Performance Limits of M-FSK With Reed–Solomon Coding and Diversity Combining

John D. Choi, *Student Member, IEEE*, Do-Sik Yoo, *Student Member, IEEE*, and Wayne E. Stark, *Fellow, IEEE*

***Abstract*—This paper examines the asymptotic ($M \rightarrow \infty$) performance of M-ary frequency-shift keying (M-FSK) in multichannels, or multiple frequency-nonselective, slowly fading channels, with coding, side information, and diversity reception. In particular, Reed–Solomon (RS) coding is considered in conjunction with the ratio-threshold test (RTT), which generates side information regarding the reliability of received symbols. The asymptotic performance of orthogonal signaling in multichannels with maximal ratio combining (MRC), postdetection equal gain combining (EGC), hybrid selection combining (H-SC), and selection combining (SC) is derived for an arbitrary statistical fading model and diversity order. The derivations reveal that coherent and noncoherent implementations of diversity combining schemes yield the same performance asymptotically. In addition, the asymptotic results are evaluated assuming a Nakagami-m fading model, and the effect of fading severity, diversity order, code rate, and side information upon the performance of the various diversity combiners is investigated. The minimum signal-to-noise ratio (SNR) required to achieve arbitrarily reliable or error-free communication, as well as the associated optimal RS code rate, are determined for various cases.**

***Index Terms*—Asymptotic performance, diversity combining, Nakagami-m fading, orthogonal modulation, Reed–Solomon coding.**

I. Introduction

THE ASYMPTOTIC performance of M-ary orthogonal modulation in an additive white Gaussian noise (AWGN) channel and its information theoretic significance in achieving the Shannon limit have been well documented. Since this seminal finding [1], the asymptotic analysis of orthogonal signaling has been extended to other channel models such as frequency-nonselective, slowly fading channels [2]–[6] and multichannels, or multiple frequency-nonselective, slowly fading channels [2], [7], [8]. These works collectively reveal that the limiting performance under such channel conditions does not achieve the infinite bandwidth AWGN channel capacity. The inclusion of channel coding and side information, however, makes arbitrarily reliable communication with orthogonal signaling feasible at finite signal-to-noise ratios (SNRs) [5], [6], [9].

Paper approved by A. Ahlen, the Editor for Modulation and Signal Design of the IEEE Communications Society. Manuscript received February 15, 2001; revised December 15, 2001 and January 15, 2002. This work was supported by the Office of Naval Research under Grant N00014-00-1-0224, and the United States Army Research Office under Grant DAAH04-96-1-0377. This paper was presented in part at the Global Telecommunications Conference, San Francisco, CA, November 27–December 1, 2000.

The authors are with the Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI 48109 USA (email: jdchoi@eecs.umich.edu; dyoo@eecs.umich.edu; stark@eecs.umich.edu).

Digital Object Identifier 10.1109/TCOMM.2002.805264

It is noted that these channel models implicitly require that for fixed rate transmissions, the orthogonal signal set be comprised of narrowband signals, as is the case for frequency-shift keying (FSK). For M-ary orthogonal modulation schemes such as pulse-position modulation or block-coded modulation, a frequency-selective channel is more appropriate because the signal bandwidth increases with respect to M.

In order to extend and generalize these previous results under a unifying framework, we examine the asymptotic performance of M-FSK in multichannels with coding, side information, and diversity reception. In particular, Reed–Solomon (RS) coding is considered in conjunction with the ratio-threshold test (RTT) [5], [9], [10], which generates side information regarding the reliability of received symbols. RTT enables the RS-bounded distance decoder to perform errors-and-erasures decoding (EED) by identifying potentially corrupted symbols, and its operation is sufficiently general such that hard-decision decoding (HDD) corresponds to a special case.

Within this framework, and employing an alternative approach to that in [8], the asymptotic performance of M-FSK in multichannels with postdetection equal gain combining (EGC) is derived and shown to be identical to that of maximal ratio combining (MRC) [2] for any statistical fading model and diversity order. These derivations are then applied to the cases of hybrid selection combining (H-SC) [11]–[14] and selection combining (SC) [12], [14]–[16], and it is shown that the performance of noncoherent implementations of these schemes asymptotically approaches that of their coherent counterparts as well, for arbitrary fading and diversity order.

The asymptotic derivations are next evaluated, assuming that each diversity channel undergoes independent Nakagami-m fading. The individual and coupled effects of fading severity, diversity order, RS code rate, and side information on the asymptotic performance of various diversity combining schemes is examined. The minimum required SNR for arbitrarily reliable communication and the corresponding optimal RS code rate is numerically computed for the different diversity combiners operating under various conditions. In addition, we investigate the asymptotic performance of MRC and EGC for large diversity orders and verify that these schemes achieve the Shannon limit as the number of processed and combined diversity channels approaches infinity. The asymptotic performance of the reduced-complexity schemes such as H-SC and SC, however, degrades rapidly as the diversity order grows large.

This paper is organized as follows. In Section II, the general system model is described. The asymptotic analysis of MRC, EGC, H-SC, and SC is detailed in Section III. In Section IV, the asymptotic results are evaluated for the Nakagami-m fading

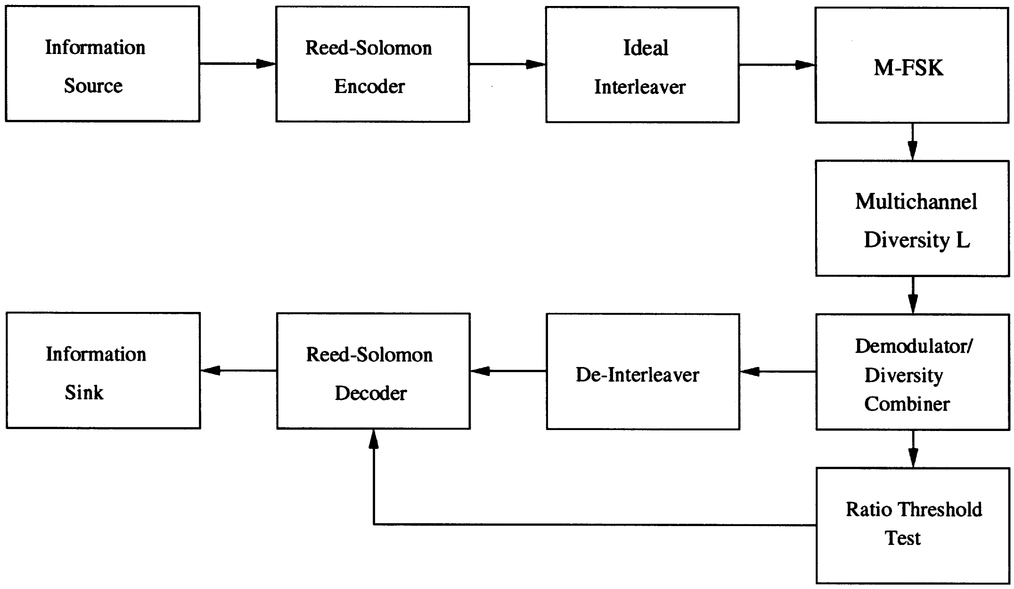


Fig. 1. Block diagram of system model.

model, and the limiting behavior of the diversity combining schemes for large diversity orders is examined. In Section V, we present the asymptotic performance of the various schemes with RS coding and side information generated by RTT. In particular, the minimum required SNR to achieve arbitrarily reliable communication and the optimal RS code rate are numerically computed. Finally, concluding remarks are made in Section VI.

II. System Model

A single-user communication system employing RS coding, multichannel signaling with M-FSK, RTT, and diversity reception is considered. The multichannel framework can accommodate time, frequency, and spatial diversity systems [14], [16], [17] so long as the underlying assumptions regarding the channels remain valid. In order to simplify the description of the system model, we will assume that the receiver utilizes multiple antennas to achieve diversity, although the results are equally applicable to appropriately designed time and frequency diversity systems for finite diversity orders.

As depicted in the block diagram (Fig. 1), the information source is first encoded by a (n, k) RS code where $n = M-1$ and the code rate is $r = k/n$. After ideal interleaving, a code symbol comprised of $\log_2 M$ bits is mapped to one of M orthogonal signals to be transmitted. The ith transmitted signal is given by

$$S_i(t) = \mathrm{Re}\left[s_i(t)e^{j2\pi f_c t}\right], \; i = 1, 2, \ldots, M, 0 \le t \le T \quad (1)$$

where f_c denotes the carrier frequency, T the symbol duration, and $s_i(t)$ the equivalent lowpass representation. The bandpass signals $\{S_i(t)\}$ are equally likely to be transmitted and possess the same energy $E = 1/2\int_0^T |s_i(t)|^2 dt$. The frequency separation between adjacent signals is chosen to be $1/T$, such that orthogonality is maintained after both coherent and noncoherent demodulation.

