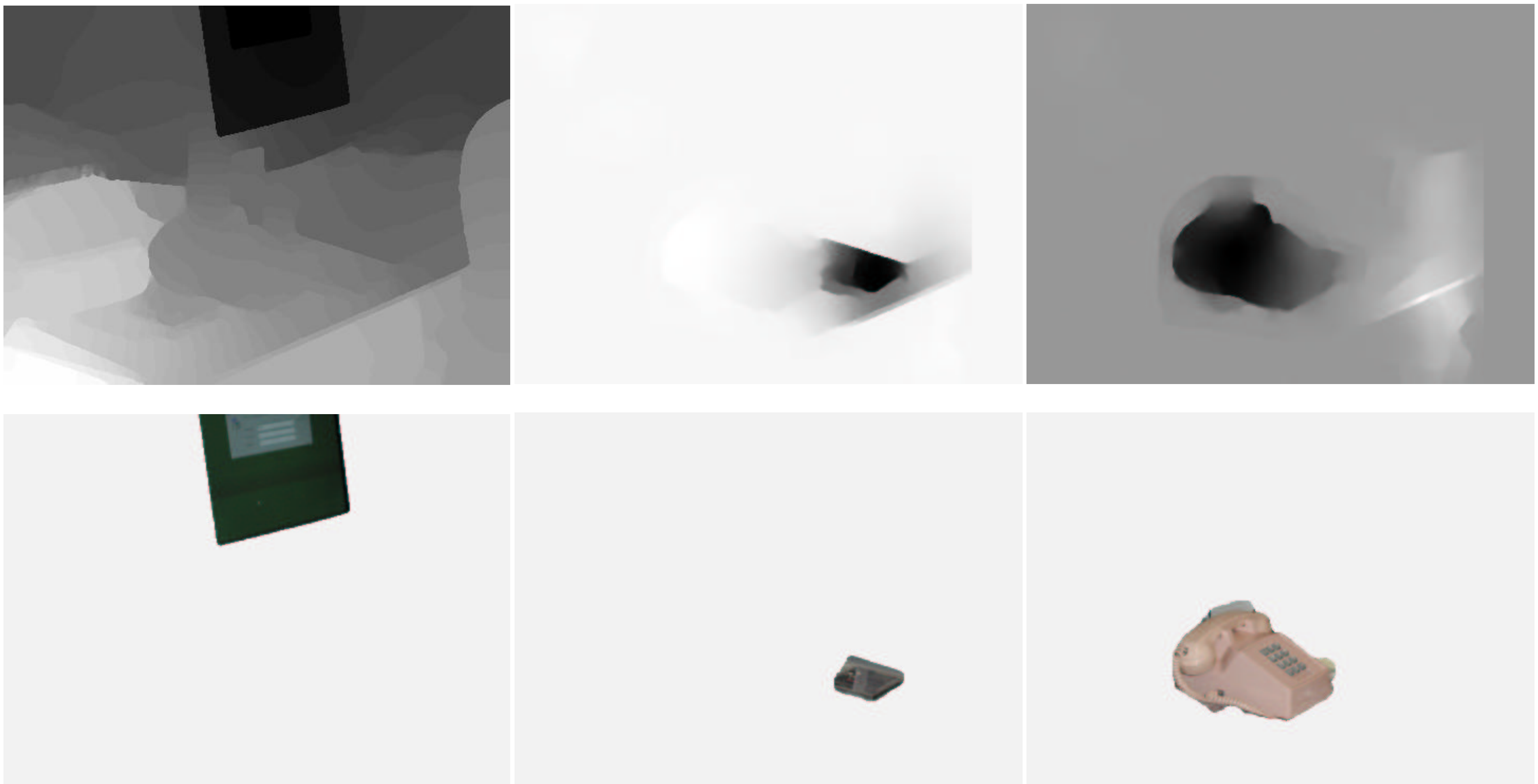


$$\Gamma_{\mathbb{U}}^{K} = \{\mathbb{U}_1, \ldots, \mathbb{U}_K\}, \qquad \Gamma_{\mathbb{V}}^{K} = \{\mathbb{V}_1, \ldots, \mathbb{V}_K\}$$

