Structural Correspondence as a Contour Grouping Problem

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Boston College

Mathematical Methods in Biomedical Image Analysis (MMBIA)

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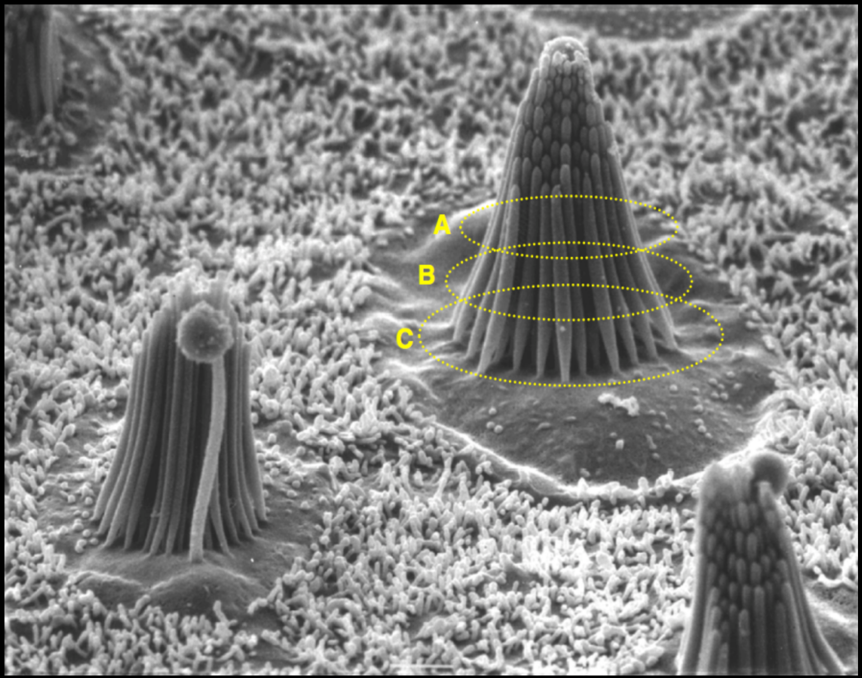
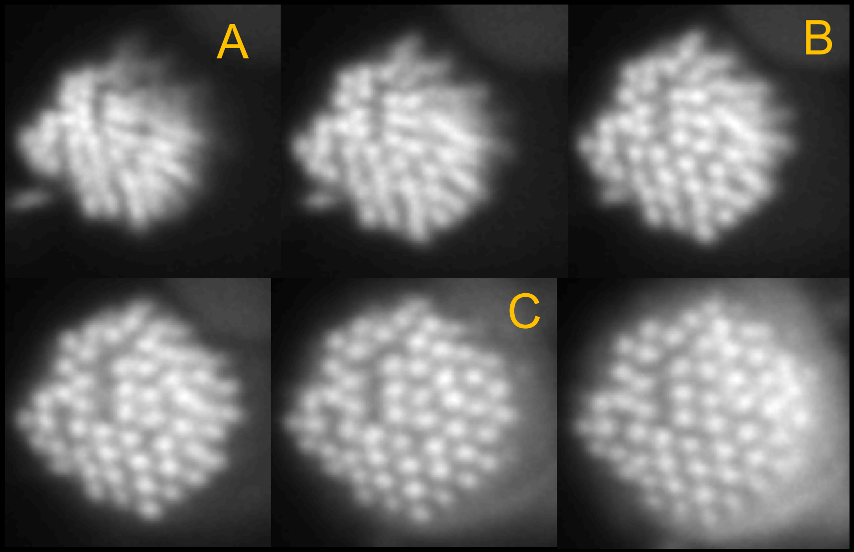
Extracting Tubular Structures: Finding Correspondence throughout Image Stacks



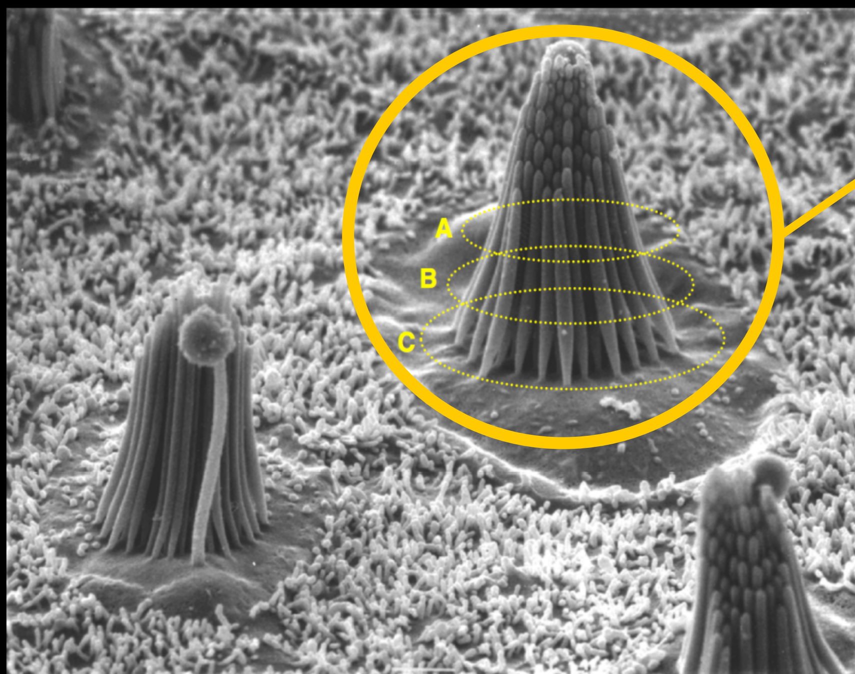
Haircell bundles of the inner ear



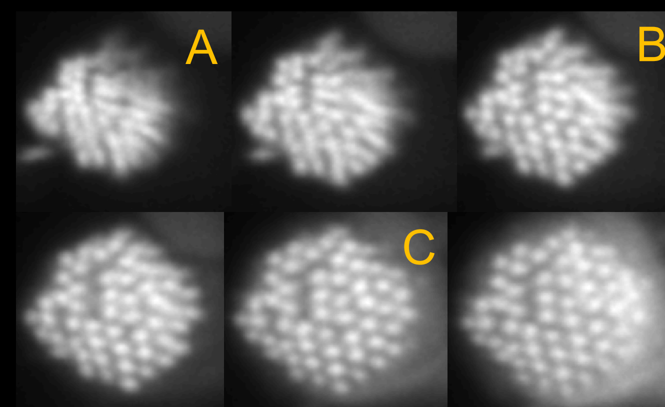
Stereocilia cross- sections

[image courtesy of M. Pathak and D. Corey at Harvard]

Extracting Tubular Structures

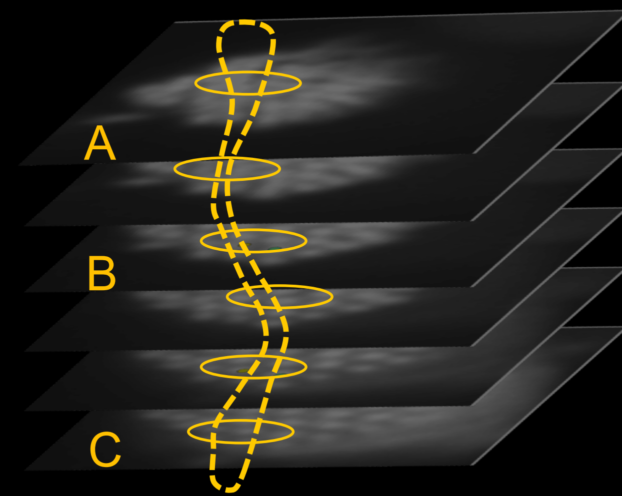


Organ Pipe Structure:
- varying lengths
- varying cross-section shapes
  (cells shifting & shrinking)



Possible Approaches

- 3D volumetric segmentation
  - naturally describes 3D tubes
  - implicit correspondence
  - pixel to pixel correspondence



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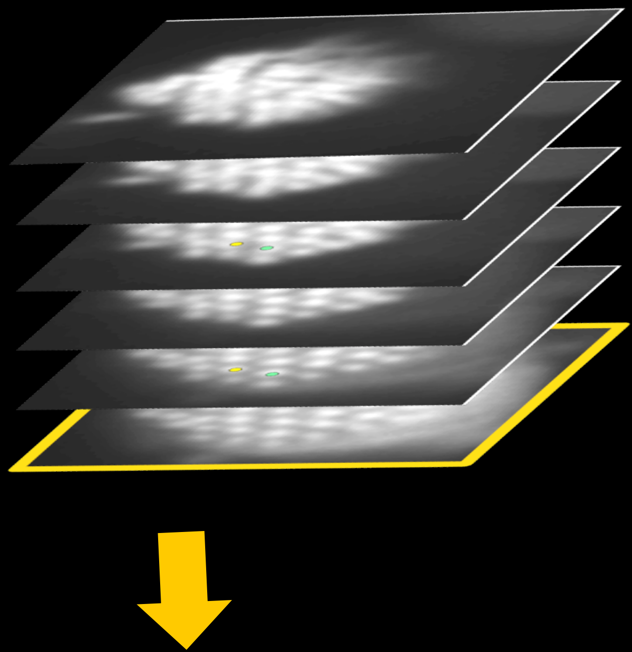
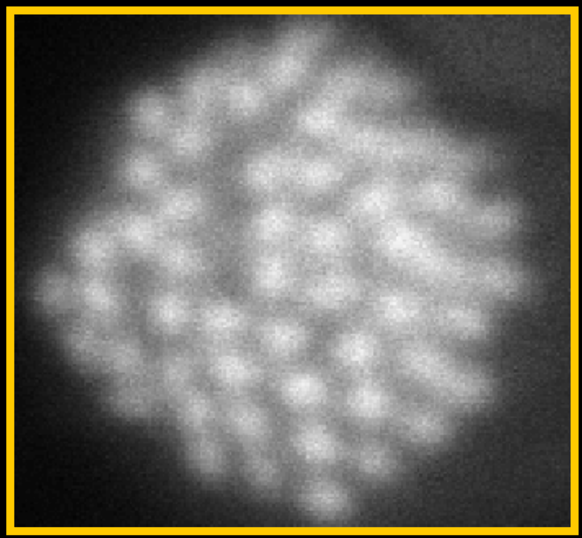


- 3D volumetric segmentation
  - naturally describes 3D tubes
  - implicit correspondence
  - pixel to pixel correspondence
- 2D segmentation + correspondence
  - explicit correspondence
  - segment to segment correspondence
  - tubes of different lengths



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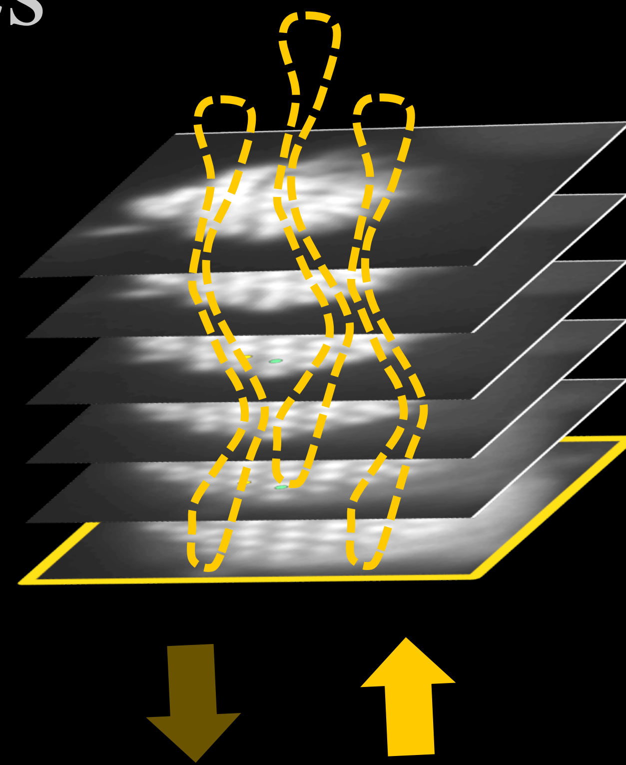

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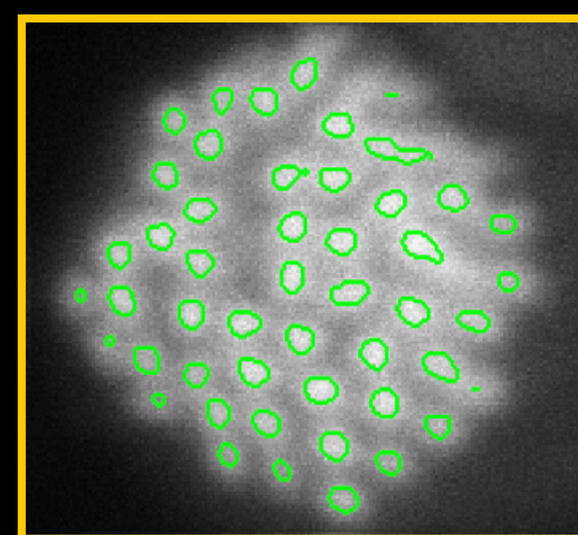