We consider a multichannel with diversity order L which is comprised of L frequency-nonselective, slowly fading channels. The receiver obtains L corrupted copies of the transmitted signal, which we assume to be the $(i = 1)$ signal, without any loss of generality. The equivalent lowpass received signal corresponding to the lth diversity channel is then given by

$$r_l(t) = \alpha_l e^{-j\phi_l} s_1(t) + n_l(t), \qquad l = 0, 1, \ldots, L-1 \quad (2)$$

where α_l and ϕ_l denote the channel-induced amplitude and phase, respectively, and $n_l(t)$ is the equivalent low-pass additive noise process. It is assumed that $\{\alpha_l\}$ and $\{\phi_l\}$ are constant over a symbol duration, the amplitudes $\{\alpha_l\}$ are continuous random variables, the phases $\{\phi_n\}$ are mutually independent, uniformly distributed over [0, 2π), and $\{n_l(t)\}$ are mutually independent, complex-valued AWGN random processes with zero mean and two-sided power spectral density $2N_0$. In addition, the random processes $\{n_l(t)\}$ and random variables $\{\alpha_l, \phi_l\}$ are all mutually independent. For generality, a full statistical description of the attenuations $\{\alpha_l\}$ is provided when evaluating the asymptotic results in Section IV.

In order to exploit the diversity inherent in the collective received signal $\{r_l(t)\}$, the receiver initially determines the correlator outputs $\{Z_{i,l}\}$ corresponding to each of the M signals for all L diversity channels

$$Z_{i,l} = \int_0^T r_l(t)s_i^*(t)\,dt, \qquad i = 1, 2, \ldots, M \; \forall l \quad (3)$$

$$= \begin{cases} 2E\alpha_l e^{-j\phi_l} + \eta_{1,l}, & i = 1 \\ \eta_{i,l}, & i \neq 1 \end{cases} \quad (4)$$

where $\eta_{i,l} = \int_0^T n_l(t)s_i^*(t)dt$ and $\{\eta_{i,l}\}$ are mutually independent, circularly symmetric, zero-mean Gaussian random variables with variance $E[\eta_{i,l}\eta_{i,l}^*] = 4EN_0$.

Because the diversity combining schemes of interest manipulate these correlator outputs $\{Z_{i,l}\}$ differently, their specific operation will be summarized in the next section. Nevertheless, all diversity combiners yield decision statistics $\{Z_i\}$ which are then compared using RTT. This technique generates side information about the reliability of the received symbol by comparing the ratio of the largest to the second-largest decision statistic. Whenever this ratio does not exceed a fixed, finite threshold ($\gamma > 1$), RTT declares a symbol erasure in an attempt to improve the performance of the bounded distance RS decoder.

More specifically, RTT identifies a symbol to be erased when

$$\max_{j\neq i} Z_j \leq Z_i < \gamma \max_{j\neq i} Z_j, \text{ for any } i \in \{1,2,\ldots,M\} \quad (5)$$

and yields an error if

$$Z_i \geq \gamma \max_{j\neq i} Z_j, \text{ for some } i \neq 1. \quad (6)$$

It is also noted that errors-only decoding or HDD is just a special case of RTT in which $\gamma = 1$.

III. Asymptotic Analysis of Diversity Combining Schemes With RTT

In this section, we determine the asymptotic performance of various diversity combiners operating with RTT for an arbitrary fading model, diversity order, RS code rate, and RTT threshold. The probabilities of a received symbol being correct, erroneous, and erased for finite M are first derived (denoted $P_c^{(M)}(\gamma)$, $P_e^{(M)}(\gamma)$, and $P_{er}^{(M)}(\gamma)$, respectively) before obtaining the limiting expressions. The asymptotic results are summarized in the following theorem which encompasses all diversity combining schemes considered, including MRC and EGC, as well as coherent and noncoherent implementations of H-SC and SC.

Theorem:

$$\lim_{M\to\infty} P_c^{(M)}(\gamma) = 1 - F_A\left(\frac{\gamma^2 L N_0 \ln 2}{r E_b}\right), \ \gamma \geq 1 \quad (7)$$

$$\lim_{M\to\infty} P_e^{(M)}(\gamma) = \begin{cases} F_A\left(\frac{LN_0 \ln 2}{rE_b}\right), & \gamma = 1 \\ 0, & \gamma > 1 \end{cases} \quad (8)$$

$$\lim_{M\to\infty} P_{er}^{(M)}(\gamma) = \begin{cases} 0, & \gamma = 1 \\ F_A\left(\frac{\gamma^2 LN_0 \ln 2}{rE_b}\right), & \gamma > 1. \end{cases} \quad (9)$$

In the above expressions, $F_A(\cdot)$ denotes the cumulative distribution function (cdf) of a random variable A, which is a function of the channel attenuations and diversity order. Because A is specified by the operation of each diversity combiner, its definition will be provided in the subsections which follow. As might be expected, the asymptotic probabilities in (7)–(9) depend upon parameters such as the code rate r, the RTT threshold γ, and the ratio of the energy per information bit per diversity channel to the noise density E_b/LN_0.

In the ensuing subsections, we will describe each of the combining schemes and verify the theorem for each case.

A. MRC

The optimal linear combining technique is MRC, which yields the maximum instantaneous SNR, and hence, minimizes the probability of symbol error [17]. The performance and optimality of MRC depend upon the receiver's knowledge of the complex channel gains. Assuming that the receiver can perfectly estimate the channel attenuations $\{\alpha_l\}$ and phase shifts $\{\phi_l\}$, MRC forms the decision statistics

$$Z_i = \mathrm{Re}\left[\sum_{l=0}^{L-1} \alpha_l e^{j\phi_l} Z_{i,l}\right], \qquad i = 1,2,\ldots,M \quad (10)$$

where $Z_{i,l}$ is the correlator output corresponding to the ith symbol on the lth channel (3), (4). In effect, the optimal combiner removes the channel-induced phase shifts and appropriately weights the contributions from all L channels (according to the received signal strength) prior to summing [17].

The substitution of (4) into (10) then leads to

$$Z_1 = 2E\sum_{l=0}^{L-1}\alpha_l^2 + \sum_{l=0}^{L-1}\alpha_l \mathrm{Re}\left[e^{j\phi_l}\eta_{1,l}\right] \quad (11)$$

$$Z_i = \sum_{l=0}^{L-1}\alpha_l \mathrm{Re}\left[e^{j\phi_l}\eta_{i,l}\right], \qquad i = 2,3,\ldots,M. \quad (12)$$

By conditioning upon $\{\phi_l\}$ and $\{\alpha_l\}$, and employing the definition of $\{\eta_{i,l}\}$, it can be shown that these decision statistics are independent, Gaussian random variables with $Z_1 \sim \mathcal{N}(2Ea, \sigma^2)$ and $\{Z_i \sim \mathcal{N}(0,\sigma^2), \forall i \neq 1\}$, where $a = \sum_{l=0}^{L-1}\alpha_l^2$ and $\sigma^2 = 2EN_0 a$.

From (6) and the fact that the $\{Z_i\}$ are conditionally independent, the probability of a correct symbol is

$$P_c^{(M)}(\gamma) = P\left[Z_1 \geq \gamma \max_{i\neq 1}\{Z_i\}\middle| 1\right] \quad (13)$$

$$= \int_0^\infty \left[\int_{-\infty}^{\infty} F_{Z_2}^{M-1}\left(\frac{z}{\gamma}\middle| a\right) f_{Z_1}(z|a)dz\right] f_A(a)da \quad (14)$$

where the inner integral in (14) denotes the conditional probability $P_{c|A}^{(M)}(\gamma, a)$ given the event $A = a = \sum_{l=0}^{L-1}\alpha_l^2$. We assume throughout this section that the probability density function (pdf) $f_A(a)$ is bounded above for all a.

Prior to deriving the asymptotic expression for $P_c^{(M)}(\gamma)$, the energy per symbol per diversity channel, E, is expressed in terms of the energy per information bit, E_b, as follows:

$$E = \frac{E_b}{L} r \log_2 M. \quad (15)$$

The first step of the asymptotic analysis involves interchanging the limit and integration by applying Lebesgue's dominated convergence theorem [18]

$$\lim_{M\to\infty} P_c^{(M)}(\gamma) = \lim_{M\to\infty}\int_0^\infty P_{c|A}^{(M)}(\gamma,a) f_A(a)da \quad (16)$$

$$= \int_0^\infty \left[\lim_{M\to\infty} P_{c|A}^{(M)}(\gamma,a)\right] f_A(a)da. \quad (17)$$

Substituting the related distributions of the decision statistics into (14) and employing (15), we obtain (18) as shown at the bottom of the next page, where $\beta_a = arE_b/(LN_0 \ln 2)$ and $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-t^2/2}\,dt$.

