

# Orthogonal Convolutional Neural Networks

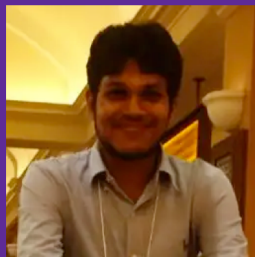
Jiayun Wang



Yubei Chen



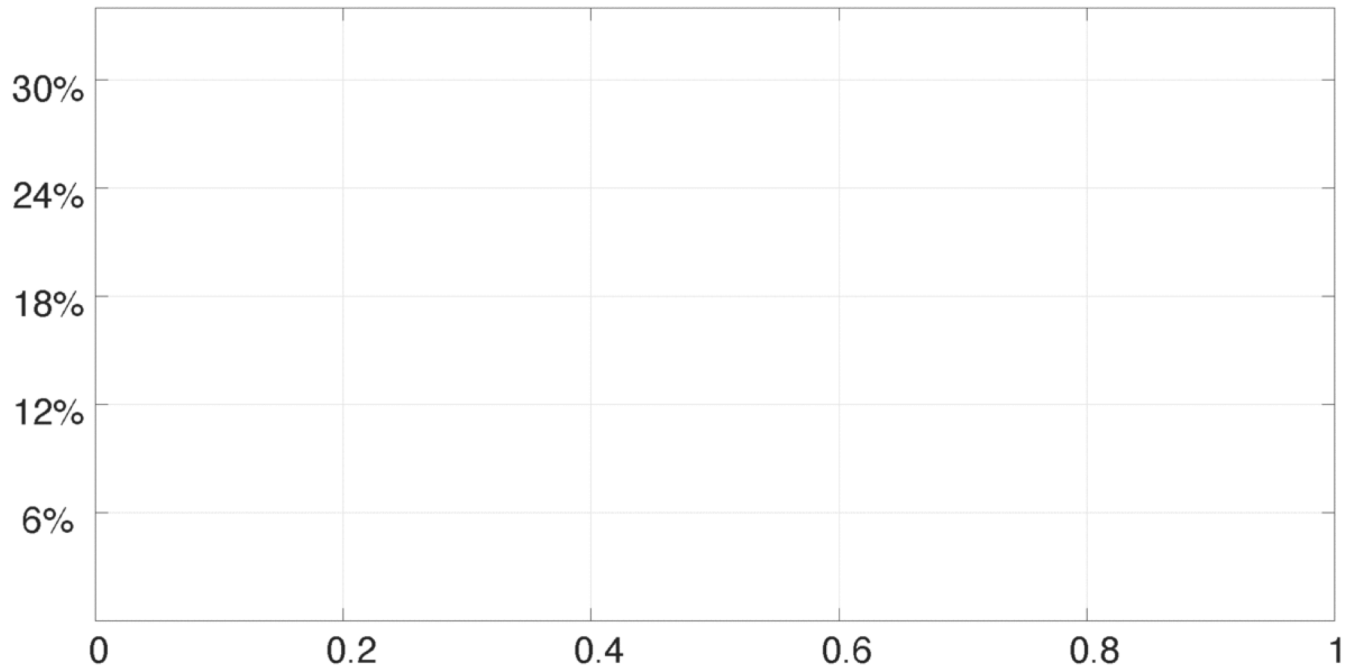
