Frequency-Hopped Spread Spectrum in the Presence of a Follower Partial-Band Jammer

Amer A. Hassan, Wayne E. Stark, Member, IEEE, and John E. Hershey

Abstract—A countermeasure to a partial-band follower jammer is proposed for frequency-hopped spread-spectrum communications. This technique randomizes the transmission technique used by the transmitter (and receiver). Either the information is carried by $M$ tones which are transmitted in a frequency slot, or by $M$ frequency slots which contain signal energy. As a countermeasure, the jammer randomizes between jamming the same frequency slot being used by the communicator, or jamming a subset of the slots not being used by the communicator. The performance for randomized strategies for the communicator and jammer is investigated. It is shown that the proposed technique enhances the system’s performance.

I. INTRODUCTION

FREQUENCY-hopped spread-spectrum (FHSS) systems are used extensively in military communications to neutralize the effects of various types of intentional jamming and fading [1], [2]. Jamming models considered in such systems include partial-band multiple-tone jamming and partial-band Gaussian noise jamming. In these cases, the jammer can choose the fraction of bandwidth jammed to cause the bit error rate to be inversely proportional to signal-to-noise ratio at the receiver, as opposed to exponentially decreasing function of signal-to-noise ratio. This detrimental effect is neutralized by proper channel coding. The standard FHSS will suffer significantly in the presence of a follower jammer even with coding. A follower jammer has the capability to determine which portion of the spread-spectrum bandwidth is being used during some time interval $\tau$, and transmits its jamming signal in that portion of the spectrum. This strategy is effective (large error probability) if $\tau$ plus the differential propagation delay is much smaller than the communicator’s symbol interval. An obvious way to protect against such interference is fast hopping which prevents the follower jammer from having sufficient time to determine the communicator’s frequency and transmit interfering signals. The greater the hopping rate, the more protected the FH system is against a follow jammer; however, system limitations might not allow fast hopping. There is also a penalty incurred in subdividing a signal into several frequency hopped elements because the energy from these separate elements is usually combined noncoherently. Another jam-resistant approach, in the case of $M$-FSK modulation, is to use $M$ distinct frequency synthesizers at the transmitter and $M$ distinct frequency synthesizers at the receiver to individually hop the $M$-ary symbols, which requires complex hardware.

A new transmission technique was proposed in [6], which enhances the strategy available to the communicator in the presence of a multiple tone follower jammer. In this paper, we evaluate a similar scheme for FH spread-spectrum in partial-band Gaussian noise follower jammer. This scheme utilizes randomized decisions by the transmitter and the receiver to lure the jammer into helping the communicator’s performance part of the time. It is shown that the proposed technique improves the performance. The system model is described in Section II. Performance analysis is given in Section III. In Section IV we present numerical results and conclusions.

II. SYSTEM MODEL

Consider a frequency-hopped spread-spectrum (FHSS) communication system. The total spread bandwidth is divided into $q$ frequency slots. Each frequency slot can be used to transmit one of $M$ orthogonal signals of duration $T$, called the signaling interval. The transmitter, the receiver, and the channel are described as follows.

- **Transmitter/Receiver:** During each signaling interval, the transmitter/receiver operates in one of two modes—conventional or unconventional.

  The conventional mode is selected by the transmitter and the receiver pseudorandomly with probability $p_c$. In this case, the transmitter transmits one of $M$ tones of duration $T$ (i.e., within one of the $q$ frequency slots), and log $M$ information bits are conveyed (all log’s are to base 2). The receiver consists of a dehopper followed by noncoherent matched filters. The filter corresponding to the largest output is taken to be the transmitted symbol.