As noted in [2], the derivation of (18) mirrors that of the asymptotic probability of a correct symbol for coherent M-ary orthogonal modulation in AWGN [19], [20]. It follows from [2] and [20] that

$$\lim_{M\to\infty} P_{c|A}^{(M)}(\gamma,a) = \begin{cases} 0, & \frac{\beta_a}{\gamma^2} < 1 \\ 1, & \frac{\beta_a}{\gamma^2} > 1 \end{cases} \quad (19)$$

where the case in which $(\beta_a/\gamma^2 = 1)$ is omitted because $P[\beta_a/\gamma^2 = 1] = 0$. Thus, the asymptotic probability of a correct symbol (17) simplifies to the expression provided earlier in (7)

$$\lim_{M\to\infty} P_c^{(M)}(\gamma) = \int_0^\infty \mathrm{I}\left(a > \frac{\gamma^2 L N_0 \ln 2}{r E_b}\right) f_A(a)da \tag{20}$$

$$= 1 - F_A\left(\frac{\gamma^2 L N_0 \ln 2}{r E_b}\right) \tag{21}$$

where $\mathrm{I}(\cdot)$ in (20) represents the indicator function.[1]

We next seek the asymptotic probability of a symbol error. As before, the expression for the symbol error probability for finite M immediately follows from (6) and the decision statistics $\{Z_i\}$ being conditionally independent, as shown in (22) at the bottom of the page, where the inner integral is simply the conditional probability $P_{e|A}^{(M)}(\gamma, a)$. In order to obtain the desired asymptotic result, we first observe the relation $f_{Z_2}(\gamma z|a) = f_{Z_2}(z|a)e^{-(\gamma^2-1)z^2/2\sigma^2}$ from the specifications accompanying (12). The conditional probability of symbol error (22) can thus be rewritten as shown in (23)–(25) at the bottom of the page, where the last step involves integration by parts.

[1] $\mathrm{I}(x) = 1$, if x; $\mathrm{I}(x) = 0$, otherwise.

For the case of $\gamma = 1$, we obtain from (25) the following simplification:

$$P_{e|A}^{(M)}(1, a) = 1 - \int_{-\infty}^{\infty} F_{Z_2}^{M-1}(z|a) f_{Z_1}(z|a)dz \tag{26}$$

$$= 1 - P_{c|A}^{(M)}(1, a). \tag{27}$$

Substituting (27) into (22) and taking the limit, we obtain the expected result

$$\lim_{M\to\infty} P_e^{(M)}(1) = 1 - \lim_{M\to\infty} P_c^{(M)}(1) \tag{28}$$

$$= F_A\left(\frac{L N_0 \ln 2}{r E_b}\right). \tag{29}$$

If $\gamma > 1$, the first term in (25) equals zero and the conditional symbol error probability becomes (30), as shown at the bottom of the page. By invoking the dominated convergence theorem, the derivation of the asymptotic symbol error probability devolves, in effect, into a derivation of the asymptotic conditional symbol error probability as in (17). Hence, only the limit of (30) is required to complete the derivation.

$$\lim_{M\to\infty} P_{c|A}^{(M)}(\gamma, a) = \lim_{M\to\infty} \int_{-\infty}^{\infty} \left(1 - Q\left(\frac{z}{\gamma}\right)\right)^{M-1} \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}\left(z - \sqrt{2\beta_a \ln M}\right)^2\right) dz \tag{18}$$

$$\begin{aligned} P_e^{(M)}(\gamma) &= P\left[\bigcup_{i=2}^{M}\left\{Z_i \geq \gamma \max_{j\neq i}\{Z_j\}\right\}\Big|1\right] \\ &= \int_0^\infty \left[\int_{-\infty}^{\infty} (M-1) F_{Z_1}\left(\frac{z}{\gamma}|a\right) F_{Z_2}^{M-2}\left(\frac{z}{\gamma}|a\right) \cdot f_{Z_2}(z|a)dz\right] f_A(a)da \end{aligned} \tag{22}$$

$$P_{e|A}^{(M)}(\gamma, a) = \int_{-\infty}^{\infty} \gamma F_{Z_1}(z|a)(M-1) F_{Z_2}^{M-2}(z|a) f_{Z_2}(\gamma z|a)dz \tag{23}$$

$$= \int_{-\infty}^{\infty} \gamma F_{Z_1}(z|a) e^{-(\gamma^2-1)z^2/2\sigma^2} \frac{d}{dz}\left\{F_{Z_2}^{M-1}(z|a)\right\} dz \tag{24}$$

$$\begin{aligned} &= \gamma F_{Z_1}(z|a) e^{-(\gamma^2-1)z^2/2\sigma^2} F_{Z_2}^{M-1}(z|a)\Big|_{-\infty}^{\infty} \\ &\quad - \int_{-\infty}^{\infty} \gamma F_{Z_2}^{M-1}(z|a) e^{-(\gamma^2-1)z^2/2\sigma^2}\left[f_{Z_1}(z|a) - \frac{(\gamma^2-1)z}{\sigma^2} F_{Z_1}(z|a)\right] dz \end{aligned} \tag{25}$$

$$P_{e|A}^{(M)}(\gamma, a) = \int_{-\infty}^{\infty} \gamma F_{Z_2}^{M-1}(z|a) e^{-(\gamma^2-1)z^2/2\sigma^2} \cdot \left[\frac{(\gamma^2-1)z}{\sigma^2} F_{Z_1}(z|a) - f_{Z_1}(z|a)\right] dz. \tag{30}$$

It can be shown that as $M \to \infty$, $P_{e|A}^{(M)}(\gamma, a) \to 0$ (see Appendix A), and consequently, the asymptotic probability of symbol error for $\gamma > 1$ is

$$\lim_{M\to\infty} P_e^{(M)}(\gamma) = \int_0^\infty \left[\lim_{M\to\infty} P_{e|A}^{(M)}(\gamma, a)\right] f_A(a)da \quad (31)$$
$$=0. \quad (32)$$

Finally, the probability of symbol erasure for finite M is given by

$$P_{er}^{(M)}(\gamma) = 1 - P_c^{(M)}(\gamma) - P_e^{(M)}(\gamma). \quad (33)$$

Taking the limit of the above expression and applying previous results from (21), (29), and (32), we obtain the asymptotic probability of symbol erasure

$$\lim_{M\to\infty} P_{er}^{(M)}(\gamma) = \begin{cases} 0, & \gamma = 1 \\ F_A\left(\frac{\gamma^2 L N_0 \ln 2}{rE_b}\right), & \gamma > 1 \end{cases} \quad (34)$$

thereby completing the proof of the theorem for the case of MRC. As in [5], RTT converts all symbol errors into erasures for $\gamma > 1$ and large M, and the threshold γ which minimizes the symbol erasure probability is $\gamma \approx 1$.

B. EGC

The analog of MRC for noncoherent reception is square-law combining or postdetection EGC, which is optimal when the complex channel gains remain unknown and the channel attenuations $\{\alpha_l\}$ are independent, Rayleigh-distributed random variables [16]. The decision statistics for EGC are generated by summing the squared envelopes of the correlator outputs $\{Z_{i,l}\}$ (4)

$$Z_1 = \sum_{l=0}^{L-1} \left|2E\alpha_l e^{-j\phi_l} + \eta_{1,l}\right|^2 \quad (35)$$
$$Z_i = \sum_{l=0}^{L-1} \left|\eta_{i,l}\right|^2, \qquad i = 2, 3, \ldots, M \quad (36)$$

where $\{\eta_{i,l}\}$ is as previously defined in (4). Unlike the derivation for MRC, the asymptotic analysis of orthogonal signaling with noncoherent reception in AWGN cannot be easily generalized to cases of diversity reception with EGC.

In order to facilitate the asymptotic analysis of EGC, we modify the decision statistics by taking the square root of $\{Z_i\}$ and define the new statistics to be $\tilde{Z}_i = \sqrt{Z_i}$, $i = 1, 2, \ldots, M$. The resulting performance is unchanged because the square-root function monotonically increases with respect to its positive argument. The motivation for this modification is to transform Z_1 and $\{Z_i, \forall i \neq 1\}$ from noncentral chi-square and central chi-square random variables with $2L$ degrees of freedom (DOF) into generalized Ricean and Rayleigh random variables, respectively, when conditioned upon $\{\alpha_l\}$ and $\{\phi_l\}$. The asymptotic analysis of EGC will consequently resemble the problem formulation encountered in single channel ($L = 1$) noncoherent systems employing envelope detection [6], [9].

