Energy and Bandwidth Efficiency in Wireless Networks

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Outline

• Introduction/Background
• Device/Physical Layer/Network Layer Models
• Performance Measure
• Numerical Results
Communication Problem

Data Rate $R$

Source $\rightarrow$ Encoder $\rightarrow$ Mod $\rightarrow$ PA

Power $P$, Bandwidth $W$

Noise $\rightarrow$ +

Sink $\leftarrow$ Decoder $\leftarrow$ Demod
Energy-Bandwidth Efficiency

• Shannon showed there is a fundamental tradeoff between energy efficiency and bandwidth efficiency for reliable communications

• $R$: Data rate (bits/second)
• $P$: Power (Joules/sec)
• $N_0$: Noise power spectral density (Watts/Hz)
• $W$: Bandwidth (Hz)

$$R < W \log_2 (1 + \frac{P}{N_0 W})$$
Shannon’s Result

\[ E_b = \frac{P}{R} \]

\[ \frac{R}{W} < \log_2 \left( 1 + \frac{E_b}{N_0} \frac{R}{W} \right) \]

\[ \frac{E_b}{N_0} > \frac{2^{\frac{R}{W}} - 1}{R/W} \]

- \( R/W \): Bandwidth efficiency (bits/second/Hz)
- \( E_b/N_0 \): Received energy per information bit-to-noise power spectral density ratio (dB)
Shannon’s Result for Binary Input (BPSK)

• When the signal alphabet is restricted to binary the capacity changes.

\[
\frac{R}{W} < 1 - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-\beta)^2}{2}} \log_2(1 + e^{-2y\beta}) dy
\]

\[
\beta = \frac{E_b}{N_0} \frac{R}{W}
\]
Energy-Bandwidth efficiency tradeoff

![Diagram showing the relationship between $E_b/N_0$ (dB) and $R/W$ for binary and unquantized inputs.](image-url)
Energy-Bandwidth Efficiency Tradeoff

![Graph showing the tradeoff between energy and bandwidth efficiency for unquantized and binary inputs.](Image)
Shannon’s Assumption

- Linear amplifier (Ideal, 100% efficient)
- Point-to-point link
- No receiver processing energy
- Infinite delay
Relaxed Assumptions

- Nonlinear amplifier (energy efficiency dependent of drive level)
- Multihop network (take into account propagation)
- Receiver processing energy
Power Amplifier

![Graphs showing the relationship between input power and power output for a power amplifier.]
Power Amplifier

- Power amplifier is most energy efficient when driven into saturation (large output power).
- Power amplifier is least energy efficient at low input drive levels (low output power)
Propagation Characteristics

\[ P_r = P_t \left( \frac{\lambda_c}{4\pi d_e} \right)^2 4G_t G_r \sin^2 \left( \frac{2\pi h_t h_r}{\lambda_c d_e} \right) \approx \frac{P_t G_t G_r h_t^2 h_r^2}{d_e^4} \]
Propagation Characteristics

• Amount of energy necessary to go a distance $d_e$ increases as $d_e^4$. 
Performance Measure

• Without Spatial Reuse
  – Energy Efficiency
  – Bandwidth Efficiency
  – Transport Efficiency
Goal

• We want to find the relation between energy consumption and bandwidth efficiency for a network taking into account amplifier characteristics, propagation characteristics, receiver processing energy.
Goal

• We want to optimize over amplifier drive level and the distance between nodes in routing packets from the source to the destination.
An Illustrative Example:

- Longer link distance =&gt; lower received SNR
- Lower data rate for each hop
- Less processing energy usage
- More efficient amplifier operation

- Shorter link distance =&gt; higher received SNR
- Higher data rate for each hop
- More processing energy usage
- Less efficient amplifier operation

Choose route to

- Minimize energy usage
- Maximize bandwidth efficiency
RX Processing Energy

- Assume a fixed power consumption of receiver ($P_{rp}$).
- Lower rate codes => receiver is on for a longer period of time for a given number of information bits.
- Large number of hops => large amount of receiver energy consumption.
System Model

- Assumptions
- Power Amplifier Model
- Signal Attenuation Model
Power Amplifier Model

- Linear at low input power levels.
- Saturation at high power levels.
- Constant amount of power turned into heat.
- $P_h = 35\text{mW}$, $P_{\text{sat}} = 75\text{mW}$,
- $\rho = 50$ (17 dB), $P_1 = 1.5\text{mW}$.
Power Amplifier Model

- Radiated Power

\[
fo(P_{in}) = \begin{cases} 
\rho P_{in}, & 0 \leq P_{in} \leq P_1 \\
Psat, & P_1 < P_{in} \leq P_{max}
\end{cases}
\]

- Consumed Power

\[
f_c(P_{in}) = fo(P_{in}) + P_h
\]
Propagation Model

Inverse power law

\[ P_r = \beta \frac{P_o}{d^n} \]

For numerical results \( \beta=1, \eta=4 \).
Energy Consumption

- Encoder $K$ information bits mapped into $N$ coded bits, rate $R=K/N$.
- $k$ hops between transmitter and receiver
- $E_p$ energy per coded bit

\[
E_t = k(E_{tx,b} + E_{rcvr,b}) = kN(f_c(P_{in})T_s + E_p) \frac{1}{K}
\]
Bandwidth Efficiency