Bias linking patches with their pixels

How Object Knowledge Helps Segmentation

pixel only	pixel w/ ROI	pixel-patch



541s	150s	110s

How Segmentation Helps Object Detection

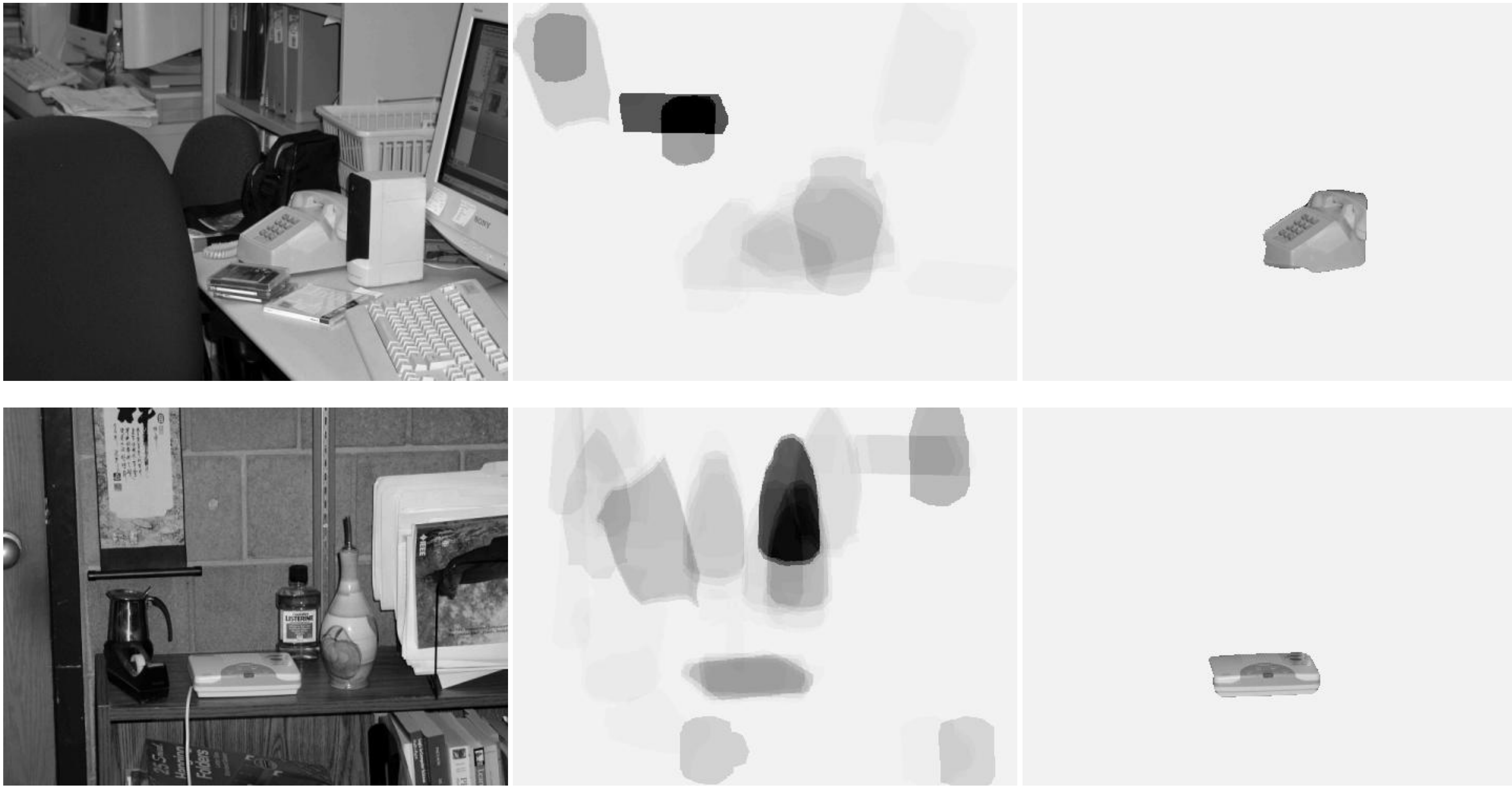


image | patch density | segmentation

