Performance Limits of Reed–Solomon Coded CDMA with Orthogonal Signaling in a Rayleigh-Fading Channel

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***Abstract*—The asymptotic performance of Reed–Solomon (RS)-coded M-ary orthogonal signaling with ratio-threshold test (RTT) type demodulation in a Rayleigh-fading channel is considered. We show that the minimum $\bar{E}_b/N_0$ needed for error-free communication is $e \ln 2$ (2.75 dB) with RTT, and 4.79 (6.8 dB) with hard decisions. The optimum code rate that minimizes the required $\bar{E}_b/N_0$ is e^{-1} with RTT and 0.46 with hard decision, and the optimum ratio threshold approaches 1 for large M. Next, we investigate the fundamental limit in direct-sequence spread-spectrum multiple-access (DS/SSMA) system employing an M-ary orthogonal code of length $N = Mm$, which is obtained by spreading every row of an $M \times M$ Hadamard matrix with a user-specific random sequence of length N. We derive the minimum $\bar{E}_b/N_0$ for error-free communication as a function of the number of users, the optimum code rate that minimizes $\bar{E}_b/N_0$, and the maximum limit on the total information transmission rate. Then, we consider a multirate DS/SSMA system, where a population of users simultaneously transmit at different power levels a variety of traffic types of different information rates. We derive the minimum required $\bar{E}_b/N_0$ and the optimum code rate for each traffic type.**

***Index Terms*— DS/SSMA, multirate DS/SSMA, orthogonal modulation, ratio threshold test, Reed–Solomon code.**

I. INTRODUCTION

THE USE OF side information permits identification and erasure of symbols that have been impaired by channel effects such as fading, jamming, background noise, etc. Since more erasures can be corrected than errors, it is advantageous to determine the reliability of the received symbols and erase unreliable symbols prior to the decoding process. There are a number of methods for generating side information, and their performances have been analyzed in [1]–[3].

In this paper we consider the ratio threshold test (RTT) [3] as a method for generating information on the quality of the channel. RTT declares an erasure whenever the ratio of the largest to the second largest of the energy detector outputs does not exceed a fixed threshold greater than one. Hard decisions can be considered as a special case of RTT with ratio threshold set to one. Performance analysis of RTT technique has been made in various channels [3]–[6]. In this paper we investigate the asymptotic performance of Reed–Solomon (RS)-coded orthogonal signaling with RTT in Rayleigh-fading channel. We derive the asymptotic probabilities of symbol erasure and symbol error for large M, and the minimum $\bar{E}_b/N_0$ for error-free communication. We also derive the optimum code rate and the optimum ratio threshold that minimize the required $\bar{E}_b/N_0$, and examine the power gain that RTT provides over hard decisions.

Next, we consider an RS-coded direct-sequence spread-spectrum multiple-access (DS/SSMA) system, where a stream of $\log_2 M$ bits is encoded by an M-ary orthogonal code of length $N = Mm$, where m is a positive integer. The orthogonal code is obtained by spreading every row of an $M \times M$ Hadamard matrix with a user-specific random sequence of length N. In [7] a number of different coding schemes for DS/SSMA system are analyzed and compared. In this paper we derive the minimum $\bar{E}_b/N_0$ for error-free communication as a function of the number of users, the optimum code rate that minimizes the required $\bar{E}_b/N_0$, and the maximum total information transmission rate. Then, we extend to a multirate DS/SSMA system, where a population of users simultaneously transmit at different power levels a variety of traffic types such as interactive data, digital voice, and video. These traffic types have different information rates, and are encoded (RS) with different code rates and spread to a fixed bandwidth. We find the minimum $\bar{E}_b/N_0$ for error-free communication for each traffic type, and the optimum code rate that minimizes the required $\bar{E}_b/N_0$ in terms of power ratios, number of users, and spreading gains.

The rest of this paper is organized as follows. In Section II we derive the probabilities of symbol erasure and symbol error for orthogonal modulation with noncoherent detection and RTT. Then, we find their asymptotic probabilities for large M. In Section III we derive the minimum $\bar{E}_b/N_0$ for error-free communication, and the optimum code rate that minimizes the required $\bar{E}_b/N_0$. In Section IV we consider a DS/SSMA system employing an M-ary orthogonal code. We derive the minimum $\bar{E}_b/N_0$ for error-free communication as a function of the number of users, the optimum code rate that minimizes the required $\bar{E}_b/N_0$, and the maximum limit on the total

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information transmission rate. In Section V we investigate the fundamental limit in multirate DS/SSMA system. In Section VI we present our conclusions.

II. Probabilities of Symbol Erasure and Symbol Error

Let

$$S_i(t) = A\cos(2\pi f_i t), \qquad i = 1, 2, \cdots, M,\ 0 \le t \le T \quad (1)$$

be the transmitted signal, where A is the signal amplitude and f_i is the ith tone frequency. We assume that the tone frequencies $\{f_i\}$ are chosen such that the signals $\{S_i(t), i = 1, 2, \cdots, M\}$ are noncoherently orthogonal, i.e., $|f_{i+1} - f_i| = 1/T$. The received signal $r(t)$ at the input of the energy detector given that $S_1(t)$ is transmitted is

$$r(t) = gA\cos(2\pi f_1 t + \theta) + n(t), \qquad 0 \le t \le T \quad (2)$$

where g is Rayleigh distributed, θ is uniformly distributed over $[0, 2\pi]$, and $n(t)$ is the white Gaussian noise of one-sided spectral density N_0. We assume that g and θ are independent, and are also independent of $n(t)$. Then, the noncoherent[1] mth energy detector output E_m, given that $S_1(t)$ is transmitted, is given in (3) and (4), shown at the bottom of the page, where

$$n_{c,m} \triangleq \sqrt{\frac{2}{T}}\int_0^T n(t)\cos(2\pi f_m t)\,dt, \qquad m = 1, 2, \cdots, M \quad (5)$$

$$n_{s,m} \triangleq \sqrt{\frac{2}{T}}\int_0^T n(t)\sin(2\pi f_m t)\,dt, \qquad m = 1, 2, \cdots, M \quad (6)$$

are independent Gaussian random variables each with mean zero and variance $N_0/2$. Since g is Rayleigh distributed and θ is uniformly distributed over $[0, 2\pi]$, $g\cos\theta$ is a Gaussian random variable with mean zero and variance $E[g^2]/2$. Thus, the conditional probability density function $P_{E_m}(z \mid 1)$ of E_m, given that $S_1(t)$ is transmitted, is

$$P_{E_m}(z \mid 1) = \begin{cases} \dfrac{1}{2\sigma_1^2}e^{-z/2\sigma_1^2}, & m = 1 \\ \dfrac{1}{2\sigma_0^2}e^{-z/2\sigma_0^2}, & m \ne 1 \end{cases} \quad (7)$$

where

$$2\sigma_i^2 = \begin{cases} \bar{E}_s + N_0, & i = 1 \\ N_0, & i = 0 \end{cases} \quad (8)$$

where $\bar{E}_s = E[g^2]A^2T/2$ is the average code symbol energy. If an RS code of rate r is used, then the average information bit energy $\bar{E}_b$ is $\bar{E}_s/(r\log_2 M)$.

