Grouping with Directed Relationships

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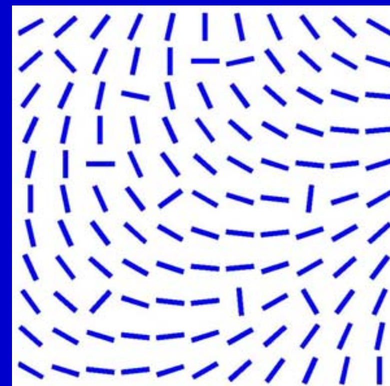
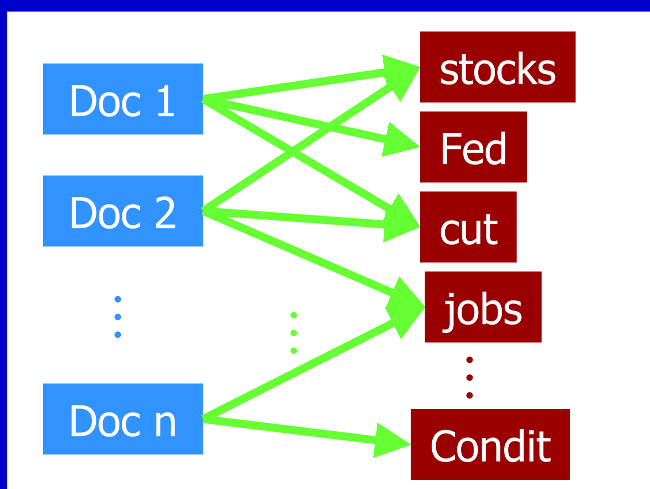
Grouping: finding global structures

coherence

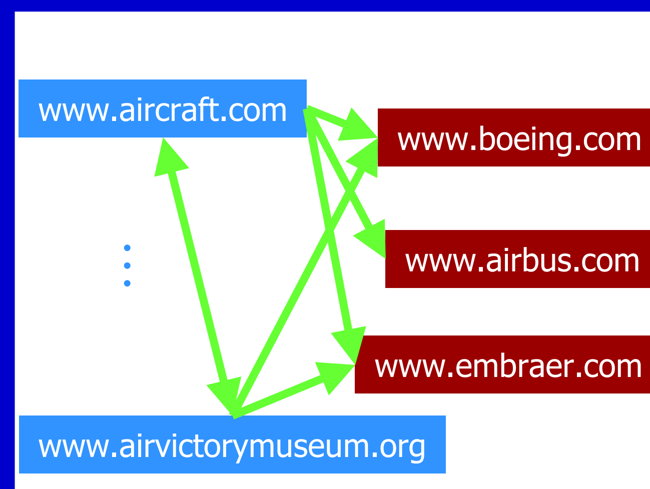


salience

figure-ground



document clustering

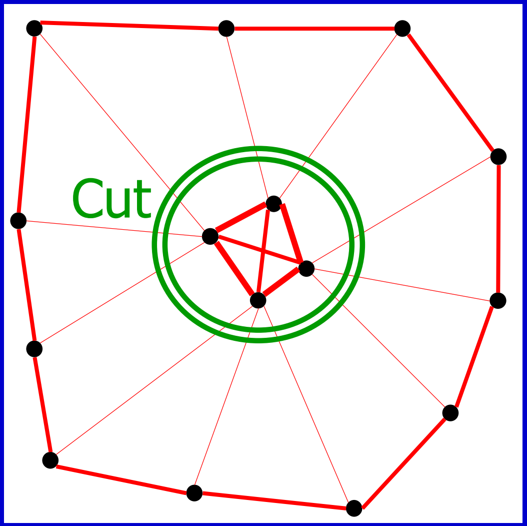


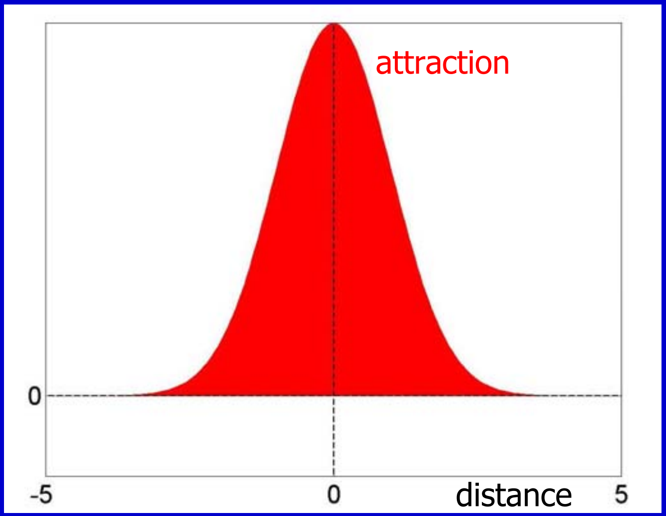
cyber community

Outline of the talk

- Motivation
- Model
- Examples
- Conclusions

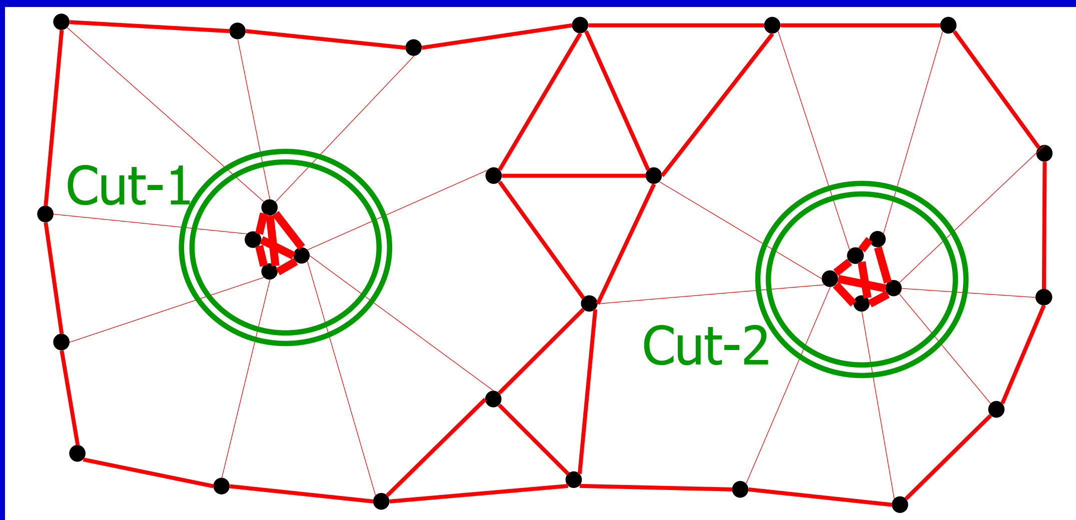
Motivation: grouping with pairwise similarity





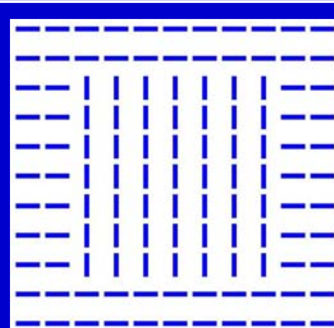
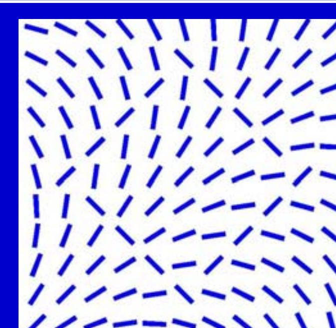
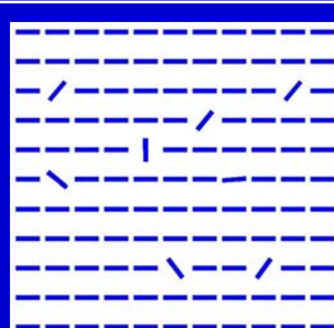
- Separation cost: pairwise similarity = attraction
- Attraction unites elements who have **common friends**