When Does Our Method Fail

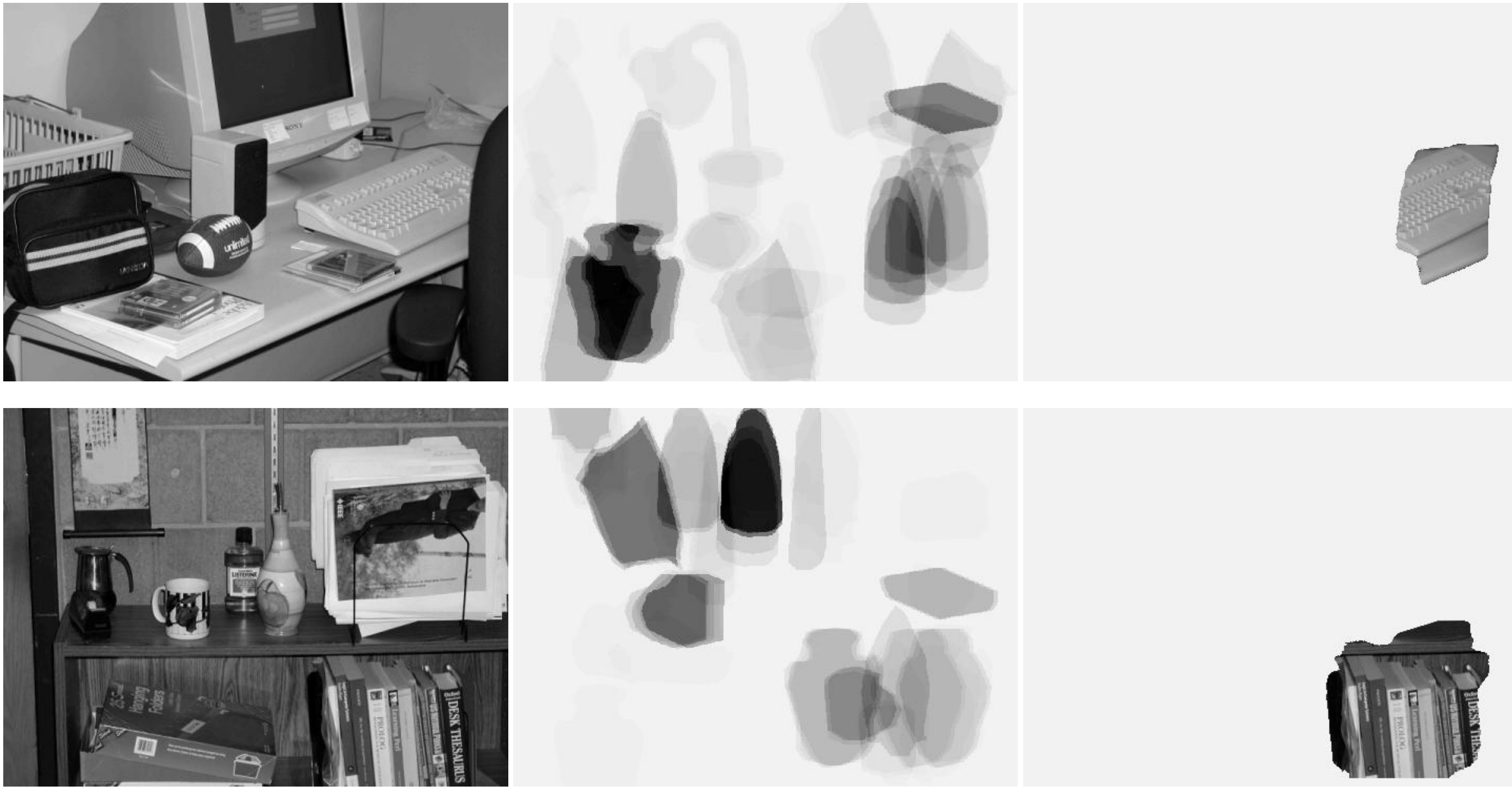
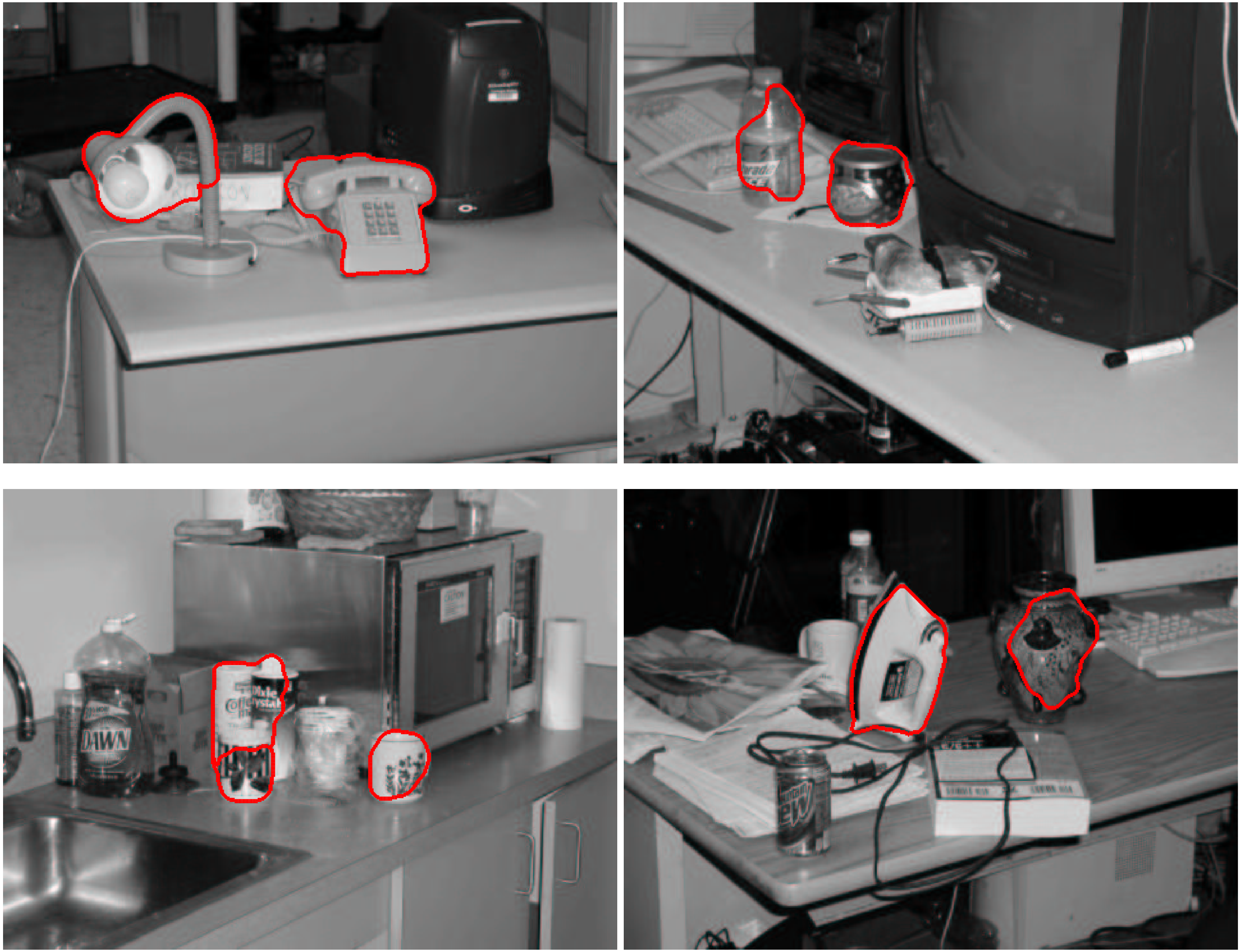