Thus, the set of decision statistics $\{\tilde{Z}_i\}$ are conditionally independent random variables with the following distributions:

$$f_{\tilde{Z}_1}(z|a) = \frac{z^L}{\tilde{\sigma}^2 s^{L-1}} \exp\left(-\frac{z^2 + s^2}{2\tilde{\sigma}^2}\right) I_{L-1}\left(\frac{zs}{\tilde{\sigma}^2}\right), \; z \geq 0 \quad (37)$$

$$f_{\tilde{Z}_i}(z|a) = \frac{z^{2L-1}}{2^{L-1}\tilde{\sigma}^{2L}(L-1)!} \exp\left(-\frac{z^2}{2\tilde{\sigma}^2}\right), \; z \geq 0, \; i \neq 1 \quad (38)$$

where $I_{L-1}(\cdot)$ is a $(L-1)$th-order modified Bessel function of the first kind, $\tilde{\sigma}^2 = 2EN_0$ is the conditional variance of the Gaussian components, and $s^2 = 4E^2 a$ is the noncentrality parameter.

The general approach taken in deriving the asymptotic probability of a correct symbol for MRC (15)–(17) can be applied here as well. From (14), (37), and (38), the conditional probability of a correct symbol for EGC is shown in (39) and (40) at the bottom of the page, where (40) results from the substitution $u = (z - s)/\tilde{\sigma}$.

Taking the limit and applying the dominated convergence theorem, we obtain the following simplifications, the details of which are specified in Appendix B.

$$\lim_{M\to\infty} P_{c|A}^{(M)}(\gamma, a)$$
$$= \int_{-\infty}^{\infty} \mathrm{I}\left(a > \frac{\gamma^2 L N_0 \ln 2}{rE_b}\right) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du \quad (41)$$
$$= \mathrm{I}\left(a > \frac{\gamma^2 L N_0 \ln 2}{rE_b}\right). \quad (42)$$

As can be seen, the generalized Ricean pdf in (39) reduces to the Gaussian pdf in the limit (41), which then becomes unity upon integration (42). The resulting expression for the asymptotic conditional probability $P_{c|A}(\gamma, a)$ is identical to that derived for MRC (19), (20), thereby proving that the probability of a correct symbol for EGC converges to that of MRC asymptotically. Because the previously outlined steps for deriving the remaining asymptotic probabilities can be applied to EGC as well, we can easily confirm that the asymptotic probabilities of symbol error and erasure are also identical for EGC and MRC.

$$P_{c|A}^{(M)}(\gamma, a) = \int_0^\infty \left[1 - e^{-z^2/2\gamma^2\tilde{\sigma}^2} \sum_{l=0}^{L-1} \frac{1}{l!} \left(\frac{z^2}{2\gamma^2\tilde{\sigma}^2}\right)^l\right]^{M-1} f_{\tilde{Z}_1}(z|a)\, dz \quad (39)$$

$$= \int_{-s/\tilde{\sigma}}^{\infty} \left[1 - e^{-(u+s/\tilde{\sigma})^2/2\gamma^2} \sum_{l=0}^{L-1} \frac{1}{l!} \left(\frac{(u+s/\tilde{\sigma})^2}{2\gamma^2}\right)^l\right]^{M-1} \cdot \frac{(u\tilde{\sigma} + s)^L}{\tilde{\sigma} s^{L-1}} e^{-1/2\{(u+s/\tilde{\sigma})^2 + (s/\tilde{\sigma})^2\}} I_{L-1}\left(\frac{(u+s/\tilde{\sigma})}{\tilde{\sigma}/s}\right) du \quad (40)$$

C. SC and H-SC

The least complex diversity combining technique is SC, which can be implemented with either coherent (C-SC) or noncoherent (NC-SC) reception and selects the correlator output with the largest SNR [15], [16]. Because the complexity of MRC increases proportionately with the number of combined diversity channels, emphasis has been recently placed on H-SC schemes which process L_c of the L available channels, where $L_c \in \{2, 3, \ldots, L-1\}$ [11]–[14]. In particular, these techniques combine the L_c correlator outputs with the largest instantaneous SNR employing either MRC (SC/MRC-L_c) or EGC (SC/EGC-L_c).

Because the MRC and EGC asymptotic derivations can accommodate any statistical characterization of the channel attenuations $\{\alpha_l\}$, the asymptotic analysis of the H-SC schemes follow the same framework. The only necessary modification is defining the attenuations to be ordered such that $\alpha_0 \geq \alpha_1 \geq \cdots \geq \alpha_{L-1}$ and replacing L with L_c in the equations. Hence, the resulting expressions for the asymptotic probabilities of SC/MRC-L_c and SC/EGC-L_c will be identical to those of MRC and EGC (21), (29), (32), (34), with the notable exception being the definition of the random variable A, which now represents the sum of L_c squared ordered statistics.

As illustrated in [12] and [14], the main impediment to analyzing hybrid schemes is averaging the conditional probability with respect to the pdf of $\sum_{l=0}^{L_c-1} \alpha_l^2$, which, while straightforward, becomes cumbersome for $L_c \geq 3$.

Although SC has been ignored in previous asymptotic work, we note that the asymptotic performance of C-SC is identical to that of NC-SC by invoking the classical results in [2] and [20] and applying the previous derivations for MRC and EGC.

IV. Asymptotic Results With Nakagami-m Fading

The asymptotic analysis presented thus far does not depend upon a particular statistical characterization of the channel attenuations. In order to make a performance comparison among the various diversity combining schemes, we consider the case in which $\{\alpha_l\}$ are independent, identically distributed (i.i.d.) Nakagami-m random variables [4], [6], [17], [21].

The pdf corresponding to the sum of L squared, i.i.d. Nakagami-m random variables is given by

$$f_A(a) = \left(\frac{m}{\Omega}\right)^{mL} \frac{a^{mL-1}}{\Gamma(mL)} \exp\left(-\frac{ma}{\Omega}\right), \qquad a \geq 0 \quad (43)$$

where $\Omega = E[\alpha_l^2], \forall l$, and m is the Nakagami parameter which is defined as $m = \Omega^2 / E[(\alpha_l^2 - \Omega)^2] \geq 1/2, \forall l$ [22]. The Nakagami parameter inversely reflects the severity of the fading with $m = 1$ corresponding to Rayleigh fading, and $m \to \infty$, which is the nonfading case.

Prior to evaluating the asymptotic probabilities with respect to the specified distribution, we note that (43) must be modified appropriately for the SC and H-SC schemes (Section III-C). The derivation of the cdf of the sum of the two largest squared attenuations for SC/EGC-2 and SC/MRC-2 parallels the treatment in [14] and is, consequently, omitted.

Because the asymptotic probabilities of symbol error and erasure are either zero or functions of the asymptotic probability of a correct symbol, we present only the asymptotic probabilities of a correct symbol for the various diversity combining schemes.

MRC, EGC:

$$\lim_{M\to\infty} P_c^{(M)}(\gamma) = 1 - P\left(mL, \frac{\gamma^2 mLN_0 \ln 2}{r\overline{E}_b}\right). \quad (44)$$

C-SC, NC-SC:

$$\lim_{M\to\infty} P_c^{(M)}(\gamma) = 1 - \left[P\left(m, \frac{\gamma^2 mLN_0 \ln 2}{r\overline{E}_b}\right)\right]^L. \quad (45)$$

SC/MRC-2, SC/EGC-2:

$$\lim_{M\to\infty} P_c^{(M)}(\gamma) = 1 - \int_0^{\gamma^2 mLN_0 \ln 2/2r\overline{E}_b} g(y)dy. \quad (46)$$

In the above expressions, $\overline{E}_b = E_b\,\Omega$ represents the average, total received energy per information bit for all diversity combiners, and $P(\cdot,\cdot)$ is the normalized incomplete gamma function [4]

$$P(v, \varepsilon) = \frac{1}{\Gamma(v)} \int_0^{\varepsilon} t^{v-1} e^{-t} dt. \quad (47)$$

Furthermore, the integrand in (46) is given by the following:

$$g(y) = L(L-1)P(m,y)^{L-2} \frac{y^{m-1}e^{-y}}{\Gamma(m)} \cdot \left[P\left(m, \frac{\gamma^2 mLN_0 \ln 2}{r\overline{E}_b} - y\right) - P(m,y)\right]. \quad (48)$$

The asymptotic probability of a correct symbol for MRC and EGC (44) provides the most insight among the given expressions (44)–(46) because of its analytical simplicity. This particular asymptotic probability depends only on the product mL as opposed to m and L individually. As a result, we can easily compare the asymptotic performance of systems with different diversity orders and channel fading characteristics. The asymptotic behavior of a Nakagami-m fading channel with diversity order L, for instance, is equivalent to that of a Rayleigh fading channel with diversity order mL.

