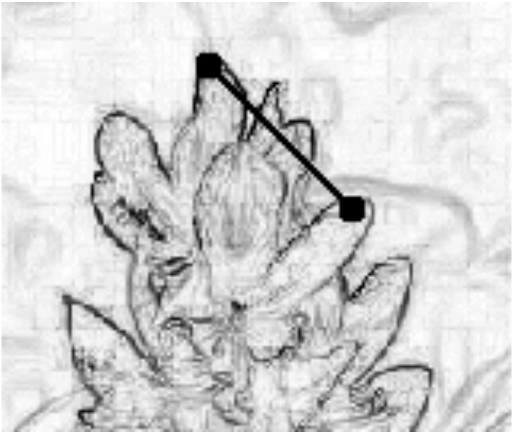
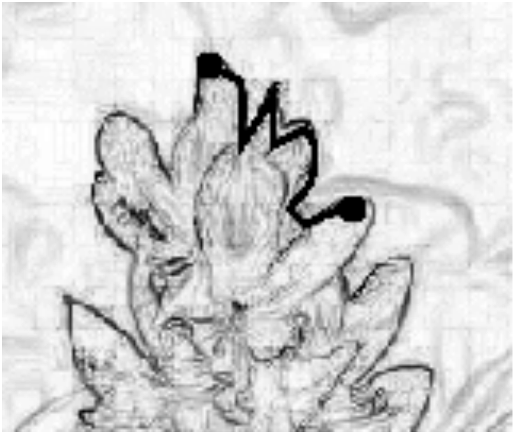
A Unifying View of Contour Length Bias Correction

Christina Pavlopoulou and Stella X. Yu
Computer Science Department
Boston College

The Length Bias Problem

- Criteria used in practice **intrinsically** favor short segments.





- Inability to model geometrically complex boundaries.
- Solutions:
  - user input
  - additional features
  - stronger priors
  - alternative criteria (mean ratio)

Contributions

- Explain the bias current criteria suffer from.
- Unify existing approaches under a single framework for correcting the length bias.

Unbiased Criterion

- Original snakes criterion is not biased towards short boundaries.

$$E[C(s)] = \underbrace{\int_{C(s)} \frac{1}{2}(\alpha|C'(s)|^2 + \beta|C''(s)|^2)ds}_{\text{prior}} - \underbrace{\lambda \int_{C(s)} \|\nabla I\| ds}_{\text{data term}}$$

- Strong image discontinuities obtain negative cost and are encouraged in the solution.

- However, functional may become **ill-posed** (minimum is -infinity).

Discrete Case

Discretized criterion:

$$E[C] = \sum_{i=1}^{n} \{ d(c_{i+1}, c_i) - \lambda \left\| \nabla I \right\|_{c_i} \}$$

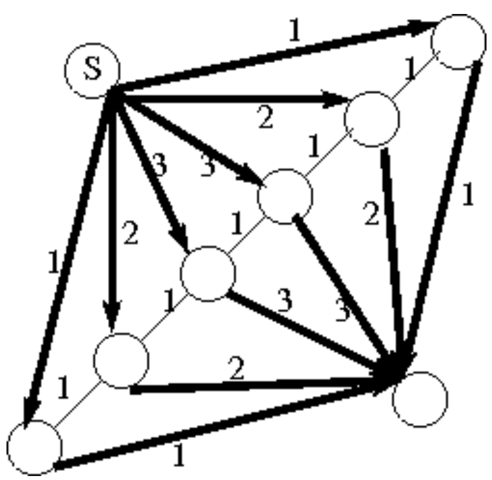
Can be optimized globally
with dynamic programming:

$$w(u, v) = d(u, v) - \lambda f(u, v)$$

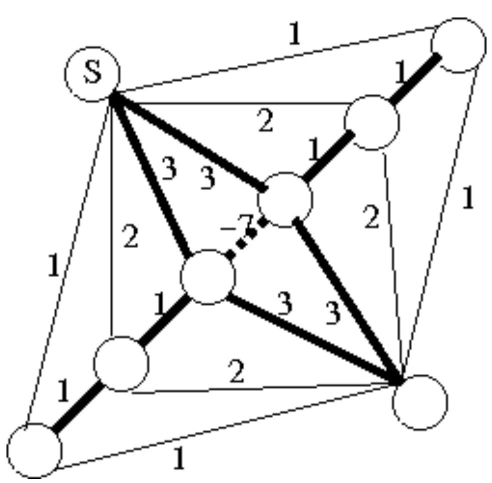
- Becomes ill-posed when there are negatively-weighted cycles.

The “black hole” effect

- For negatively weighted cycles the problem is ill-posed.
- Removing negative cycles is a hard problem !



shortest paths from source S for graph with no negative cycles



a negative cycle acts as *black hole* in the energy landscape; all shortest paths are forced to include the cycle.