Motivation: similarity grouping is not enough

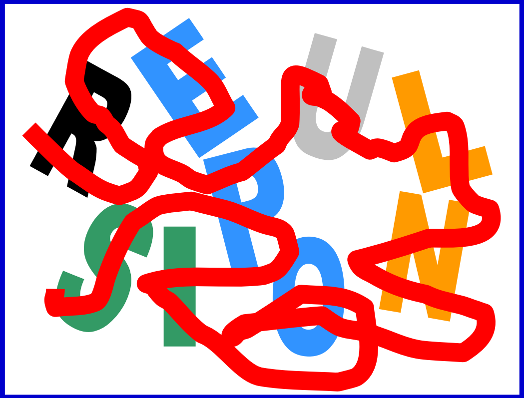


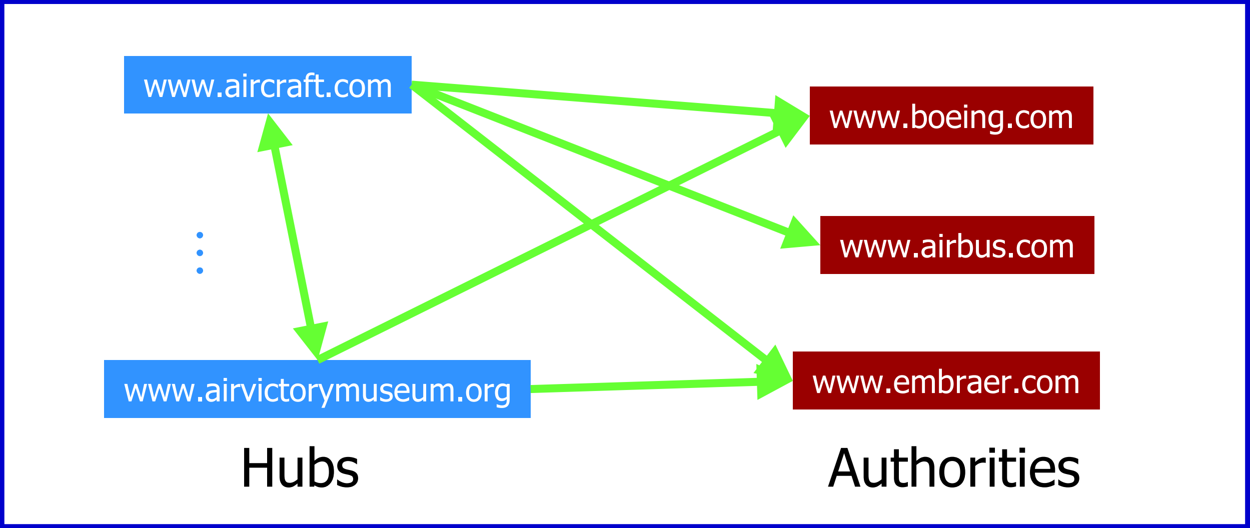
- Cost-1 + Cost-2 > min (Cost-1, Cost-2)
- Cannot unite elements who have **<u>common enemies</u>**

Motivation: similarity vs. dissimilarity

	Coherent ground	Incoherent ground
Coherent figure	 Yes	 Bad
Incoherent figure	 No	

Motivation: grouping with ordering

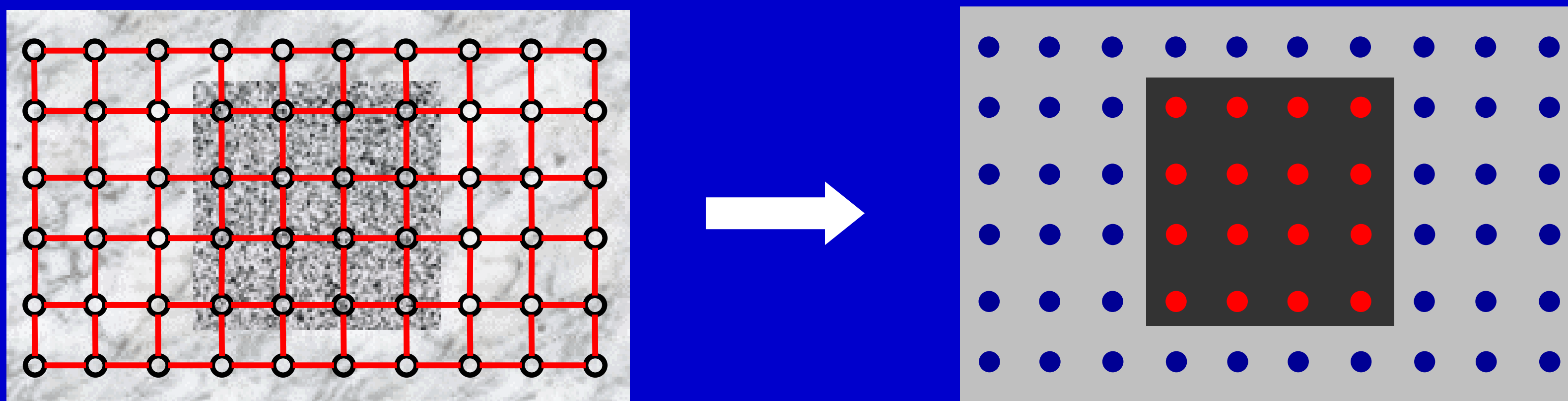




Model

- Goal
  - ➔ Similarity grouping, dissimilarity grouping, figure-ground in one step
- Representation
  - ➔ A pair of directed graphs for any pairwise relationships
- Criteria
  - ➔ Generalized normalized cuts
- Energy formulation
  - ➔ Rayleigh quotients of Hermitian matrices
- Solution
  - ➔ Phase plane partitioning

Review: segmentation in a graph framework



- G=(V, E, W)
  - ➔ V: each node denotes a pixel
  - ➔ E: each edge denotes a pixel-pixel relationship
  - ➔ W: each weight measures pairwise similarity
- Segmentation = node partitioning
  - ➔ break V into disjoint sets V_1 , V_2

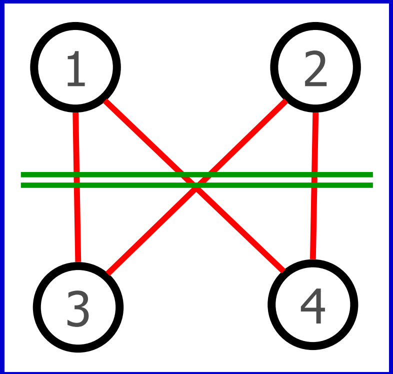
[Shi & Malik, 97]

[Puzicha et al, 98]

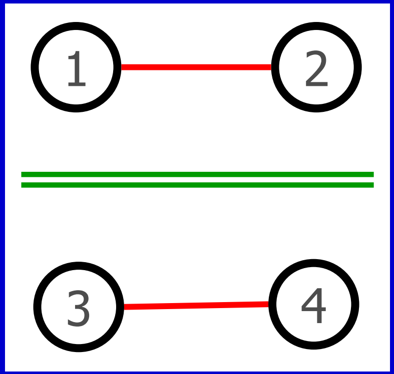
[Perona & Freeman, 99]

⋮

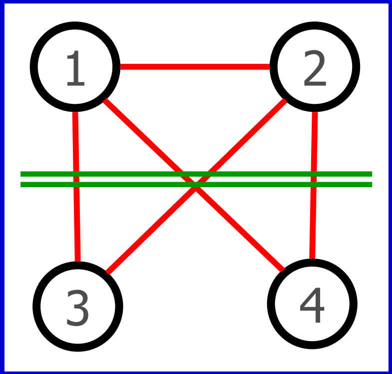
Review: cuts, associations, degrees



Cuts:
between-group similarity



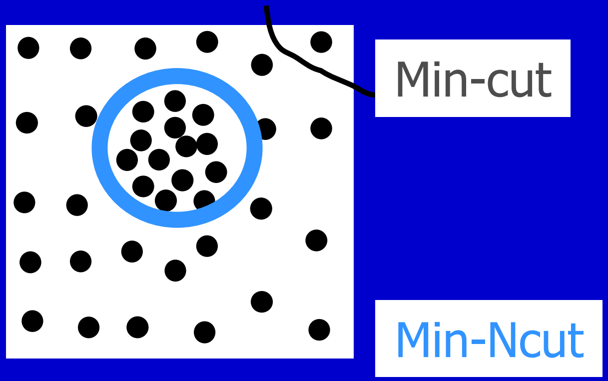
Associations:
within-group similarity



Degrees:
total similarity

Review: criteria and properties

- Two goals
  - ➔ Maximize normalized associations
  - ➔ Minimize normalized cuts
- Duality to achieve both goals at the same time
- Normalization for larger structures