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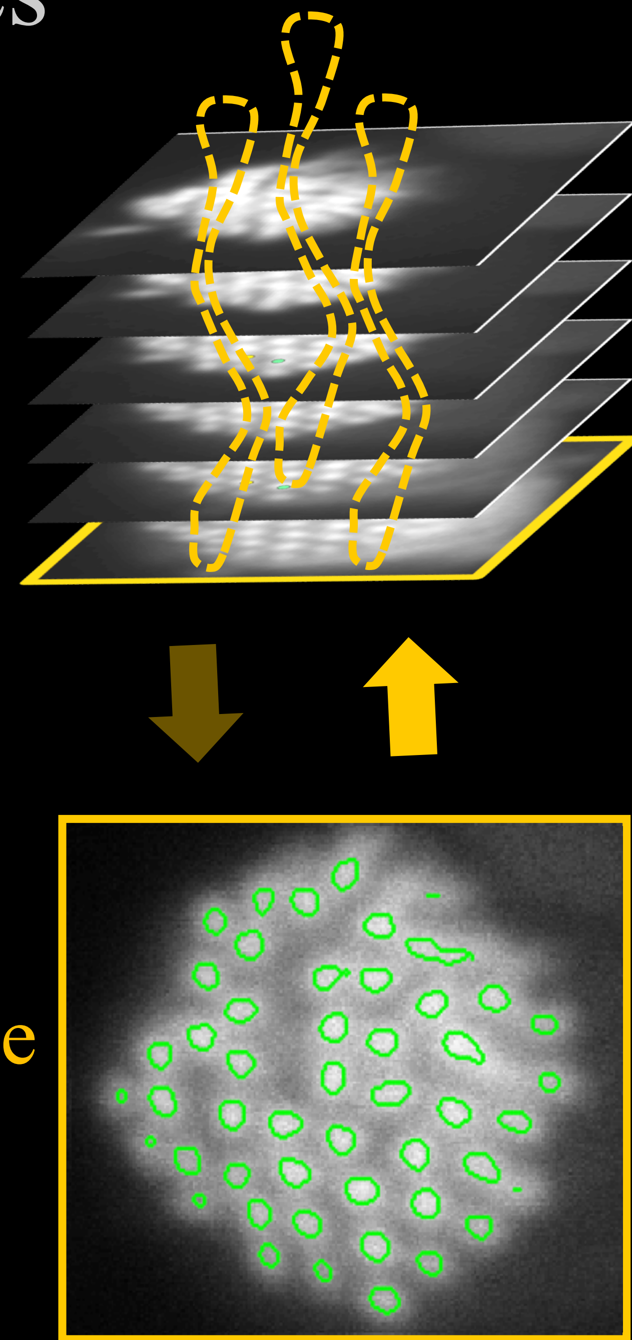
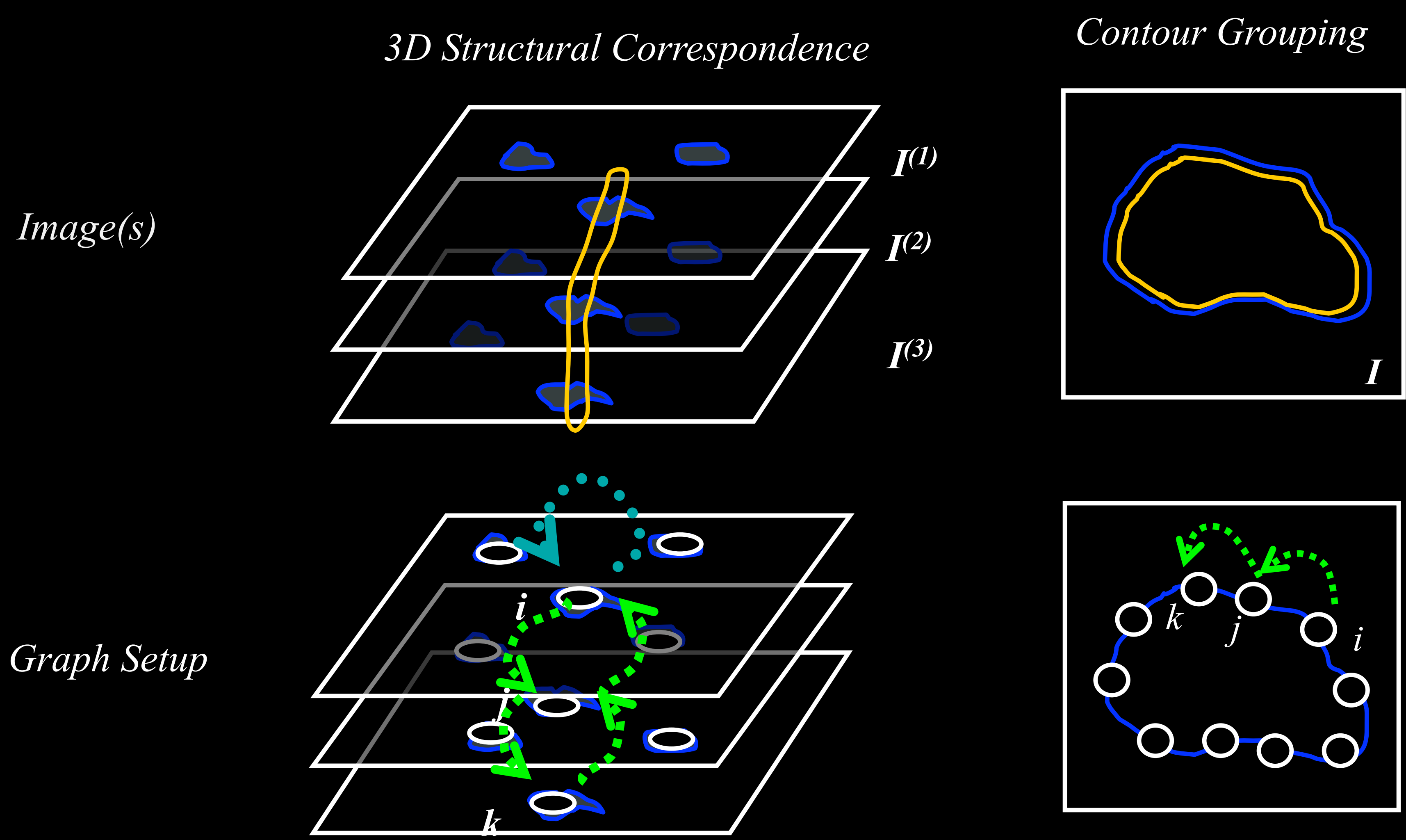


3D Correspondence as 2D Contour Grouping



Spectral-Graph Approach for Contour Grouping

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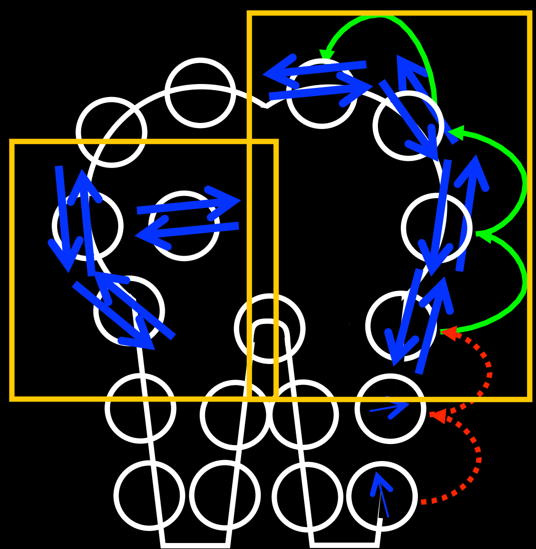
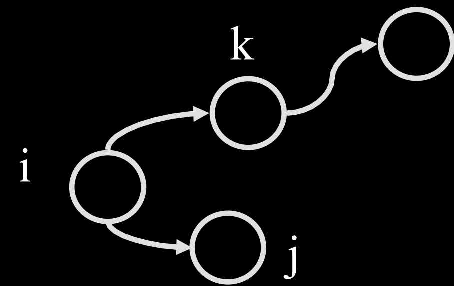
Spectral-Graph Approach for Contour Grouping

1D Line Graph



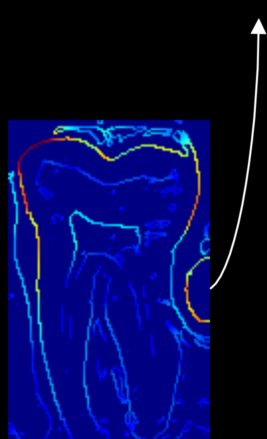


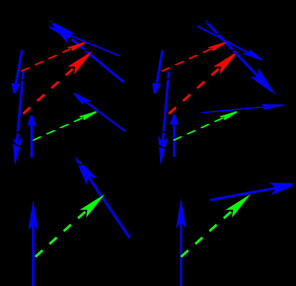
Random Walk

$$\vec{P} = D^{-1}W$$

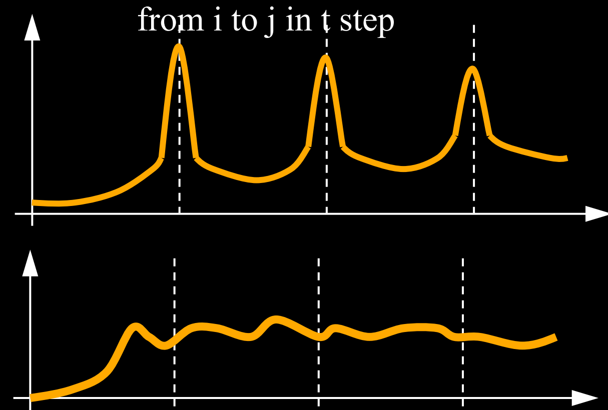
$$D = \mathrm{diag}(W \cdot \mathbf{1})$$

$$G^{\text{contour}} = < V^{\text{contour}}, W^{\text{contour}} >$$





Elastic Energy

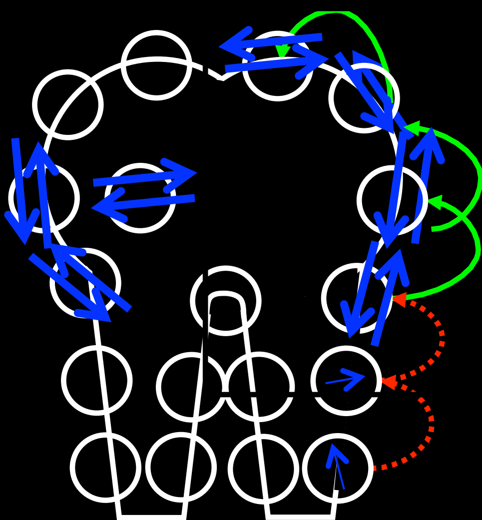


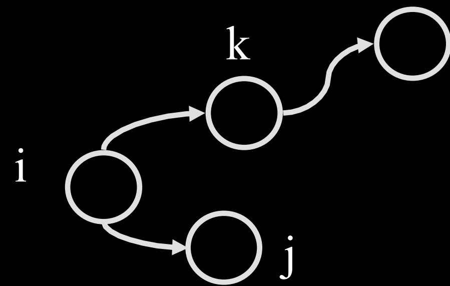
Peakiness of returning probability

[Zhu et al. `07]

Spectral-Graph Approach for Contour Grouping

1D Line Graph





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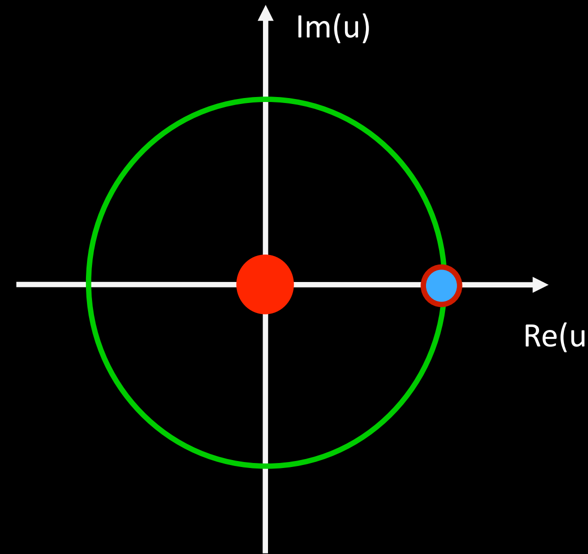
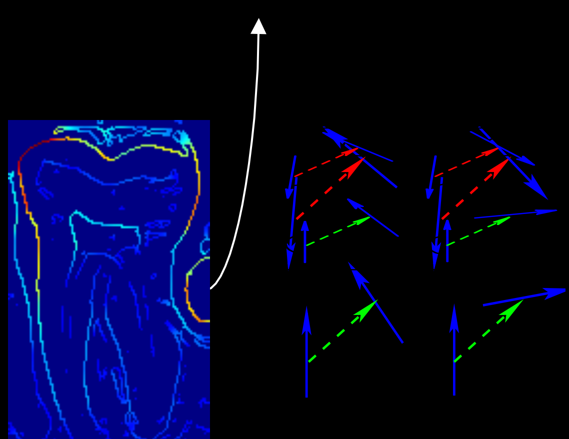


Complex Embedding

$$G^{\text{contour}} =< V^{\text{contour}}, W^{\text{contour}} >$$



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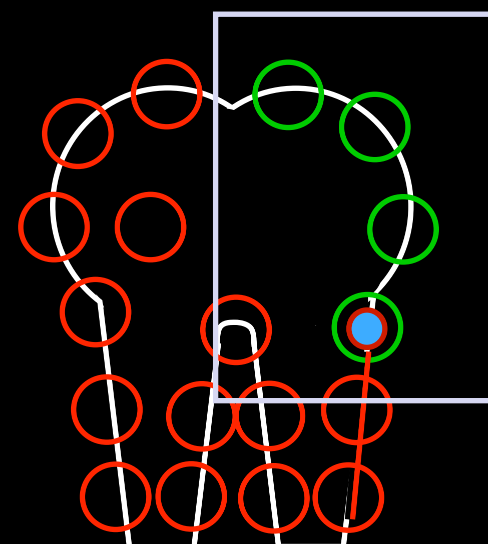
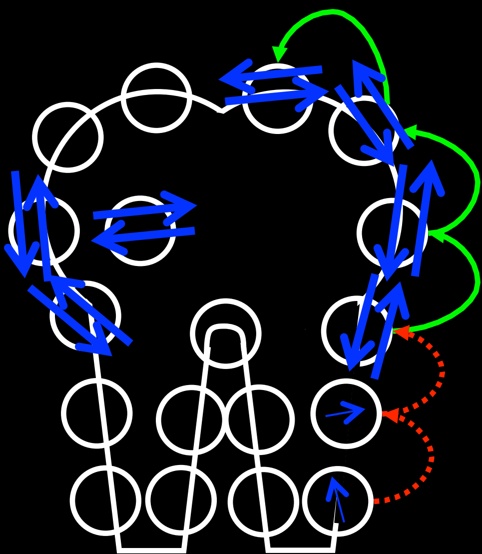


Contours

[Zhu et al. `07]

Spectral-Graph Approach for Contour Grouping

1D Line Graph



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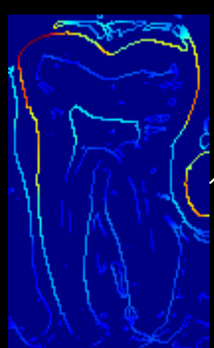

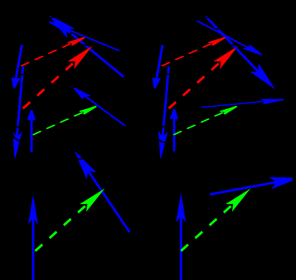

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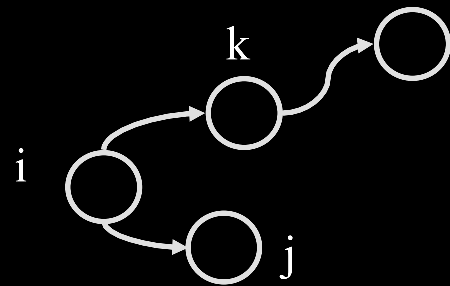

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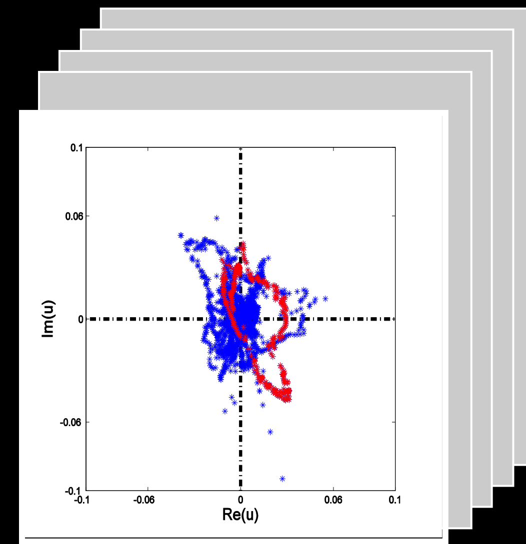