Explanation of Length Bias

- To remove the negative cycles, weights are converted to positive by adding a constant M:

$$w_M(u,v) = w(u,v) + M$$

- Does not preserve the optima of the objective.
- Results in an additional smoothing term:

$$E_M(\mathbf{C}) = \sum_{(u,v)\in C} \{d(u,v) - \lambda f(u,v)\} + nM$$

Bias Correction

- Optimal way of converting negative weights to positive requires graphs with no negative cycles.
- Seek weights of the form:

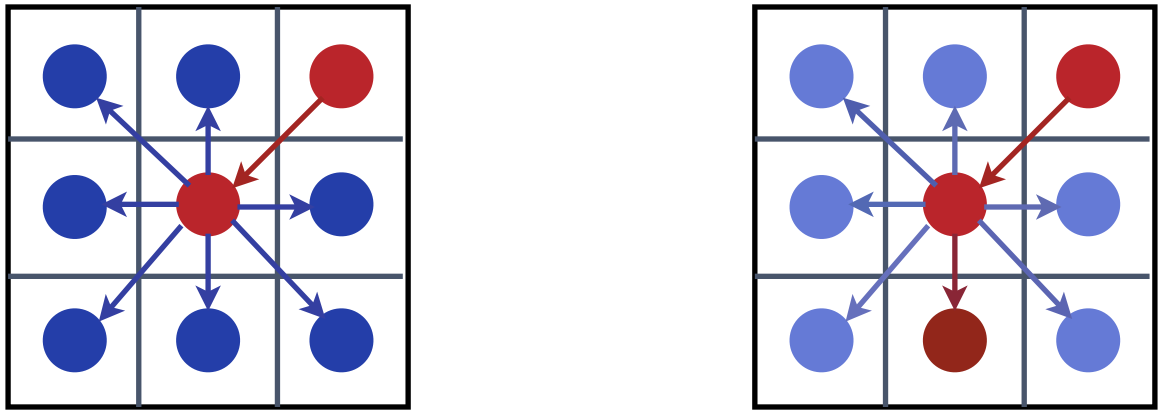
$$\hat{w}(u, v) = w_M(u, v) - \alpha(u, v)$$

- Existing approaches provide different choices for $\alpha(u, v)$

Local Bias Correction

- Weight transformation:

$$w^{+}(u,v) = w_M(u,v) - \max_{w} w_M(u,w)$$



- Similar approaches:
  - non-maximum suppression (Mortensen 2004)
  - piecewise boundary extension (Mortensen 2001)

Probabilistic Criterion (Pavlopoulou, Yu, 2009)

- Best contour delineates strong discontinuities and is distinct in its vicinity (enforced by probability of observations) :

$$E[C,O] = \log P(O|C) + \log P(C) - \log P(O)$$

- Weights produced by this criterion are of the form:

$$\hat{w}(u,v_i) = w_M(u,v_i) - \log \sum_{j \neq i} \exp^{-w_M(u,v_j)}$$

- The log-sum-exp term behaves like the max term in the local bias correction approach.

Ratio Weight Cycles (Jermyn, Ishikawa, 2001)

- Normalize by length of contour:

$$w(C) = \frac{\sum_e w(e)}{\sum_e n(e)}$$

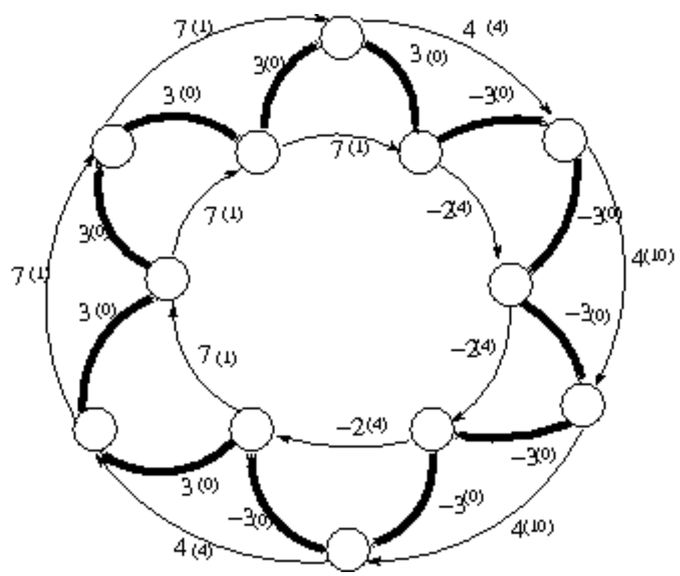
- Equivalent to finding zero cost cycles:

