SurReal: Fréchet Mean and Distance Transform for Complex-Valued Deep Learning

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Overview

1. New complex-valued deep learning theory that handles scaling ambiguity with equivariance and invariance properties on a manifold.
2. Sur-real experimental validation with significant performance gain (94% → 98%) at a fraction (8%) of the baseline model size.

New Deep Learning Theory

1. New manifold representation $\mathbf{R}^+ \times \mathbf{SO}(2)$.

$$a + ib \overset{F}{\mapsto} (r, R(\theta)),$$
$$r = (a+ib) = \sqrt{a^2+b^2}$$
$$\theta = \arg(a+ib) = \text{atan2}(b,a)$$
$$R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}.$$

2. New group action G for complex scaling: the product of planar rotation and scaling.
3. New convolution that is equivariant to G: weighted Fréchet Mean filtering:

$$\text{wFM}(\{\mathbf{z}_i\},\{w_i\}) = \arg\min_{\mathbf{m}\in\mathbf{C}} \sum_{i=1}^{K} w_i d^2(\mathbf{z}_i, \mathbf{m}),$$
$$\text{where } \sum_{i=1}^{K} w_i = 1, \quad w_i \in (0,1], \forall i$$
$$d(\mathbf{z}_1,\mathbf{z}_2) = \sqrt{\log^2\left(\frac{r_2}{r_1}\right) + \|\text{logm}\left(R_1^{-1}R_2\right)\|^2}.$$

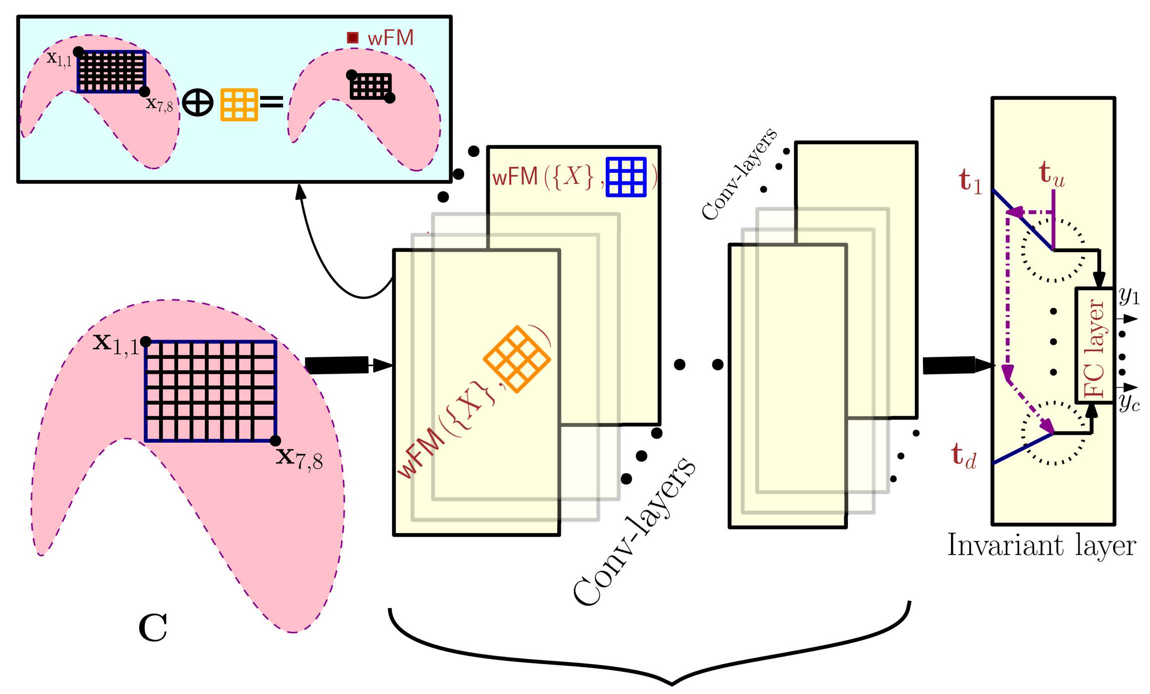
4. New fully connected layer operator that is invariant to G: distance to the wFM.