  The unconventional mode is selected with probability $1-p_c$. In this case, the transmitter randomly chooses one of the $M$ tones and transmits it in one of $r$ frequency slots, where the set of $r$ frequency slots are selected pseudorandomly. The $M$ tones which are transmitted do not carry any information, but it is the presence or absence of energy in the $r$ selected frequency slots that conveys information. In this case, log $r$ information bits are transmitted. The choice of $r$ depends on the system tradeoffs (usually data rate, complexity, and throughput), but we will assume that $r = M$ throughout the paper. The receiver consists of a bank of radiometers measuring the energy in the $r$ frequency slots. The slot with the largest energy is chosen as the transmitted symbol.
• Channel: The source of interference considered here is a partial-band Gaussian follower jammer. Let \( W_{sa} \) be the total spread-spectrum bandwidth, and \( J \) be the total power available to the follower jammer; then \( N_{j} = J/W_{sa} \) is the effective noise power spectral density. The jammer concentrates all the available power on a fraction \( \rho \) of the spread-spectrum bandwidth, where we take \( \rho = s/q \), \( s = 1, 2, \ldots, q \). Two modes of operation are also available to the jammer — conventional and unconventional. The jammer chooses to operate in the conventional mode with probability \( p_{j} \). In this case, the jammer injects all his power in the transmitter’s hop. The unconventional mode is chosen with probability \( 1 - p_{j} \). In this case, the jammer does not jam the transmitter's hop, but randomly jams a subset of the other \( q - 1 \) frequency slots. Note that the jammer randomizes his decision based on his knowledge of the communications system.

To gain a brief insight into system tradeoffs and the efficacy of the proposed technique, consider the event in which the transmitter is operating in the unconventional mode, and the jammer selects the conventional mode. This event occurs with a certain probability. In this case, the jammer is helping the communicator by adding more energy into his frequency slot. The overall effect (including other events with certain probabilities) is an improvement in system’s performance.

III. SYSTEM PERFORMANCE

The analysis below involves several simplifying assumptions to allow us to focus on the issues of interest. We neglect receiver noise under the assumption that it is dominated by the jamming interference, which is a typical assumption when analyzing communications systems in a jamming environment. Let \( Z_{C} \) be a random variable that takes values in \( \{0, 1\} \) such that \( Z_{C} = 0 \) indicates that the transmitter/receiver are in the conventional mode, and \( Z_{C} = 1 \) indicates the unconventional mode. Similarly,

\[
Z_{j} = \begin{cases} 
0 & \text{if jammer is in conventional mode,} \\
1 & \text{if jammer is in unconventional mode.}
\end{cases}
\]

Also, let \( E_{a} \) be the event that an information symbol is in error, and let \( p_{b} \) be the bit error rate. Then,

\[
P(E_{a}) = \sum_{i,j} P(E_{a}, Z_{C} = i, Z_{j} = j) = \sum_{i,j} P(E_{a}|Z_{C} = i, Z_{j} = j) \cdot P(Z_{C} = i, Z_{j} = j).
\]

We assume that \( Z_{j} \) and \( Z_{C} \) are independent random variables. Thus,

\[
P(Z_{C} = i, Z_{j} = j) = P(Z_{C} = i)P(Z_{j} = j),
\]

where \( P(Z_{C} = 0) = 1 - P(Z_{C} = 1) = p_{b} \), and \( P(Z_{j} = 0) = 1 - P(Z_{j} = 1) = p_{j} \).

Consider first the case of both communicator and jammer operating in conventional modes \( (Z_{C} = Z_{j} = 0) \). The (Gaussian) jamming noise density in the (one) slot jammed is then \( qN_{j} \), and the resulting error probability with noncoherent reception is

\[
P(E_{a}|Z_{C} = 0, Z_{j} = 0) = \sum_{n=1}^{M-1} \frac{(-1)^{n+1}}{n+1} \binom{M-1}{n} \cdot \exp\left(-\frac{n \log ME_{b}}{(n+1)qN_{j}}\right).
\]

Next consider the case of conventional mode for the communicator and unconventional mode for the jammer. In this case, it is easy to see that (due to the absence of thermal noise)

\[
P(E_{a}|Z_{C} = 0, Z_{j} = 1) = 0.
\]

Similarly,

\[
P(E_{a}|Z_{C} = 1, Z_{j} = 0) = 0.
\]

Finally consider the unconventional mode for both communicator and jammer. In this case, the receiver uses an energy detector or a radiometer to decide on the hop used (representing the \( M \)-ary symbol). If the jammer signal \( j(t) \) hops into one of these \( M \) frequency slots, the input to the radiometer is a stationary Gaussian process with double-sided power spectral density \( N_{e}/2 = qN_{j}/2s \), where \( s \) is the number of frequency slots jammed. In this case, the output of the energy detector \( W \) is given by

\[
W = \int_{0}^{T} j^{2}(t) \, dt.
\]