Complex Embedding

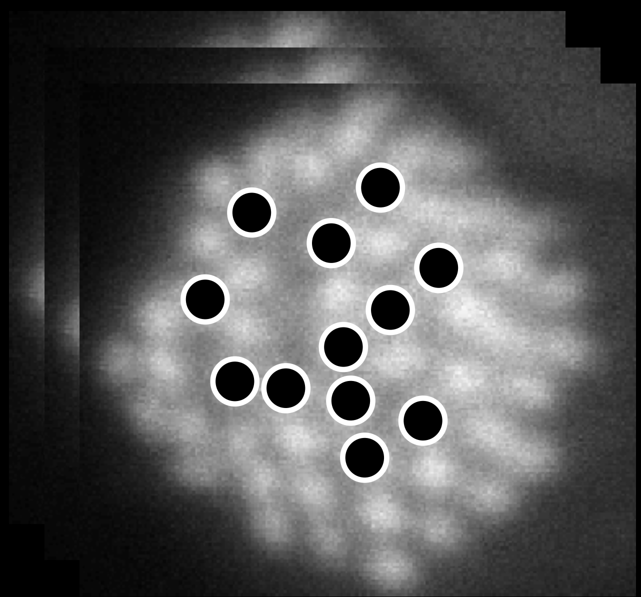
Discretization

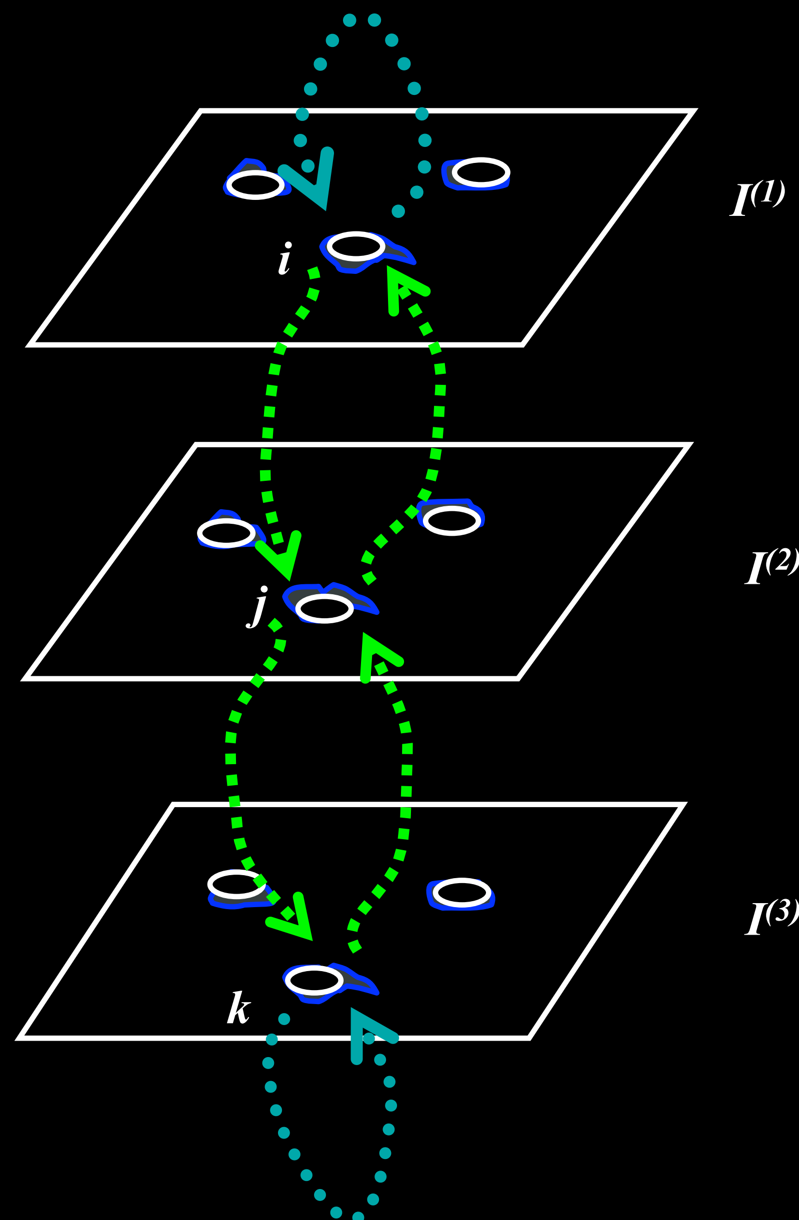
Contours

[Zhu et al. `07]

Structural Correspondence Graph Setup

$$G^{\text{structure}} = < V^{\text{structure}}, W^{\text{structure}} >$$





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Structural Correspondence Graph Setup

$$G^{\text{structure}} =< V^{\text{structure}}, W^{\text{structure}} >$$

$$w_{i \to j} = \begin{cases} \xi(i,j) + (\psi_{i \to j})\mathtt{i} & i \neq j \\ \xi(i,i) + \mathtt{i} & i = j = 1, n \\ \xi(i,i) * 0.1 + \mathtt{i} & i = j = 2, \ldots, n-1 \end{cases}$$

Bending:

$$\xi(i,j) = \exp\left(-|d_j - d_i| / \sigma\right)$$

d_i, d_j positions of node i and j

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d_i, d_j positions of node i and j

Jumping between stacks:

$$\psi_{i \to j} = t - s$$

$$i \in I^{(s)}, j \in I^{(t)}$$

$$s, t \in 1, \ldots, n$$

ψ number steps taken

Structural Correspondence Graph Setup

$$G^{\text{structure}} = < V^{\text{structure}}, W^{\text{structure}} >$$

returning link

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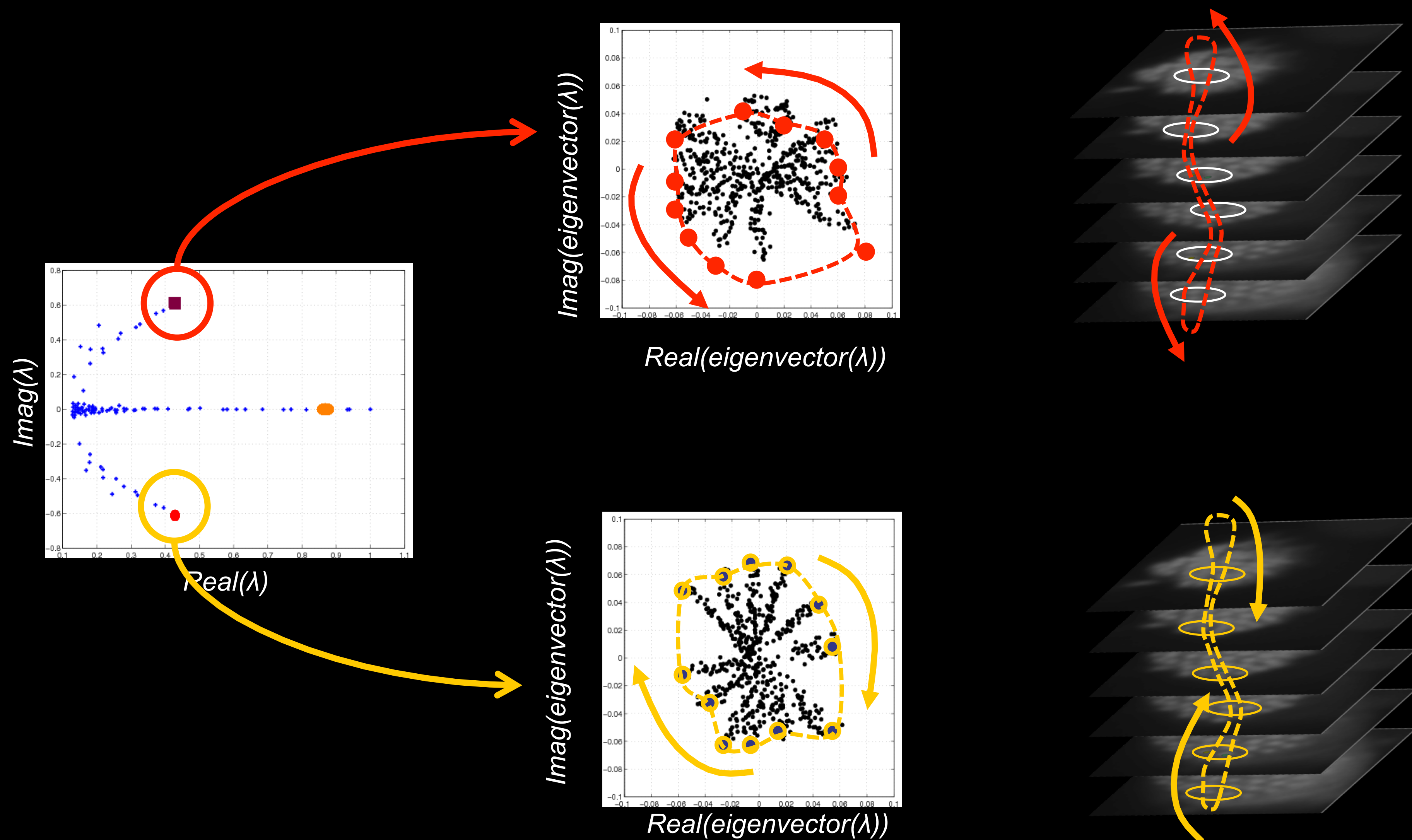
Structural Correspondence Embedding Space



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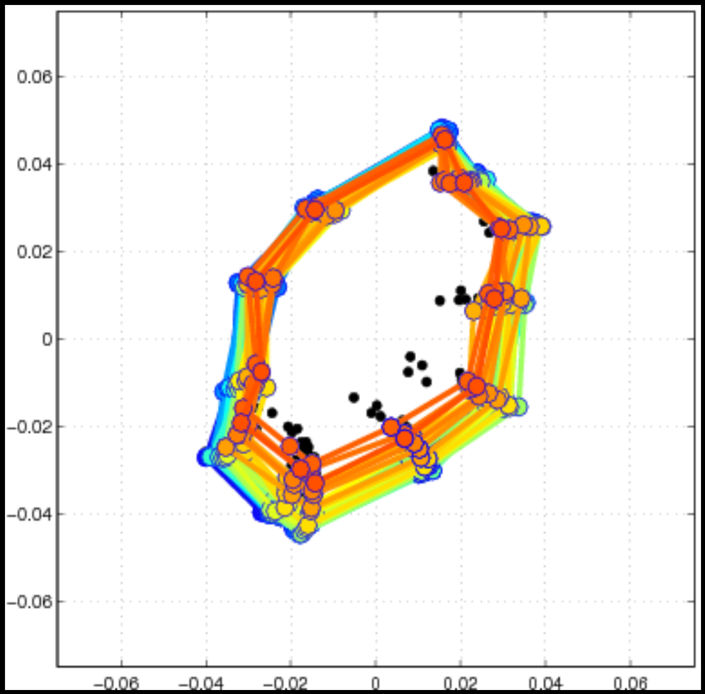
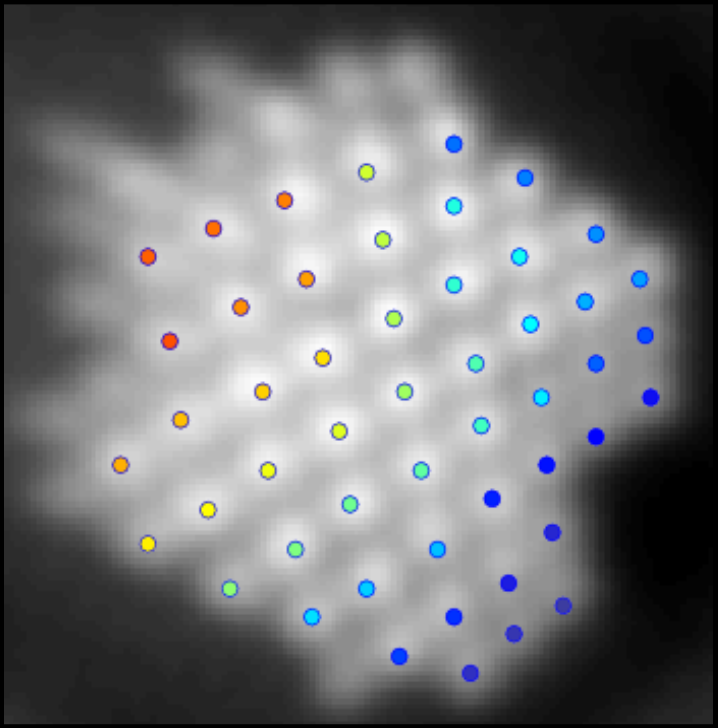
Structural Correspondence: Shifting Cells

complex eigenvector



cycles of length 6



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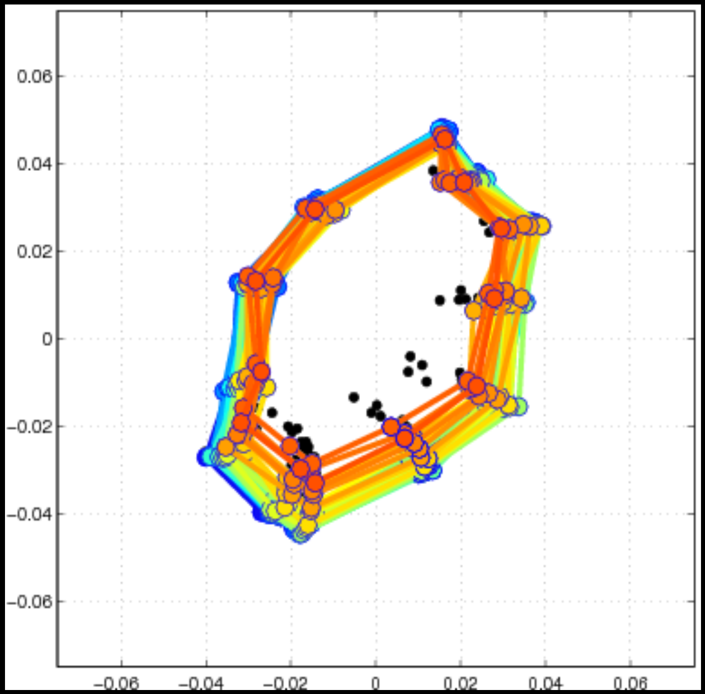

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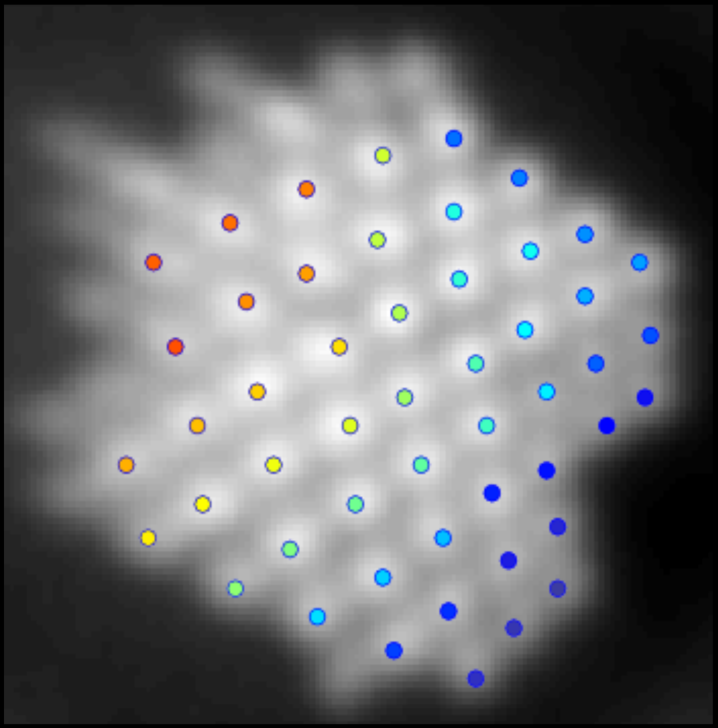

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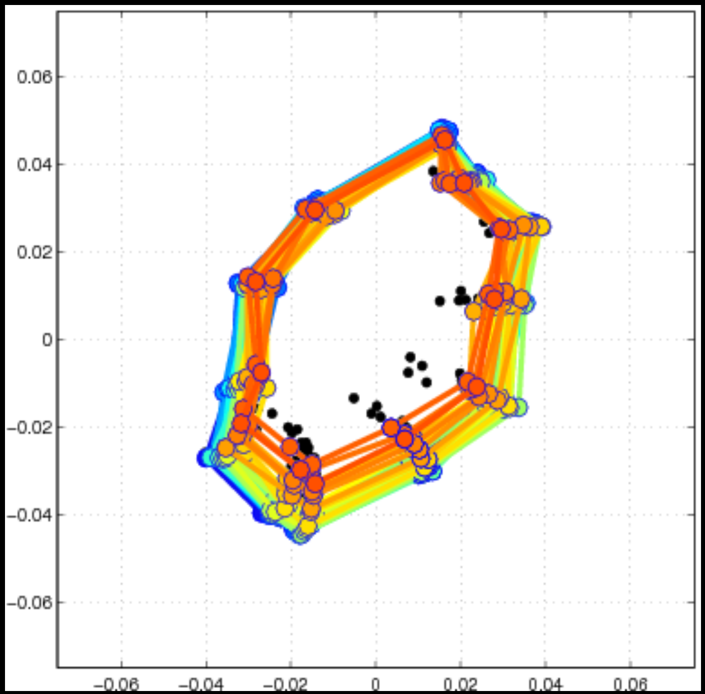