Review: energy formulation

$$X_l(u) = \begin{cases} 1, & u \in V_l \\ 0, & u \notin V_l \end{cases}$$

$$X_1 = \begin{bmatrix} 1_{n_1 \times 1} \\ 0_{n_2 \times 1} \end{bmatrix}, \quad X_2 = \begin{bmatrix} 0_{n_1 \times 1} \\ 1_{n_2 \times 1} \end{bmatrix}$$

- Variables: indicators

$$Nassoc(X_1, X_2) = \sum_{t=1}^{2} \frac{X_t^T W X_t}{X_t^T D X_t}$$

- Energy functions

$$y = (1-\alpha) X_1 - \alpha X_2, \quad \alpha = \frac{\deg(V_1)}{\deg(V)}$$

- Change of variables

$$Nassoc(y) = \frac{y^T W y}{y^T D y}, \qquad s.t. \quad y^T D 1 = 0$$

- Rayleigh quotient

$$W \; y_{opt} = \lambda_2 \; D \; y_{opt}$$

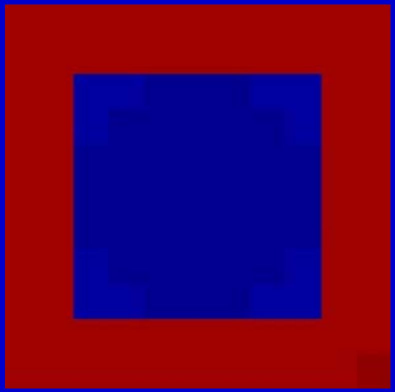
- Solution: eigenvector

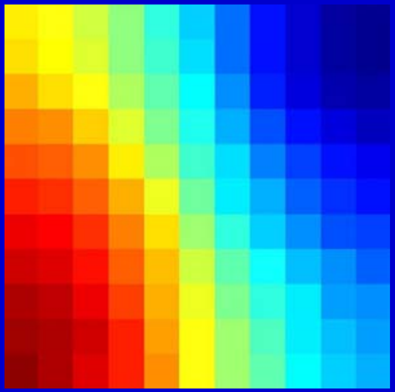
Review: interpretation of the eigensolution

- The derivation holds so long as $X_1 + X_2 = 1$

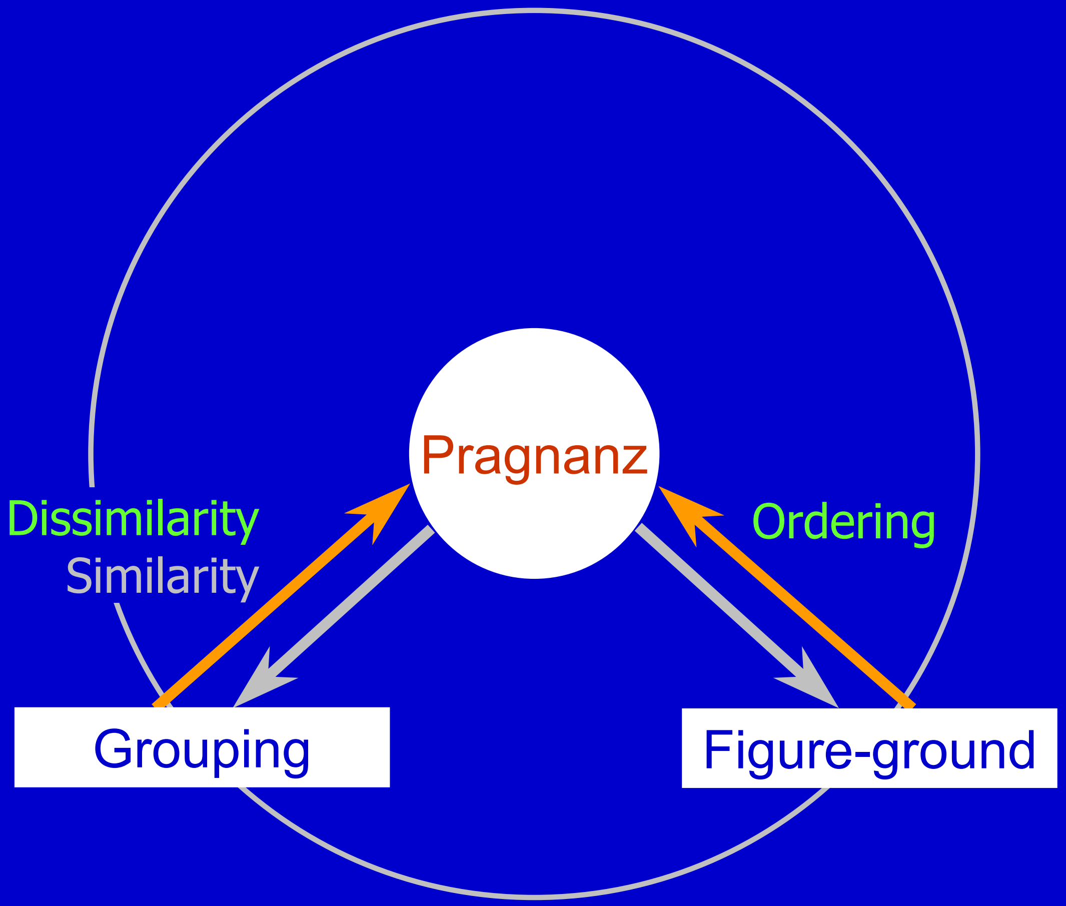
$$y = X_1 - \alpha$$

- The eigenvector solution is a linear transformation (scaled and offset version) of the probabilistic membership indicator for one group.
- If y is well separated, then two groups are well defined; otherwise, the separation is ambiguous