$$\mathbf{m} = \text{wFM}(\{\mathbf{t}_i\},\{v_i\})$$
$$u_i = d(\mathbf{t}_i, \mathbf{m}).$$

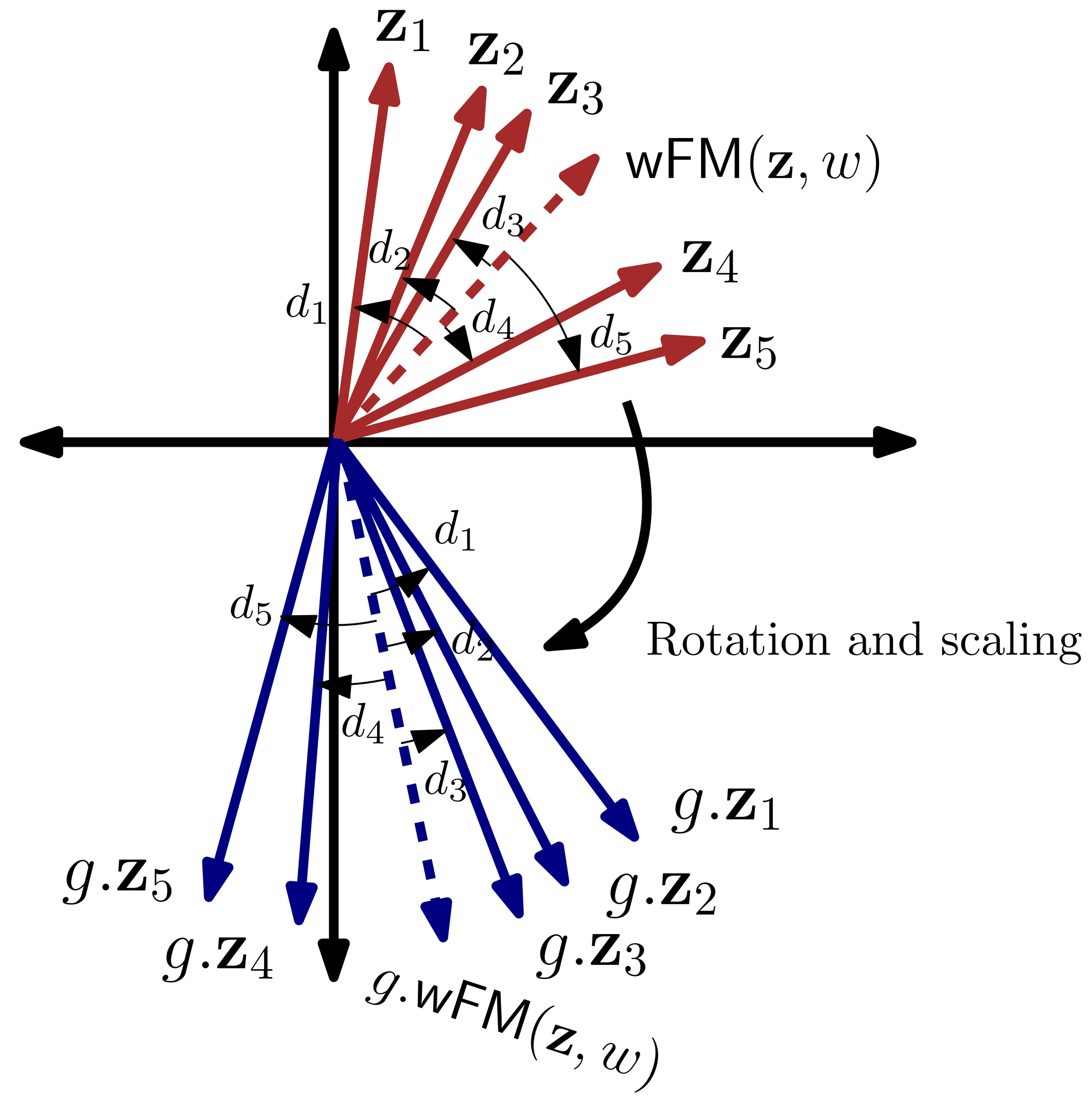
5. New nonlinear activation function: ReLU in the tangent space, log/exp maps back to manifold.