RTT generates an erasure if

$$\max_{k \ne m} E_k \le E_m < \gamma \max_{k \ne m} E_k \quad (9)$$

for some parameter $\gamma \ge 1$, and each $m \in \{1, 2, \cdots, M\}$, and produces an error if

$$E_m \ge \gamma \max_{k \ne m} E_k \quad (10)$$

for some $m \ne 1$, when $S_1(t)$ is transmitted. Notice that conventional hard decisions corresponds to the special case of $\gamma = 1$. Because of the symmetry of the signal set, the probability of symbol error is independent of the transmitted signal if each signal is transmitted equally likely. Thus, for our analysis we assume that $S_1(t)$ is transmitted without loss of generality. Then, the probability of symbol error $p_e(\gamma)$ is

$$p_e(\gamma) = P\left\{\bigcup_{m=2}^{M} E_m \ge \gamma \max_{k \ne m} E_k \mid 1\right\} \quad (11)$$

$$= (M-1)\int_0^\infty \left(1 - e^{-z/2\gamma\sigma_1^2}\right) \times \left(1 - e^{-z/2\gamma\sigma_0^2}\right)^{M-2} \cdot \frac{1}{2\sigma_0^2}e^{-z/2\sigma_0^2}\,dz \quad (12)$$

$$= \frac{M-1}{\gamma}\sum_{k=0}^{M-2}\binom{M-2}{k} \times \frac{(-1)^k}{\left(1 + \frac{k}{\gamma}\right)\left[\left(1 + \frac{k}{\gamma}\right)\cdot\left(\frac{\bar{E}_s}{N_0} + 1\right) + \frac{1}{\gamma}\right]}. \quad (13)$$

Notice that if we let $\gamma = 1$ in (13), then we get the well-known probability of symbol error formula with hard decisions in Rayleigh-fading channel [8]. The probability of correct decision $p_c(\gamma)$ is

$$p_c(\gamma) = P\{E_1 \ge \gamma \max_{m \ne 1} E_m \mid 1\} \quad (14)$$

$$= \int_0^\infty \left(1 - e^{-z/2\gamma\sigma_0^2}\right)^{M-1} \cdot \frac{1}{2\sigma_1^2}e^{-z/2\sigma_1^2}\,dz \quad (15)$$

$$= \sum_{k=0}^{M-1}\binom{M-1}{k}\frac{(-1)^k}{1 + \dfrac{k}{\gamma}\left(\dfrac{\bar{E}_s}{N_0} + 1\right)} \quad (16)$$

whereas the probability of symbol erasure $p_{\mathrm{er}}(\gamma)$ is

$$p_{\mathrm{er}}(\gamma) = 1 - p_c(\gamma) - p_e(\gamma). \quad (17)$$

[1] As M increases, the performance of noncoherent demodulation approaches that of coherent demodulation with a significant advantage in receiver complexity [12].

$$E_m = \left[\left(\sqrt{\frac{2}{T}}\int_0^T r(t)\cos(2\pi f_m t)\,dt\right)^2 + \left(\sqrt{\frac{2}{T}}\int_0^T r(t)\sin(2\pi f_m t)\,dt\right)^2\right] \quad (3)$$

$$= \begin{cases} \left(\sqrt{\dfrac{A^2T}{2}}g\cos\theta + n_{c,1}\right)^2 + \left(\sqrt{\dfrac{A^2T}{2}}g\sin\theta + n_{s,1}\right)^2, & m = 1 \\ (n_{c,m})^2 + (n_{s,m})^2, & m \ne 1 \end{cases} \quad (4)$$

A. Limit Analysis

In this subsection we find the following asymptotic probabilities: $\lim_{M\to\infty} p_e(\gamma)$, $\lim_{M\to\infty} p_c(\gamma)$, and $\lim_{M\to\infty} p_{\rm er}(\gamma)$.

Proposition 1:

$$\lim_{M\to\infty} p_c(\gamma) = 2^{-\gamma N_0/r\bar{E}_b}. \tag{18}$$

Proof: Letting $u = z/\sigma_1^2$ in (15) gives

$$\lim_{M\to\infty} p_c(\gamma) = \lim_{M\to\infty} \int_0^\infty (1-M^{-au})^{M-1}\frac{1}{2}e^{-u/2}\,du \tag{19}$$

where

$$\begin{aligned} a &\triangleq \left(\frac{r\bar{E}_b}{N_0} + \frac{1}{\log_2 M}\right)\cdot\frac{1}{\gamma 2\ln 2} \qquad (20)\\ &\to \frac{r\bar{E}_b}{N_0}\cdot\frac{1}{\gamma 2\ln 2} \end{aligned}$$

as $M\to\infty$. If we let $L \triangleq (1-M^{-au})^{M-1}$, then

$$\begin{aligned} \ln L &= (M-1)\ln(1-M^{-au}) \qquad (21)\\ &= (M-1)\left(-M^{-au} - \frac{1}{2}M^{-2au} - \frac{1}{3}M^{-3au} - \cdots\right) \qquad (22)\\ &\sim -M^{(1-au)}, \qquad \text{for large } M \qquad (23) \end{aligned}$$

implying

$$\lim_{M\to\infty} L = \begin{cases} 0, & u < 1/a\\ 1, & u > 1/a. \end{cases} \tag{24}$$

Therefore, we get

$$\begin{aligned} \lim_{M\to\infty} p_c(\gamma) &= \int_{1/a}^\infty \frac{1}{2}e^{-u/2}\,du \qquad (25)\\ &= 2^{-\gamma N_0/r\bar{E}_b}. \qquad (26) \end{aligned}$$

Proposition 2:

$$\lim_{M\to\infty} p_e(\gamma) = \begin{cases} 0, & \gamma > 1\\ 1-2^{-(N_0/r\bar{E}_b)}, & \gamma = 1. \end{cases} \tag{27}$$

Proof: Letting $u = z/\sigma_1^2$ in (12) gives

$$\begin{aligned} \lim_{M\to\infty} p_e(\gamma) = \lim_{M\to\infty} b\int_0^\infty &(M-1)M^{-bu}(1-e^{-u/2\gamma})\\ &\times (1-M^{-bu/\gamma})^{M-2}\ln M\,du \qquad (28) \end{aligned}$$

where

$$b \triangleq \frac{r\bar{E}_b/N_0}{2\ln 2}. \tag{29}$$

If we let

$$L \triangleq (M-1)M^{-bu}(1-M^{-bu/\gamma})^{M-2}\ln M \tag{30}$$

then for large M

$$\begin{aligned} \ln L &= (1-bu)\ln M + \ln(\ln M) + (M-2)\ln(1-M^{-bu/\gamma}) \qquad (31)\\ &= \ln(\ln M) + (1-bu)\ln M + (M-2)\\ &\quad\times\left[-M^{-bu/\gamma} - \frac{1}{2}M^{-2bu/\gamma} - \frac{1}{3}M^{-3bu/\gamma} - \cdots\right]. \qquad (32) \end{aligned}$$

Since the $-(M-2)M^{-bu/\gamma}$ term in (32) predominates over all others

$$\lim_{M\to\infty} \ln L = -\infty, \qquad \text{if } 1 - bu/\gamma > 0 \tag{33}$$

implying

$$\lim_{M\to\infty} L = 0, \qquad \text{if } u < \gamma/b. \tag{34}$$

Thus, we can write

$$\lim_{M\to\infty} p_e(\gamma) = \lim_{M\to\infty} b\int_{\gamma/b}^\infty (M-1)M^{-bu}\left(1-e^{-\frac{u}{2\gamma}}\right)\ln M\,du. \tag{35}$$

It is to be noted that the term $(1-M^{-bu/\gamma})^{M-2}$ in (28) has in effect been eliminated from the integrand, but has been compensated for by increasing the lower limit from zero to γ/b. The compensation is exact as M approaches infinity. Performing integration in (35) gives

$$\begin{aligned} \lim_{M\to\infty} p_e(\gamma) &= \lim_{M\to\infty} M^{-(\gamma-1)}\left[1 - \frac{e^{-1/2b}b\ln M}{1/2\gamma + b\ln M}\right] \qquad (36)\\ &= \begin{cases} 0, & \gamma > 1\\ 1-2^{-(N_0/r\bar{E}_b)}, & \gamma = 1. \end{cases} \qquad (37) \end{aligned}$$

Notice that $\gamma = 1$ and $r = 1$ in (37) yields the same result given in [9]. The asymptotic probability of symbol erasure can be obtained by applying Propositions 1 and 2 in (17), which results in

$$\lim_{M\to\infty} p_{\rm er}(\gamma) = \begin{cases} 1-2^{-\gamma(N_0/r\bar{E}_b)}, & \gamma > 1\\ 0, & \gamma = 1. \end{cases} \tag{38}$$

Thus, for large M, all errors become erasures if $\gamma > 1$.

III. Coded Performance

In this section we derive the minimum $\bar{E}_b/N_0$ required for error-free communication with RTT and errors-and-erasures decoding. We present the optimum ratio threshold and the optimum code rate that minimize the required $\bar{E}_b/N_0$. This is compared to receivers that do not erase (hard decisions) and use errors-only decoding.