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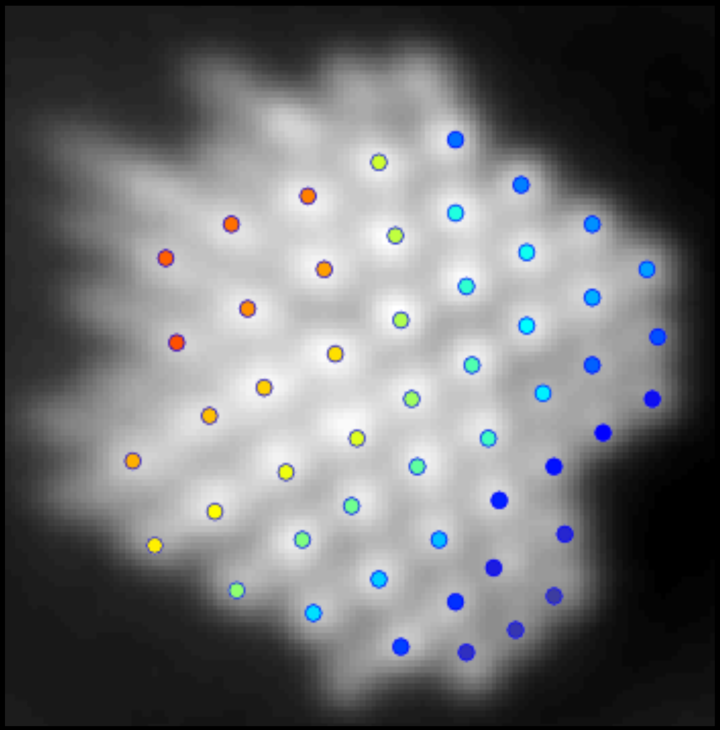

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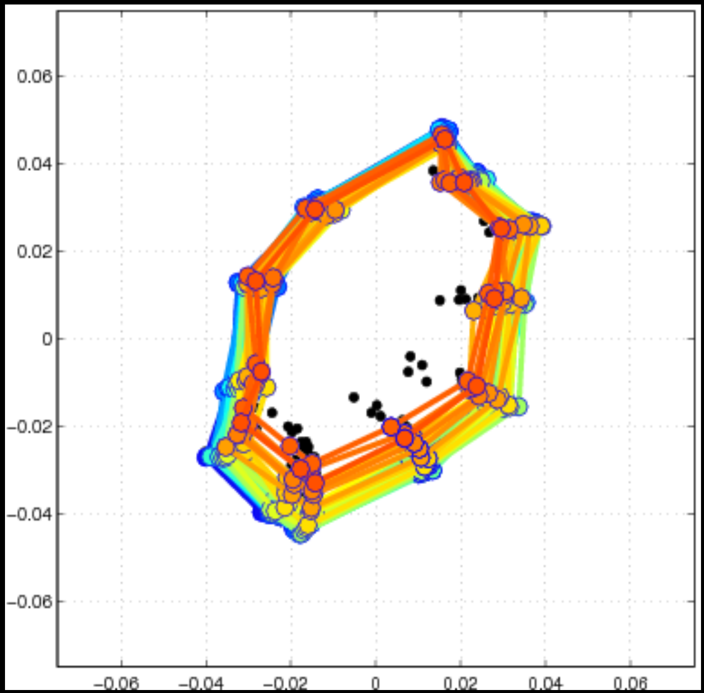

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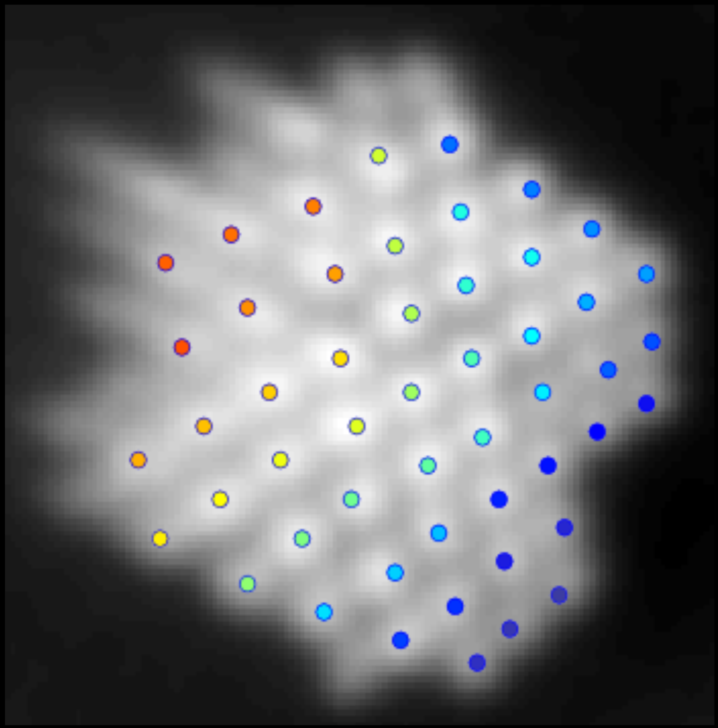

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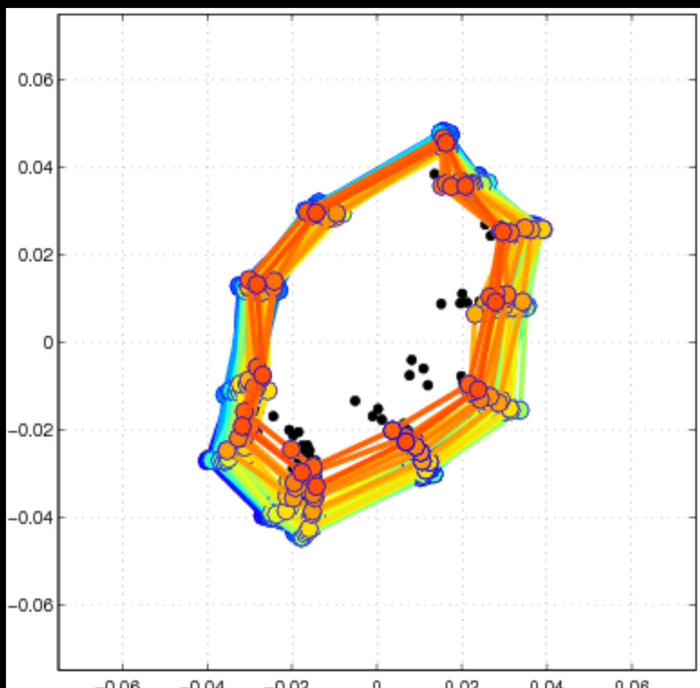

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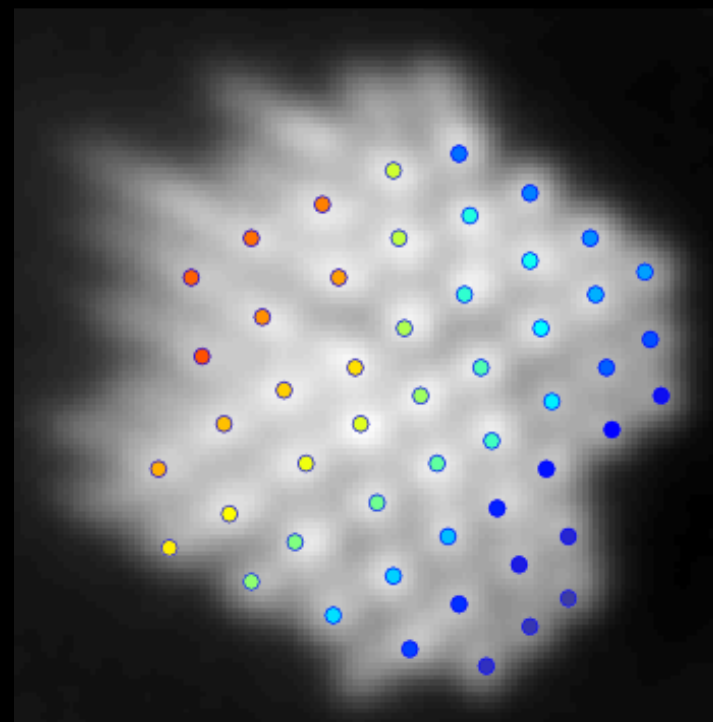

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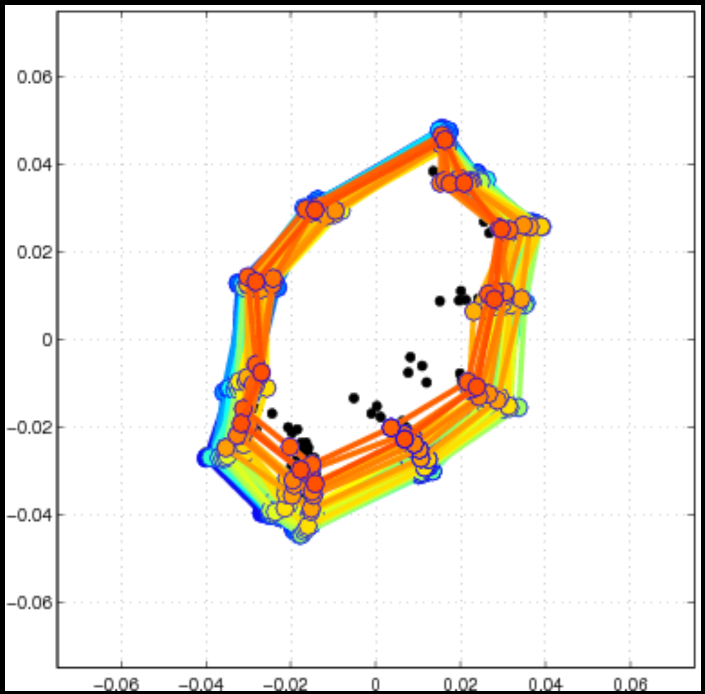

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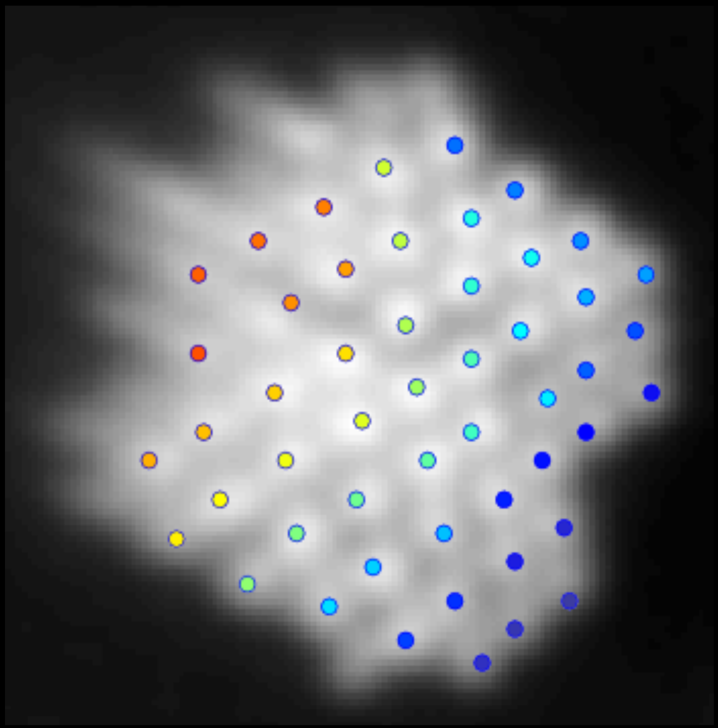

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Structural Correspondence: Shifting Cells

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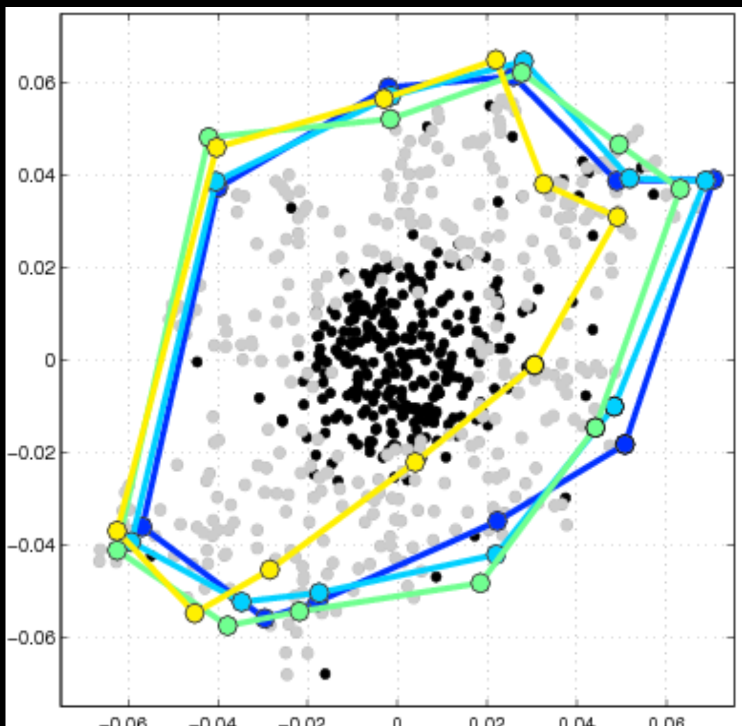
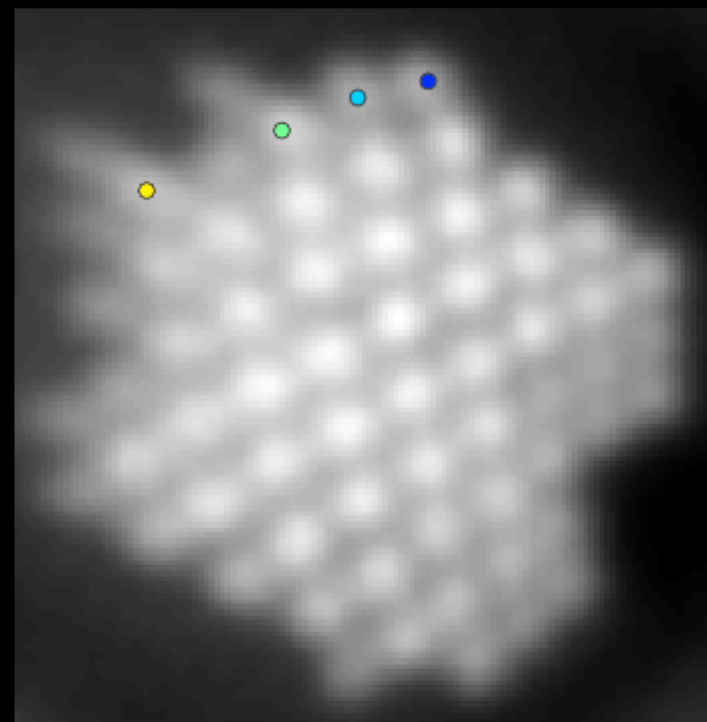