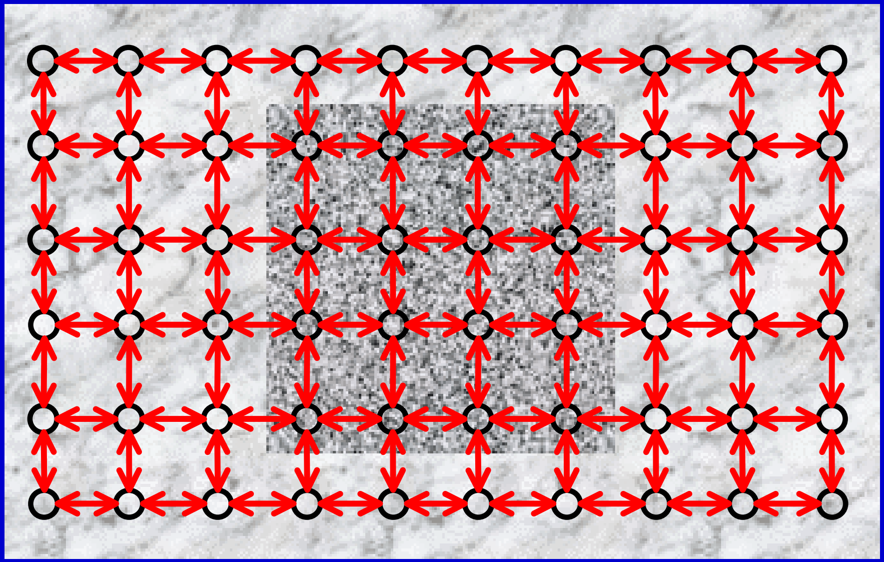


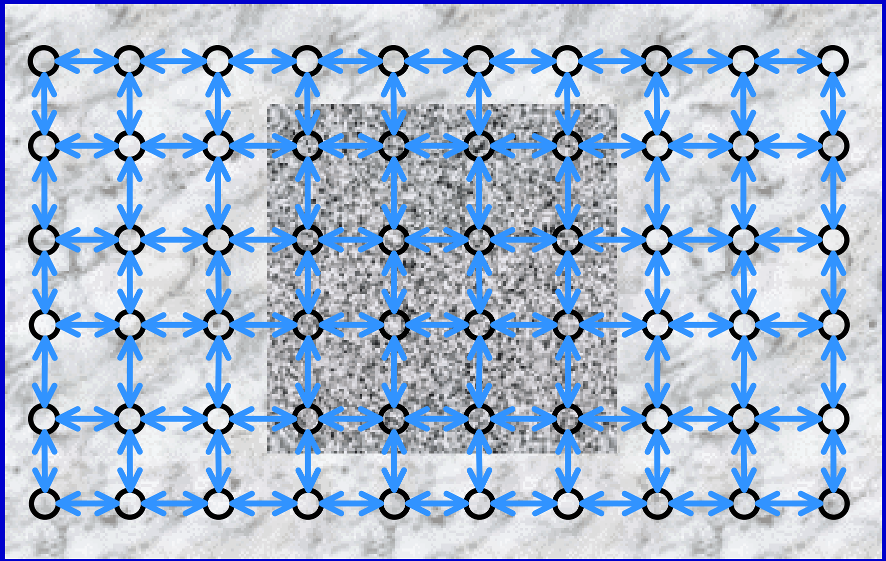


Model: goal


Pragnanz
Dissimilarity
Similarity
Ordering
Grouping
Figure-ground

Model: representation

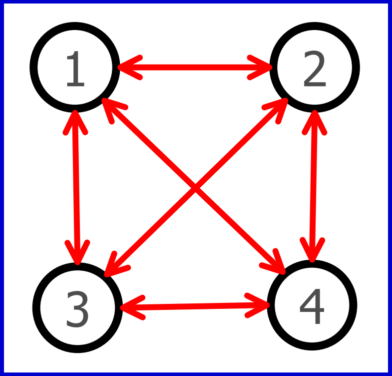




- G=(V, E, A)
- A asymmetric
- Separation cost

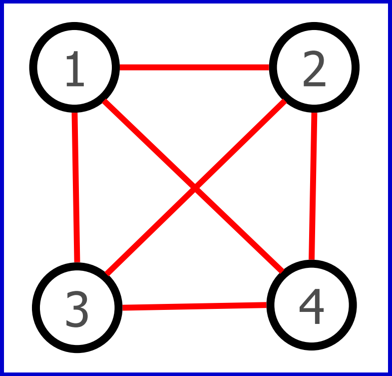
- G=(V, E, R)
- R asymmetric
- Separation gain

Model: attraction, repulsion, difference flow



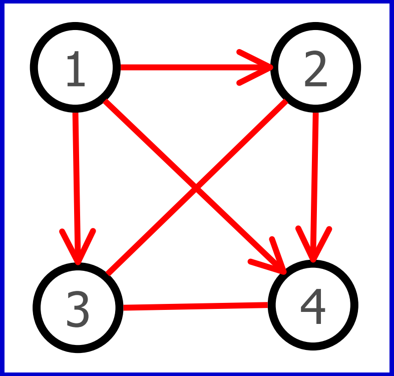
A

=

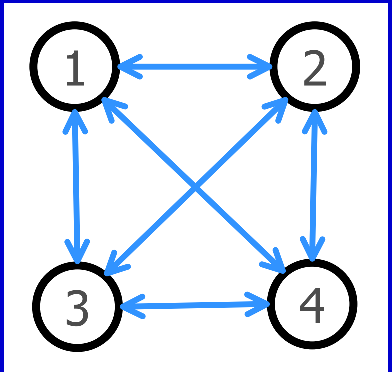


$A_u = A + A^T$

+

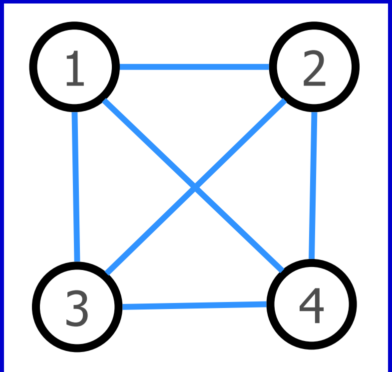


$A_d = A - A^T$



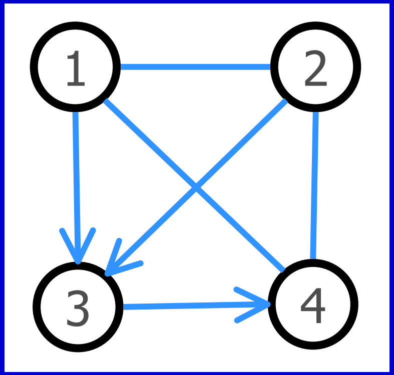
R

=



$R_u = R + R^T$

+

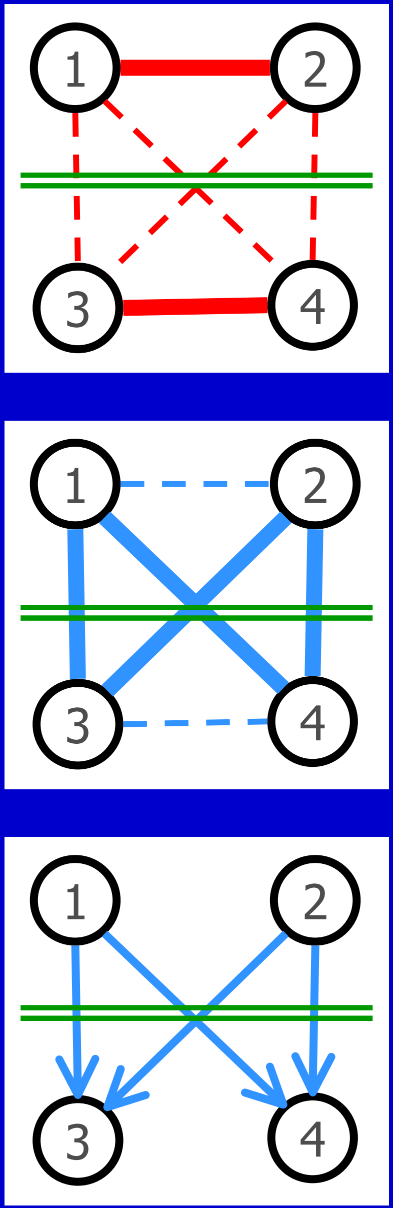


$R_d = R - R^T$

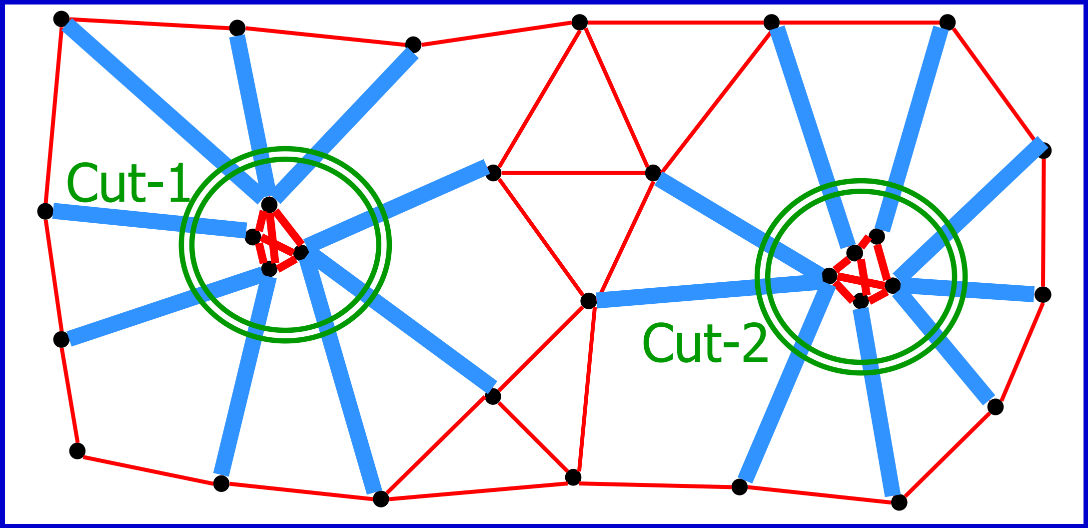
Model: criteria

- Maximize normalized
  - within-region similarity
  - between-region dissimilarity
  - figure-to-ground order