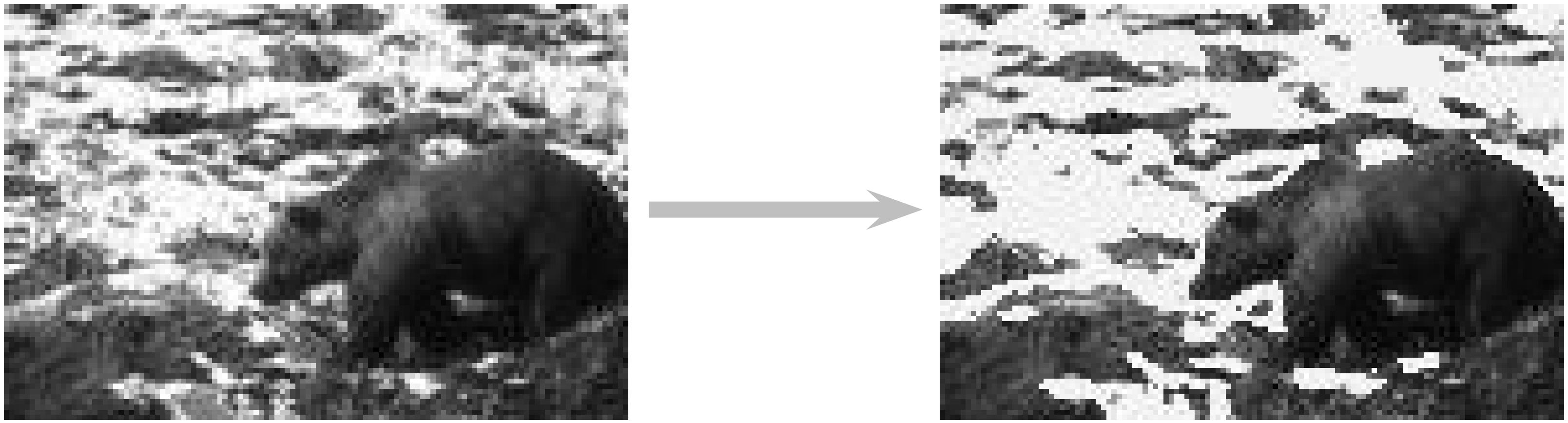
image patch density segmentation

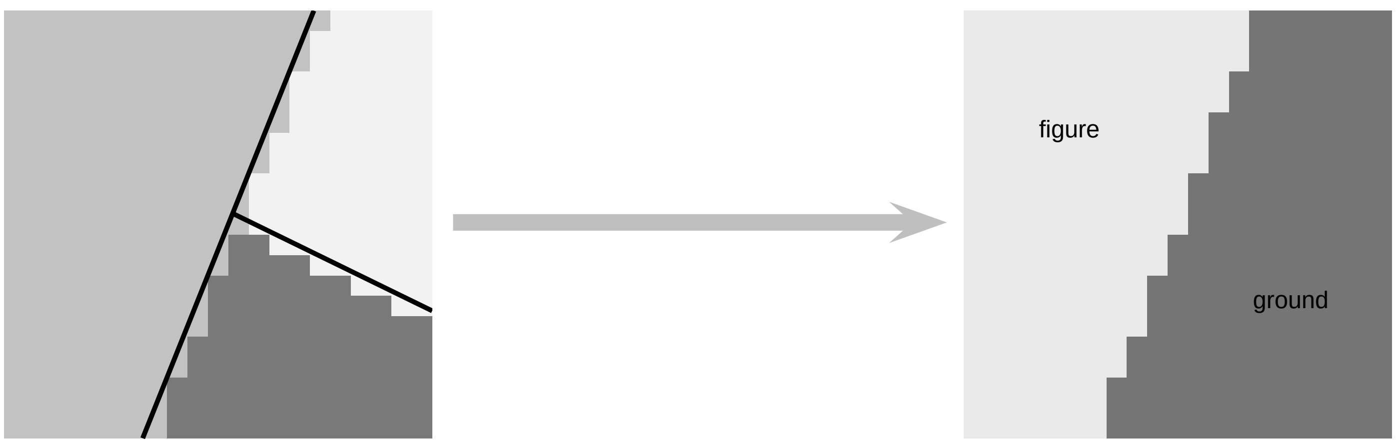
Equally Applicable to Multiple Objects



Contributions to Perceptual Organization