Let Z_i, $i = 1, 2, \cdots, n$ be

$$Z_i = \begin{cases} 0, & \text{if the RTT ouput is correct}\\ 1, & \text{if the RTT ouput is an erasure}\\ 2, & \text{if the RTT ouput is in error.} \end{cases} \tag{39}$$

As the (n, k) RS code can correct any set of t symbol errors and e symbol erasures provided $2t + e \le n - k$ [10], the probability of not decoding correctly P_E with RTT and errors-and-erasures decoding is

$$P_E = \sum_{j=n-k+1}^{n} P\left(\sum_{i=1}^{n} Z_i = j\right). \tag{40}$$

In this paper we assume that the code symbol size is M, and thus a large M implies a large block length n, because of the property $n = M - 1$ with RS codes. An ideal interleaver/deinterleaver is assumed in order to randomize error bursts caused by long fades. Thus, $\{Z_i\}, i = 1, 2, \cdots, n$ are assumed to be independent. By the central limit theorem

[11], as the code length n becomes large, the distribution of $\sum_{i=1}^{n} Z_i$ approaches a normal distribution with

$$E\left[\sum_{i=1}^{n} Z_i\right] = n(p_{\rm er}(\gamma) + 2p_e(\gamma)) \tag{41}$$

and

$$\begin{aligned}\mathrm{var}\left[\sum_{i=1}^{n} Z_i\right] &= E\left[\left(\sum_{i=1}^{n} Z_i\right)^2\right] - E^2\left[\sum_{i=1}^{n} Z_i\right] &(42)\\ &= n[1 - p_{\rm er}(\gamma) - (1 - p_{\rm er}(\gamma) - 2p_e(\gamma))^2]. &(43)\end{aligned}$$

Thus, as $n, k \to \infty$ while $r \triangleq k/n$ is fixed

$$\begin{aligned}\lim_{n,k\to\infty} P_E &= \lim_{n,k\to\infty} \int_\eta^\infty \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\,du &(44)\\ &= \begin{cases} 0, & r < 1 - p_{\rm er}(\gamma) - 2p_e(\gamma) \\ 0.5, & r = 1 - p_{\rm er}(\gamma) - 2p_e(\gamma) \\ 1, & r > 1 - p_{\rm er}(\gamma) - 2p_e(\gamma) \end{cases} &(45)\end{aligned}$$

where

$$\begin{aligned}\eta &\triangleq \frac{n - k + 1 - E\left[\sum_{i=1}^{n} Z_i\right]}{\sqrt{\mathrm{var}\left[\sum_{i=1}^{n} Z_i\right]}} &(46)\\ &= \frac{\sqrt{n}(1 - r - p_{\rm er}(\gamma) - 2p_e(\gamma))}{\sqrt{1 - p_{\rm er}(\gamma) - (1 - p_{\rm er}(\gamma) - 2p_e(\gamma))^2}}. &(47)\end{aligned}$$

Equation (45) shows that error-free communication is possible with RTT and errors-and-erasures decoding provided

$$\begin{aligned} r &< 1 - p_{\rm er}(\gamma) - 2p_e(\gamma) &(48)\\ &= \begin{cases} 2^{-\gamma(N_0/r\bar{E}_b)}, & \gamma > 1 \\ 2 \cdot 2^{-(N_0/r\bar{E}_b)} - 1, & \gamma = 1 \end{cases} &(49)\end{aligned}$$

or equivalently

$$\begin{aligned}\frac{\bar{E}_b}{N_0} &> \begin{cases} \dfrac{\gamma}{r\log_2(1/r)}, & \gamma > 1 \\ \dfrac{1}{r[1 - \log_2(1+r)]}, & \gamma = 1 \end{cases} &(50)\\ &\triangleq (\bar{E}_b/N_0)_{\min}. &(51)\end{aligned}$$

The right-hand side of (49) is the maximum possible code rate for error-free communication, and is called the achievable rate. Inspection of (50) reveals that the optimum ratio threshold that minimizes $(\bar{E}_b/N_0)_{\min}$ is arbitrarily close to one, but is slightly greater than one. Thus, the minimum $\bar{E}_b/N_0$ required for error-free communication with RTT can be arbitrarily close to (but is slightly greater than) $1/[r\log_2(1/r)]$. This is because we want to choose γ as small as possible but greater than one. We will ignore this small difference. Fig. 1 shows $(\bar{E}_b/N_0)_{\min}$ versus code rate r. The optimum code rate $r_{\rm opt}$ that minimizes $(\bar{E}_b/N_0)_{\min}$ can be found from (50) as

$$r_{\rm opt} = \begin{cases} e^{-1}, & \gamma > 1 \\ 0.46, & \gamma = 1. \end{cases} \tag{52}$$

For values of r given in (52), the $(\bar{E}_b/N_0)_{\min}$ is

$$(\bar{E}_b/N_0)_{\min} = \begin{cases} e\ln 2\ (= 2.75\ \text{dB}), & \text{RTT} \\ 4.79\ (= 6.80\ \text{dB}), & \text{hard decisions.} \end{cases} \tag{53}$$

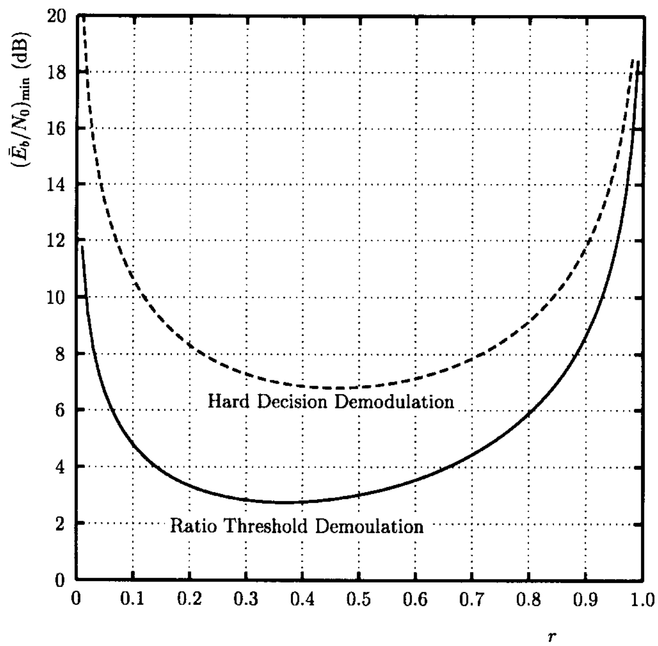

Fig. 1. $(\bar{E}_b/N_0)_{\min}$ versus code rate r.

This indicates that RTT provides a gain of 4.05 dB over hard decisions ($\gamma = 1$). The power gain G_p that RTT provides over hard decisions as a function of r is obtained from (50) as

$$G_p = \frac{\log_2(1/r)}{\gamma[1 - \log_2(1+r)]}. \tag{54}$$

The discontinuity of $(\bar{E}_b/N_0)_{\min}$ as the ratio threshold approaches one can be explained as follows. Equations (37) and (38) indicate that all errors become erasures if $\gamma > 1$ for large M. Since the RS code can correct twice as many erasures as errors, we may obtain a power gain by setting $\gamma > 1$. In addition, (38) indicates that the erasure probability decreases (or the correct probability increases) as γ is decreased. The implication is that it is better to erase unreliable symbols and correct erasures rather than to make a hard decision and correct errors. Figs. 2 and 3 show the required $\bar{E}_b/N_0$ for the probability of not decoding correctly P_E to be 10^{-5} versus code rate r for $M = 16$–128 with RTT (Fig. 2) and hard decisions (Fig. 3). We find that the optimal code rate for finite M converges to the asymptotic value, and the convergence rate is faster with the optimal ratio threshold test. Also, the required $\bar{E}_b/N_0$ at the asymptotic optimum rate is very close to the actual minimum $\bar{E}_b/N_0$.