$$(r,R) \mapsto (\exp(\text{ReLU}(\log(r))), \text{expm}(\text{ReLU}(\text{logm}(R)))$$

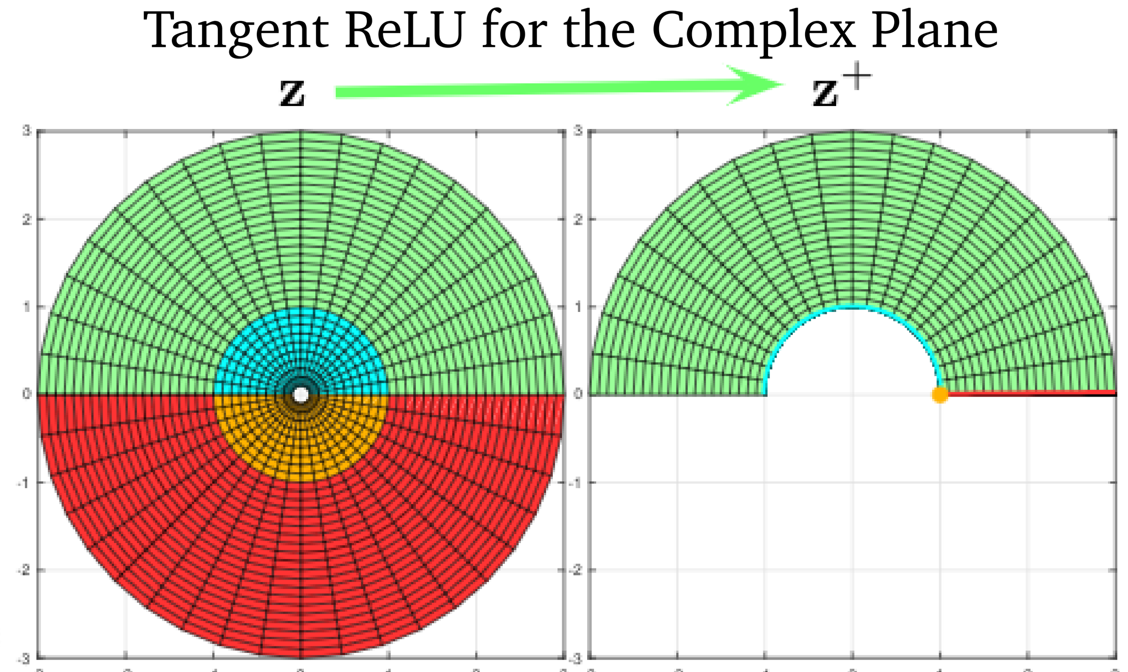
Schematic of Our CNN



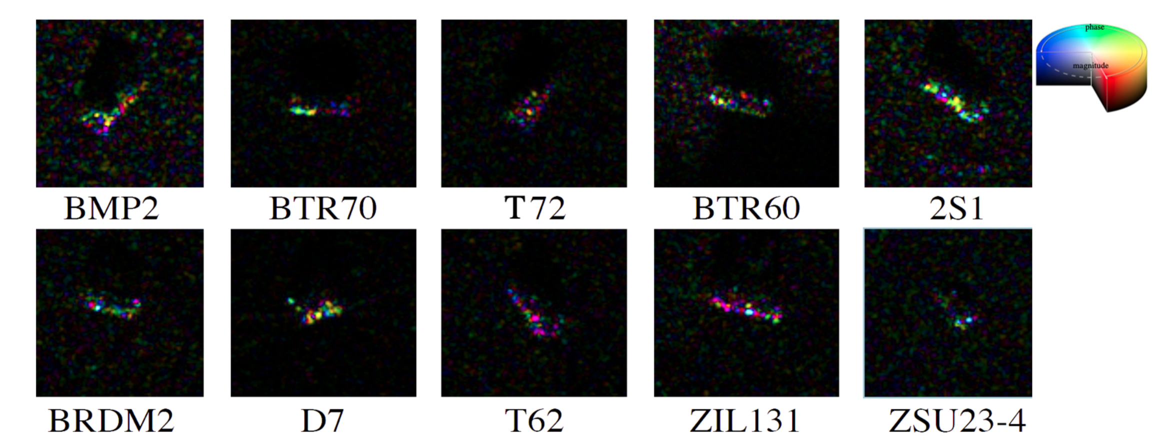
Equivariance of Fréchet Mean Filtering
Invariance of Distance Transform