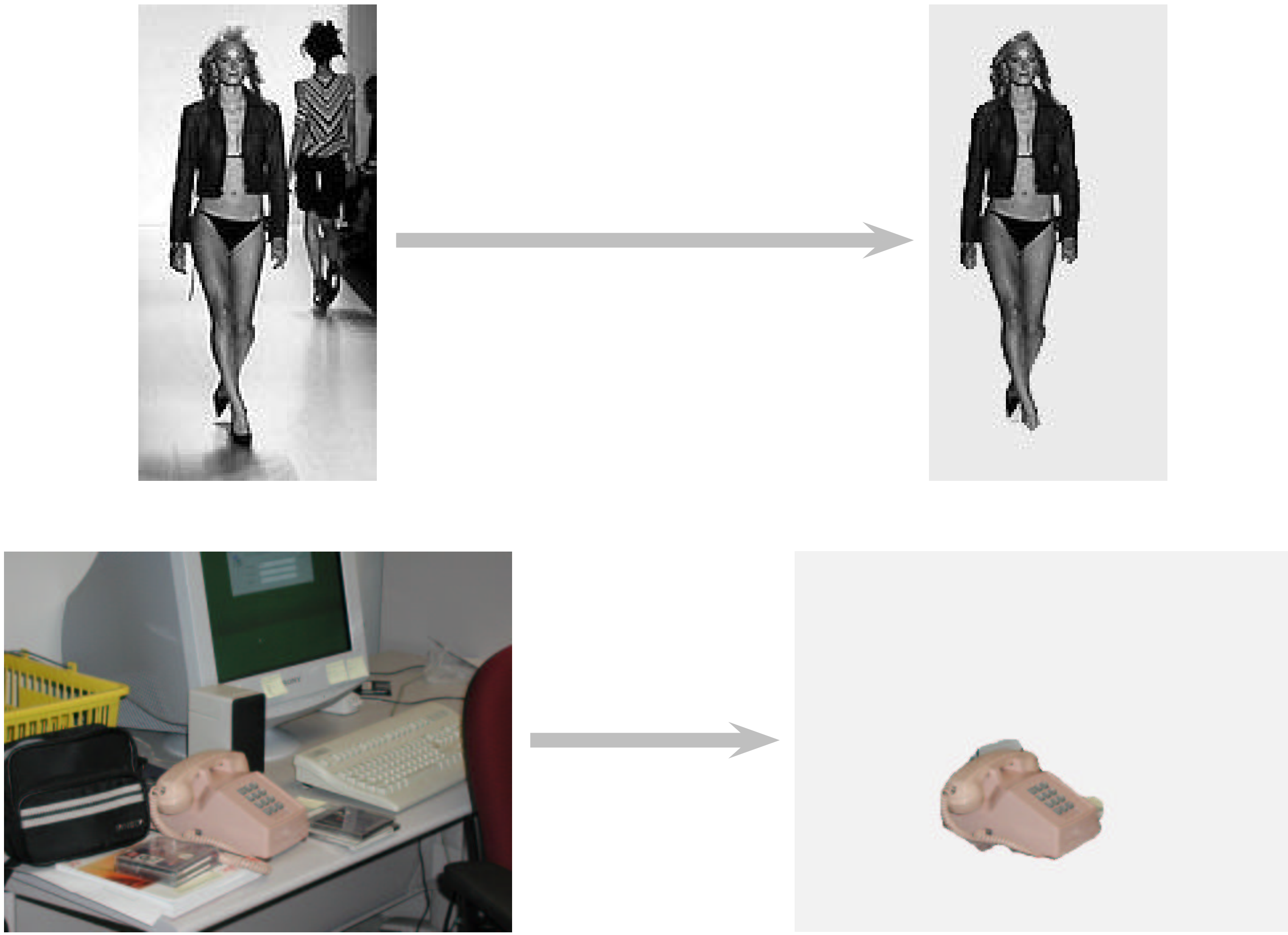
1. grouping and figure-ground in one framework





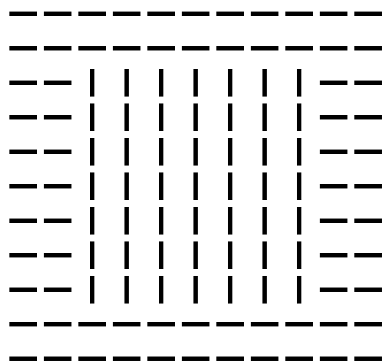
Contributions to Perceptual Organization