cycles of length 5



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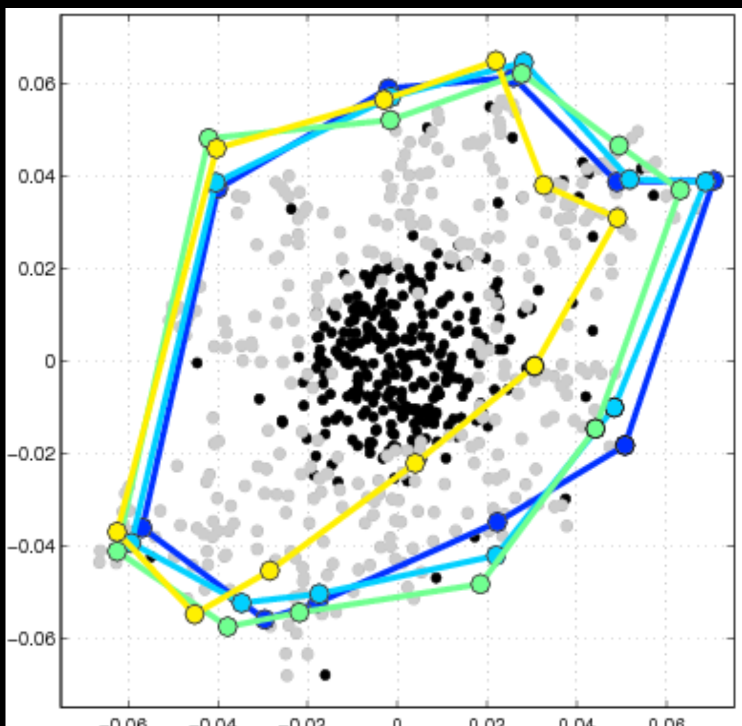
Structural Correspondence Embedding Space

complex eigenvector



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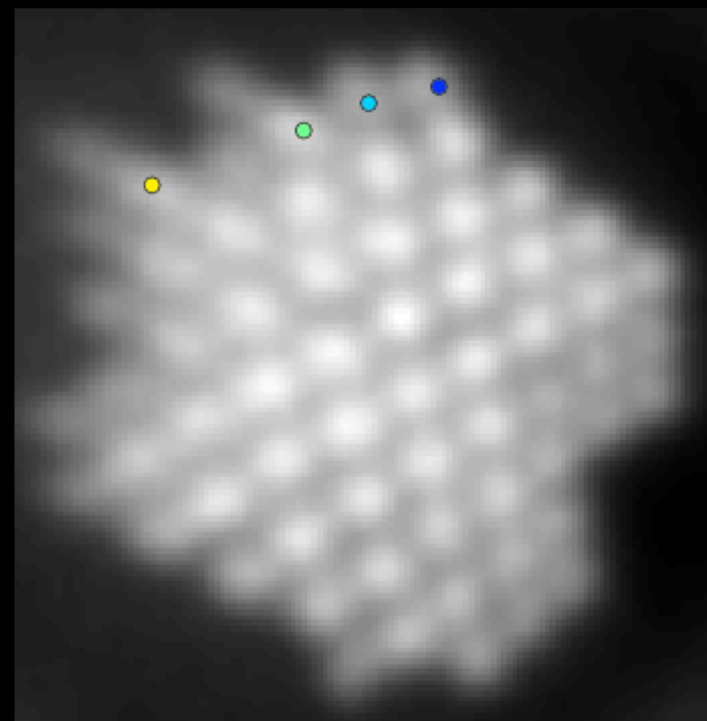
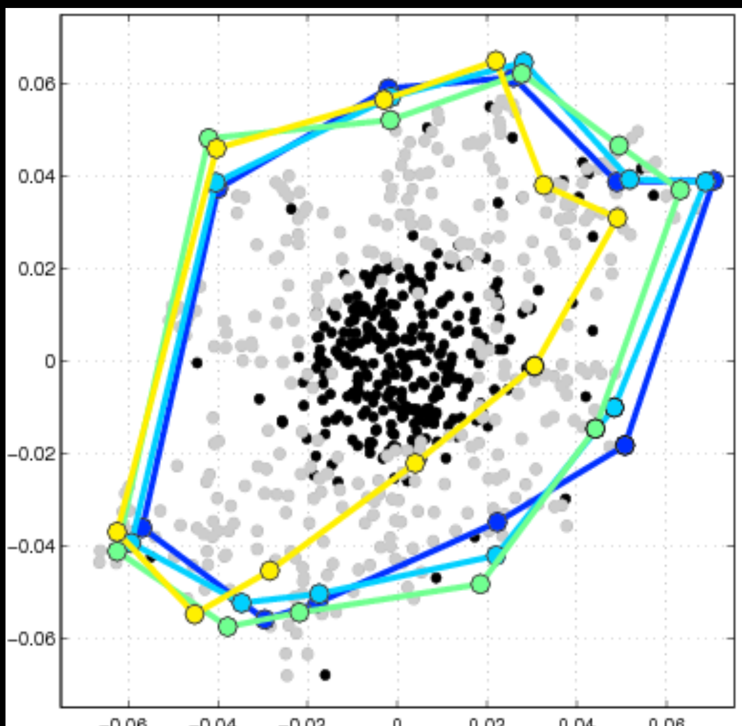


Structural Correspondence: Shifting Cells

complex eigenvector



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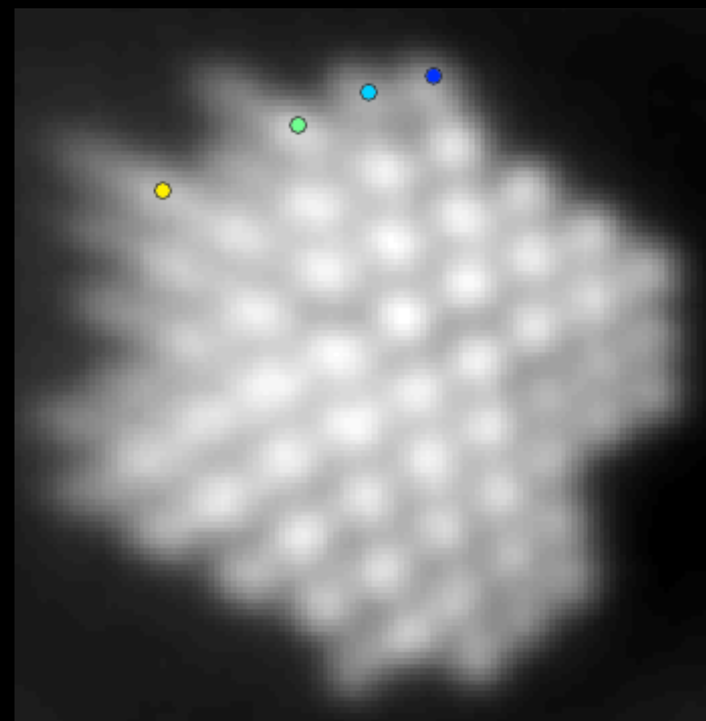

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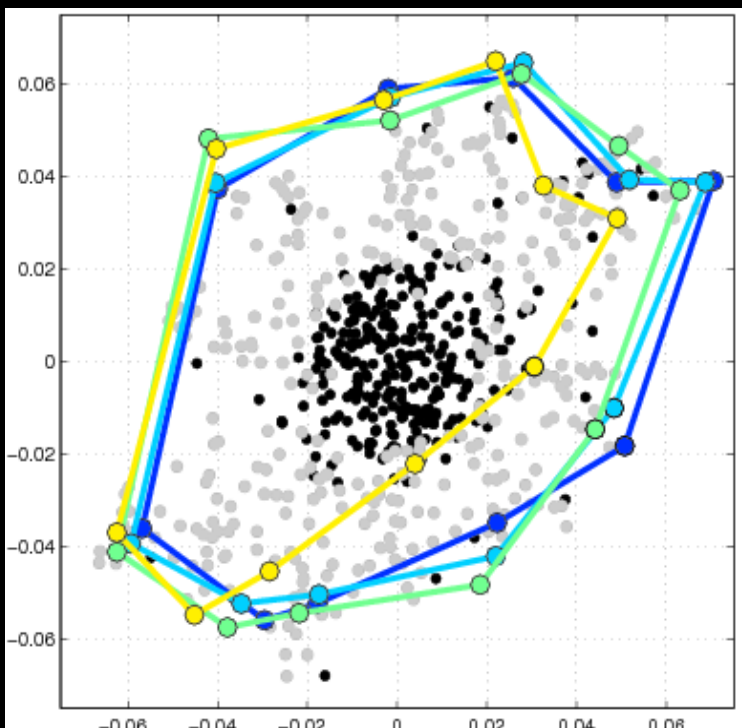

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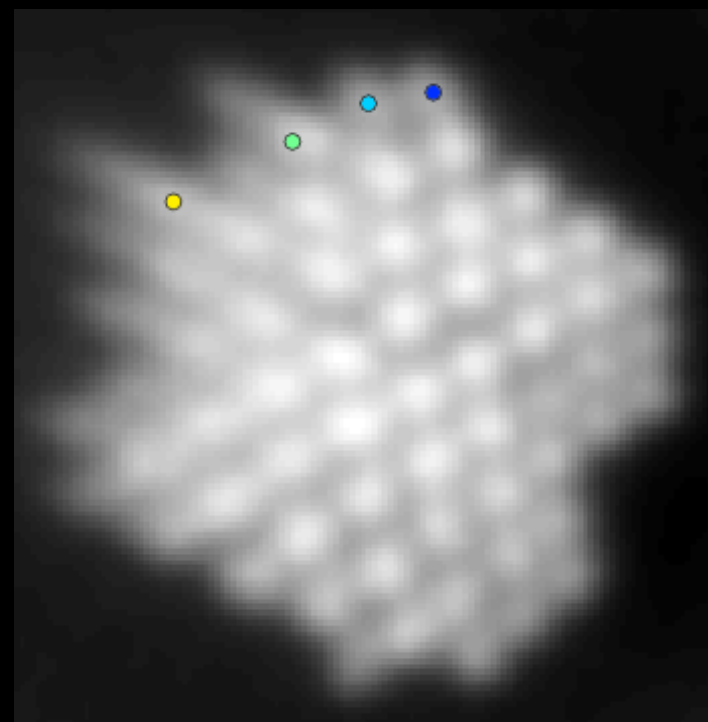

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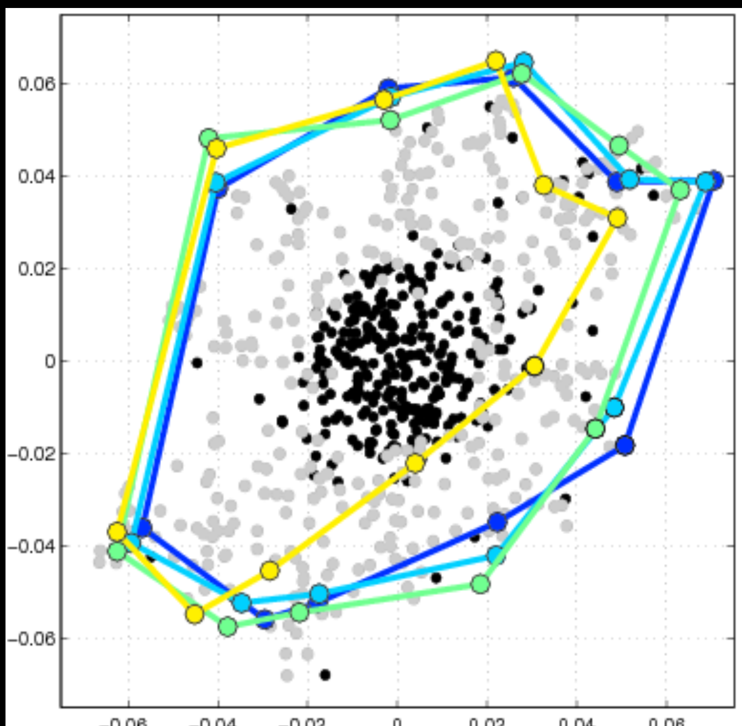

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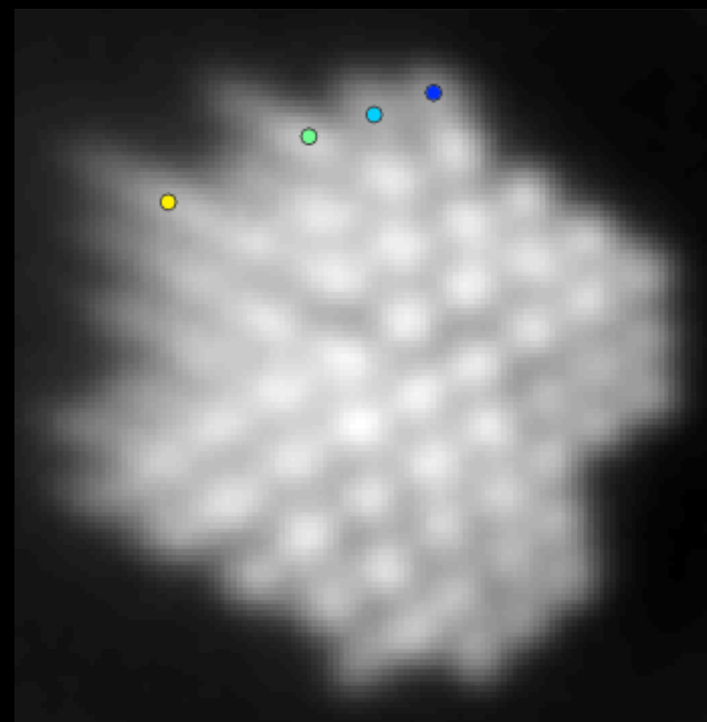

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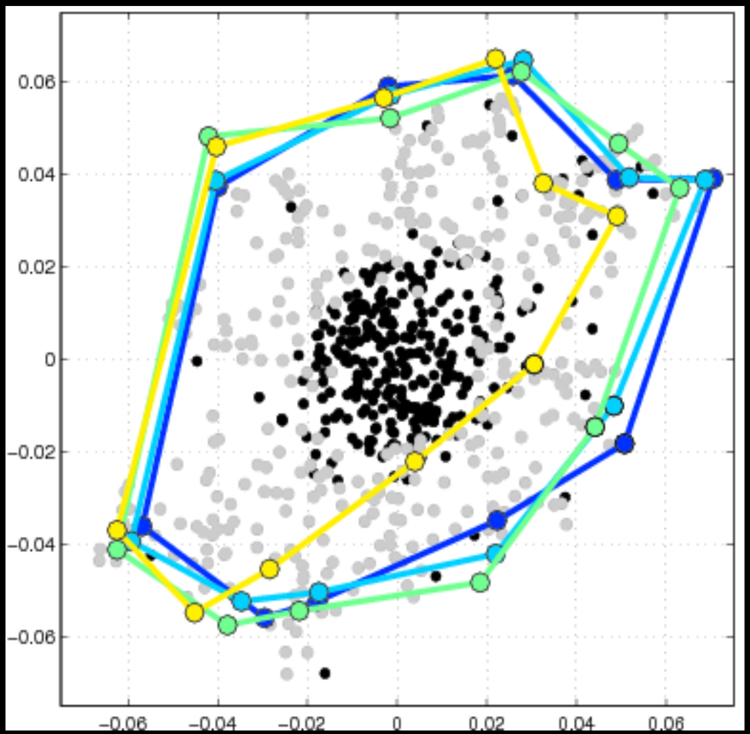