IV. DS/SSMA WITH ORTHOGONAL MODULATION

In this section we consider an RS-coded DS/SSMA system, where a stream of $\log_2 M$ bits from RS encoder output is encoded by an M-ary orthogonal code of length $N = Mm$, where m is a positive integer. The orthogonal code is obtained by spreading every row of an $M \times M$ Hadamard matrix with a user-specific random sequence of length N. Fig. 4 shows the orthogonal modulator, where $\underline{\mathbf{d}}_i = (d_{i,0}, d_{i,1}, \cdots, d_{i,M-1})$ is the ith row in the Hadamard matrix, and $\underline{\mathbf{p}}^{(k)}$ is the random sequence of user k. We assume the sequence $\underline{\mathbf{c}}_i^{(k)} = \underline{\mathbf{d}}_i\underline{\mathbf{p}}^{(k)} =$

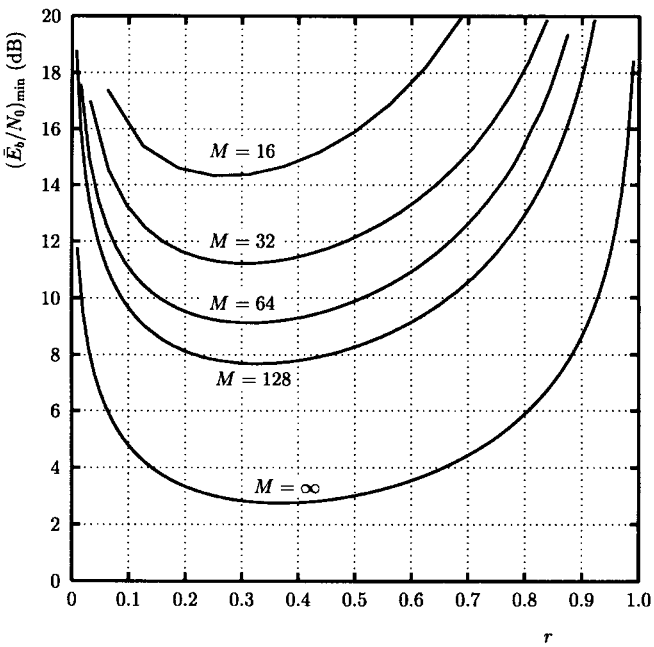

Fig. 2. Required $\bar{E}_b/N_0$ for $P_E = 10^{-5}$ versus code rate r: optimum RTT.

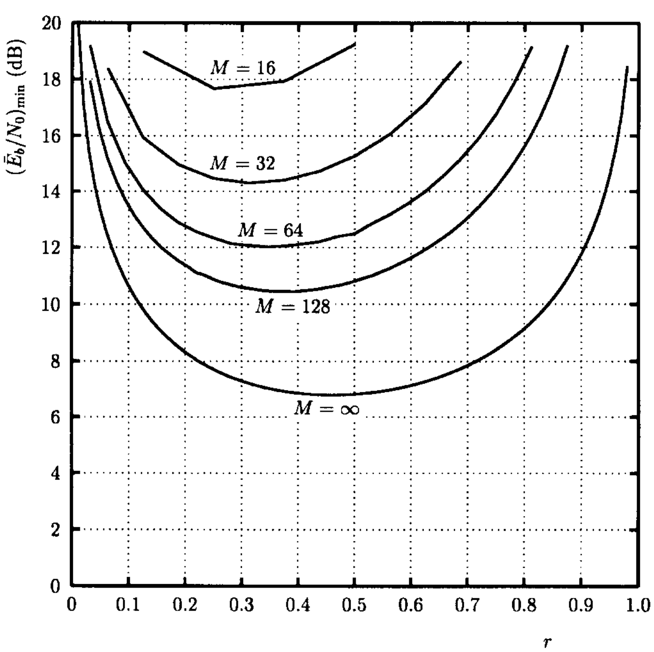

Fig. 3. Required $\bar{E}_b/N_0$ for $P_E = 10^{-5}$ versus code rate r: hard decision demodulation.

$(c_{i,0}^{(k)}, c_{i,1}^{(k)}, \cdots, c_{i,N-1}^{(k)})$, $i = 1, 2, \cdots, M$ is a random sequence with $P(c_{i,l}^{(k)} = 1) = P(c_{i,l}^{(k)} = -1) = 1/2$, for all i and l, and

$$\langle \underline{\mathbf{c}}_i^{(k)}, \underline{\mathbf{c}}_j^{(k)} \rangle = \frac{1}{N} \sum_{l=0}^{N-1} c_{i,l}^{(k)} c_{j,l}^{(k)} \tag{55}$$

$$= \begin{cases} 1, & i = j \\ 0, & i \neq j. \end{cases} \tag{56}$$

That is, $\{\underline{\mathbf{c}}_1^{(k)}, \underline{\mathbf{c}}_2^{(k)}, \cdots, \underline{\mathbf{c}}_M^{(k)}\}$ is an orthogonal code of length N. The encoder output is binary phase-shift-keying (BPSK) modulated. Then the modulator output of user k, $X^{(k)}(t)$ is

$$X^{(k)}(t) = Ad^{(k)}(t)p^{(k)}(t)\cos(2\pi f_c t + \phi^{(k)}) \tag{57}$$

where A is the signal magnitude, $d^{(k)}(t)$ is the Hadamard encoder output of user k, $p^{(k)}(t)$ is the random sequence of user k, f_c is the carrier frequency, and $\phi^{(k)}$ is the carrier phase of user k. The chip waveform $p_{T_c}(t)$ is a rectangular pulse, $p_{T_c}(t) = 1$ for $t \in [0, T_c)$ and zero elsewhere, where T_c is the chip duration. We assume there are K users, each transmitting $\log_2 M$ channel bits per T seconds independently.

We assume the channel is modeled as a frequency-nonselective Rayleigh-fading channel with multiple-access interference and background noise. This model is pessimistic in the sense that we do not assume that we can resolve the multipath signals and make a diversity combining. The received signal $r(t)$ is

$$\begin{aligned} r(t) = &\sum_{k=1}^{K} g^{(k)} A d^{(k)}(t - \tau^{(k)}) p^{(k)}(t - \tau^{(k)}) \\ &\times \cos[2\pi f_c (t - \tau^{(k)}) + \phi^{(k)} + \zeta^{(k)}] + n(t) \end{aligned} \tag{58}$$

where channel gains $\{g^{(k)}, k = 1, 2, \cdots, K\}$ are independent and identically (Rayleigh) distributed, the channel-induced phase deviation $\zeta^{(k)}$ of user k is assumed to be uniformly distributed over $[0, 2\pi]$, and the path delay $\tau^{(k)}$ of user k is assumed to be an integer multiple of T_c (i.e., chip synchronous). $n(t)$ is a Gaussian random process with mean zero and two-sided power spectral density $N_0/2$.

The noncoherent demodulator is shown in Fig. 5, where $d_i(t) = \sum_{m=0}^{M-1} d_{i,m} p_{T/M}(t - mT/M)$, and the noncoherent detector output R_i, $i = 1, 2, \cdots, M$ of the reference user (user 1) given that $\underline{\mathbf{d}}_1$ is transmitted by the reference user (i.e., $r(t) = g^{(1)} A d_1 (t - \tau^{(1)}) p^{(1)}(t - \tau^{(1)}) \cos(2\pi f_c (t - \tau^{(1)}) + \phi^{(1)} + \zeta^{(1)}) + \sum_{k=2}^{K} g^{(k)} A d^{(k)}(t - \tau^{(k)}) p^{(k)}(t - \tau^{(k)}) \cos(2\pi f_c (t - \tau^{(k)}) + \phi^{(k)} + \zeta^{(k)}) + n(t))$ and that acquisition/tracking is perfect (i.e. $\tau^{(1)} = 0$), is

$$\begin{aligned} R_1 = &\left[\left(\sqrt{\frac{2}{T}} \int_0^T r(t) d_1(t) p^{(1)}(t) \cos(2\pi f_c t)\, dt \right)^2 \right. \\ &\left. + \left(\sqrt{\frac{2}{T}} \int_0^T r(t) d_1(t) p^{(1)}(t) \sin(2\pi f_c t)\, dt \right)^2 \right] \\ = &\left[\left(\sqrt{\frac{A^2 T}{2}} g^{(1)} \cos\theta^{(1)} \right. \right. \\ &\left. + \sum_{k=2}^{K} \sqrt{\frac{A^2 T}{2}} X_{k,1} g^{(k)} \cos\theta^{(k)} + \eta_{c,1} \right)^2 \\ &+ \left(\sqrt{\frac{A^2 T}{2}} g^{(1)} \sin\theta^{(1)} \right. \\ &\left. \left. + \sum_{k=2}^{K} \sqrt{\frac{A^2 T}{2}} X_{k,1} g^{(k)} \sin\theta^{(k)} + \eta_{s,1} \right)^2 \right] \end{aligned} \tag{59}$$