$$\hat{w}(C) = w(e) - \lambda n(e) = 0$$

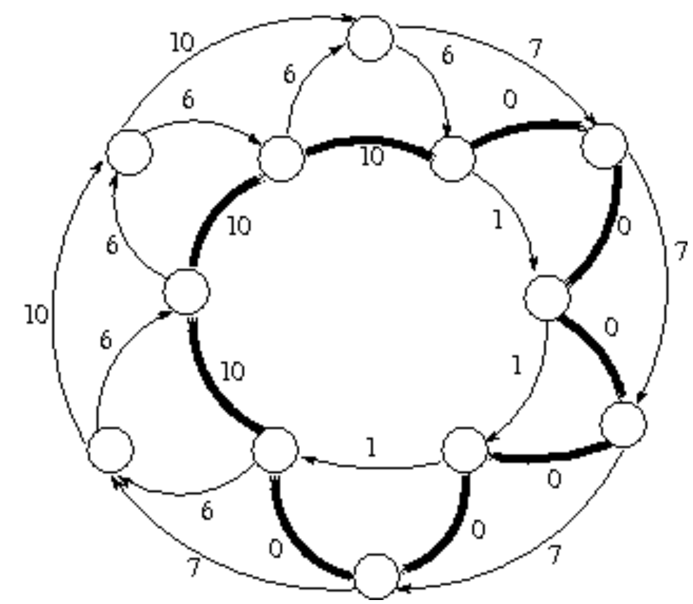
- Find maximum λ so that negative cycles are not created.

- Employed to find salient cycles. Does not admit user interaction.

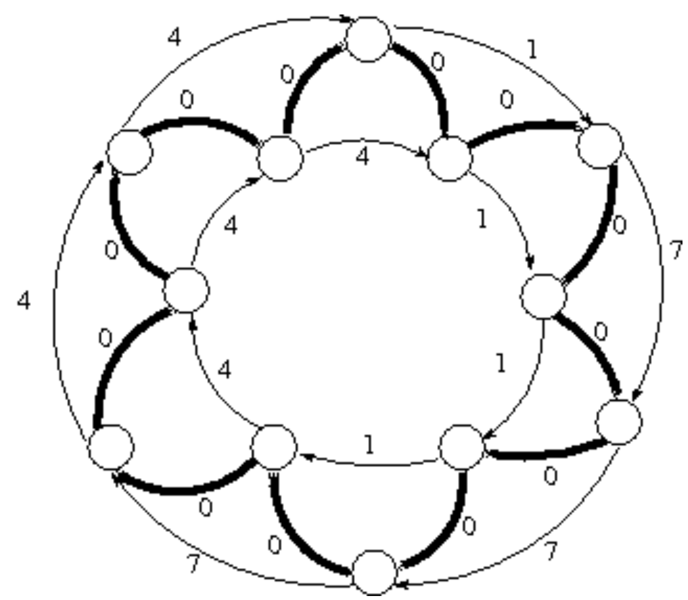
Results: Synthetic Examples



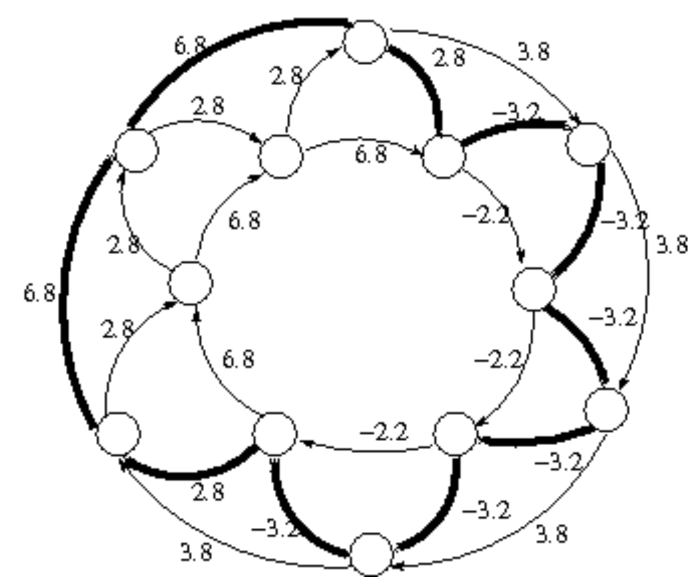
optimal



biased (constant added)



locally corrected



mean ratio

Contour Completion

- Key points were selected based on gradient magnitude.
- Shortest paths were computed among key points (distanced more than a threshold).
- Weights were computed based on gradient magnitude.
- Biased criterion connects key points via shortest boundary segments.

Results: Contour Completion

Conclusions

- Original energy contour criterion is unbiased but ill-posed.
- Adding a constant results in bias (significant for geometrically complex boundaries).
- Current approaches provide different criteria of removing the bias from each edge weight.