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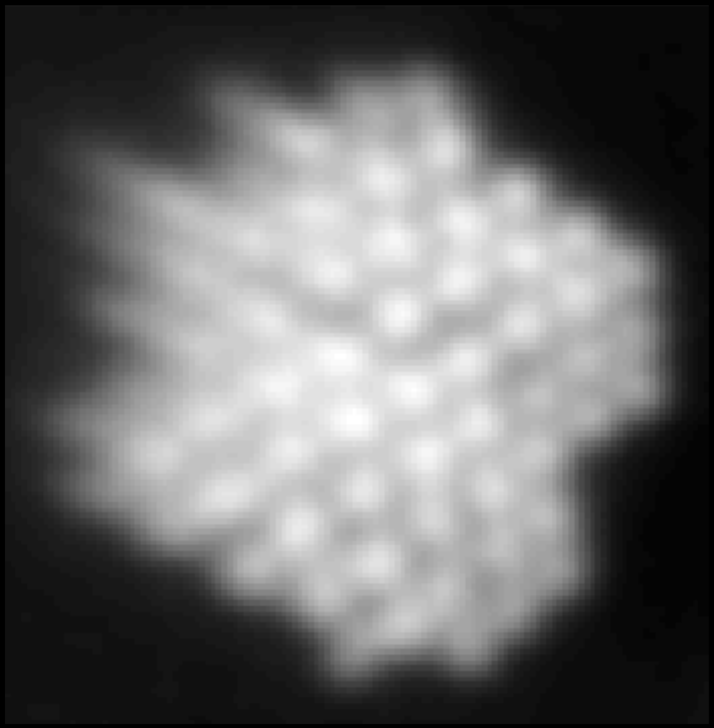

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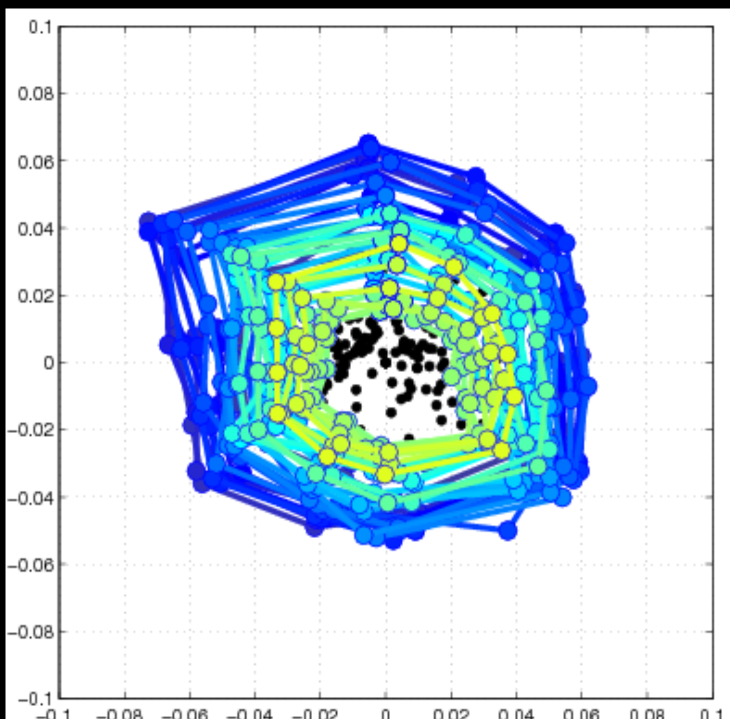
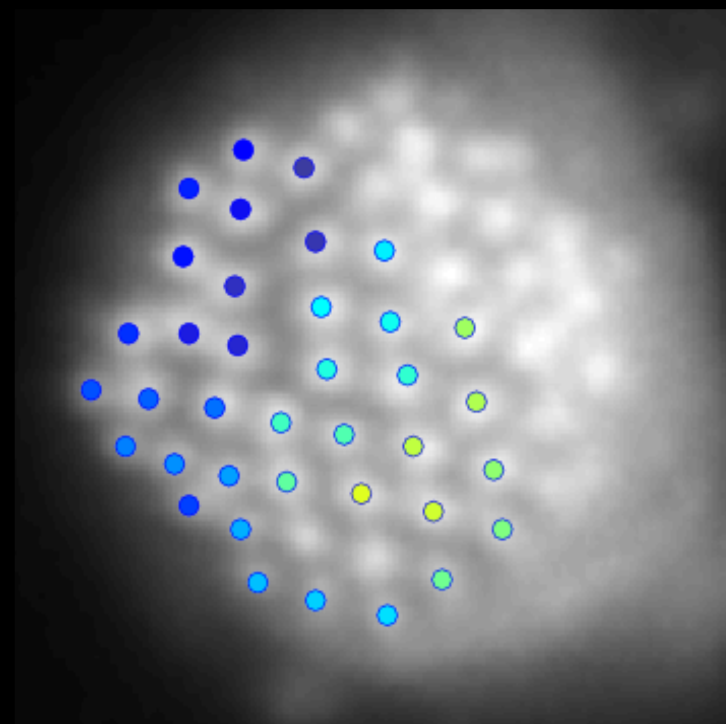
Structural Correspondence: Shrinking Cells

complex eigenvector



cycles of length 6



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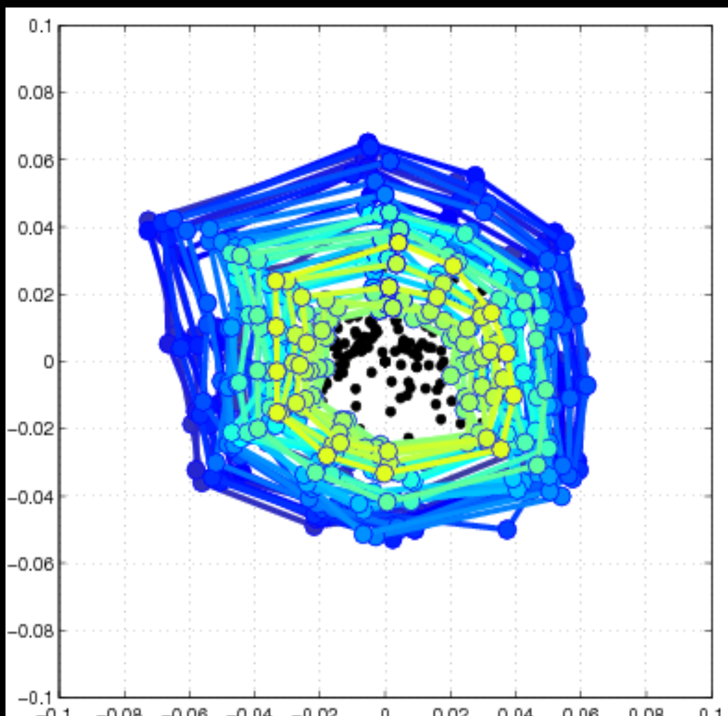
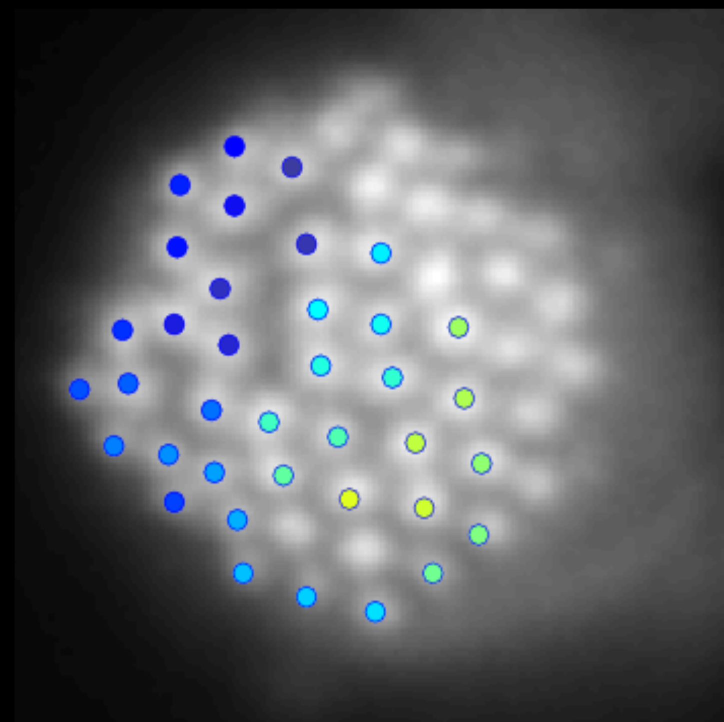
Structural Correspondence: Shrinking Cells

complex eigenvector

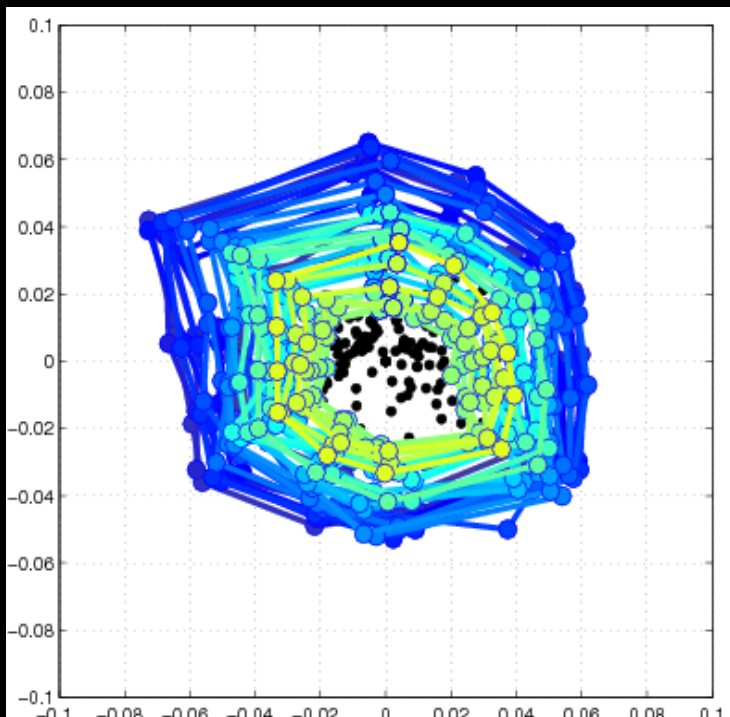


cycles of length 6



Structural Correspondence: Shrinking Cells

complex eigenvector



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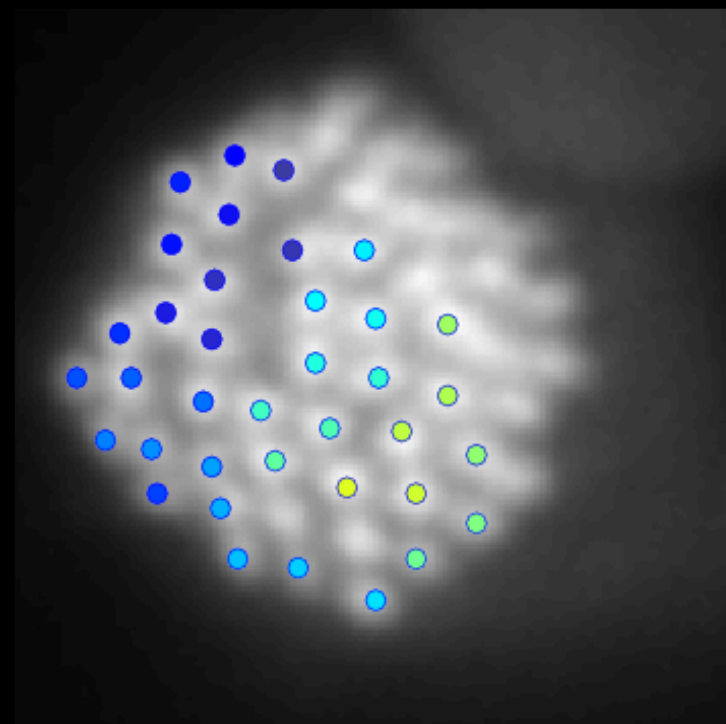

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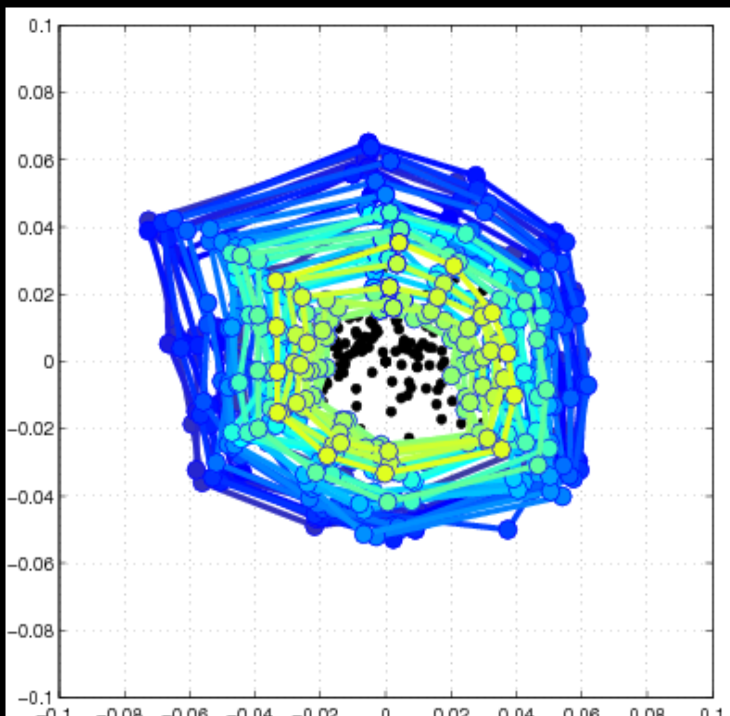
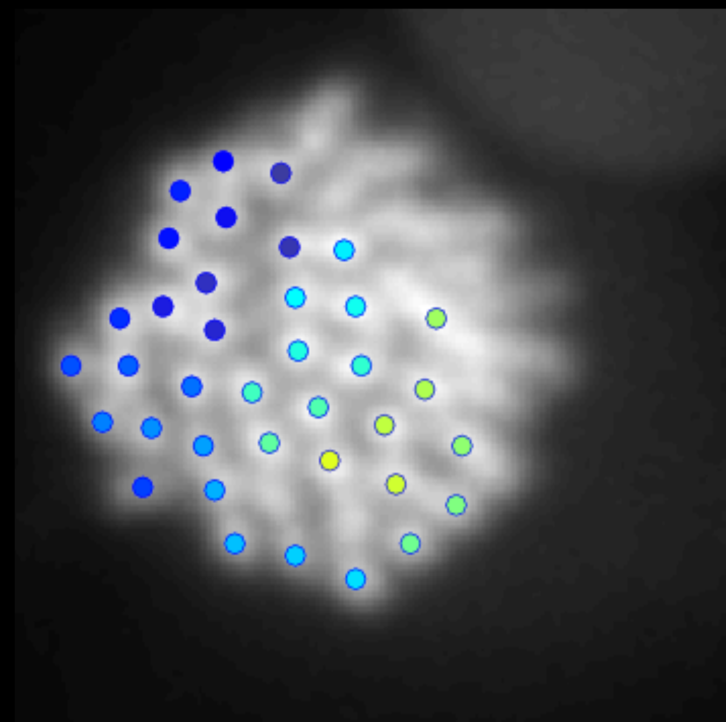
Structural Correspondence: Shrinking Cells