2. grouping integrated with spatial and object attention

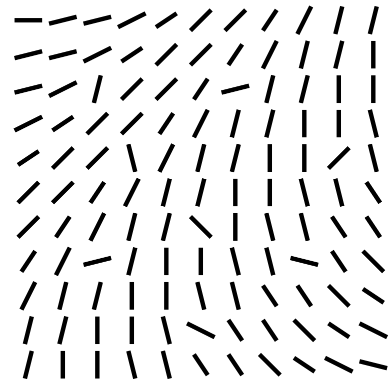


Contributions to Graph Theory

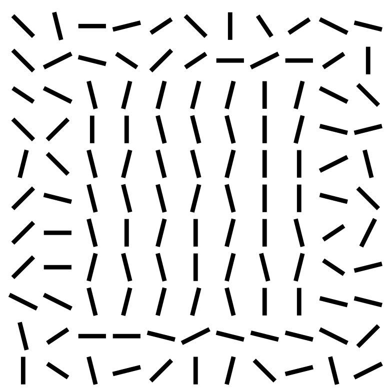
1. new grouping cues



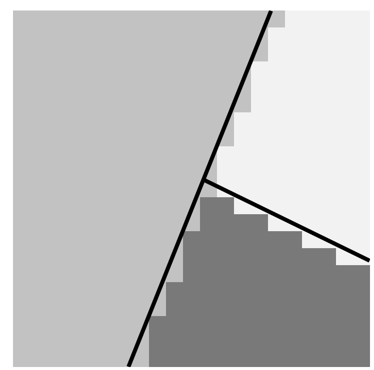
attraction



repulsion



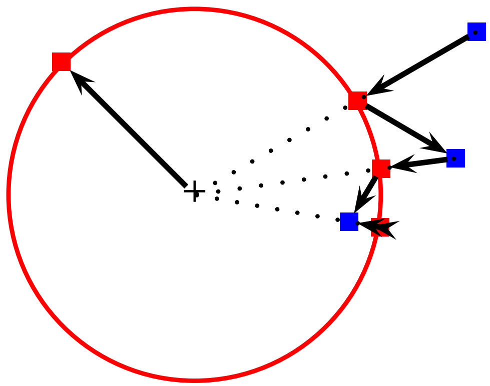
regularization



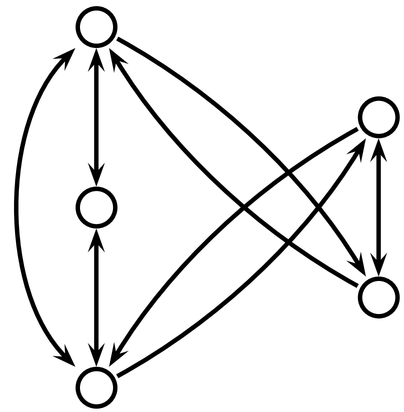
depth

Contributions to Graph Theory

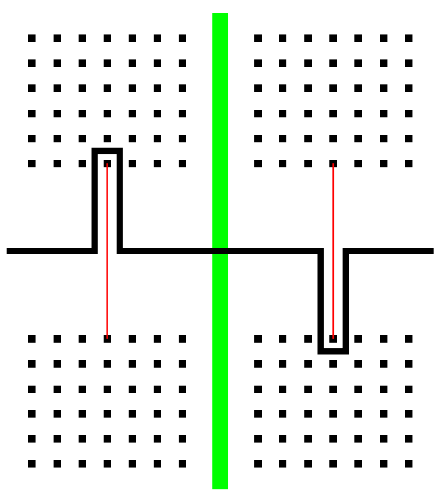
2. new graph partitioning techniques



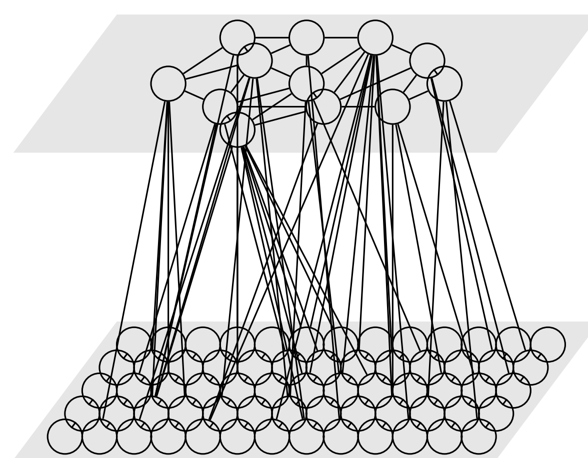
K-way cuts



directed cuts



biased cuts



joint cuts

Future Work

1. Automatic selection of the number of classes.
2. A model-based view on spectral clustering.
3. A criterion for comparing two segmentations.
4. Closing a feedback loop.
5. Object representation.
6. Scaling up.

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  Shyjan Mahamud, David Tolliver
  Jing Xiao, Vandi Verma

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  NSF LIS 9720350
  NSF CAREER 9984706
  NIH EY 08098