- Minimize normalized
  - between-region similarity
  - within-region dissimilarity
  - ground-to-figure order

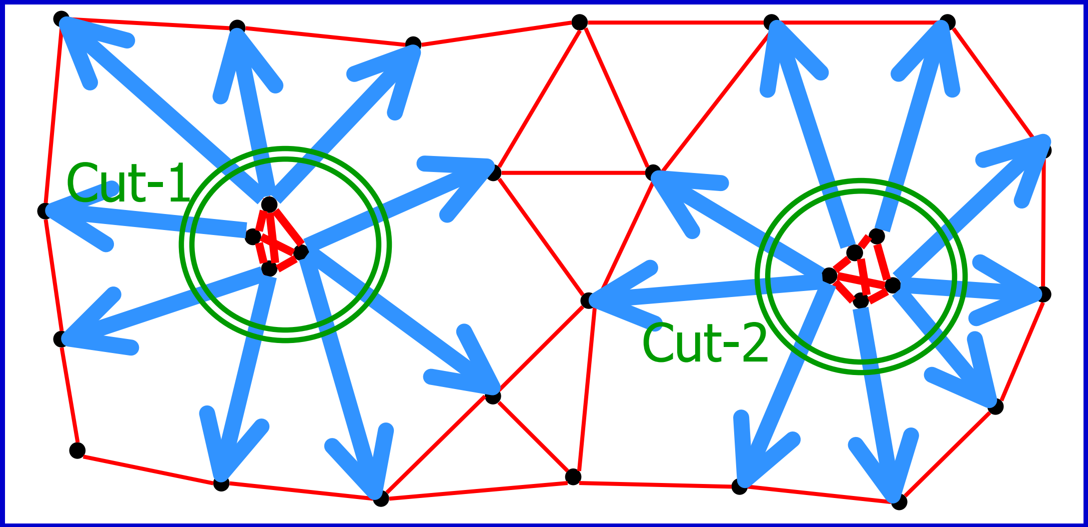


Model: is repulsion what we want?



- Cost-1 + Cost-2 < min (Cost-1 + Gain-2, Cost-2 + Gain-1)
- Repulsion unites elements who have **<u>common enemies</u>**

Model: is difference flow what we want?



- Cut (V1, V2) $\neq$ Cut (V2, V1)
- Ordered partitioning

Model: energy formulation

$$X_l(u) = \begin{cases} 1, & u \in V_l \\ 0, & u \notin V_l \end{cases}$$

- Variables: group indicators

$$U = A_u - R_u + D_R$$

$$V = A_d + R_d$$

$$D = D_{A_u} + D_{R_u}$$

- Weight matrices and degree matrix

- Energy functions of indicator variables

$$Nassoc(X_1, X_2) = \sum_{t=1}^{2} \frac{X_t^{\ T} U X_t}{X_t^T D X_t} + \frac{2 X_1^{\ T} V X_2}{\sqrt{X_1^T D X_1 \cdot X_2^T D X_2}}$$

Model: energy formulation

$$Nassoc(X_1, X_2) = \sum_{t=1}^{2} \frac{X_t{}^T U X_t}{X_t^T D X_t} + \frac{2X_1{}^T V X_2}{\sqrt{X_1^T D X_1 \cdot X_2^T D X_2}}$$

$$z = \sqrt{1-\alpha}\, X_1 - i\sqrt{\alpha}\, X_2, \quad \alpha = \frac{\deg(V_1)}{\deg(V)}$$

$$W = U + i \cdot V$$

$$Nassoc(z) = \frac{z^H W z}{z^H D z}$$

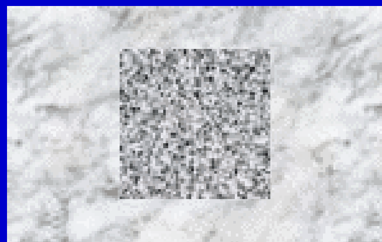
$$s.t. \quad z^{2T} D 1 = 0$$

- Change of variables
- Generalized affinity
- Rayleigh quotient

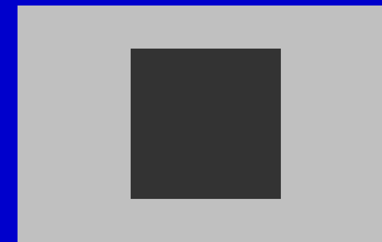
Model: three aspects of solutions

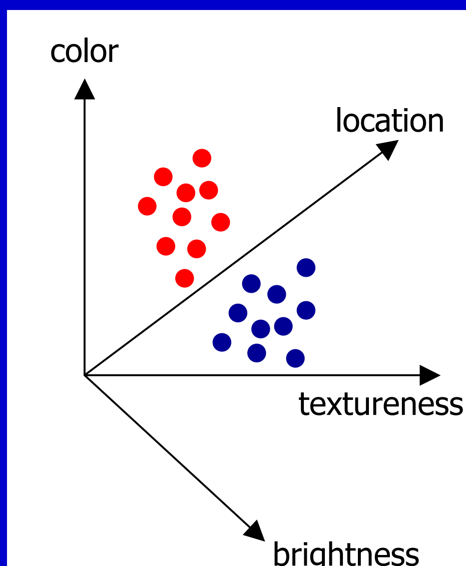
- Efficient solutions in the continuous domain:
  - ➔ Eigenvector corresponding to the largest eigenvalue of (W, D)
- Little increase in complexity
  - ➔ Hermitian matrices and sparse matrix eigendecomposition
- Interpretation of complex-valued solutions
  - ➔ Phase plane partitioning

Model: segmentation as embedding



Assign region identity.





$$z_{opt} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$$

grouping

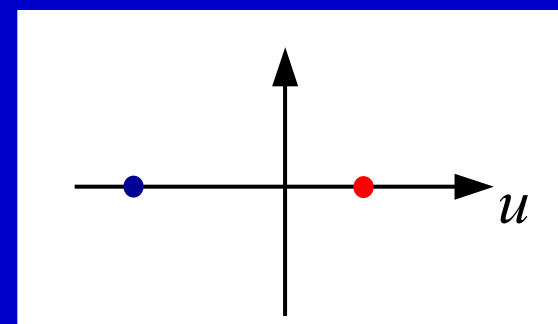
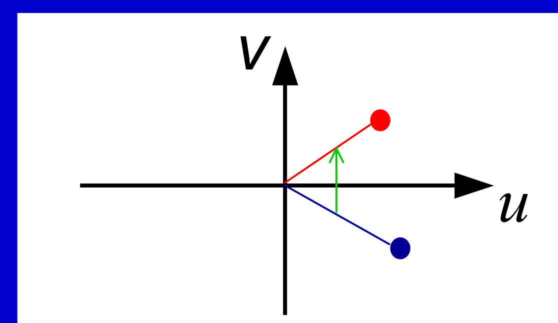


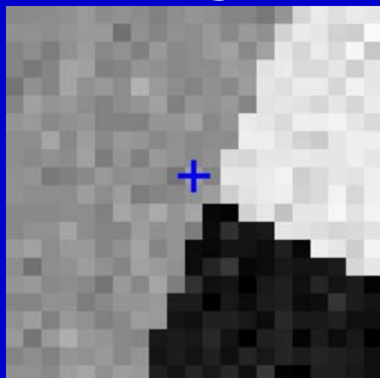
figure-ground

$$z_{opt} = \begin{bmatrix} u_1 + i \cdot v_1 \\ \vdots \\ u_n + i \cdot v_n \end{bmatrix}$$