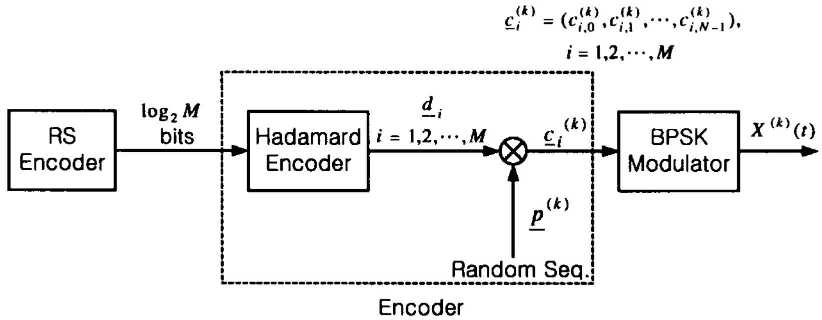

Fig. 4. Orthogonal modulator of user k.

and

$$\begin{aligned} R_i &= \left[\left(\sqrt{\frac{2}{T}}\int_0^T r(t)d_i(t)p^{(1)}(t)\cos(2\pi f_c t)\,dt\right)^2\right. \\ &\quad \left. + \left(\sqrt{\frac{2}{T}}\int_0^T r(t)d_i(t)p^{(1)}(t)\sin(2\pi f_c t)\,dt\right)^2\right] \\ &= \left[\left(\sum_{k=2}^{K}\sqrt{\frac{A^2T}{2}}X_{k,i}g^{(k)}\cos\theta^{(k)} + \eta_{c,i}\right)^2\right. \\ &\quad \left. + \left(\sum_{k=2}^{K}\sqrt{\frac{A^2T}{2}}X_{k,i}g^{(k)}\sin\theta^{(k)} + \eta_{s,i}\right)^2\right] \end{aligned} \tag{60}$$

for $i = 2, 3, \cdots, M$, where

$$\theta^{(k)} = 2\pi f_c \tau^{(k)} - \phi^{(k)} - \zeta^{(k)} \tag{61}$$

$$X_{k,i} = \frac{1}{T}\int_0^T d^{(k)}(t-\tau^{(k)})p^{(k)}(t-\tau^{(k)})d_i(t)p^{(1)}(t)\,dt \tag{62}$$

$$\eta_{c,i} = \sqrt{\frac{2}{T}}\int_0^T n(t)d_i(t)p^{(1)}(t)\cos(2\pi f_c t)\,dt \tag{63}$$

$$\eta_{s,i} = \sqrt{\frac{2}{T}}\int_0^T n(t)d_i(t)p^{(1)}(t)\sin(2\pi f_c t)\,dt. \tag{64}$$

As the multiple access interference term in (59) and (60) is the sum of $(K-1)$ independent terms, we model it by a Gaussian random variable. Thus, it follows from (59) and (60), and the fact $E[(X_{k,i})^2] = 1/N$ that the conditional probability density function $P_{R_i}(r \mid 1)$ of R_i, given $\underline{\mathbf{d}}_1$ is transmitted by the reference user, is

$$P_{R_i}(r \mid 1) = \begin{cases} \dfrac{1}{2\sigma_1^2}e^{-r/2\sigma_1^2}, & i = 1 \\ \dfrac{1}{2\sigma_0^2}e^{-r/2\sigma_0^2}, & i = 2, 3, \cdots, M \end{cases} \tag{65}$$

where

$$2\sigma_j^2 = \begin{cases} \bar{E}_s + (K-1)\bar{E}_s/N + N_0, & j = 1 \\ (K-1)\bar{E}_s/N + N_0, & j = 0 \end{cases} \tag{66}$$

and $\bar{E}_s = E[(g^{(k)})^2]A^2T/2$. If we assume that the path delay $\tau^{(k)}$ in (58) is uniformly distributed over $[0, T)$, then a factor of 2/3 should be multiplied in the multiple access interference term in (66). Equation (66) indicates that the equivalent noise spectral density N_e is

$$\begin{aligned} N_e &= (K-1)\bar{E}_s/N + N_0 \quad (67) \\ &= r\bar{E}_b(K-1)\log_2 M/(Mm) + N_0. \quad (68) \end{aligned}$$

The minimum $\bar{E}_b/N_0$ for error-free communication is found by replacing N_0 in (50) by N_e, which yields

$$\frac{\bar{E}_b}{N_0} > \begin{cases} \dfrac{1}{r\left[\frac{1}{\gamma}\log_2(1/r) - R_b\right]}, & \gamma > 1 \\ \dfrac{1}{r[1-\log_2(1+r) - R_b]}, & \gamma = 1 \end{cases} \tag{69}$$

$$\triangleq (\bar{E}_b/N_0)_{\min} \tag{70}$$

where

$$R_b = (K-1)\log_2 M/(Mm). \tag{71}$$

For large K (where $K-1 \sim K$), R_b can be interpreted as the total channel transmission rate, and thus rR_b represents the total information transmission rate in information bits/channel chip. For finite K, R_b becomes negligible as M becomes large. However, if K grows as $Mm/\log_2 M$, then R_b approaches a nonzero value. The optimum code rate r_{opt} that minimizes $(\bar{E}_b/N_0)_{\min}$ is found from (69) as

$$r_{\text{opt}} = e^{-1}2^{-\gamma R_b} \tag{72}$$

with RTT, and that with hard decisions ($\gamma = 1$) is given in Table I.

When the optimum code rate and the optimum ratio threshold ($\gamma_{\text{opt}} \to 1$) are used, $(\bar{E}_b/N_0)_{\min}$ with RTT is

$$(\bar{E}_b/N_0)_{\min} = 2^{R_b}e\ln 2 \tag{73}$$

and that with hard decisions is given in Table I. Note that the required minimum $\bar{E}_b/N_0$ grows exponentially with the total channel transmission rate R_b. Table I indicates that the power gain that RTT provides over hard decisions is more significant with larger R_b.

The maximum limit on the total information transmission rate rR_b for error-free communication is found from (69) as

$$rR_b < \begin{cases} r\log_2\dfrac{1}{r} - \dfrac{N_0}{\bar{E}_b}, & \text{RTT} \\ r[1-\log_2(1+r)] - \dfrac{N_0}{\bar{E}_b}, & \text{hard decisions} \end{cases} \tag{74}$$

$$\triangleq (rR_b)_{\max}. \tag{75}$$

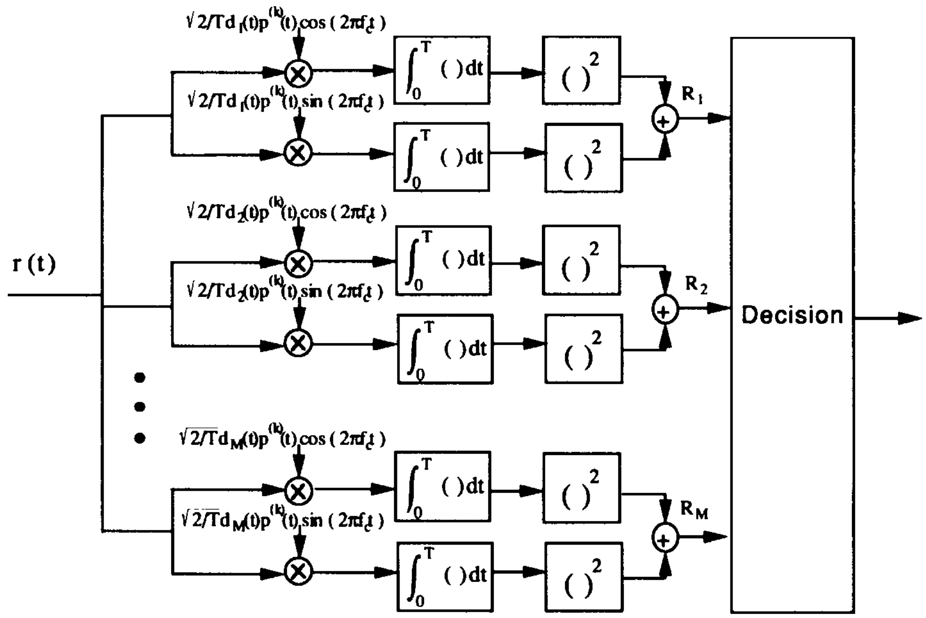

Fig. 5. Noncoherent demodulator of user k.