Rudrasis Chakraborty



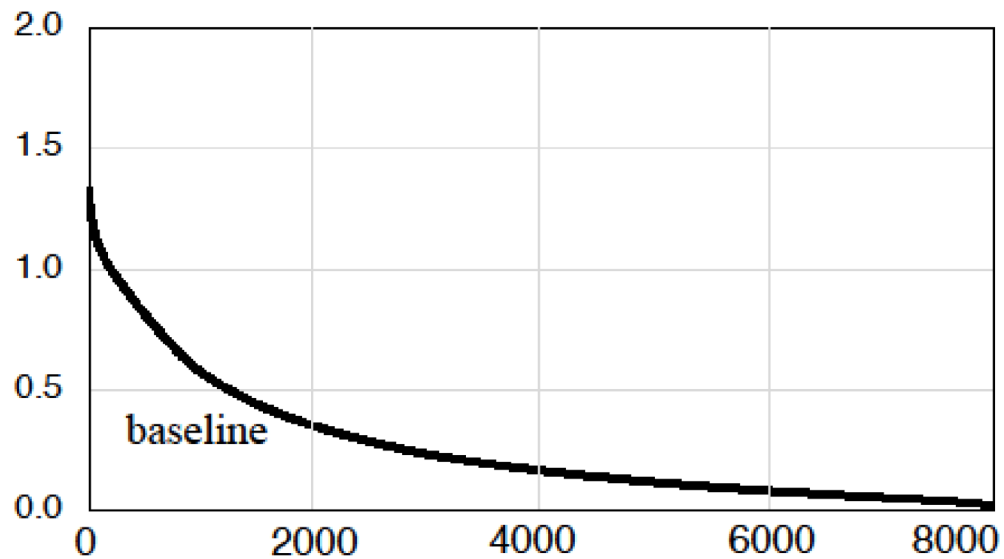
Stella X. Yu



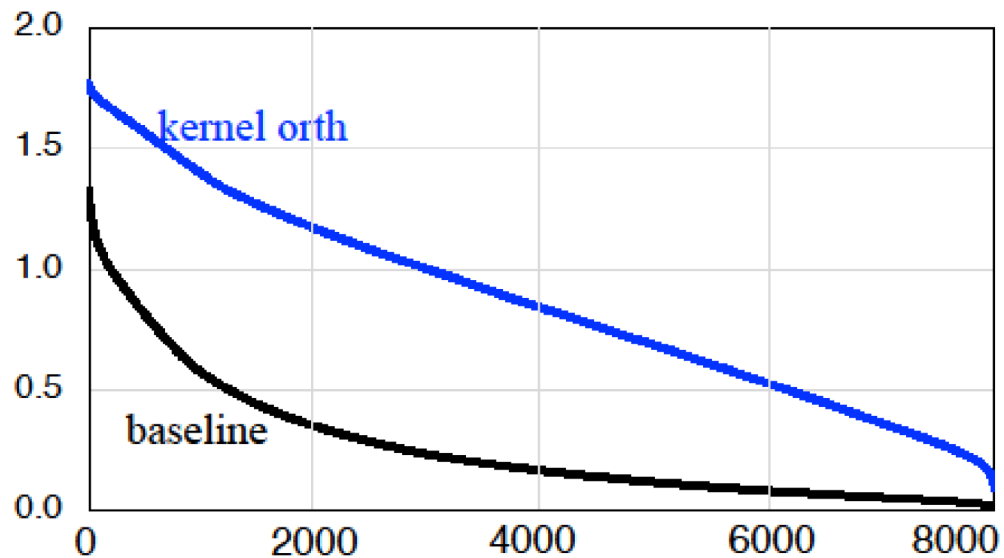
# Filter similarity increases with depth