complex eigenvector



cycles of length 6



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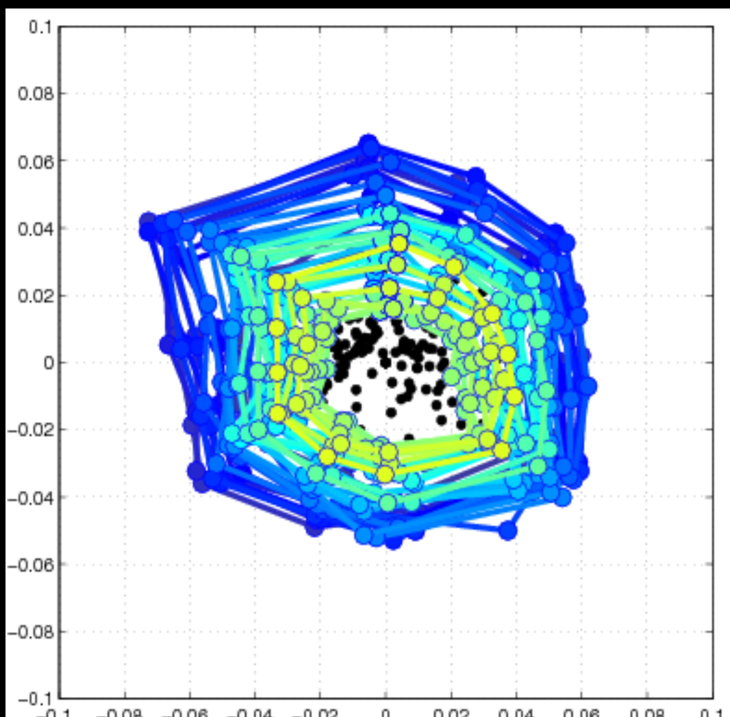
Structural Correspondence: Shrinking Cells

complex eigenvector



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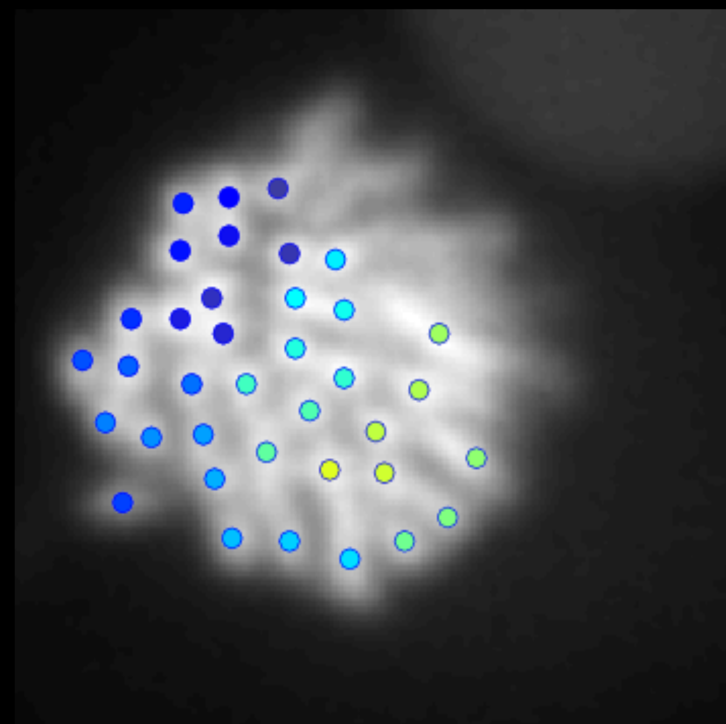

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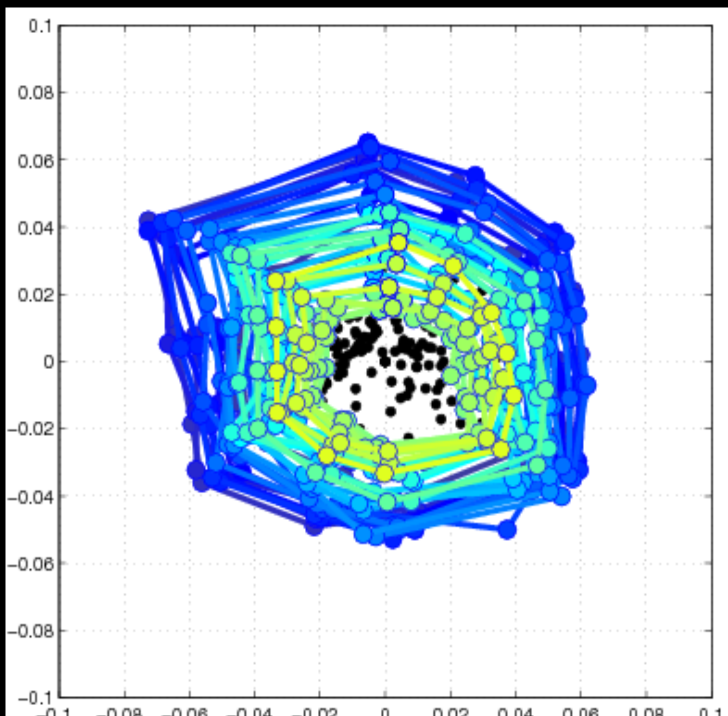
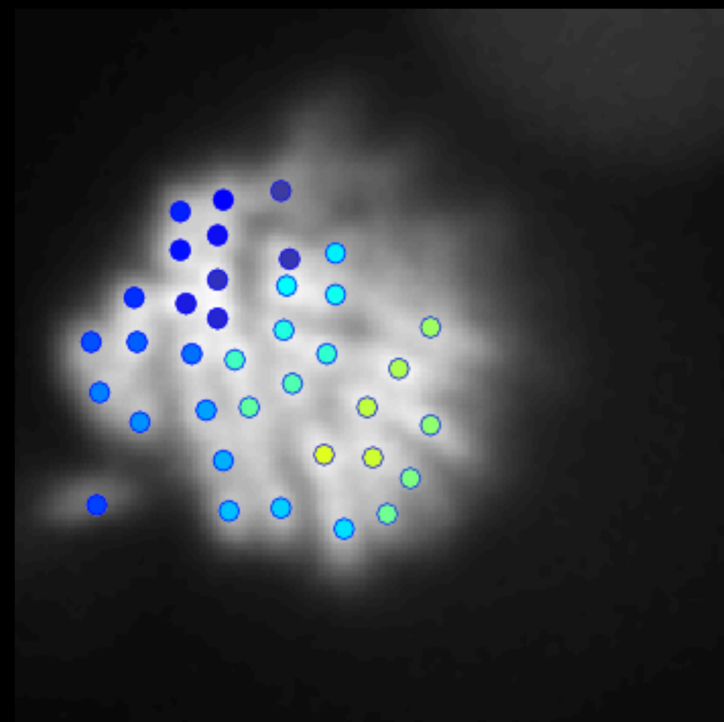
Structural Correspondence: Shrinking Cells

complex eigenvector

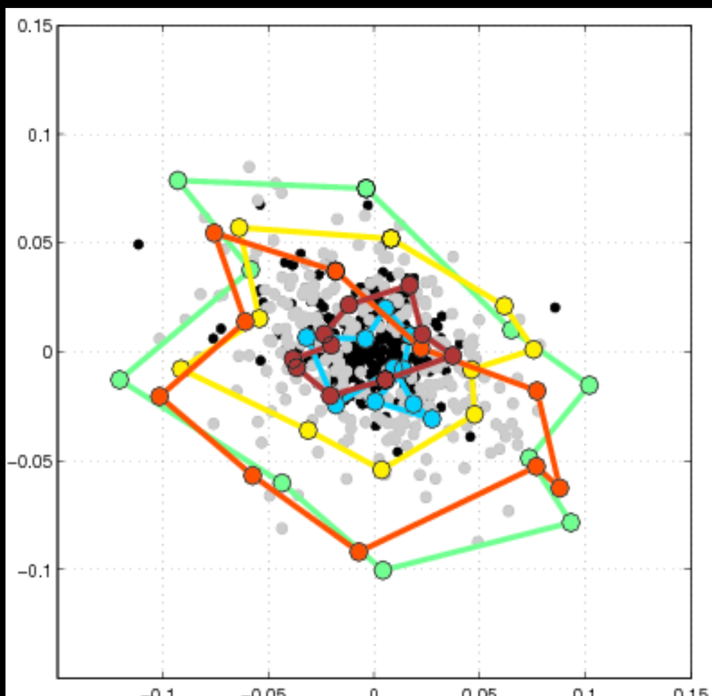


cycles of length 6

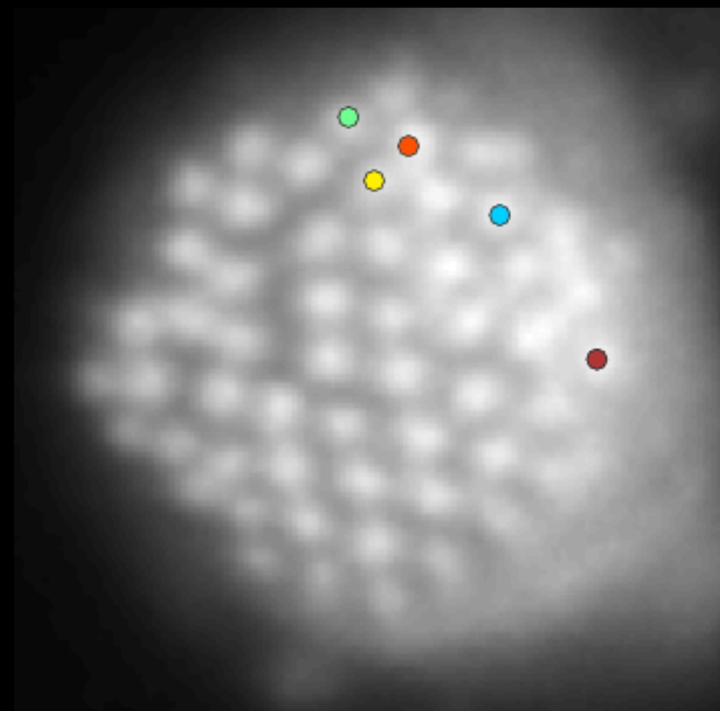


Structural Correspondence: Shrinking Cells

complex eigenvector

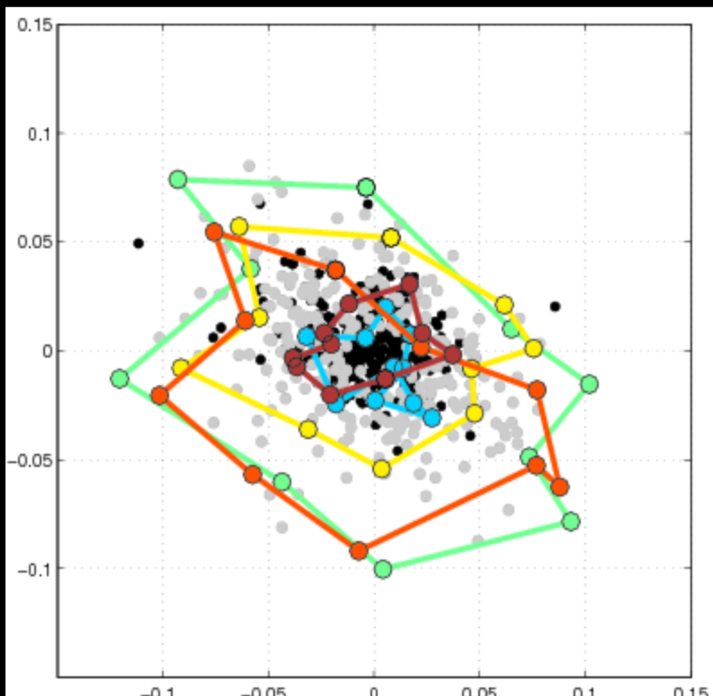


cycles of length 5

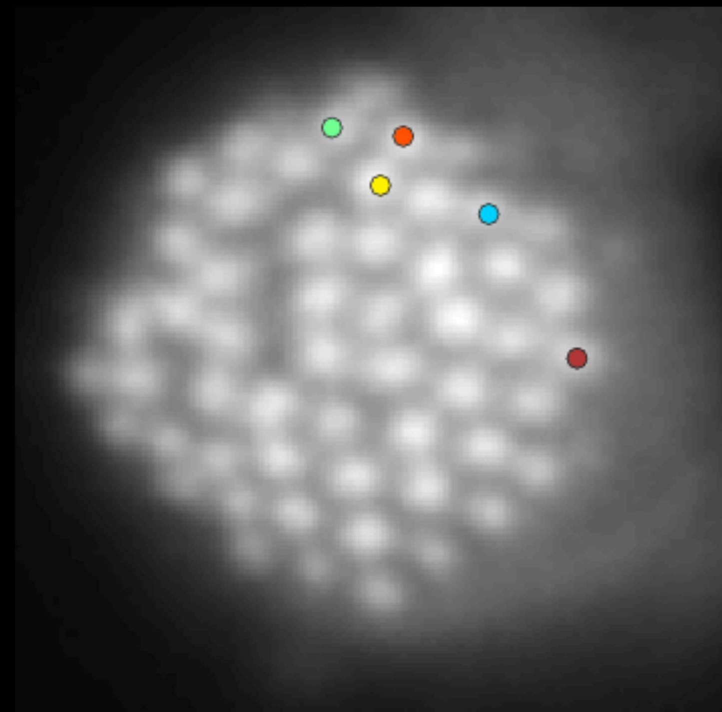


Structural Correspondence: Shrinking Cells

complex eigenvector



cycles of length 5



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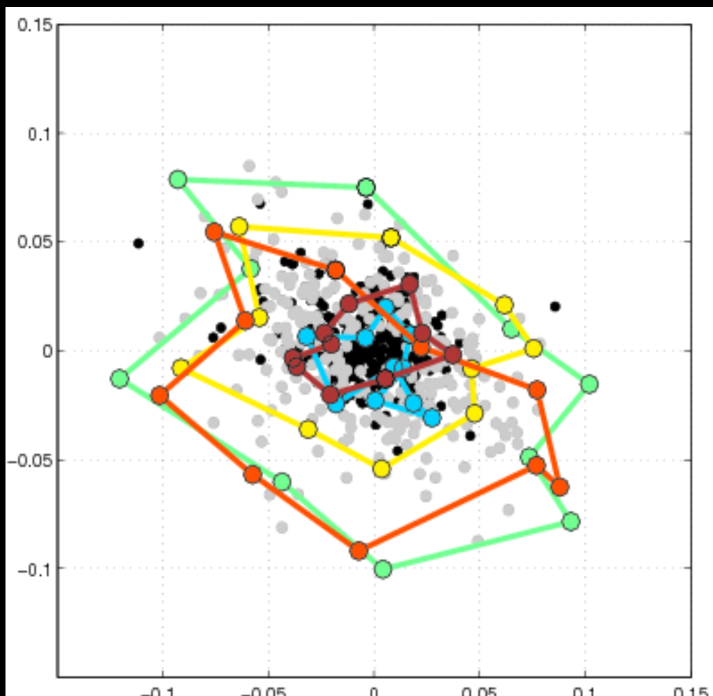
Structural Correspondence: Shrinking Cells

complex eigenvector



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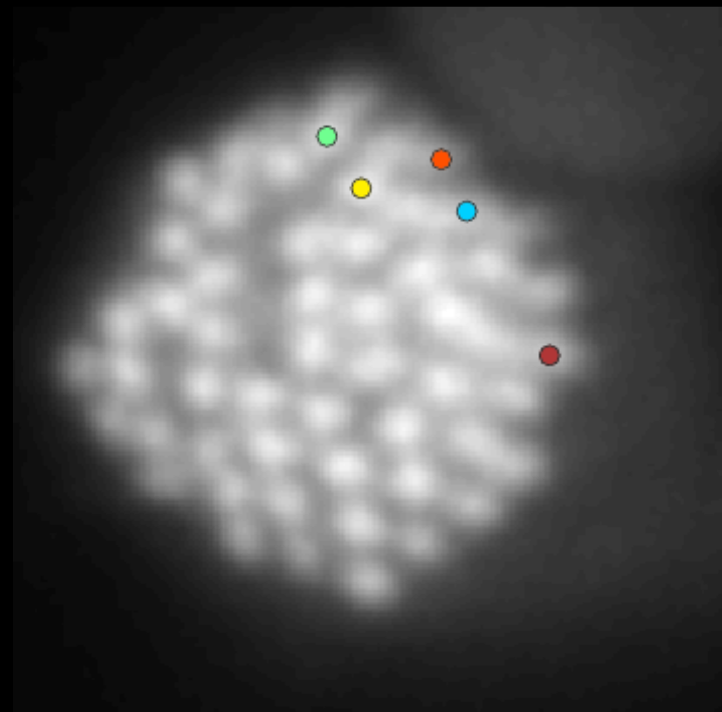