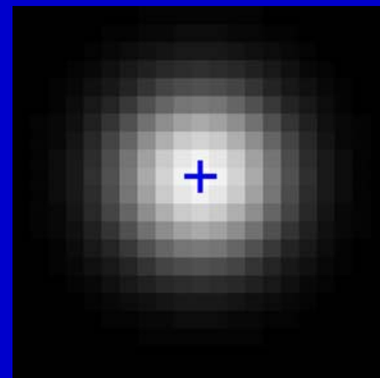


Results: difference flow for relative depth

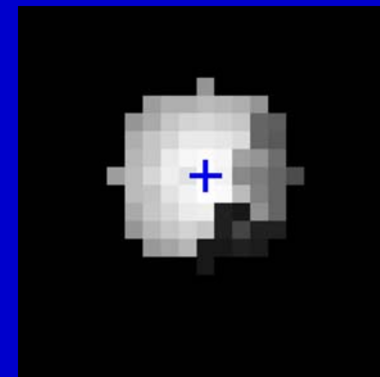
image



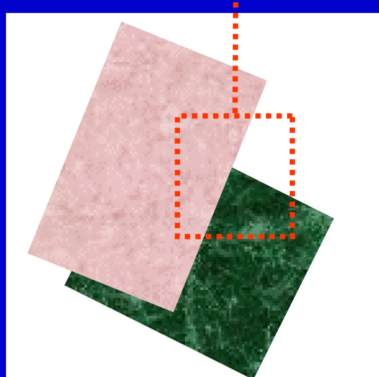
A by proxmity



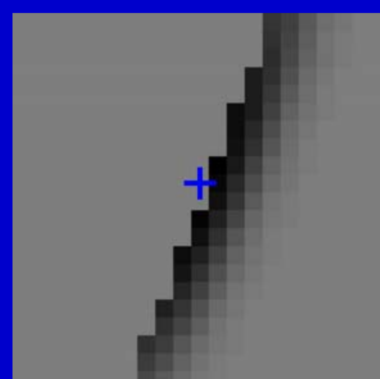
A by brightness similarity



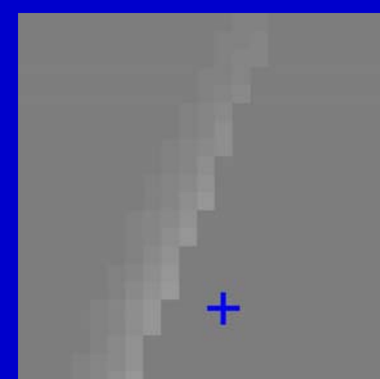
Relative depth
from T-junctions



R_d for a figure pixel

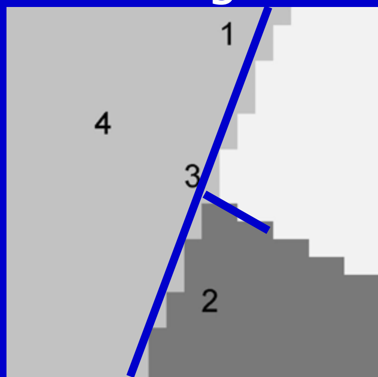


R_d for a ground pixel

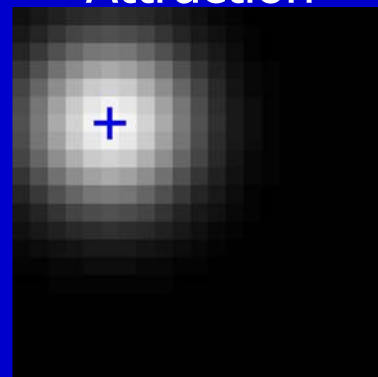


Results: interaction of attraction and repulsion

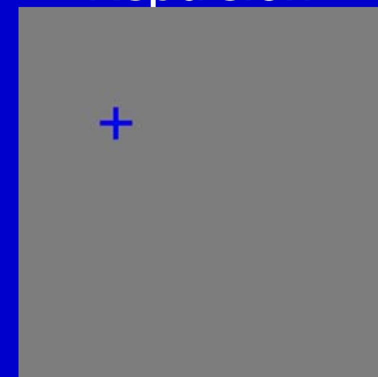
Image



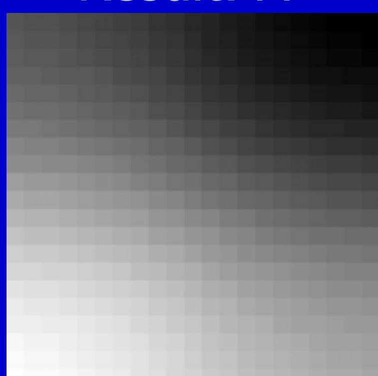
Attraction



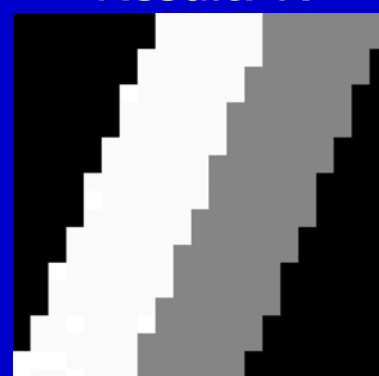
Repulsion