TABLE I
$r_{\rm opt}$ AND $(\bar{E}_b/N_0)_{\min}$ VERSUS R_b; RTT AND HARD DECISION

R_b	RTT r_{opt}	RTT $(\overline{E_b}/N_0)_{min}[dB]$	HD r_{opt}	HD $(\overline{E_b}/N_0)_{min}[dB]$
0.0	0.368	2.751	0.46	6.802
0.1	0.343	3.052	0.40	7.803
0.2	0.320	3.353	0.35	8.912
0.3	0.299	3.654	0.29	10.156
0.4	0.279	3.955	0.24	11.579
0.5	0.260	4.256	0.20	13.243
0.6	0.243	4.557	0.15	15.264
0.7	0.226	4.858	0.11	17.841
0.8	0.211	5.159	0.07	21.446

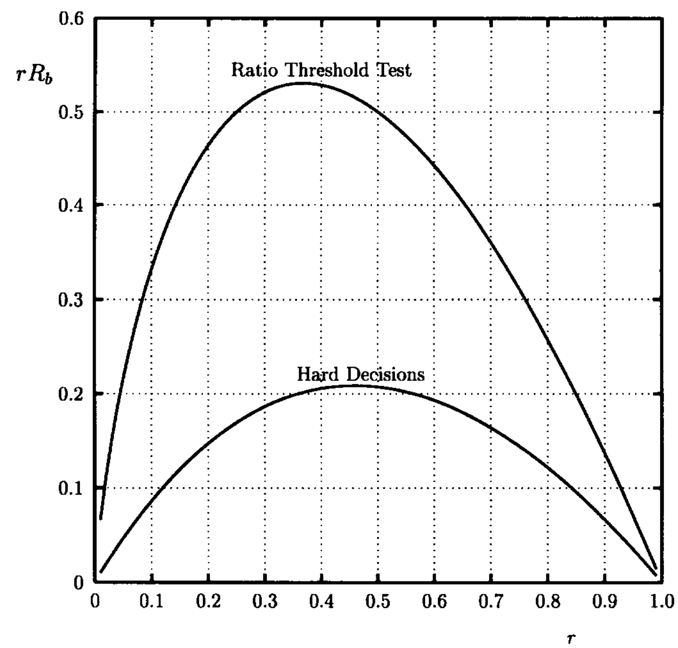

Fig. 6. Throughput rR_b as a function of code rate r.

The thoughput as a function of the code rate is shown in Fig. 6. If we optimize the right-hand side of (74) over the code rate r, we get the same optimum code rate as in (52). Then, the maximum total information transmission rate $(rR_b)_{\max}$ in information bits/channel chip is

$$(rR_b)_{\max} = \begin{cases} e^{-1}\log_2 e - \dfrac{N_0}{\bar{E}_b}, & \text{RTT} \\ 0.209 - \dfrac{N_0}{\bar{E}_b}, & \text{hard decisions} \end{cases} \tag{76}$$

$$\rightarrow \begin{cases} e^{-1}\log_2 e \ (= 0.531), & \text{RTT} \\ 0.209, & \text{hard decisions} \end{cases} \tag{77}$$

for $\bar{E}_b/N_0 \gg 1$, i.e., in an interference-limited system. This indicates that RTT provides an increase in total information transmission rate by a factor of 2.5 over hard decisions.

V. MULTIRATE DS/SSMA

We consider a multirate DS/SSMA system, where a population of users simultaneously transmit a variety of traffic types such as interactive data, digital voice, and video. These traffic types have different information transmission rates and performance requirements. In this paper we restrict attention to the case of two traffic types, but it is straightforward to generalize the result to the case of more than two traffic types.

We assume that there are K_1 users transmitting type-1 information at a rate of R_1 bits/s with power P_1, and K_2 users transmitting type-2 information at a rate of R_2 bits/s with power P_2. A stream of $\log_2 M$ information bits from type-j information source is encoded by an RS encoder of rate r_j and then encoded by an M-ary orthogonal code of length $N_j = Mm_j$, for some positive integer m_j, and is sent during $T_j = (r_j \log_2 M)/R_j$ seconds. We assume that the chip duration is T_c for both traffic types, so that the spreading code length N_j is T_j/T_c. That is, we spread all signals to the same bandwidth, regardless of their source bit rate. Thus, the positive integer m_j should be $\lfloor (r_j \log_2 M)/(MR_jT_c) \rfloor$, where $\lfloor x \rfloor$ is the largest integer not exceeding x. In this section we find the minimum $\bar{E}_b/N_0$ for error-free communication, and the optimum code rate that minimizes the required $\bar{E}_b/N_0$, for both types of information.

Let $X_j^{(k)}(t) = \sqrt{2P_j} d_j^{(k)}(t) p_j^{(k)}(t) \cos(2\pi f_c t + \phi_j^{(k)})$, $j = 1, 2$ be the transmitted signal of user k transmitting type-j information, where $d_j^{(k)}(t)$ is the Hadamard encoder output, $p_j^{(k)}(t)$ is the random sequence, and $\phi_j^{(k)}$ is the carrier phase, all of user k transmitting type-j information. Then the received signal $r(t)$ is

$$\begin{aligned} r(t) = &\sum_{k=1}^{K_1} g_1^{(k)} \sqrt{2P_1} d_1^{(k)}\big(t - \tau_1^{(k)}\big) p_1^{(k)}\big(t - \tau_1^{(k)}\big) \\ &\times \cos\big[2\pi f_c\big(t - \tau_1^{(k)}\big) + \phi_1^{(k)} + \zeta_1^{(k)}\big] \\ &+ \sum_{m=1}^{K_2} g_2^{(m)} \sqrt{2P_2} d_2^{(m)}\big(t - \tau_2^{(m)}\big) p_2^{(m)}\big(t - \tau_2^{(m)}\big) \\ &\times \cos\big[2\pi f_c\big(t - \tau_2^{(m)}\big) + \phi_2^{(m)} + \zeta_2^{(m)}\big] + n(t) \end{aligned} \tag{78}$$

where $\{g_1^{(k)}\}$ and $\{g_2^{(m)}\}$ are channel gains, $\{\tau_1^{(k)}\}$ and $\{\tau_2^{(m)}\}$ are path delays, and $\{\zeta_1^{(k)}\}$ and $\{\zeta_2^{(m)}\}$ are channel induced phase deviations. Note that there are two indexes, one corresponding to the information types (subscripts) and the other corresponding to the user numbers (superscripts). The noncoherent detector output $R_{1,i}$, $i = 1, 2, \cdots, M$ of the reference user (user 1) transmitting type-1 information, given that $\underline{\mathbf{d}}_1$ is transmitted by the reference user and that acquisition/tracking is perfect (i.e., $\tau_1^{(1)} = 0$), is given in (79) and (80), shown at the bottom of the page, where

$$\theta_j^{(k)} = 2\pi f_c \tau_j^{(k)} - \phi_j^{(k)} - \zeta_j^{(k)} \tag{81}$$

$$\begin{aligned} X_{l,n}^{(k)}(j) = &\frac{1}{T_j} \int_0^{T_j} d_l^{(k)}\big(t - \tau_l^{(k)}\big) p_l^{(k)}\big(t - \tau_l^{(k)}\big) \\ &\times d_n(t) p_j^{(1)}(t)\, dt, \end{aligned} \tag{82}$$

$$\eta_c(j,i) = \sqrt{\frac{2}{T_j}} \int_0^{T_j} n(t) d_i(t) p_j^{(1)}(t) \cos(2\pi f_c t)\, dt \tag{83}$$

$$\eta_s(j,i) = \sqrt{\frac{2}{T_j}} \int_0^{T_j} n(t) d_i(t) p_j^{(1)}(t) \sin(2\pi f_c t)\, dt. \tag{84}$$