It can be shown (see [4]) that the probability density function of \( W \) is given by

\[
p_{W}(w) = \begin{cases} 
\frac{(N_{e}w)^{M-1}}{2^{M-1}(M-1)!} \exp\left(-\frac{N_{e}w}{4}\right), & w \geq 0, \\
0, & \text{otherwise}.
\end{cases}
\]

Obviously, the output of the energy detector when only the transmitted signal is present is \( \Gamma_{v} = E_{b} \log M \). Let \( H^{(j)} \) be the event the jammer hops into exactly \( j \) of the transmitter’s set of \( M \) designated frequency slots. Note that \( j \) must be less than \( q \) since the jammer (in the unconventional mode) does not jam the hop being used by the transmitter. Also, \( j \) must be at least 1, otherwise the jammer does not transmit at all and the error probability will be zero. The output of the radiometers affected by the jammer is a sequence of independent and identically distributed random variables \( \{W_{1}, \ldots, W_{j}\} \). Then

\[
P(E_{a}|Z_{C} = 1, Z_{j} = 1) = \sum_{j=1}^{\min(M-1,s)} \cdot P\left(E_{a}|Z_{C} = 1, Z_{j} = 1, H^{(j)}\right)P\left(H^{(j)}\right),
\]

where

\[
P\left(E_{a}|Z_{C} = 1, Z_{j} = 1, H^{(j)}\right) = 1 - f_{j}(E_{b}/N_{e}),
\]

and

\[
f_{j}(x) = \frac{1}{\sqrt{2\pi} \sigma_{j}(x)} \exp\left(-\frac{(x-E_{b}/N_{e})^{2}}{2\sigma_{j}(x)^{2}}\right).
\]
and
\[
\begin{align*}
  f_j(E_b/N_e) &= P \left\{ \bigcap_{i=1}^j (W_i < E_b \log M) \right\} \\
  &= \prod_{i=1}^j P[W_i < \Gamma_x] \\
  &= [P[w_i < \Gamma_x]^j \\
  &= \left[ 1 - P[W_i > \Gamma_x] \right]^j \\
  &= \left[ 1 - \frac{1}{2^{M(M-1)!}} \int_{\Gamma_x}^{\infty} p_W(w) \, dw \right]^j \\
  &= \left[ 1 - \exp \left\{ -\frac{\Gamma_x}{N_e} \sum_{i=0}^{M-1} \frac{1}{i!} \left( \frac{\Gamma_x}{N_e} \right)^i \right\} \right]^j.
\end{align*}
\]

Also, a simple combinatorial argument shows
\[
P(H^{(j)}) = \binom{M-1}{j} \binom{q-s}{s-j} \frac{q^{-s}}{s^j}.
\]

Therefore, the symbol error rate is given by
\[
P(E_s) = \alpha p_j \beta [1 - p_j] (1 - p_e)
\]
where
\[
\alpha = \sum_{n=1}^{M-1} \frac{(-1)^{n+1}}{n+1} \binom{M-1}{n} \exp \left\{ -\frac{n \log ME_b}{n+1} qN_j \right\},
\]
and
\[
\beta = \sum_{j=1}^{\min(M-1,s)} \binom{M-1}{j} \binom{q-s}{s-j} \left[ 1 - f_j(E_b/N_e) \right].
\]

The bit error probability \( p_b \) is determined from the symbol error probability by
\[
p_b = \frac{M}{2(M-1)} P(E_s).
\]

Notice that \( \alpha \) does not depend on \( s \), and \( \beta \) depends on \( s \).

We then numerically choose \( s \) to maximize \( \beta \) (and, therefore, maximize the bit error probability) for a fixed \( E_b/N_j \).