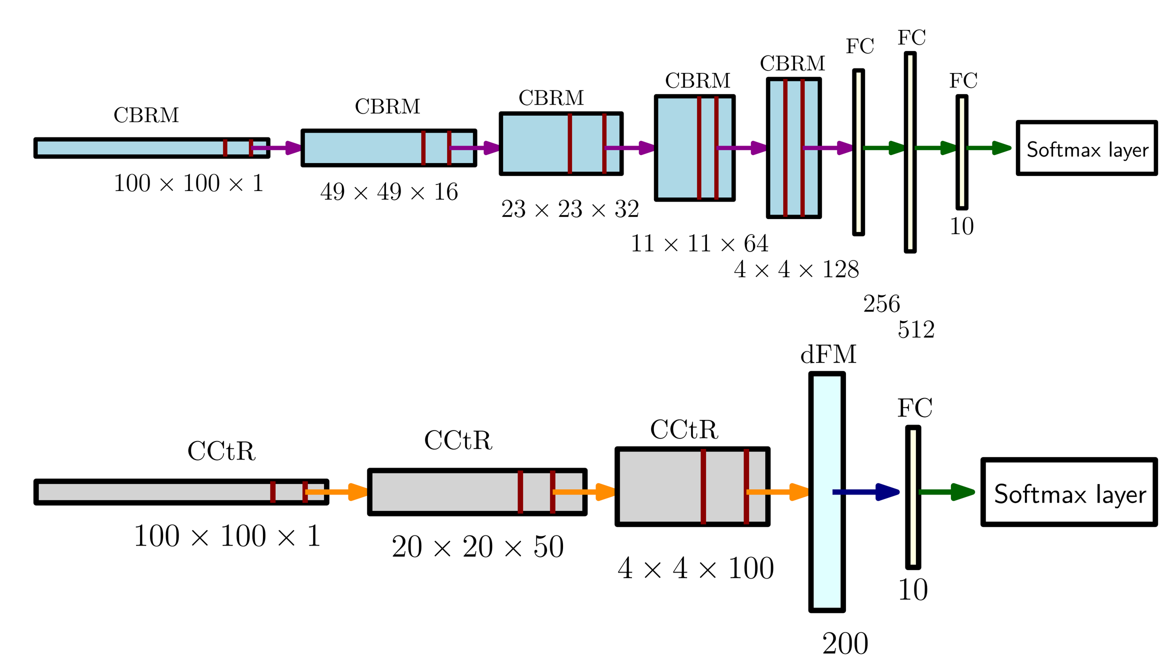


Tangent ReLU for the Complex Plane

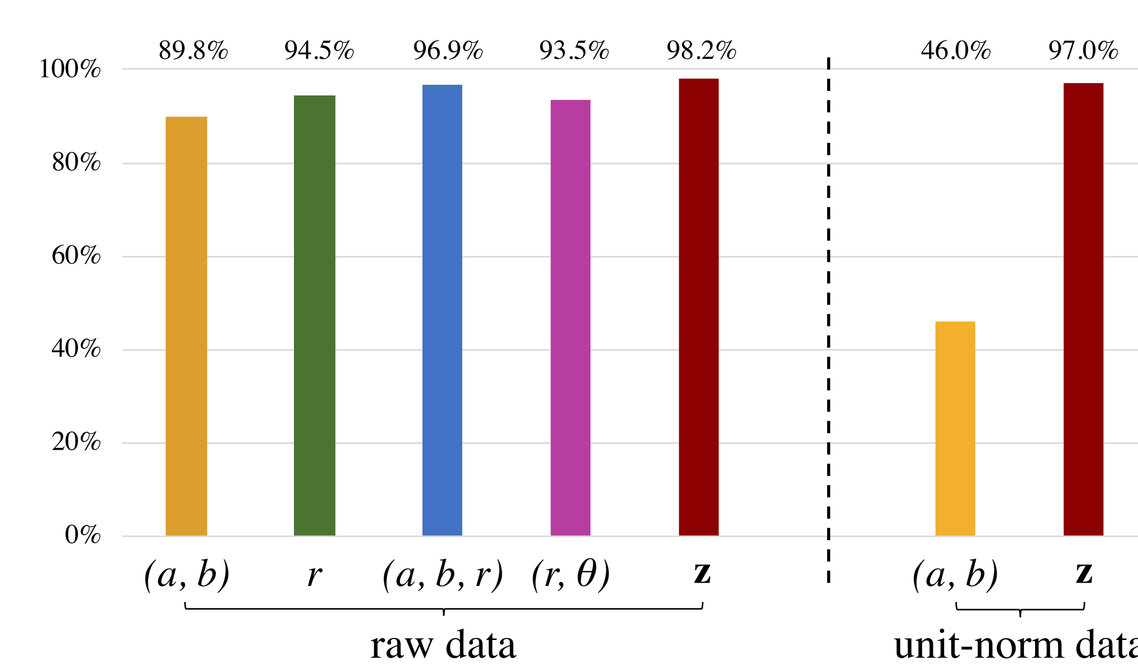


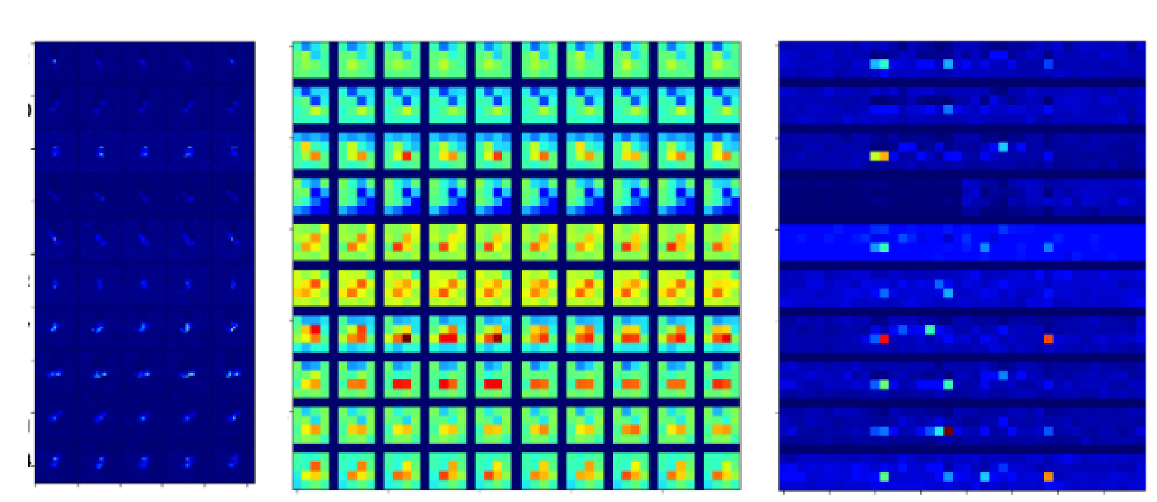
Our Results on MSTAR





CNN model	domain representation	# parameters
real	(a,b)	530,170
real	r	530,026
real	(a,b,r)	530,314
real	(r,θ)	530,170
complex	$\mathbf{z}$	**44,826**





Our Results on RadioML