# A typical conv layer has highly irregular spectrum



# Kernel orthogonality is widely used as a regularization



Saxe et al. 2014

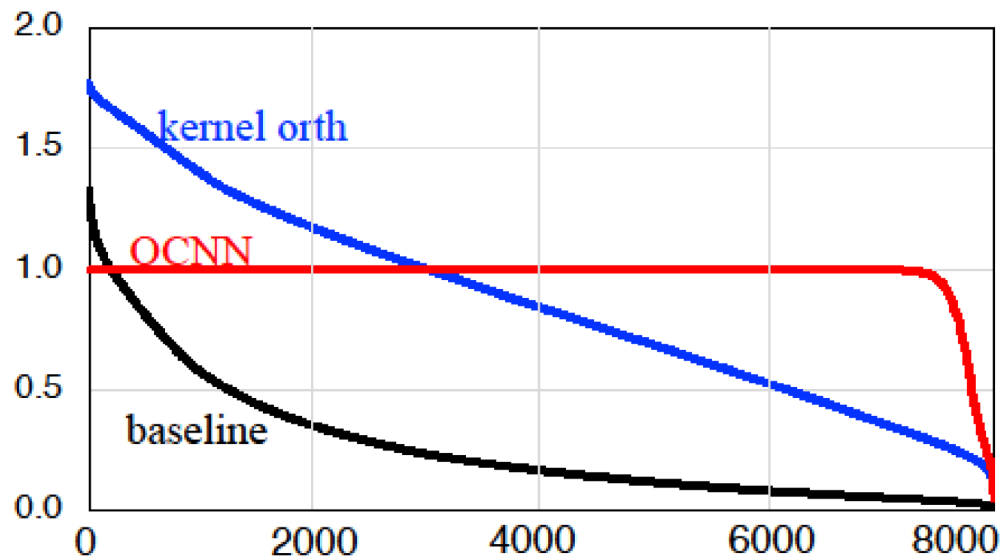
Dorobantu et al. 2016

Rodriguez et al. 2017

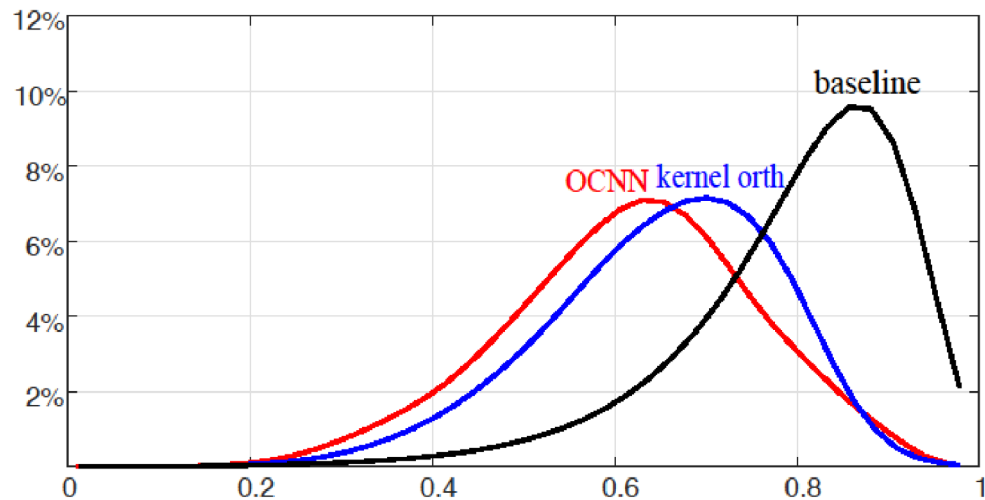
Bansal et al. 2018

...