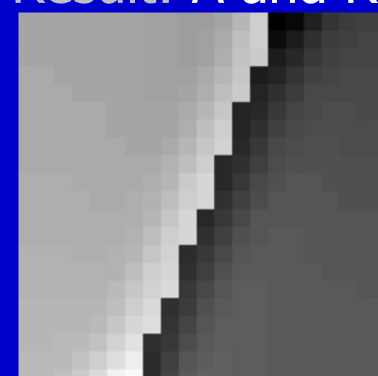
Result: A



Result: R

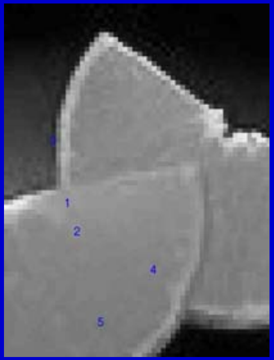


Result: A and R

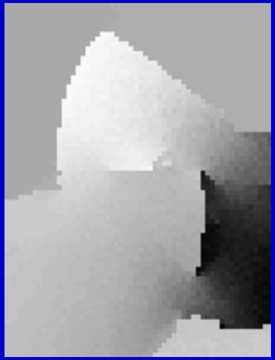


Results: figure-ground segregation

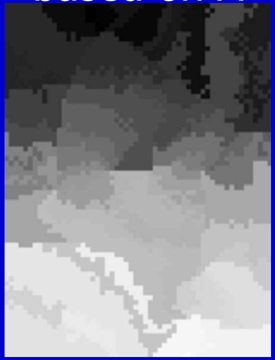
Image



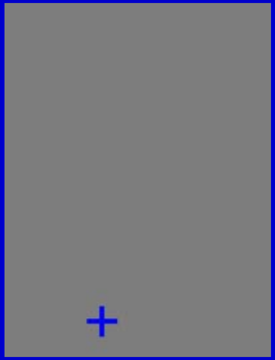
Solutions: A



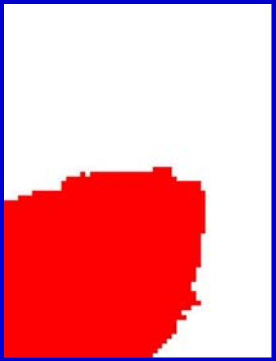
Oversegmentation based on A



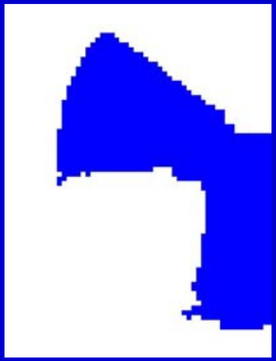
R



Figure

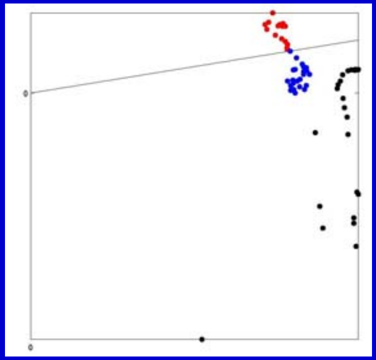


Ground 1



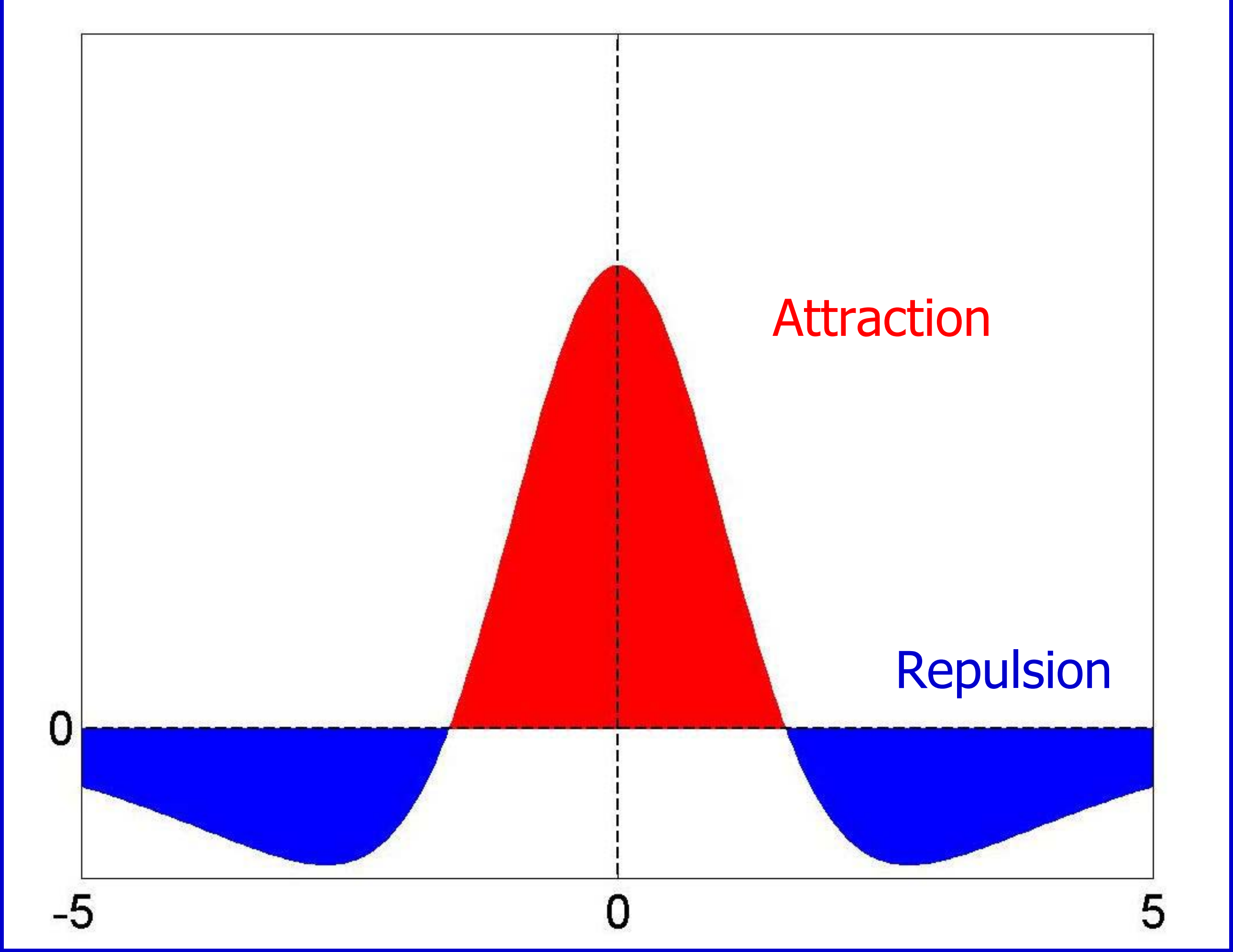
Ground 2

Phase plot

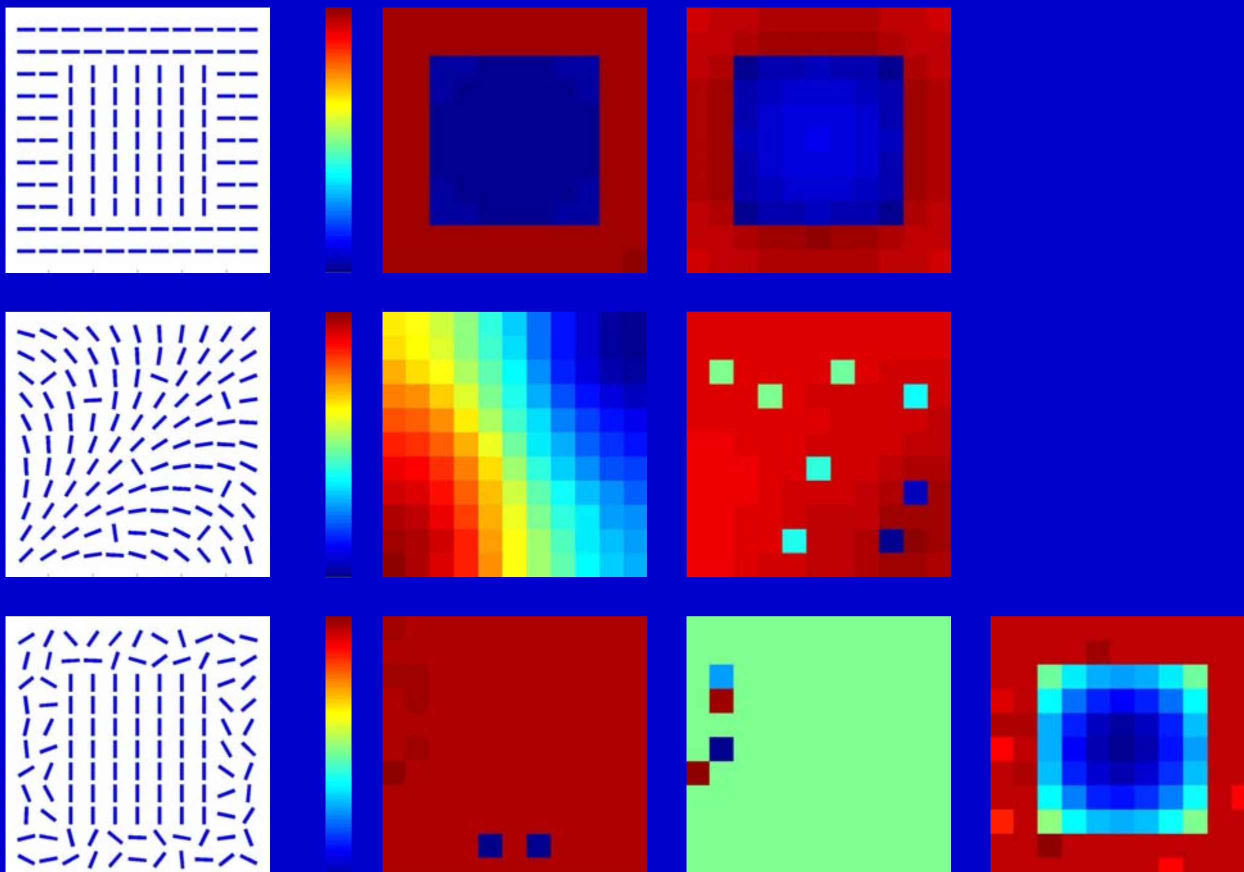


Results: from Gaussian to Mexican hat

$$U = A_u - R_u + D_R$$



Results: popout



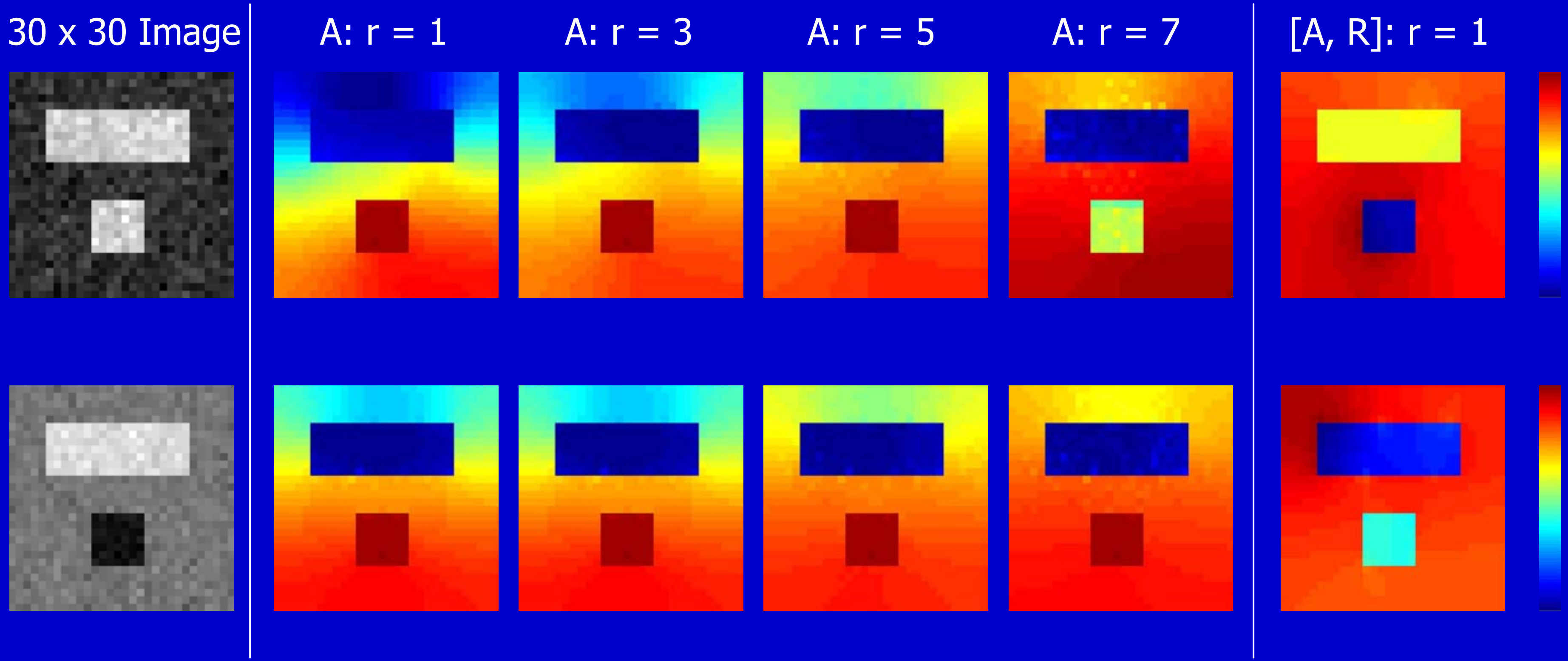