In addition, previous results regarding the performance of orthogonal signaling with a large alphabet size and large SNR [3], [4] can be generalized to the case of diversity reception with MRC or EGC in Nakagami-m fading. It can be shown that the asymptotic probability of symbol error varies as the inverse mLth power of $\overline{E}_b/N_0$ whenever $\overline{E}_b/N_0$ is large.

A. Asymptotic Performance for Large Diversity Order

The asymptotic probabilities in (44)–(46) indicate that M-FSK with finite diversity order in a Nakagami-m fading environment does not achieve the infinite bandwidth AWGN channel capacity. We next examine the asymptotic performance of the diversity combining schemes as the diversity order approaches infinity.

For the particular case of MRC and EGC, the asymptotic probability of a correct symbol as $L \to \infty$ for fixed m behaves in the same manner as if $m \to \infty$ for fixed L. As a result, the derivation below complements previous asymptotic work involving Nakagami-m fading with no diversity [6].

By rewriting (44), the asymptotic probability of a correct symbol can be expressed as

$$\lim_{M\to\infty} P_c^{(M)}(\gamma) = 1-\int_0^\infty \mathrm{I}\left(t<\frac{\gamma^2 mLN_0\ln 2}{r\overline{E}_b}\right)\frac{t^{mL-1}e^{-t}}{\Gamma(mL)}dt \quad (49)$$

$$\simeq 1-\int_0^\infty \mathrm{I}\left(u<\frac{\gamma^2 N_0\ln 2}{r\overline{E}_b}\right)\sqrt{\frac{mL}{2\pi}}e^{mL(1+\ln u-u)}\cdot e^{-\ln u}du \quad (50)$$

where in (50) we have made the substitution $u=t/mL$ and the assumption that L is large, thereby permitting Stirling's approximation [23].

In order to obtain the limiting behavior of (50) as $L\to\infty$, we employ informal arguments as in [6] and [24] to obtain the following simplifications:

$$\lim_{L\to\infty}\left\{\lim_{M\to\infty} P_c^{(M)}(\gamma)\right\} = 1-\int_0^\infty \mathrm{I}\left(u<\frac{\gamma^2 N_0\ln 2}{r\overline{E}_b}\right)\delta(u-1)du \quad (51)$$

$$= \mathrm{I}\left(\frac{\overline{E}_b}{N_0}>\frac{\gamma^2\ln 2}{r}\right). \quad (52)$$

Thus, MRC and EGC asymptotically approach the AWGN channel capacity (when $r=1$ and hence, $\gamma=1$) in Nakagami-m fading as the diversity order goes to infinity. This result corroborates the conclusions drawn in [24] which examined the performance of MRC in Rayleigh fading from an alternative framework, as well as those in [8].

Next, we determine whether the SC schemes, C-SC and NC-SC, exhibit the same behavior as MRC and EGC when the number of processed diversity channels grows large. Taking the limit of the asymptotic probability of a correct symbol in (45), we obtain the following result:

$$\lim_{L\to\infty}\left\{\lim_{M\to\infty} P_c^{(M)}(\gamma)\right\} = 1-\lim_{L\to\infty}\left[P\left(m,\frac{\gamma^2 mLN_0\ln 2}{r\overline{E}_b}\right)\right]^L \quad (53)$$

$$=0 \quad (54)$$

which contrasts sharply with the asymptotic performance of MRC and EGC for large diversity orders (see Appendix C for derivation). In order to determine which L minimizes the required SNR at a target BER, the asymptotic expressions for SC must be evaluated numerically.

V. Asymptotic Performance With RS Coding

As discussed in the previous section, the various diversity combining schemes do not achieve capacity in the absence of $m\to\infty$ or $L\to\infty$. These schemes do achieve arbitrarily reliable communication, however, when RS coding is incorporated and additional performance gains are possible by employing EED (EED, RTT: $\gamma>1$).

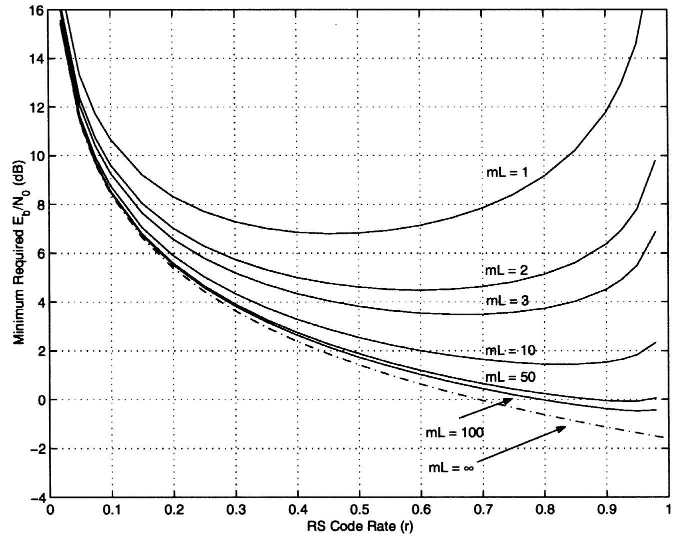


Fig. 2. Minimum required $\overline{E}_b/N_0$ for arbitrarily reliable communication for MRC and EGC with HDD in Nakagami-m fading ($mL = 1,2,3,10,50,100,\infty$).

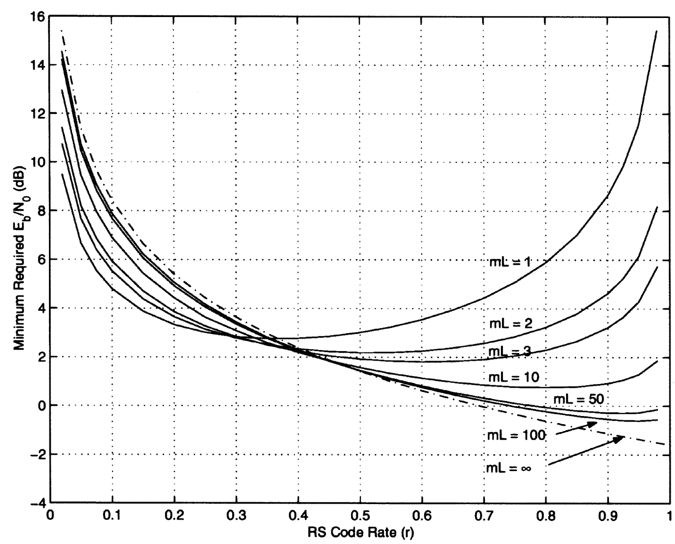


Fig. 3. Minimum required $\overline{E}_b/N_0$ for arbitrarily reliable communication for MRC and EGC with EED in Nakagami-m fading ($mL = 1,2,3,10,50,100,\infty$).

The asymptotic performance of M-FSK in multichannels with diversity reception, RS coding, and HDD or EED can be derived from the asymptotic probabilities specified earlier. It is well known that an (n,k) RS code can correct any collection of t errors and e erasures as long as the relation $2t+e\le n-k$ is satisfied [17]. Furthermore, the assumption of ideal interleaving ensures that the probabilities of error and erasure are independent from received symbol to symbol. Because the block length tends toward infinity as $M\to\infty$, the probability of a codeword error approaches zero when the following condition holds [5], [9]:

$$r<1-\lim_{M\to\infty}P_{er}^{(M)}(\gamma)-2\cdot\lim_{M\to\infty}P_e^{(M)}(\gamma), \qquad \gamma\ge 1. \quad (55)$$

This constraint imposes the maximum possible code rate or equivalently, the minimum $\overline{E}_b/N_0$, necessary to achieve arbitrarily reliable communication. By substituting in the asymp-

TABLE I
MINIMUM REQUIRED SNR (IN DECIBELS) FOR ARBITRARILY RELIABLE COMMUNICATION AT OPTIMAL RS CODE RATE WITH HDD IN NAKAGAMI-m FADING

Diversity Combining Scheme	Nakagami parameter (m), Diversity order (L)											
	0.5				1.0				2.0			
	2	3	4	10	2	3	4	10	2	3	4	10
C-SC, NC-SC	7.82	7.13	6.90	7.26	5.91	5.92	6.10	7.39	4.71	5.19	5.66	7.63
SC/MRC-2, SC/EGC-2	N/A	5.64	5.16	4.98	N/A	4.04	4.00	4.89	N/A	2.99	3.29	4.98
MRC, EGC	6.88	5.33	4.48	2.47	4.48	3.48	2.89	1.43	2.89	2.17	1.73	0.65

TABLE II
MINIMUM REQUIRED SNR (IN DECIBELS) FOR ARBITRARILY RELIABLE COMMUNICATION AT OPTIMAL RS CODE RATE WITH EED IN NAKAGAMI-m FADING