# OCNN can do even better

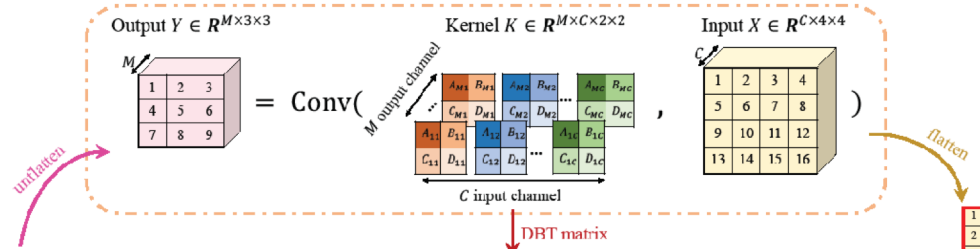


# Filter diversity improvement with OCNN



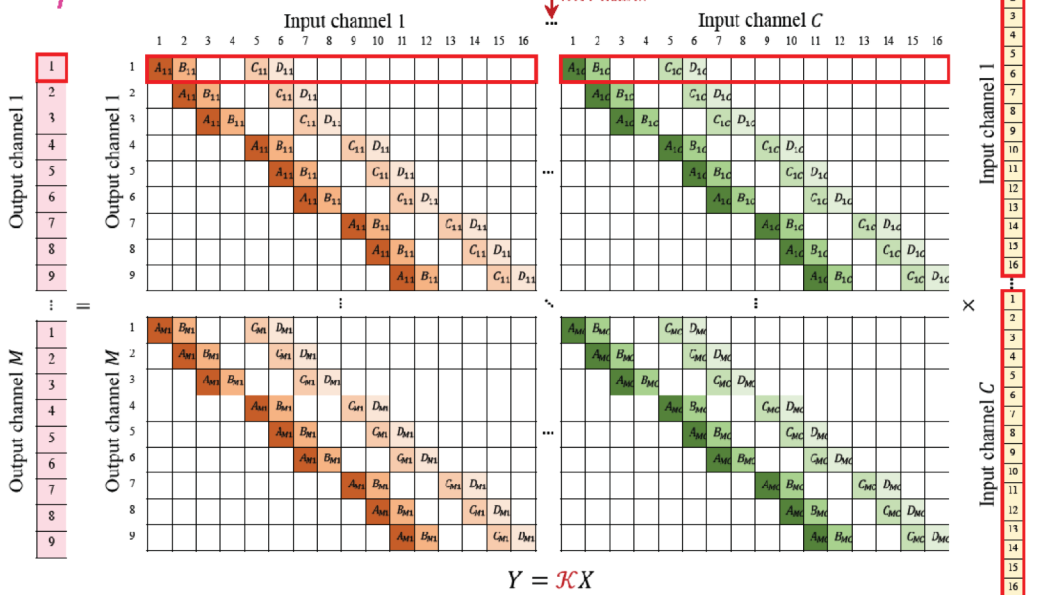
# Convolution is an efficient matrix-vector multiplication

$$Y = \text{Conv}(K, X), \text{ stride } 1$$



Convolution:

$$Y = K * X$$



Matrix-vector form:

$$K \rightarrow \mathcal{K}, Y = \mathcal{K}X$$

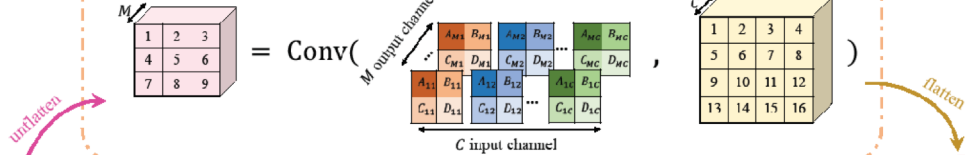
# Convolution is an efficient matrix-vector multiplication

$$Y = \text{Conv}(K, X), \text{ stride } 1$$

Output  $Y \in \mathbb{R}^{M \times 3 \times 3}$

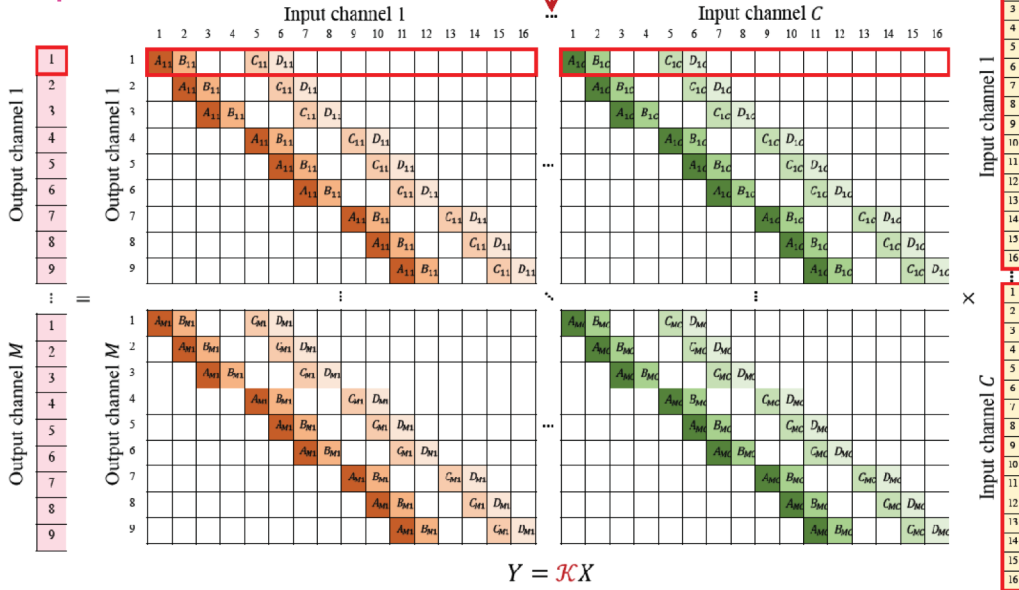
Kernel  $K \in \mathbb{R}^{M \times C \times 2 \times 2}$