Attraction to bind similar elements

Repulsion to bind dissimilar elements

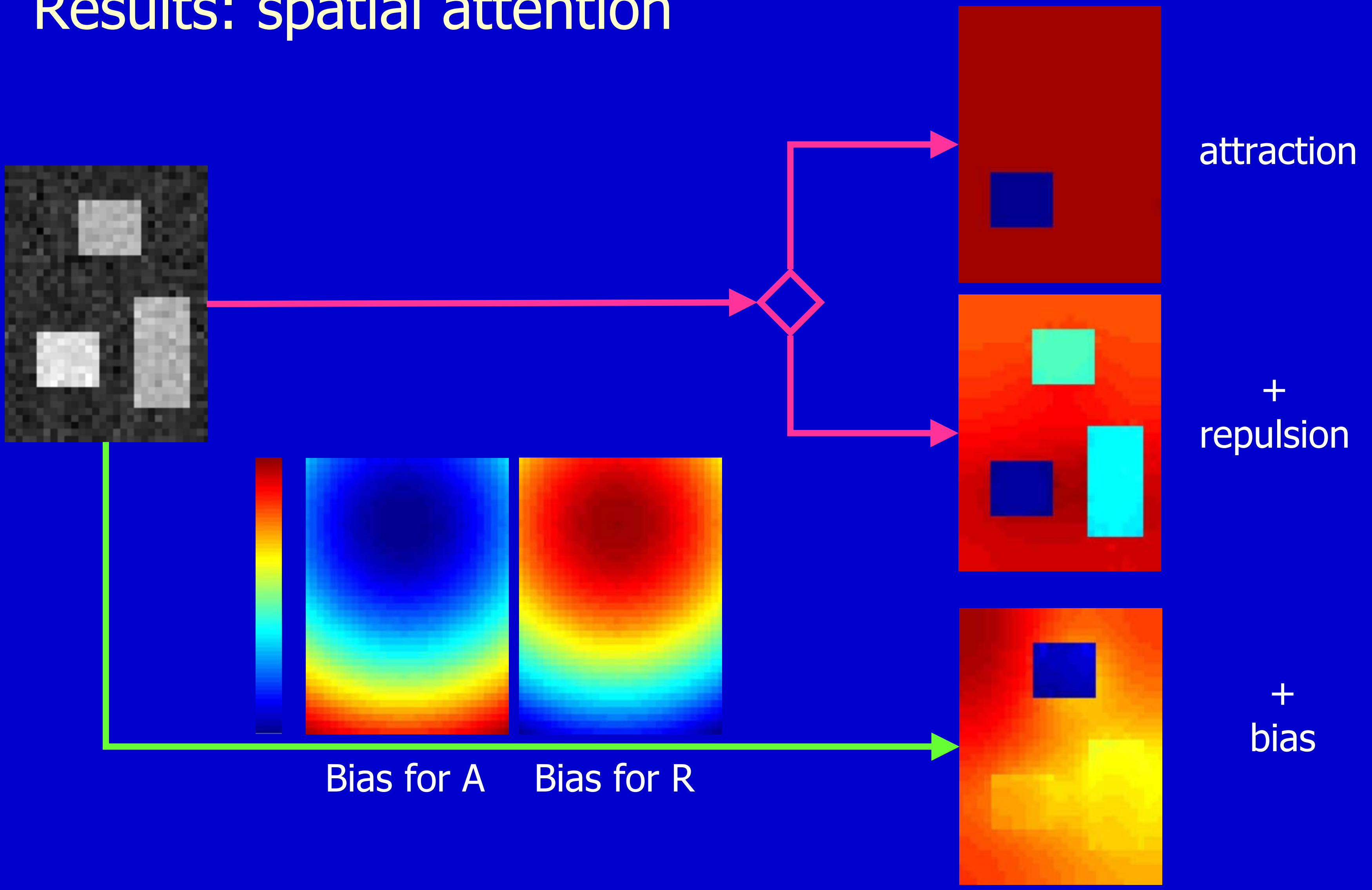
Regularization to equalize

Stimuli Attraction +Repulsion +Regularization

Results: computational efficiency



Results: spatial attention



Conclusions

- Pairwise relationships
  - ➔ Attraction: similarity grouping
  - ➔ Repulsion: dissimilarity grouping
  - ➔ Difference flow: relative ordering
- Advantages of repulsive and ordinal relationships:
  - ➔ Complementary
  - ➔ Computational efficiency
  - ➔ Treat 2D and 3D configuration cues equally
- Figure-ground organization

	Coherent ground	Incoherent ground
Coherent figure	Attraction	+Regularization
Incoherent figure	+Repulsion	