Similarly, the noncoherent detector output $R_{2,i}$, $i = 1, 2, \cdots, M$ of the reference user transmitting type-2 information, given that $\underline{\mathbf{d}}_1$ is transmitted by the reference user, is

$$\begin{aligned} R_{2,i} &= \Bigg[\Bigg(\sqrt{\frac{2}{T_2}} \int_0^{T_2} r(t) d_i(t) p_2^{(1)}(t) \cos(2\pi f_c t)\, dt\Bigg)^2 \\ &\quad + \Bigg(\sqrt{\frac{2}{T_2}} \int_0^{T_2} r(t) d_i(t) p_2^{(1)}(t) \sin(2\pi f_c t)\, dt\Bigg)^2\Bigg] \end{aligned} \tag{85}$$

$$R_{1,i} = \Bigg[\Bigg(\sqrt{\frac{2}{T_1}} \int_0^{T_1} r(t) d_i(t) p_1^{(1)}(t) \cos(2\pi f_c t)\, dt\Bigg)^2 + \Bigg(\sqrt{\frac{2}{T_1}} \int_0^{T_1} r(t) d_i(t) p_1^{(1)}(t) \sin(2\pi f_c t)\, dt\Bigg)^2\Bigg] \tag{79}$$

$$= \begin{cases} \Bigg[\Bigg(\sqrt{P_1T_1} g_1^{(1)} \cos\theta_1^{(1)} + \sqrt{P_1T_1} \sum_{k=2}^{K_1} g_1^{(k)} \cos\theta_1^{(k)} X_{1,1}^{(k)}(1) \\ \quad + \sqrt{P_2T_1} \sum_{m=1}^{K_2} g_2^{(m)} \cos\theta_2^{(m)} X_{2,1}^{(m)}(1) + \eta_c(1,1)\Bigg)^2 \\ \quad + \Bigg(\sqrt{P_1T_1} g_1^{(1)} \sin\theta_1^{(1)} + \sqrt{P_1T_1} \sum_{k=2}^{K_1} g_1^{(k)} \sin\theta_1^{(k)} X_{1,1}^{(k)}(1) \\ \quad + \sqrt{P_2T_1} \sum_{m=1}^{K_2} g_2^{(m)} \sin\theta_2^{(m)} X_{2,1}^{(m)}(1) + \eta_s(1,1)\Bigg)^2\Bigg], & i = 1 \\ \Bigg[\Bigg(\sqrt{P_1T_1} \sum_{k=2}^{K_1} g_1^{(k)} \cos\theta_1^{(k)} X_{1,i}^{(k)}(1) \\ \quad + \sqrt{P_2T_1} \sum_{m=1}^{K_2} g_2^{(m)} \cos\theta_2^{(m)} X_{2,i}^{(m)}(1) + \eta_c(1,i)\Bigg)^2 \\ \quad + \Bigg(\sqrt{P_1T_1} \sum_{k=2}^{K_1} g_1^{(k)} \sin\theta_1^{(k)} X_{1,i}^{(k)}(1) \\ \quad + \sqrt{P_2T_1} \sum_{m=1}^{K_2} g_2^{(m)} \sin\theta_2^{(m)} X_{2,i}^{(m)}(1) + \eta_s(1,i)\Bigg)^2\Bigg], & i = 2, 3, \cdots, M \end{cases} \tag{80}$$

$$
= \begin{cases} \Bigg[\Bigg(\sqrt{P_2T_2}g_2^{(1)}\cos\theta_2^{(1)}+\sqrt{P_2T_2}\sum_{m=2}^{K_2}g_2^{(m)}\cos\theta_2^{(m)}X_{2,1}^{(m)}(2) \\ \quad +\sqrt{P_1T_2}\sum_{k=1}^{K_1}g_1^{(k)}\cos\theta_1^{(k)}X_{1,1}^{(k)}(2)+\eta_c(2,1)\Bigg)^2 \\ \quad +\Bigg(\sqrt{P_2T_2}g_2^{(1)}\sin\theta_2^{(1)}+\sqrt{P_2T_2}\sum_{m=2}^{K_2}g_2^{(m)}\sin\theta_2^{(m)}X_{2,1}^{(m)}(2) \\ \quad +\sqrt{P_1T_2}\sum_{k=1}^{K_1}g_1^{(k)}\sin\theta_1^{(k)}X_{1,1}^{(k)}(2)+\eta_s(2,1)\Bigg)^2\Bigg], \quad i=1 \\ \Bigg[\Bigg(\sqrt{P_2T_2}\sum_{m=2}^{K_2}g_2^{(m)}\cos\theta_2^{(m)}X_{2,i}^{(m)}(2) \\ \quad +\sqrt{P_1T_2}\sum_{k=1}^{K_1}g_1^{(k)}\cos\theta_1^{(k)}X_{2,i}^{(k)}(2)+\eta_c(2,i)\Bigg)^2 \\ \quad +\Bigg(\sqrt{P_2T_2}\sum_{m=2}^{K_2}g_2^{(m)}\sin\theta_2^{(m)}X_{2,i}^{(m)}(2) \\ \quad +\sqrt{P_1T_2}\sum_{k=1}^{K_1}g_1^{(k)}\sin\theta_1^{(k)}X_{2,i}^{(k)}(2)+\eta_s(2,i)\Bigg)^2\Bigg], \\ \qquad\qquad i=2,3,\cdots,M. \end{cases} \tag{86}
$$

If we assume that the multiple-access interference is Gaussian, then it follows from (80) and (86), and the fact that $E[(X_{l,n}^{(k)}(j))^2] = 1/N_j$, for all k, l, n, and j, that the conditional probability density function $P_{R_{j,i}}(r \mid 1)$ of $R_{j,i}$, given that $\underline{\mathbf{d}}_1$ is transmitted by the reference user transmitting type-j information, is

$$
P_{R_{j,i}}(r \mid 1) = \begin{cases} \dfrac{1}{2\sigma_1^2}e^{-r/2\sigma_1^2}, & i=1 \\ \dfrac{1}{2\sigma_0^2}e^{-r/2\sigma_0^2}, & i=2,3,\cdots,M \end{cases} \tag{87}
$$

where $2\sigma_1^2$ is given in (88), shown at the bottom of the page, and

$$
2\sigma_0^2 = \begin{cases} (K_1-1)\bar{g^2}P_1T_1/N_1+K_2\bar{g^2}P_2T_1/N_1+N_0, & j=1 \\ K_1\bar{g^2}P_1T_2/N_2+(K_2-1)\bar{g^2}P_2T_2/N_2+N_0, & j=2 \end{cases} \tag{89}
$$

where $\bar{g^2} = E[(g_j^{(k)})^2]$, for all j and k. If we let $\bar{E}_{s,j} = \bar{g^2}P_jT_j$, and $\bar{E}_{b,j} = \bar{E}_{s,j}/(r_j\log_2 M)$, then the equivalent noise spectral density $N_{e,j}$ for type-j information is given in (90), shown at the bottom of the page. Then, the minimum $\bar{E}_{b,j}/N_{e,j}$ for error-free communication for type-j information traffic is found by replacing $\bar{E}_b$ by $\bar{E}_{b,j}$, and N_0 by $N_{e,j}$ in (50), which yields

$$
\frac{\bar{E}_{b,j}}{N_{e,j}} > \begin{cases} \dfrac{\gamma}{r_j\log_2(1/r_j)}, & \gamma>1 \\ \dfrac{1}{r_j[1-\log_2(1+r_j)]}, & \gamma=1 \end{cases} \tag{91}
$$

or equivalently

$$
\frac{\bar{E}_{b,1}}{N_0} > \begin{cases} \dfrac{1}{r_1\left[\frac{1}{\gamma}\log_2(1/r_1)-\frac{P_2}{P_1}R_{2,1}-R_{1,1}\right]}, & \gamma>1 \\ \dfrac{1}{r_1\left[1-\log_2(1+r_1)-\frac{P_2}{P_1}R_{2,1}-R_{1,1}\right]}, & \gamma=1 \end{cases} \tag{92}
$$