Input  $X \in \mathbb{R}^{C \times 4 \times 4}$



Convolution:

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Matrix-vector form:

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# Convolution is an efficient matrix-vector multiplication

$$Y = \text{Conv}(K, X), \text{ stride } 1$$

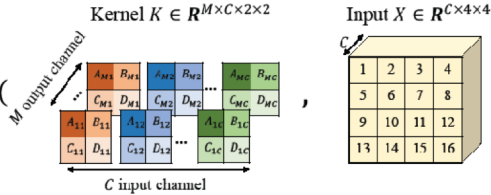
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Input  $X \in \mathbb{R}^{C \times 4 \times 4}$

1	2	3
4	5	6
7	8	9

= Conv(

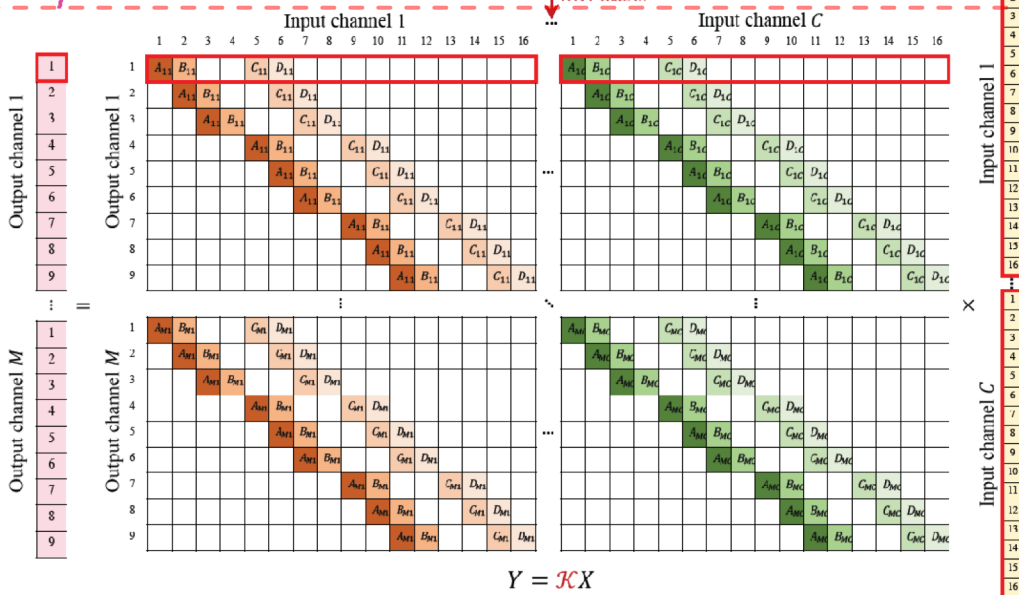


Convolution:

$$Y = K * X$$

unflatten

flatten

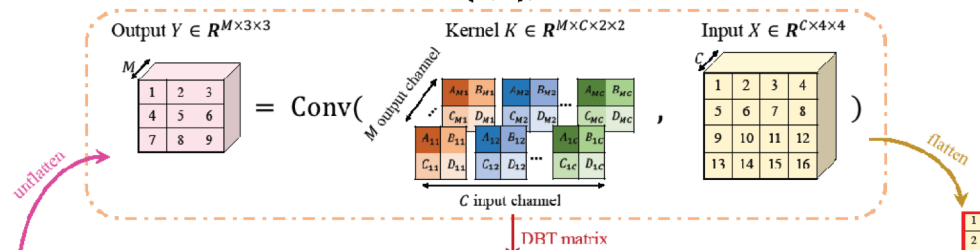


Matrix-vector form:

$$K \rightarrow \mathcal{K}, Y = \mathcal{K}X$$

# Orthogonal convolution or orthogonal kernel?

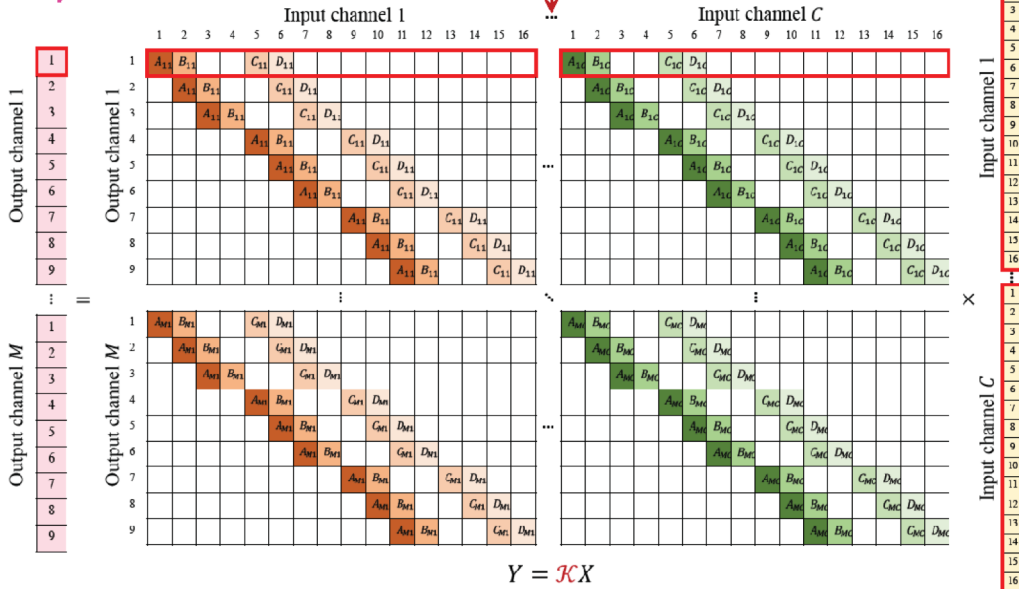
$$Y = \text{Conv}(K, X), \text{ stride } 1$$



Convolution:

$$Y = K * X$$

kernel orthogonality:  $KK^T = I$



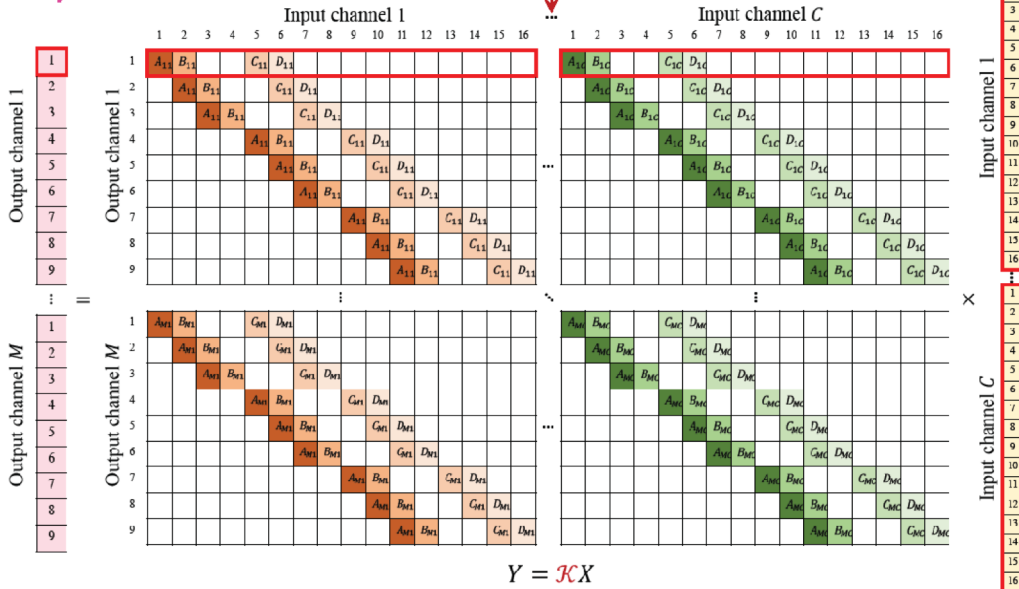
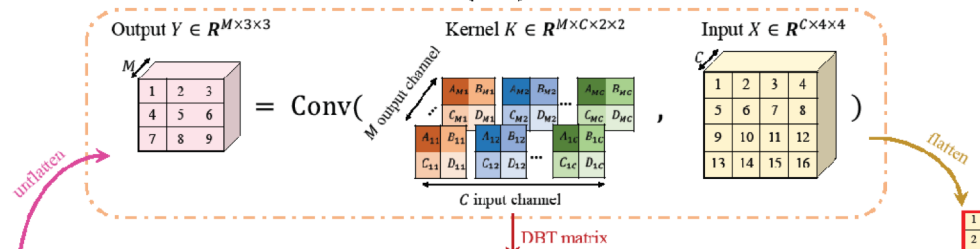
Matrix-vector form:

