Characteristic Curves, Sweet Points, and Lead Time Control of Re-entrant Lines

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Abstract

The characteristic curve (CC) of a production system is a function describing the behavior of production lead time (*LT*) vs. throughput (*TP*). In systems with unlimited buffers, this function has a knee-type shape. Operating the system below the knee is not efficient, since *TP* can be increased without an appreciable increase in *LT*. Operating above the knee is also counterproductive – *LT* becomes extremely large without a significant increase of *TP*. Thus, the desirable operating point is at the knee, which is why it is often referred to in practice as the 'sweet point'. In this paper, an analytical/empirical method for calculating CCs of single-product re-entrant lines without batch processing is developed using the so-called bottleneck workcenter model of systems at hand. Based on this method, the position of the sweet point is quantified and open/closed-loop control of job release policies, which ensure the operation at the desired CC point (including the sweet point) are provided. The development is carried out in terms of the First Buffer First Served and Last Buffer First Served dispatch policies, although CCs for other policies can be investigated in a similar manner.

Keywords: Re-entrant lines, Lead time, Throughput, Characteristic curves, Sweet points, Open- and closed-loop job release policies, FBFS and LBFS dispatch.

1 Introduction

1.1 Motivation and goal

Production systems with infinite buffers typically have the lead time (LT) vs. throughput (TP) characteristic curve (CC) with a knee-type shape as illustrated in Figure 1.1. Here LT is the average time a job (e.g., lot) spends in the system, being processed or waiting for processing (LT is sometimes referred to as the production cycle-time or flow-time), and TP is the average number of jobs produced by the system per unit of time. In the steady state of system operation, the throughput equals to the release rate (RR), which is the number of jobs released into the system per unit of time. So, the CC of Figure 1.1 can be understood as LT vs. RR or, in different notations, as $LT = F(RR)$, where $F(\cdot)$ is a function defined by the production system, i.e., its structure (e.g., serial, cellular, or re-entrant), the number of machines in the system, their reliability models (e.g., Bernoulli, exponential, Weibull, etc.), and, in the case of re-entrant lines, dispatch policies (i.e., priorities of servicing buffers containing jobs at various stages of their processing).

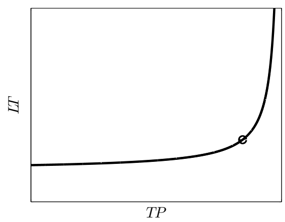


Figure 1.1: Characteristic curve of production system with infinite buffers

The shape of $F(RR)$ implies that operating the system below the knee (indicated by the black dot in Figure 1.1) is not efficient since RR (and, thus, TP) can be increased without an appreciable increase of LT. Operating the system above the knee is also counterproductive, since LT (and, thus, work-in-process, WIP) becomes extremely large without a significant increase of TP. Thus, the desirable operating point is at the knee, which is why it is often referred to in practice as the *sweet point*. In this paper, we use the terms 'sweet point' and 'knee' interchangeably.

Operating a production system at or close to the knee requires the knowledge of function $F(RR)$. For *serial* production lines, this function has been investigated analytically in [1] and [2] for machines having the Bernoulli and exponential reliability models, respectively. *Cellular* lines with

Bernoulli machines have been analyzed in [3]. The goal of the current paper is to investigate this function for *re-entrant* lines with exponential and non-exponential machines and, on this basis, develop open- and closed-loop job release policies, which ensure the sweet point operation or the operation at any desired point of CC. Note that re-entrant lines, which are widely used in semiconductor industry, are particularly known for having excessively long *LT*; hence, providing a method for its evaluation and control is an important industrial problem. Clearly, it is of theoretical interest as well.

1.2 System considered

The development reported in this paper is carried out using the bottleneck workcenter model of a re-entrant line (BNWC model) introduced and investigated in [4–7] and further explored in [8]. This model (see Figure 1.2) considers in details BNWC (consisting of M machines and N buffers), while all other workcenters are modeled as fixed time delays, $T_0, T_1, \ldots, T_N$. In addition, the BNWC model is defined by dispatch policies employed. The current paper considers two of such policies: First Buffer First Served (FBFS) and Last Buffer First Served (LBFS), although other dispatch policies can be explored in a similar manner.