Solving, analytically, the above optimization for general \( M \) is nontrivial. However, for the case \( M = 2 \), analytical results can be obtained. If we define \( \alpha = E_b/N_j \), then we have
\[
\beta = \frac{s}{q-1} e^{-\frac{a_s}{q}} \left( 1 + \frac{as}{q} \right), \quad 1 \leq s \leq q - 1.
\]

We need to find \( \rho = s/q \) which maximizes the above expression. Treating \( s \) as a continuous variable, we can easily perform the optimization over \( s \). Differentiating the above expression with respect to \( s \) and equating it to zero, we get
\[
\frac{d\beta(s)}{ds} = \frac{1}{q-1} e^{-\frac{a_s}{q}} \left( 1 + \frac{as}{q} - \frac{a_s^2}{q^2} \right) = 0.
\]

where the valid solution to the above equation is \( as/q = (1 + \sqrt{5})/2 \) (the other being negative). Since the above derivative is increasing for all \( 0 < as/q < (1 + \sqrt{5})/2 \) and decreasing for all \( as/q > (1 + \sqrt{5})/2 \), the global maximum is achieved at \( as/q = (1 + \sqrt{5})/2 \). Therefore, for \( M = 2 \), the optimum fraction of band jammed (in the unconventional mode) is
\[
\rho = \left\{ \begin{array}{ll}
  \frac{q-1}{q}, & E_b/N_j \leq \frac{0.1800}{E_b/N_j} \leq \frac{1.6180}{q-1} \\
  \frac{0.8000}{E_b/N_j} \leq \frac{q-1}{q}, & E_b/N_j \geq \frac{1.6180}{q-1}.
\end{array} \right.
\]

The resulting \( \beta \) is
\[
\beta = \left\{ \begin{array}{ll}
  \frac{q-M}{q-s-j} \approx \rho^j (1 - \rho)^{M-1-j}, & 0 < \rho = s/q \leq \frac{q-M}{q-s-j} \leq \frac{M-1}{M-2},
\end{array} \right.
\]

where \( \rho = s/q \). Using this approximation in (10) yields
\[
\beta = e^{-\rho} \left[ 1 - (1 - \rho)^{M-1-j} \right].
\]

Now we notice that
\[
f_j(E_b/N_j) \approx [1 - g_M(\rho p)]^j,
\]
where
\[
g_M(\rho p) = e^{-\rho} \sum_{i=0}^{M-1} \frac{(\rho p)^i}{i!}.
\]

and assume \( s >> M - 1 \), the above expression simplifies to
\[
\beta = \left\{ \begin{array}{ll}
  \sum_{j=1}^{M-1} \binom{M-1}{j} (1 - \rho)^{M-1-j} \left[ 1 - [1 - g_M(\rho p)]^j \right], & 0 < \rho = s/q \leq 1,
\end{array} \right.
\]

we can simplify the above to
\[
\beta = 1 - (1 - \rho)^{M-1} - [\rho [1 - g_M(\rho p)] + (1 - \rho)]^{M-1} + (1 - \rho)^{M-1} = 1 - (1 - \rho g_M(\rho p))^{M-1}.
\]

Finding the worst case \( \rho \) now is not too difficult. In Appendix A we show that \( \beta(\rho) \) has a unique maximum. The worst case \( \rho \) has the following form:
\[
\rho = \left\{ \begin{array}{ll}
  1, & E_b/N_j \leq A_M \\
  \frac{0.1800}{E_b/N_j}, & E_b/N_j \geq A_M,
\end{array} \right.
\]

where \( A_M = B_M / \log_2 M \) and \( B_M \) is the solution to
\[
\frac{\partial g_M(x)}{\partial x} = -g_M(x).
\]
The resulting form for $\beta$ is

$$
\beta = \begin{cases} 
1 - (1 - g_M(a))^{M-1}, & E_b/N_J \leq A_M \\
1 - \left(1 - \frac{A_M}{E_b/N_J} g_M(A_M)\right)^{M-1}, & E_b/N_J \geq A_M.
\end{cases}
$$

(11)

In Table I, we list the constants $A_M$ and $A_M g_M(A_M)$.