- $B_{\text{eff}} = \frac{\text{expected number of correctly decoded (end-to-end) bits}}{\text{per channel use.}}$

- $P_s(R, E_b/N_0) = \text{probability of packet success per hop which depends on the code rate } R \text{ and the received SNR.}$
Physical Layer Models

- Threshold Model

\[
P_s(R, \frac{E_b}{N_0}) = \begin{cases} 
0, & \frac{E_b}{N_0} < \Gamma \\
1, & \frac{E_b}{N_0} > \Gamma 
\end{cases}
\]

- Coded Model
- Uncoded Model

\[P_e = 1 - P_s\]
Physical Layer Model (AWGN)
Transport Efficiency

• Often it is desirable to have a single measure of a network performance.
• A measure of performance capturing energy use and bandwidth efficiency is the transport efficiency.

\[ \mu(k, R, P_{in}) = \frac{B_{eff}}{E_t} \]

• Transport efficiency is the bits/second/Hz possible per unit energy.
• The transport efficiency depends on the number of hops, the code rate and the operation of the amplifier.
Energy-Bandwidth Efficiency (Single Hop)

\[ E_p = 0.25\mu J/\text{symbol} \text{ at 50K symbols per second corresponds to 125 mWatts receiver processing power} \]
Energy-Bandwidth Efficiency (multi-hop)

![Graph depicting the relationship between bandwidth efficiency and consumed signal energy-to-noise ratio. The graph shows multiple curves for different hop counts (hop=1, hop=2, hop=3, hop=4), with markers indicating maximum and minimum SNR over hops.]
Transport Efficiency vs. Distance

AWGN Capacity Model w/o Spatial Reuse, $E_p = 0.25(\mu J/symbol)$

- Increasing number of hops
- $k=1$
- $k=2$
- $k=3$
AWGN Capacity Model w/o Spatial Reuse, $E_p = 0.25 \mu J$/symbol

Optimum number of hops

Distance (m)

Optimum code rate

Optimization number of hops
Conclusion Without Spatial Reuse

- Transport efficiency decreases only inverse linear with distance for any power propagation law, amplifier characteristic, coding/modulation technique.

\[
\max_{k, P_{in}, R} \mu(k, R, P_{in}) = \frac{\delta}{d_e}
\]

- The constant depends on the coding, the propagation model, the amplifier model.
- Same results holds for any functional dependence of error probability on SNR, any amplifier model, propagation characteristics.
Spatial Reuse (Linear Case)
Spatial Reuse
Spatial Reuse

- $L=$ minimum hop separation for concurrent transmissions.
- $\Omega =$ number of simultaneous transmissions.

$\Omega \approx \frac{k}{L}$

- Accounting for interference from two other transmissions with $L=3$ yields

$$SINR = \frac{\beta P_{out} T_s d^{-\eta}}{N_0 + \beta P_{out} T_s [(2d)^{-\eta} + (4d)^{-\eta}]}$$
Energy-Bandwidth Efficiency

AWGN Capacity Model w Spatial Reuse at 4000m, $E_p = 0.25 (\mu J/\text{symbol})$

Bandwidth Efficiency (bps/Hz) vs. Consumed signal energy-to-noise ratio, $\text{SNR}_c$ (dB)

- Max. $\mu$ over hops
- $\text{Max. } \mu$
- Min. $\text{SNR}_c$
- $\text{hop}=7$
- $\text{hop}=6$
- $\text{hop}=5$
- $\text{hop}=4$
- $\text{hop}=3$
- $\text{hop}=2$
- $\text{hop}=1$
Numerical Result

AWGN Capacity Model w Spatial Reuse, $E_p = 0.25(\mu J/$symbol)

Optimal Transport Efficiency

Increasing number of hops

$k=1$

$k=2$

Distance (m)
Transport Efficiency vs. Distance

Max. Transport Efficiency for pathloss, $\eta$ with $E_p = 25(\mu J/\text{symbol})$

Optimal Transport Efficiency

Distance (m)

- w/o sp reuse, $\eta = 2.5$
- w/o sp reuse, $\eta = 3$
- w/o sp reuse, $\eta = 3.5$
- w/o sp reuse, $\eta = 4$
- w sp reuse, $\eta = 2.5$
- w sp reuse, $\eta = 3$
- w sp reuse, $\eta = 3.5$
- w sp reuse, $\eta = 4$
Optimization Parameters with Spatial Reuse

AWGN Capacity Model w Spatial Reuse, $E_p = 0.25(\mu J/$symbol$)$

- Optimum number of hops
- Optimum code rate
Comparison of Optimum Number of Hops

AWGN Capacity Model, $E_p = 0.25(\mu J/symbol)$

- without spatial reuse
- with spatial reuse

Optimum number of hops vs. Distance (m)
Conclusions

• The tradeoff between energy and bandwidth efficiency for wireless networks has been quantified incorporating amplifier model inefficiency, propagation and network routing.
• Results indicate relatively short distances, high rate coding are desirable.
• Analysis technique easily applicable to fading and other modulation techniques as well as to specific codes.
• Results might change (lower code rates) if time/frequency selective fading is included but the fundamental relationship with distance does not change.
Conclusions

- There are many extensions necessary
  - Include MAC layer
  - Include spatial distribution of nodes as opposed to infinite density of node
  - Include mobility (energy to update routing path)
  - Find practical ways to achieve performance limits.