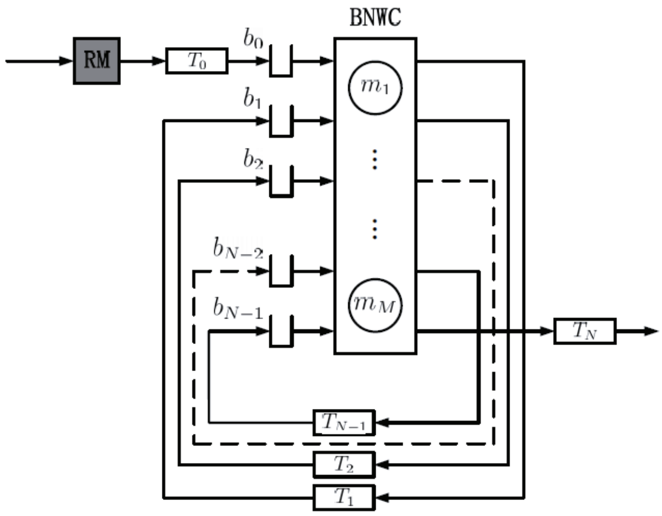


Figure 1.2: BNWC model of re-entrant line with release mechanism

To model the job release, we associate with this system a job release mechanism (RM, shown in Figure 1.2 in gray). Various types of RMs can be considered. Initially, we assume that RM is another machine (referred to as the *release machine*), which releases a job during a machine cycle-time with a certain probability. Then, we generalize the results to deterministic, once-per-hour or once-per-shift, release policies.

Given this system, operating in a single-product regime without batch processing, the current paper develops a method for calculating its CC, the sweet point, and open/closed-loop job release policies to ensure operation at the desired point of the CC. Specifically, the contributions of this paper are the following:

- An analytical expression of CC as a function of the BNWC structure (i.e., the number of machines and buffers, the machines' reliability characteristics, and job release rate) as well as two parameters identified empirically (using either simulations or operating the system under two different job release rates).
- An analytical expression of the sweet point, defined as the point of CC with the largest curvature.
- A quantification of LT's, which are feasible (i.e., achievable) in a given system.
- An analytical expression for a job release rate, which ensures the desired feasible LT as a function of machine parameters.
- If the machine parameters are not known precisely, a closed-loop job release policy, which ensures operation close to the desired point of the CC (even if open-loop release leads to instability).
- While the above results are initially obtained for exponential machines, it is shown that they hold for machines obeying the Weibull, gamma, and log-normal reliability models as well.

1.3 On the Validity of BNWC Model in the Problem of Lead Time Investigation

Although investigating the validity of the BNWC model of re-entrant lines is beyond the scope of this paper, in this subsection, we provide an example, which quantifies the conditions when the complete and the BNWC models exhibit a similar lead time behavior.

Consider the systems with three workcenters and a job release machine shown in Figure 1.3. Assume that all the machines are exponential, and the machines in each workcenter with multiple machines have identical efficiency: e_1 in $WC1$ and e_3 in $WC3$. The efficiency of the release machine and the machine in $WC2$ are denoted as e_0 and e_2, respectively. Also, assume for simplicity that the cycle time of all machines is the same, τ (in min).

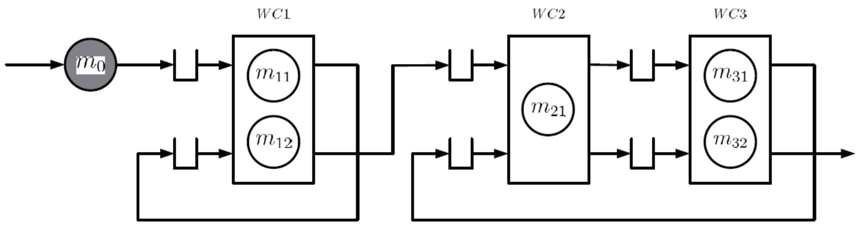


Figure 1.3: Re-entrant line with three workcenters

Given this system, introduce the notion of WC *workload* as

$$\rho_{WCi} = \frac{N_{WCi}}{M_{WCi}} \frac{e_0}{e_i}, \; i = 1, 2, 3, \tag{1.1}$$

where N_{WCi} and M_{WCi} are the number of buffers and machines in workcenter i, respectively. A workcenter with the largest ρ_{WCi} is referred to as the BNWC and its *severity* is defined as

$$S = \frac{\text{largest } \rho_{WCi}}{\text{second largest } \rho_{WCi}}. \tag{1.2}$$

If, for instance, $e_1 = e_3 = 0.85$, $e_2 = 0.99$ and $e_0 = 0.4$, then $\rho_{WC1} = \rho_{WC3} = 0.47$, $\rho_{WC2} = 0.81$, and the BNWC is $WC2$ with $S = 1.72$. If, on the other hand, $e_1 = e_3 = 0.99$, $e_2 = 0.85$ and $e_0 = 0.4$, then $\rho_{WC1} = \rho_{WC3} = 0.40$ and $\rho_{WC2} = 0.94$, and the BNWC is again $WC2$, but with $S = 2.33$.

In view of the above, the BNWC model of the system in Figure 1.3 is that in Figure 1.4, where $T_0 = 2$ min, and $T_1 = T_2 = 1$ min, assuming that $\tau = 1$ min.