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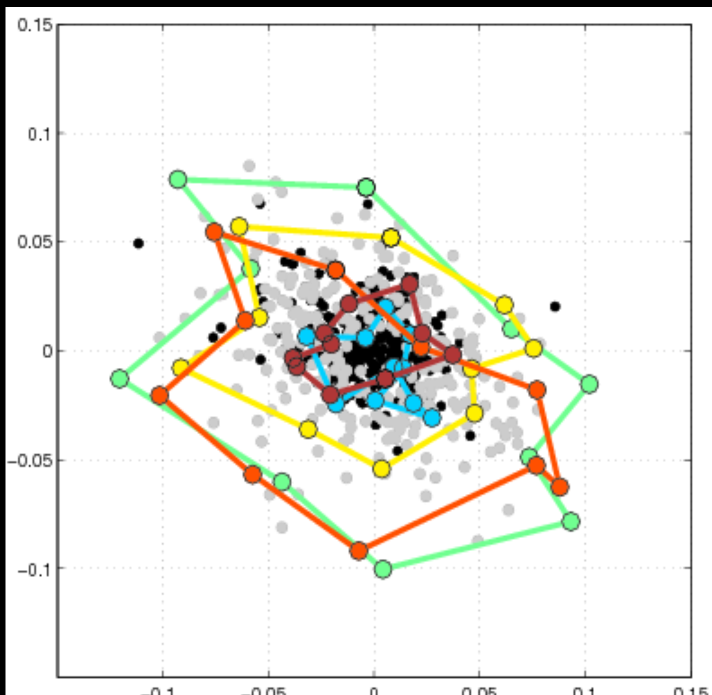
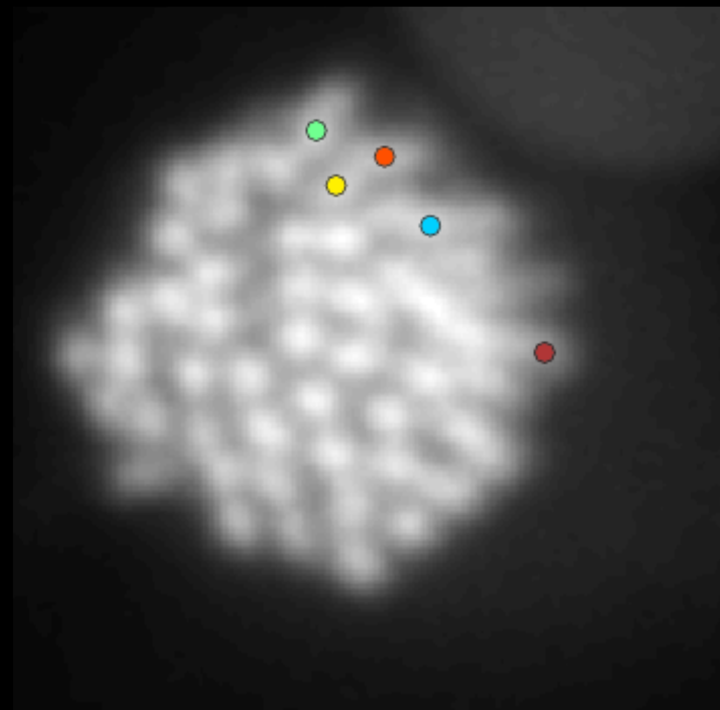
Structural Correspondence: Shrinking Cells

complex eigenvector



cycles of length 5



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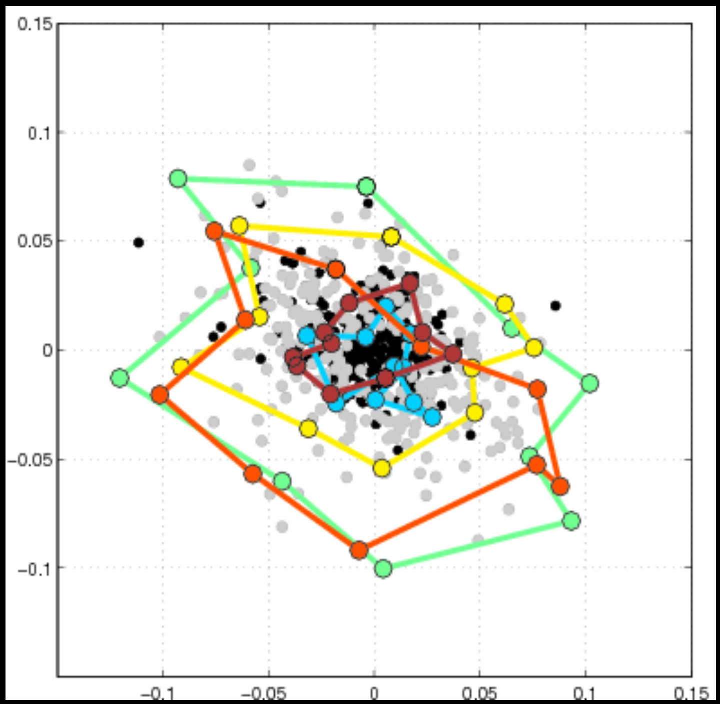
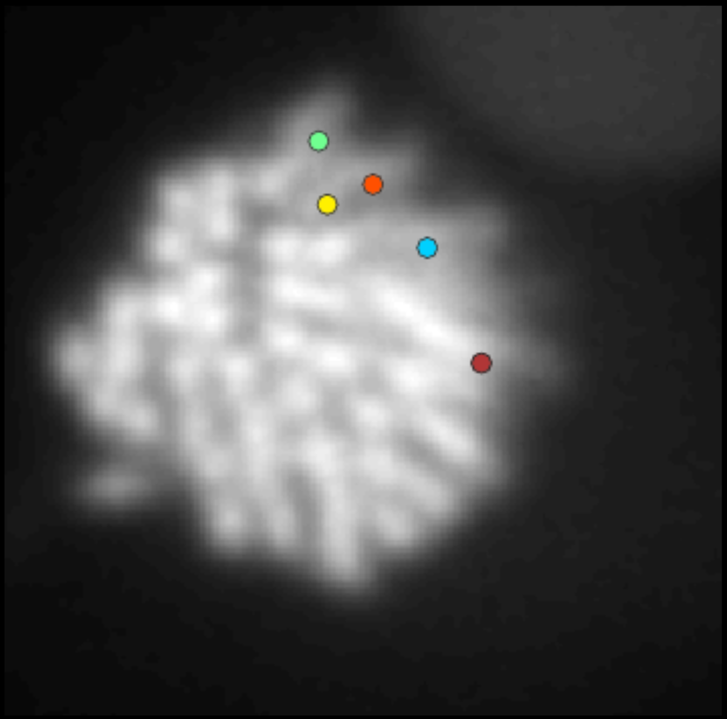
Structural Correspondence: Shrinking Cells

complex eigenvector



cycles of length 5



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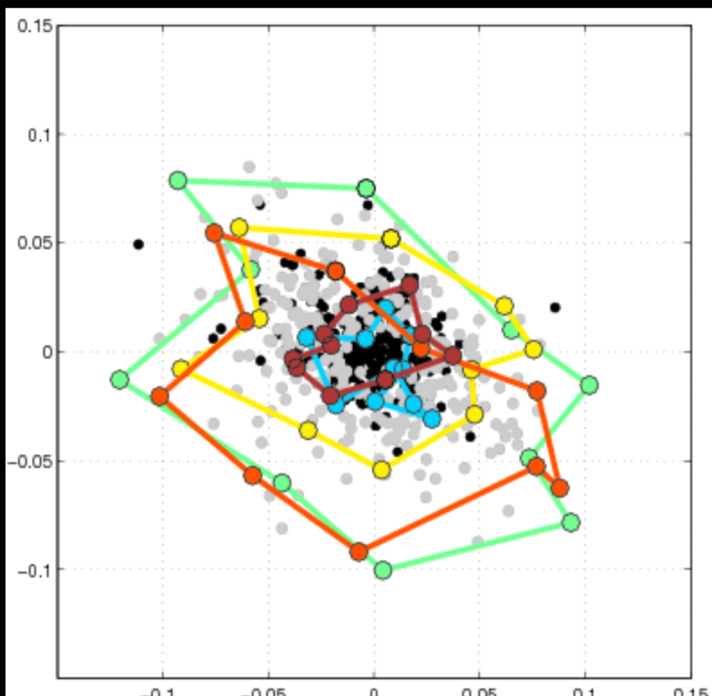
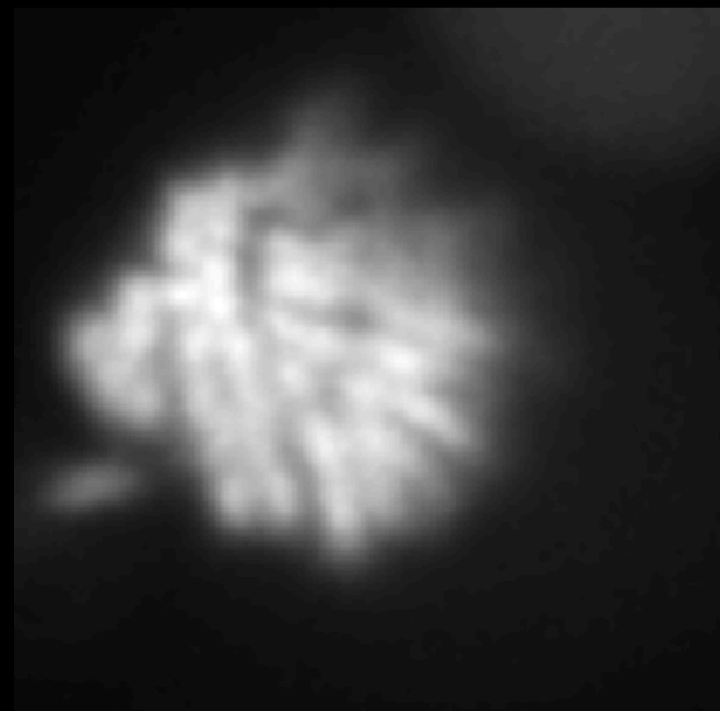
Structural Correspondence: Shrinking Cells

complex eigenvector



cycles of length 5



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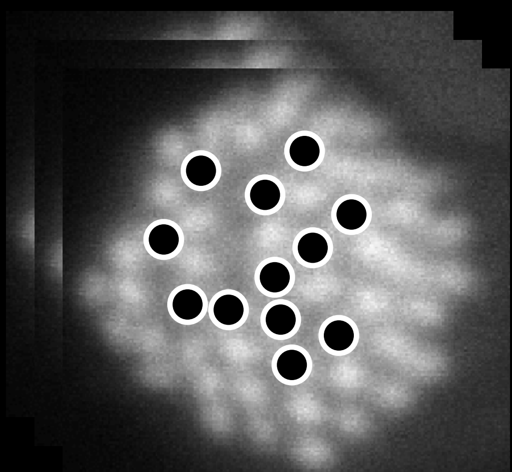
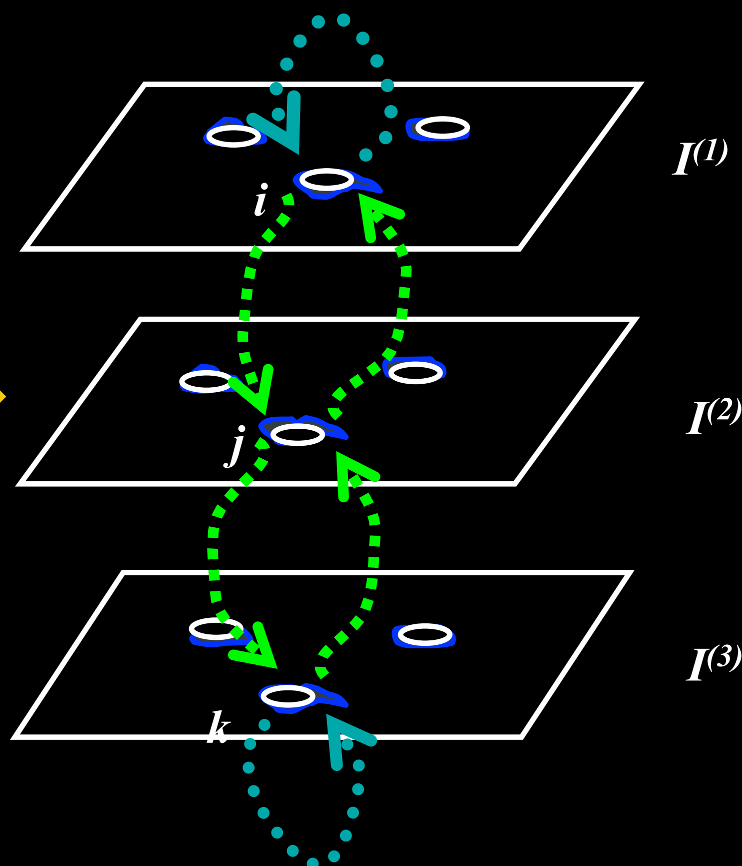
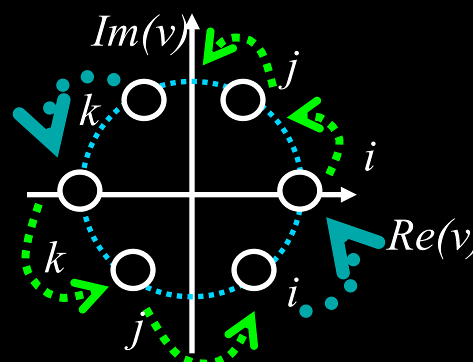
Spectral Graph Partitioning Framework for Structural Correspondences

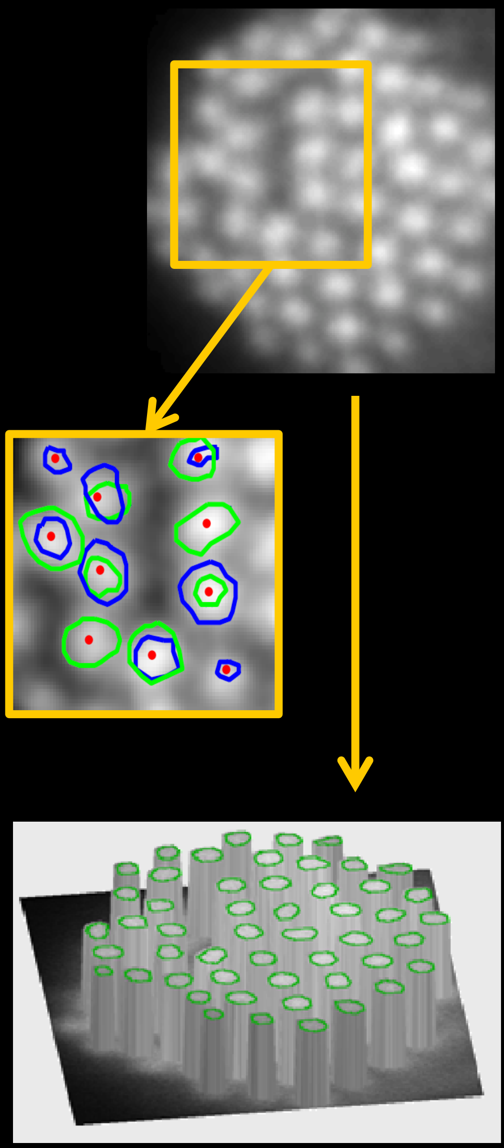
Different contour lengths encoded in eigenvectors of different magnitude







In pratice, how do we find the nodes?



[CVPR 2010]