$$K \rightarrow \mathcal{K} Y = \mathcal{K} X$$

conv orthogonality:  $\mathcal{K}\mathcal{K}^T = I$

# Orthogonal convolution or orthogonal kernel?

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Convolution:

$$Y = K * X$$

kernel orthogonality:  $KK^T = I$

Matrix-vector form:

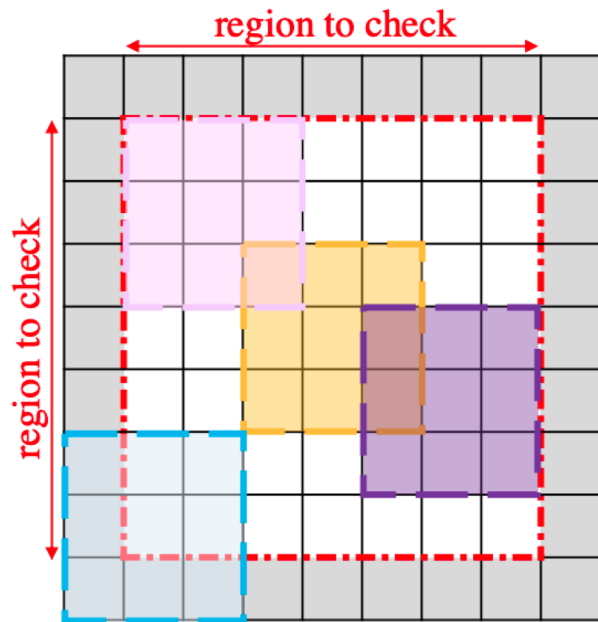
$$K \rightarrow \mathcal{K} Y = \mathcal{K}X$$

conv orthogonality:  $\mathcal{K}\mathcal{K}^T = I$

$$\mathcal{K}\mathcal{K}^T = I \Rightarrow KK^T = I$$

$$\mathcal{K}\mathcal{K}^T = I \not\Leftarrow KK^T = I$$

# A fast algorithm for orthogonal convolution



- Kernel Orthogonality:

$$\begin{cases} \text{Conv}(K, K, \text{padding} = 0) = I_{r0} \\ \text{Conv}(K^T, K^T, \text{padding} = 0) = I_{c0} \end{cases}$$

- Convolutional Orthogonality:

$$\text{Conv}(K, K, \text{padding} = P, \text{stride} = S) = I_{r0}$$

Same # parameters and test time, only 9% more training time

# Universal improvements

	Task	Metric	Gain
Image Classification	CIFAR100	classification accuracy	3%
	ImageNet	classification accuracy	1%
	semi-supervised learning	classification accuracy	3%
Feature Quality	fine-grained image retrieval	kNN classification accuracy	3%
	unsupervised image inpainting	PSNR	4.3
	image generation	FID	1.3
	deep metric learning	NMI	1.2
Robustness	black box attack	attack time	7x less



Thanks