<table>
<thead>
<tr>
<th>$M$</th>
<th>$A_M$</th>
<th>$A_M g_M(A_M)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.618</td>
<td>0.83996</td>
</tr>
<tr>
<td>4</td>
<td>1.473</td>
<td>0.97119</td>
</tr>
<tr>
<td>8</td>
<td>1.935</td>
<td>1.49065</td>
</tr>
<tr>
<td>16</td>
<td>2.983</td>
<td>2.53345</td>
</tr>
<tr>
<td>32</td>
<td>4.989</td>
<td>4.49974</td>
</tr>
</tbody>
</table>

For $E_b/N_J$ large, (11) simplifies to

$$
\beta = \frac{(M - 1)A_M g(A_M)}{E_b/N_J}.
$$

Notice that for $M = 2$, (11) simplifies to the expression derived earlier, except for a factor of $(q/(q - 1))$ which is very near 1 for large $q$.

Now consider the case where the transmitter (and thus the receiver) wants to choose the fraction of time spent in the conventional mode to minimize the bit error rate, based on its knowledge of $p_j$, while the jammer is interested in choosing its strategy $p_j$ based on its knowledge of $p_c$. In this case, one is interested in the solution to the game $\max_{p_j} \min_{p_c} p_b(p_j, p_c)$ viewed by the jammer, and $\min_{p_c} \max_{p_j} p_b(p_j, p_c)$ viewed by the transmitter/receiver. We prove the following Theorem in Appendix B.

**Theorem:**

$$
\max_{p_j} \min_{p_c} p_b(p_j, p_c) = \min_{p_c} \max_{p_j} p_b(p_j, p_c)
$$

$$
= \frac{M}{2(M - 1)} (\alpha \beta) = \frac{M}{2(M - 1)} (\alpha + \beta)
$$

for $\alpha$ and $\beta$ given above. The following corollary is of interest.

**Corollary:**

$$
p_0 = \min \{\alpha, \beta\}.
$$

This says that the above strategy can only improve the performance. For $M = 2$, we have

$$
p_0 = \begin{cases} 
\alpha / \left(\frac{q - 1}{q} \frac{E_b}{N_J} \alpha + 1\right), & E_b/N_J \leq 1.618 \frac{q - 1}{q - 1} \\
\alpha / \left(\frac{q - 1}{q} E_b/N_J \alpha + 1\right), & E_b/N_J \geq 1.618 \frac{q - 1}{q - 1}.
\end{cases}
$$

IV. NUMERICAL RESULTS AND CONCLUSIONS

Figs. 1–9 show the performance in terms of bit error rate of the proposed system, where conv. and unconv. refer to the conventional and unconventional modes (simultaneously) for the transmitter/receiver and the jammer, respectively. Also shown is the bit error rate in equilibrium, that is, the solution to the game. Fig. 1 shows the performance for $q = 100$ and $M = 2$, where there is a gain of 9 dB of the equilibrium strategy over the conventional strategy at bit error rate of $10^{-2}$. Notice that the bit error rate in the unconventional mode deviates from the inverse linear relationship as predicted analytically for large $E_b/N_J$. This is due to the course quantization of the optimized parameter $\mu$, since we are optimizing with respect to $s$ being an integer, and $q = 100$ is small, the quantization error dominates above a certain threshold. This unpredicted behavior almost disappears for $M = 2$ in Figs. 4 and 7 where $q$ is large, and therefore the quantization error is less.

In Fig. 5, for the same number of frequency slots and bit error rate as above, for $M = 32$ the gain is more than 5 dB for equilibrium over unconventional. Figs. 4–6 show
the performance for $q = 500$ and $M = 2, 8, 32$, respectively. The gain is even larger than the $q = 100$ case. Figs. 7–9 show similar behavior for $q = 1000$ and $M = 2, 8, 32$, respectively. The approximations for large $q$ in (11) seem to
be very good approximations for $q = 500$, $M = 16$, and $q = 1000$, $M = 2, 16, 32$, provided $E_b/N_0$ is not too large so that the optimal fraction of the band jammed results in $s$ being less than $M – 1$, as exhibited in Figs. 5 and 7–9. The approximations is poor for small $q$ or large $M$, as would be expected, and as shown in Figs. 2–3 and 6.