Diversity Combining Scheme	Nakagami parameter (m), Diversity order (L)											
	0.5				1.0				2.0			
	2	3	4	10	2	3	4	10	2	3	4	10
C-SC, NC-SC	3.71	4.11	4.47	5.95	3.54	4.15	4.64	6.56	3.31	4.12	4.76	7.09
SC/MRC-2, SC/EGC-2	N/A	2.73	2.84	3.76	N/A	2.36	2.63	4.13	N/A	1.99	2.46	4.49
MRC, EGC	2.76	2.43	2.18	1.35	2.18	1.81	1.55	0.76	1.55	1.19	0.94	0.24

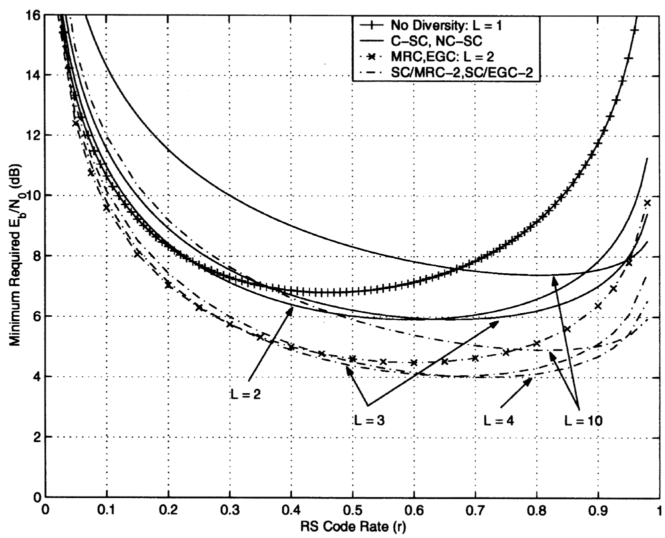


Fig. 4. Minimum required $\overline{E}_b/N_0$ for arbitrarily reliable communication for C-SC and NC-SC as well as SC-MRC-2 and SC-EGC-2 with HDD in Rayleigh fading ($m = 1$).

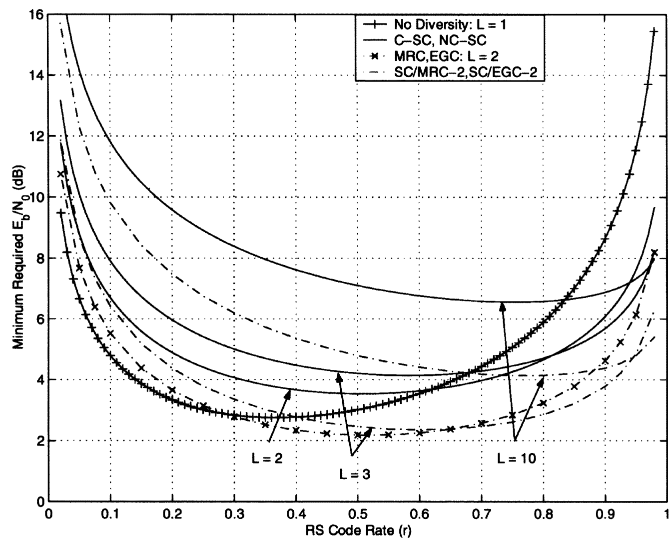


Fig. 5. Minimum required $\overline{E}_b/N_0$ for arbitrarily reliable communication for C-SC and NC-SC as well as SC-MRC-2 and SC-EGC-2 with EED in Rayleigh fading ($m = 1$).

totic symbol error and erasure probabilities (29), (32) corresponding to (44)–(46) into the above condition, the minimum required SNR for error-free communication at a fixed code rate can be evaluated numerically.

Figs. 2 and 3 depict the performance of both MRC and EGC in Nakagami-m fading with HDD and EED, respectively. As the effective diversity order mL increases, we note that the minimum required SNR decreases, the optimal code rate tends to $r = 1$, and the performance of HDD approaches that of EED. The performance degradation at low code rates for EED with improving diversity indicates that the system suffers from overcoding. If we fix the Nakagami parameter and examine the performance for various L, it is clear that increasing the number of processed diversity channels is more effective for severely faded channels (small m).

A performance comparison of MRC and EGC with H-SC ($L_c = 2$) and SC is presented in Tables I and II, which correspond to the cases of HDD and EED, respectively. These tables contain the minimum required SNR for error-free communication at the optimal code rate for a fixed m and L. Upon inspection, the gain of MRC and EGC over H-SC, and in turn, the gain of H-SC over SC, increases as the fading becomes less severe ($m \to \infty$). This trend can be attributed to the fact that combining additional diversity channels characterized by a large Nakagami parameter significantly enhances the relative performance, whereas combining severely faded channels provides only minimal benefits. We note that the performance differential between the reduced complexity schemes and MRC or EGC is the smallest for $m = 0.5$, among the three values of m considered. Also, even with only $L_c = 2$, H-SC is highly effective in bridging the performance gap between SC and MRC or EGC.

When comparing the performance of HDD to that of EED for each of the various schemes, the tables indicate that the gain provided by EED in reducing the minimum required SNR at a specific m and L is approximately the same for all the diversity combiners. The use of side information, however, particularly benefits the reduced complexity schemes when comparing the optimal performance of H-SC and SC over a range of diversity orders with that of MRC or EGC at a fixed m. EED also reduces the optimal diversity order for these schemes, with H-SC achieving the overall minimum required SNR at $L=3$ and SC at $L=2$. For large diversity orders, these numerical results for both H-SC and SC confirm the conclusions reached in the previous section regarding the declining performance of SC for large L.

In order to study the asymptotic behavior of these reduced-complexity schemes in greater detail, their performance in Rayleigh fading ($m=1$) with HDD and EED is shown in Figs. 4 and 5, respectively. (Because the asymptotic probabilities become greatly simplified for this particular distribution, they are provided in Appendix D.) From these plots as well as the tables, it is evident that the optimal diversity order for H-SC and SC increases with the degree of fading severity when HDD is employed. In addition, SC with EED performs worse than a system with no diversity ($L=1$), while H-SC with EED yields a performance loss when compared to MRC or EGC operating with only $L=2$.

The plots also reveal that the performance of H-SC and SC improves with increasing diversity order only at code rates near unity. Coupling this with the interpretation of diversity order as the inverse of a repetition code rate, the general behavior of these schemes for large diversity orders suggests that the performance degradation occurs because of overcoding.

VI. Conclusion

This paper extends and unifies previous asymptotic analysis of M-ary orthogonal modulation in frequency-nonselective, slowly fading channels by incorporating RS coding, RTT, and diversity reception. The asymptotic probabilities of a symbol being correct, erroneous, and erased are derived for various diversity combiners and arbitrary diversity order as well as a statistical fading model. Employing these derivations, the minimum required SNR for arbitrarily reliable communication with RS coding and either HDD or EED is determined assuming Nakagami-m fading for a broad range of parameters specifying the code rate, fading severity, and diversity order.

It is shown that generating side information through RTT and employing EED significantly reduces the minimum required SNR for arbitrarily reliable communication for small diversity orders and severe fading conditions. The gain provided by EED over HDD diminishes as the diversity order increases for all diversity combining schemes considered. Although increasing the number of processed diversity channels is an effective technique for MRC or EGC in conjunction with HDD, the performance of the reduced-complexity schemes generally declines, except when severe fading exists. For low diversity orders, H-SC also provides an adequate compromise between performance and complexity relative to the extreme approaches of diversity combining, namely, SC and MRC or EGC.

For a restricted set of assumptions, this work can be applied to a direct-sequence code-division multiple-access (DS-CDMA) system with M-ary orthogonal modulation and random spreading codes operating in a frequency-selective, slowly fading channel. In particular, it must be assumed that the multipath channel contains only a fixed number of paths, independent of M, and that the signal-dependent self-interference present at the correlator outputs can be neglected.

The asymptotic analysis of a frequency-hopped spread spectrum (FH-SS) system with M-FSK and diversity reception in multichannels with partial-band jamming [9], [25] represents a possible area of future work.