$$
\triangleq (\bar{E}_{b,1}/N_0)_{\min}
$$

$$
\frac{\bar{E}_{b,2}}{N_0} > \begin{cases} \dfrac{1}{r_2\left[\frac{1}{\nu}\log_2(1/r_2)-\frac{P_1}{P_2}R_{1,2}-R_{2,2}\right]}, & \gamma>1 \\ \dfrac{1}{r_2\left[1-\log_2(1+r_2)-\frac{P_1}{P_2}R_{1,2}-R_{2,2}\right]}, & \gamma=1 \end{cases} \tag{93}
$$

$$
\triangleq (\bar{E}_{b,2}/N_0)_{\min}
$$

where

$$
R_{1,1} = (K_1-1)(\log_2 M)/N_1 \tag{94}
$$

$$
R_{1,2} = K_1(\log_2 M)/N_2 \tag{95}
$$

$$
R_{2,1} = K_2(\log_2 M)/N_1 \tag{96}
$$

$$
R_{2,2} = (K_2-1)(\log_2 M)/N_2. \tag{97}
$$

Note that $R_{i,j}$ depends on the number of users K_1 and K_2, and the source information rates R_1 and R_2. For RTT, the optimum code rate $r_{\text{opt},j}$ that minimizes $\bar{E}_{b,j}/N_0$ is found from (92) and (93) as

$$
r_{\text{opt},j} = \begin{cases} e^{-1}2^{-(R_{1,1}+\frac{P_2}{P_1}R_{2,1})}, & j=1 \\ e^{-1}2^{-(R_{2,2}+\frac{P_1}{P_2}R_{1,2})}, & j=2. \end{cases} \tag{98}
$$

Notice that increasing the power of one traffic type requires to lower the code rate of the other exponentially with the power ratio of two traffic types. When the above optimum code rate and the optimum ratio threshold are used, $(\bar{E}_{b,j}/N_0)_{\min}$ with

$$
2\sigma_1^2 = \begin{cases} \bar{g^2}P_1T_1+(K_1-1)\bar{g^2}P_1T_1/N_1+K_2\bar{g^2}P_2T_1/N_1+N_0, & j=1 \\ \bar{g^2}P_2T_2+K_1\bar{g^2}P_1T_2/N_2+(K_2-1)\bar{g^2}P_2T_2/N_2+N_0, & j=2 \end{cases} \tag{88}
$$

$$
N_{e,j} = \begin{cases} (K_1-1)r_1\bar{E}_{b,1}(\log_2 M)/N_1+\frac{P_2}{P_1}K_2r_1\bar{E}_{b,1}(\log_2 M)/N_1+N_0, & j=1 \\ \frac{P_1}{P_2}K_1r_2\bar{E}_{b,2}(\log_2 M)/N_2+(K_2-1)r_2\bar{E}_{b,2}(\log_2 M)/N_2+N_0, & j=2 \end{cases}. \tag{90}
$$

RTT is

$$(\bar{E}_{b,j}/N_0)_{\min} = \begin{cases} 2^{\left(R_{1,1}+\dfrac{P_2}{P_1}R_{2,1}\right)} e\ln 2, & j=1 \\ 2^{\left(R_{2,2}+\dfrac{P_1}{P_2}R_{1,2}\right)} e\ln 2, & j=2. \end{cases} \tag{99}$$

The optimum power ratio $(P_2/P_1)_{\rm opt}$ that minimizes $(\bar{E}_{b,1}/N_0)_{\min}$ [dB] $+ (\bar{E}_{b,2}/N_0)_{\min}$ [dB] is

$$\begin{aligned}(P_2/P_1)_{\rm opt} &= \sqrt{R_{1,2}/R_{2,1}} \\ &= \sqrt{\frac{K_1 r_1 R_2}{K_2 r_2 R_1}}.\end{aligned} \tag{100}$$

VI. Conclusion

In this paper we have investigated the asymptotic performance of RS-coded M-ary orthogonal signaling with ratio threshold test (RTT) in Rayleigh-fading channel. We derived the asymptotic probabilities of symbol erasure and symbol error for large values of M, and the minimum $\bar{E}_b/N_0$ needed for error-free communication. We found that the minimum $\bar{E}_b/N_0$ needed for error-free communication is $e\ln 2$ (2.75 dB) with RTT, and 4.79 (6.8 dB) with hard decisions. The optimum code rate that minimizes the required $\bar{E}_b/N_0$ is e^{-1} with RTT and 0.46 with hard decisions, and the optimum ratio threshold that minimizes the required $\bar{E}_b/N_0$ approaches one for large M.

Next, we considered a DS/SSMA system employing an M-ary orthogonal code of length $N = Mm$, which is obtained by spreading every row of an $M \times M$ Hadamard matrix with a user-specific random sequence of length N. We showed that the optimum code rate with RTT is $e^{-1}2^{-\gamma R_b}$, where γ is the ratio threshold, and the minimum $\bar{E}_b/N_0$ needed for error-free communication is $2^{R_b}e\ln 2$, where $R_b = (K-1)\log_2 M/(Mm)$ is the total channel transmission rate. The maximum limit on the total information transmission rate in information bits/channel chip is $e^{-1}\log_2 e$ $(= 0.531)$ with RTT and 0.209 with hard decisions, in interference-limited region. It is found that the power gain that RTT provides over hard decisions is more significant with larger R_b.

Then, we extended to a multirate DS/SSMA system, where a population of users simultaneously transmit at different power levels a variety of traffic types of different information rates. We derived the minimum $\bar{E}_b/N_0$ for error-free communication, and the optimum code rate that minimizes the required $\bar{E}_b/N_0$, for each traffic type in terms of power ratio, number of users, and spreading gains. It is found that increasing the power of one traffic type requires to lower the code rate of the other exponentially with the power ratio of two traffic types. The results may easily be generalized to the case of more than two traffic types.

References

[1] A. W. Lam and D. V. Sarwate, "Comparison of two methods for generation of side information in frequency-hopping spread-spectrum multiple-access communications," in *Proc. Conf. Information Science and Systems*, Mar. 1987, pp. 426–431.

[2] C. W. Baum and M. B. Pursley, "A decision-theoretic approach to the generation of side information in frequency-hop multiple-access communications," *IEEE Trans. Commun.*, vol. 43, pp. 1768–1777, Feb./Mar./Apr./ 1995.

[3] A. J. Viterbi, "A robust ratio-threshold technique to mitigate tone and partial band jamming in coded MFSK systems," in *IEEE MILCOM Conf. Rec.*, Oct. 1982, pp. 22.4.1–22.4.5.

[4] L. F. Chang and R. J. McEliece, "A study of Viterbi's ratio threshold AJ technique," in *IEEE MILCOM Conf. Rec.*, Oct. 1984, pp. 11.2.1–11.2.5.

[5] H. M. Kwon and P. J. Lee, "Performance of Viterbi's ratio threshold against mixed jamming," in *IEEE MILCOM Conf. Rec.*, 1989, pp. 25–29.

[6] K. G. Castor and W. E. Stark, "Performance of diversity/Reed-Solomon coded SSFH communications with errors/erasures via ratio thresholding," in *Proc. Allerton Conf. Communications, Control, and Computer*, Oct. 1985, pp. 869–877.

[7] V. W. Cheng and W. E. Stark, "Performance of trellis coded direct-sequence spread-spectrum multiple-access with noncoherent reception in a fading environment," in *IEEE PIMRC Conf. Rec.*, 1996, pp. 1125–1129.

[8] J. G. Proakis, *Digital Communications*, 3rd ed. New York: McGraw-Hill, 1995, p. 790.

[9] M. V. Hegde and W. E. Stark, "Asymptotic performance of M-ary orthogonal signals in worst case partial-band and Rayleigh fading," *IEEE Trans. Commun.*, vol. 36, pp. 989–992, Aug. 1988.

[10] R. E. Blahut, *Theory and Practice of Error Control Codes*. Reading, MA: Addison-Wesley, 1983.

[11] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*. New York: McGraw-Hill, 1984.

[12] A. J. Viterbi, *Principles of Coherent Communication*. New York: McGraw-Hill, 1966.

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