We have evaluated the performance of an uncoded FHSS system in the presence of a follower jammer. The proposed technique randomizes the transmission used by the transmitter and the receiver. Either the information is carried by $M$ tones which are transmitted in a frequency slot, or by $M$ frequency slots which contain signal energy. As a counter-countermeasure, the jammer randomizes between jamming the same frequency slot being used by the communicator or jamming a subset of the slots not being used by the communicator. The performance for randomized strategies for the communicator and jammer was investigated. It is shown that the proposed technique enhances the system's performance. Numerical results suggest that coding for the system above is crucial to get an acceptable bit error rate. In this case, one can choose the receiver in the unconventional mode to be a bank of energy detectors such that if two or more frequency slots have energy above a certain threshold, the received symbol is erased. The decoder can be a bounded distance decoder that corrects errors and erasures.

**APPENDIX A**

We will show that if $g(x)$ has a unique maxima. Notice first that if $g(x)$ is a continuous nonnegative function of $x$, then it is easy to show that

$$\min_y g(x)^M = \left[ \min_x g(x) \right]^M.$$

Then it is sufficient to find

$$\arg \max_{\rho} pg_M(\rho).$$

where $g_M(\rho)$ is as defined earlier in the paper. Moreover,

$$\frac{\partial pg_M(\rho)}{\partial \rho} = e^{-\rho} \left( \sum_{i=0}^{M-1} \frac{(\rho)^i}{i!} \right) \frac{(M-\rho)^M}{(M-1)!}$$

$$\frac{\partial^2 pg_M(\rho)}{\partial \rho^2} = \frac{(\rho)^M}{(M-1)!} e^{-\rho} (\rho - M - 1).$$

Hence, $\rho \leq M + 1 \implies pg_M(\rho)$ is convex $\cap$, and $pg_M(\rho)$ has a local maxima $\rho^*$ in the interval $[0, M + 1]$. for $\rho > M + 1$, $pg_M(\rho)$ is convex $\cup$, and $pg_M(\rho) = 0$ as $\rho \to \infty$. Therefore, $\rho^*$ is a global maximum.

**APPENDIX B**

We will show the following. For $0 \leq x, y \leq 1$,

$$\min_{x} \max_{y} [xy + (1 - x)(1 - y)A] = \min_{y} \max_{x} (x(1 - (1 - x)A)y + (1 - x)A).$$

If $x \geq (A/(1 + A))$, then $y = 1$ achieves the maximum and the minimum over $x$ is achieved at $x = (A/(1 + A))$. Thus, $\min_{y \in [1/(1 + A), 1]} \max_{x} [xy + (1 - x)(1 - y)A] = (A/(1 + A))$. If $x \leq (A/(1 + A))$, then $y = 0$ achieves the maximum and the minimum over $x$ is achieved for $x = (A/(1 + A))$. Thus, $\min_{x \in [1/(1 + A), 1]} \max_{y} [xy + (1 - x)(1 - y)A] = \frac{A}{1 + A}$. Thus, $\min_{x \in [1/(1 + A), 1]} \max_{y} [xy + (1 - x)(1 - y)A] = \frac{A}{1 + A}$. A similar argument holds for $\max_{x} \min_{y} [xy + (1 - x)(1 - y)A].$

**REFERENCES**


Amer A. Hassan was born in Beirut, Lebanon, in November 1960. He received the B.S. and M.S. degrees from the University of Kansas, Lawrence, in May 1983 and July 1984, respectively, and the Ph.D. degree from the University of Michigan, Ann Arbor, in December 1981. He worked in 1985 as a Member of Technical Staff at Bell Communications Research, Redbank, NJ. Since January 1989, he has been with GE Corporate Research and Development Center in Schenectady, NY, as a Member of Technical Staff. His research interests are in the areas of communications.

John E. Hershey is a member of Technical Staff of the General Electric Corporate Research and Development Center Schenectady, NY. He is the coauthor of Data Transportation and Protection (New York: Plenum) and coeditor of Digital Signal Processing (New York: Academic). He came to GE from the BDM Corporation Boulder, CO, in 1989.

Wayne E. Stark (S’77–M’82) for a photograph and biography, see the January 1993 issue of this TRANSACTIONS, p. 209.