Appendix A
MRC: Asymptotic Conditional Probability of a Symbol Error

We recall from (30) that the conditional symbol error probability for $\gamma > 1$ is

$$P_{e|A}^{(M)}(\gamma,a)=\int_{-\infty}^{\infty}\gamma F_{Z_2}^{M-1}(z|a)e^{-(\gamma^2-1)z^2/2\sigma^2} \cdot\left[\frac{(\gamma^2-1)z}{\sigma^2}F_{Z_1}(z|a)-f_{Z_1}(z|a)\right]dz. \tag{56}$$

For all $a>0$, for all $M>2$, and for all z, there exists a real number B such that

$$e^{-(\gamma^2-1)z^2/2\sigma^2}\left|\frac{(\gamma^2-1)z}{\sigma^2}F_{Z_1}(z|a)-f_{Z_1}(z|a)\right|\leq B \tag{57}$$

and hence

$$P_{e|A}^{(M)}(\gamma,a)\leq\gamma B\int_{-\infty}^{\infty}F_{Z_2}^{M-1}(z|a)\,dz. \tag{58}$$

We then obtain the desired result by taking the limit, applying the dominated convergence theorem, and observing that $F_{Z_2}(z|a)<1$ for all z as follows:

$$\lim_{M\to\infty}P_{e|A}^{(M)}(\gamma,a)\leq\gamma B\int_{-\infty}^{\infty}\lim_{M\to\infty}F_{Z_2}^{M-1}(z|a)dz \tag{59}$$
$$=0. \tag{60}$$

Appendix B
EGC: Asymptotic Conditional Probability of Correct Symbol

For notational simplicity, we further specify the integrand in (40) as follows:

$$P_{c|A}^{(M)}(\gamma,a)=\int_{-s/\tilde{\sigma}}^{\infty}C_1^{(M)}(u)\cdot C_2^{(M)}(u)du \tag{61}$$

where

$$C_1^{(M)}(u)=\left[1-e^{-(u+s/\tilde{\sigma})^2/2\gamma^2}\sum_{l=0}^{L-1}\frac{1}{l!}\left(\frac{\left(u+\frac{s}{\tilde{\sigma}}\right)^2}{2\gamma^2}\right)^l\right]^{M-1}$$
$$C_2^{(M)}(u)=\frac{(u\tilde{\sigma}+s)^L}{\tilde{\sigma}s^{L-1}}e^{-1/2\left\{(u+s/\tilde{\sigma})^2+(s/\tilde{\sigma})^2\right\}}I_{L-1}\left(\frac{u+\frac{s}{\tilde{\sigma}}}{\frac{\tilde{\sigma}}{s}}\right).$$

Taking the limit of the conditional probability of a correct symbol (61), the following expression results from applying the dominated convergence theorem and noting that $\lim_{M\to\infty}-s/\tilde{\sigma}=-\infty$:

$$\lim_{M\to\infty}P_{c|A}^{(M)}(\gamma,a)=\int_{-\infty}^{\infty}\lim_{M\to\infty}C_1^{(M)}(u)\cdot\lim_{M\to\infty}C_2^{(M)}(u)du. \tag{62}$$

Prior to evaluating the individual limits, the following simplifications are first made for $C_1^{(M)}(u)$:

$$C_1^{(M)}(u) = \left[1 - \exp\left(-\frac{(u+\sqrt{2\beta_a \ln M})^2}{2\gamma^2}\right) \cdot \sum_{l=0}^{L-1} \frac{1}{l!}\left(\frac{(u+\sqrt{2\beta_a \ln M})^2}{2\gamma^2}\right)^l\right]^{M-1} \tag{63}$$

$$= \left[1 - e^{-\chi_M^2} \sum_{l=0}^{L-1} \frac{\chi_M^{2l}}{l!}\right]^{\exp\left(\gamma^2/\beta_a(\chi_M - u/\sqrt{2\gamma^2})^2\right)-1} \tag{64}$$

where β_a is as previously defined (18), and we have substituted $\chi_M = (u+\sqrt{2\beta_a \ln M})/\sqrt{2\gamma^2}$ in (64).

We note that if $\lim_{x\to b} g(x)/h(x) = 1$, then $\lim_{x\to b} f(x)\, g(x) = \lim_{x\to b} f(x)h(x)$, where b is an extended real number. Applying this result and observing the relation $\lim_{x\to 0} \ln(1+x)/x = 1$, the limit of the logarithm of $C_1^{(M)}(u)$ is obtained for any given u

$$\begin{aligned}
&\lim_{M\to\infty} \ln C_1^{(M)}(u)\\
&= \lim_{M\to\infty} \left(e^{\gamma^2/\beta_a(\chi_M - u/\sqrt{2\gamma^2})^2} - 1\right) \ln\left(1 - e^{-\chi_M^2}\sum_{l=0}^{L-1}\frac{\chi_M^{2l}}{l!}\right)\\
&= \lim_{M\to\infty} \left(e^{\gamma^2/\beta_a(\chi_M - u/\sqrt{2\gamma^2})^2} - 1\right) \cdot \left(-e^{-\chi_M^2}\sum_{l=0}^{L-1}\frac{\chi_M^{2l}}{l!}\right)\\
&= \begin{cases} -\infty, & \frac{\beta_a}{\gamma^2} < 1\\ 0, & \frac{\beta_a}{\gamma^2} > 1. \end{cases}
\end{aligned} \tag{65}$$

This result corresponds to the first term of the integrand provided in (41)

$$\lim_{M\to\infty} C_1^{(M)}(u) = \mathrm{I}\left(a > \frac{\gamma^2 L N_0 \ln 2}{r E_b}\right). \tag{66}$$

To complete the derivation for the asymptotic conditional probability of a correct symbol, the limit of $C_2^{(M)}(u)$ in (62) must be evaluated. To this end, we employ the asymptotic property of the nth-order modified Bessel function of the first kind [26]

$$\lim_{x\to\infty} \frac{I_n(x)}{\frac{e^x}{\sqrt{2\pi x}}} = 1. \tag{67}$$

Because the argument $(u + s/\tilde{\sigma})s/\tilde{\sigma} \to \infty$ as $M \to \infty$, we obtain the following for any given u:

$$\begin{aligned}
&\lim_{M\to\infty} C_2^{(M)}(u)\\
&= \lim_{M\to\infty} \frac{(u\tilde{\sigma}+s)^L}{\tilde{\sigma}s^{L-1}} e^{-1/2\left\{(u+s/\tilde{\sigma})^2 + (s/\tilde{\sigma})^2\right\}} I_{L-1}\left(\frac{u+\frac{s}{\tilde{\sigma}}}{\frac{\tilde{\sigma}}{s}}\right)\\
&= \lim_{M\to\infty} \frac{(u\tilde{\sigma}+s)^L e^{-1/2\left\{(u+s/\tilde{\sigma})^2 + (s/\tilde{\sigma})^2\right\}} e^{(u+s/\tilde{\sigma})s/\tilde{\sigma}}}{\tilde{\sigma}s^{L-1}\sqrt{2\pi(u+\frac{s}{\tilde{\sigma}})\frac{s}{\tilde{\sigma}}}}\\
&= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) \cdot \lim_{M\to\infty}\left(\frac{u}{\sqrt{2\beta_a \ln M}} + 1\right)^{L-1/2}\\
&= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right).
\end{aligned} \tag{68}$$

Hence, the derivation of the asymptotic conditional probability of a correct symbol is complete.

APPENDIX C
SC: LARGE DIVERSITY ORDER CASE

For notational simplicity, we redefine (53) to be

$$\lim_{L\to\infty}\left\{\lim_{M\to\infty} P_c^{(M)}(\gamma)\right\} = 1 - \lim_{L\to\infty} D^{(L)} \tag{69}$$

where

$$D^{(L)} = \left[P\left(m, \frac{\gamma^2 m L N_0 \ln 2}{r\overline{E}_b}\right)\right]^L \tag{70}$$

$$= \left(1 - \int_{L\varepsilon}^{\infty} \frac{t^{m-1}e^{-t}}{\Gamma(m)}dt\right)^L \tag{71}$$

and $\varepsilon = \gamma^2 m N_0 \ln 2/r\overline{E}_b$. Taking the limit of the logarithm of $D^{(L)}$ and applying the previous result of $\lim_{x\to 0}\ln(1+x)/x = 1$, we obtain

$$\begin{aligned}
&\lim_{L\to\infty} \ln D^{(L)}\\
&\quad= \lim_{L\to\infty} L \ln\left(1 - \int_{L\varepsilon}^{\infty}\frac{t^{m-1}e^{-t}}{\Gamma(m)}dt\right) &(72)\\
&\quad= \lim_{L\to\infty} L\left(-\int_{L\varepsilon}^{\infty}\frac{t^{m-1}e^{-t}}{\Gamma(m)}dt\right) &(73)\\
&\quad= -\lim_{L\to\infty}\int_0^{\infty} L^{m+1}e^{-L\varepsilon(u+1)}\frac{(u+1)^{m-1}\varepsilon^m}{\Gamma(m)}du &(74)\\
&\quad= 0. &(75)
\end{aligned}$$

In (74), the substitution of $u = t/(L\varepsilon) - 1$ is made, and the last step (75) results from applying the dominated convergence theorem. Employing (75) in (69), we verify the validity of (54).

APPENDIX D
ASYMPTOTIC PROBABILITIES FOR VARIOUS DIVERSITY COMBINERS IN RAYLEIGH FADING

Although Rayleigh fading ($m = 1$) represents a special case of the Nakagami-m fading model given in Section IV, the resulting expressions for the asymptotic probabilities of interest are greatly simplified. The asymptotic probability of a correct symbol for the different combining schemes are

MRC, EGC:

$$\lim_{M\to\infty} P_c^{(M)}(\gamma) = 2^{-\gamma^2 L N_0/(r\overline{E}_b)} \sum_{l=0}^{L-1} \frac{1}{l!}\left(\frac{\gamma^2 L N_0 \ln 2}{r\overline{E}_b}\right)^l.$$

C-SC, NC-SC:

$$\lim_{M\to\infty} P_c^{(M)}(\gamma) = 1 - \left(1 - 2^{-\gamma^2 L N_0/(r\overline{E}_b)}\right)^L.$$

SC/MRC-2, SC/EGC-2:

$$\lim_{M\to\infty} P_c^{(M)}(\gamma) = 1 - L(L-1)\left[\frac{1}{2}\left(1 - 2^{-\gamma^2 L N_0/(r\overline{E}_b)} \cdot \sum_{l=0}^{1} \frac{1}{l!}\left(\frac{\gamma^2 L N_0 \ln 2}{r\overline{E}_b}\right)^l\right) + \sum_{l=1}^{L-2}\binom{L-2}{l}(-1)^l W(l)\right]$$

where

$$W(l) = \frac{1}{2+l} - \frac{2^{-\gamma^2 LN_0/(r\overline{E}_b)}}{l} + \frac{2^{1-\gamma^2 LN_0(2+l)/(2r\overline{E}_b)}}{l(2+l)}.$$

ACKNOWLEDGMENT

The authors would like to thank Dr. T. Ji and Dr. M. Chu for their suggestions and insights during discussions of this research.

REFERENCES

[1] G. L. Turin, "The asymptotic behavior of ideal M-ary systems," *Proc. IRE*, vol. 47, pp. 93–94, Jan. 1959.
[2] W. C. Lindsey, "Coding for specular and scatter channels," *IEEE Trans. Commun.*, vol. COM–13, pp. 237–238, June 1965.
[3] M. V. Hedge and W. E. Stark, "Asymptotic performance of M-ary orthogonal signals in worst case partial-band interference and Rayleigh fading," *IEEE Trans. Commun.*, vol. 36, pp. 989–992, Aug. 1988.
[4] P. J. Crepeau, "Asymptotic performance of M-ary orthogonal modulation in generalized fading channels," *IEEE Trans. Commun.*, vol. 36, pp. 1246–1248, Nov. 1989.
[5] S. Kim and W. E. Stark, "Performance limits of Reed–Solomon coded CDMA with orthogonal signaling in a Rayleigh-fading channel," *IEEE Trans. Commun.*, vol. 46, pp. 1125–1134, Sept. 1998.
[6] M. Chu, "Performance and modeling of communications in channels with memory," Ph.D. dissertation, Univ. of Michigan, Ann Arbor, 1999.
[7] W. C. Lindsey, "Error probabilities for Rician fading multichannel reception of binary and N-ary signals," *IEEE Trans. Inform. Theory*, vol. IT–10, pp. 339–350, Oct. 1964.
[8] J. N. Pierce, "Ultimate performance of M-ary transmissions on fading channels," *IEEE Trans. Inform. Theory*, vol. IT–12, pp. 2–5, Jan. 1966.
[9] M. Chu and W. E. Stark, "Asymptotic performance of a coded communication system with orthogonal signaling in partial band jamming," in *Proc. MILCOM*, 1998, pp. 1003–1007.
[10] A. J. Viterbi, "A robust ratio-threshold technique to mitigate tone and partial band jamming in coded MFSK systems," in *Proc. MILCOM*, 1982, pp. 22.4.1–22.4.5.
[11] M. Z. Win and J. H. Winters, "Analysis of hybrid selection/maximal-ratio combining in Rayleigh fading," in *Proc. ICC*, 1999, pp. 6–10.
[12] M. Alouini and M. K. Simon, "Performance of coherent receivers with hybrid SC/MRC over Nakagami-m fading channels," *IEEE Trans. Veh. Technol.*, vol. 48, pp. 1155–1164, July 1999.
[13] ——, "A MFG-based performance analysis of generalized selection combining over Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 48, pp. 401–415, Mar. 2000.
[14] T. Eng, N. Kong, and L. B. Milstein, "Comparison of diversity combining techniques for Rayleigh-fading channels," *IEEE Trans. Commun.*, vol. 44, pp. 1117–1129, Sept. 1996.
[15] G. Chyi, J. G. Proakis, and C. M. Keller, "On the symbol error probability of a maximum-selection diversity reception schemes over a Rayleigh fading channel," *IEEE Trans. Commun.*, vol. 37, pp. 79–83, Jan. 1989.
[16] J. N. Pierce, "Theoretical diversity improvement in frequency-shift keying," *Proc. IRE*, vol. 46, pp. 903–910, May 1958.
[17] J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 1995.
[18] W. Rudin, *Real and Complex Analysis*. New York: McGraw-Hill, 1987.
[19] A. J. Viterbi, "On coded phase-coherent communications," *IRE Trans. Space Electron. Telemetry*, vol. 7, pp. 3–12, Mar. 1961.
[20] ——, *Principles of Coherent Communication*. New York: McGraw-Hill, 1966.
[21] P. J. Crepeau, "Uncoded and coded performance of MFSK and DPSK in Nakagami fading channels," *IEEE Trans. Commun.*, vol. 40, pp. 487–493, Mar. 1992.
[22] V. A. Aalo, O. Ugweje, and R. Sudhakar, "Performance analysis of a DS/CDMA system with noncoherent M-ary orthogonal modulation in Nakagami fading," *IEEE Trans. Veh. Technol.*, vol. 47, pp. 20–29, Feb. 1998.
[23] S. Ross, *A First Course in Probability*. New York: Macmillan, 1994.
[24] W. C. Y. Lee, "Estimate of channel capacity in Rayleigh fading environment," *IEEE Trans. Veh. Technol.*, vol. 39, pp. 187–189, Aug. 1990.
[25] C. M. Keller and M. B. Pursley, "Diversity combining for channels with fading and partial-band interference," *J. Select. Areas Commun.*, vol. SAC–5, pp. 248–259, Feb. 1987.
[26] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions*. New York: Dover, 1972.
[27] J. D. Choi, D. Yoo, and W. E. Stark, "Asymptotic performance of orthogonal signaling with coding and diversity in Rayleigh fading," in *Proc. GLOBECOM*, 2000, pp. 917–921.

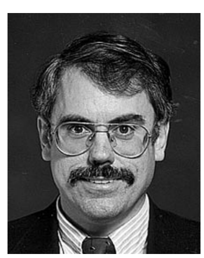
John D. Choi (S'95) received the B.S.E.E. degree from Duke University, Durham, NC, in 1997, and the M.S.E.E. degree from the University of Michigan, Ann Arbor, in 1998. He is currently pursuing the Ph.D. degree at the University of Michigan, Ann Arbor.

His main research interests are wireless communication systems and networking.

Mr. Choi is a member of Phi Beta Kappa, Tau Beta Pi, and Eta Kappa Nu.

Do-Sik Yoo (S'98) was born in Incheon, Korea, in 1968. He received the B.S. degree in electrical engineering and the M.S. degree in physics from Seoul National University, Seoul, Korea, in 1990 and 1994, respectively. He received the M.S. and Ph.D. degrees, both in electrical engineering systems, from the University of Michigan, Ann Arbor, in 1998 and 2002, respectively.

His current research interests are in wireless communication system design technologies including diversity combining, joint channel estimation and decoding, and multiuser and/or intersymbol interference treatment.



Wayne E. Stark (S'77–M'78–SM'94–F'98) received the B.S. (with highest honors), M.S., and Ph.D. degrees in electrical engineering from the University of Illinois, Urbana-Champaign, in 1978, 1979, and 1982, respectively.

Since September 1982, he has been a Faculty Member in the Department of Electrical Engineering and Computer Science at the University of Michigan, Ann Arbor, where he is currently a Professor. His research interests include the areas of coding and communication theory, especially for spread-spectrum and wireless communications networks.

Dr. Stark is a member of Eta Kappa Nu, Phi Kappa Phi, and Tau Beta Phi. He was involved in the planning and organization of the 1986 International Symposium on Information Theory, held in Ann Arbor, MI. From 1984 to 1989, he was Editor for Communication Theory of the IEEE TRANSACTIONS ON COMMUNICATIONS in the area of spread-spectrum communications. He was selected by the National Science Foundation as a 1985 Presidential Young Investigator. He is Principal Investigator of an Army Research Office Multidisciplinary University Research Initiative (MURI) Project on low-energy mobile communications.