We have investigated the lead time, LT_{exact}, of the system in Figure 1.3 and that of its BNWC approximation, LT_{BNWC}, for various values of machine efficiencies as a function of ρ_{BNWC}. Typical results are illustrated in Figure 1.5 for two dispatch policies, FBFS and LBFS. Based on the results obtained, we conclude that the BNWC model faithfully represents the lead time of a multi-

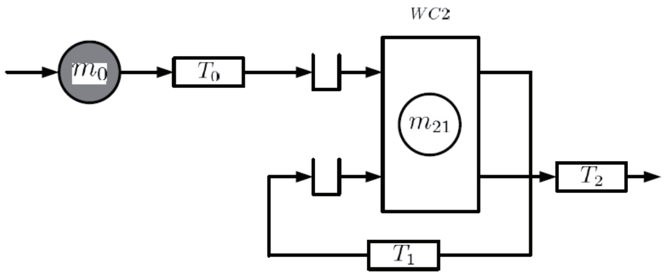
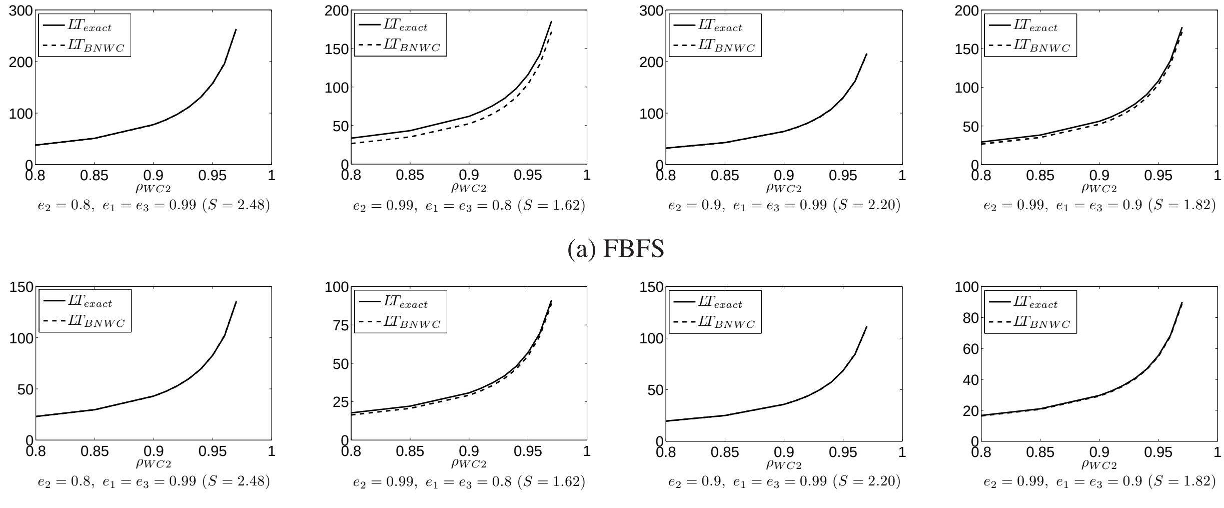


Figure 1.4: BNWC model of re-entrant line in Figure 1.3

workcenter re-entrant line if the severity of the bottleneck workcenter is sufficiently high.



(a) FBFS

(b) LBFS

Figure 1.5: Typical relationships between LT_{exact} and LT_{BNWC}

1.4 Related Literature

The literature on CC analysis in re-entrant lines can be classified into two groups. The first one is devoted to CC identification using discrete event simulations and the second to analytical approaches using queueing theory models. Below, we provide several remarks on each of these groups, with the aim to place the current paper in the framework of this literature.

The representative papers of the first group are [9–14]. The main issue here is to generate an "economical" sample of job release intensities, which would allow for an effective CC identification. Specifically, [9, 10] provide a fixed sample size strategy, which generates CC with minimal mean square error. Based on [11], where a method for improving design of experiments has been developed, the series of papers [12–14] presents effective techniques for CC evaluation in single- and multi-product re-entrant lines by judiciously selecting input traffic intensity using metamodels motivated by queueing theory expressions.

The representative publications of the second group are [15–18]. Specifically, the survey [15] states that many problems related to re-entrant lines performance analysis still remain open. References [16] and [17] provide bounds on CCs in closed tandem lines. In addition, [17] introduces the notion of critical *WIP*, which leads to system operation at the sweet point; however, methods for calculating the critical *WIP* in re-entrant lines with unreliable machines are not provided. Finally, [18] develops a capacity planning tool for design on re-entrant lines taking into account numerous practical features, including randomness in process times, random machine breakdowns, batch processing, machine setups, and multi-job production. In spite of the number of simplifying assumptions taken in the development of this tool, simulations show that it provides reasonably accurate results and is quite useful for rapid investigation of various "what if" scenarios.

The current paper has an intermediate position within this body of literature. Namely, it uses simulation or operation of the system under two levels of job release intensities, but then provides analytical expressions for CCs, sweet points, and control strategies, which ensure the desired lead time performance as a function of the system and machine parameters.

Another body of literature, which relates to the current paper, addresses the issue of job release to ensure a sufficiently small *WIP* (see [10, 19–26]). The main approaches here are kanban and CONWIP. It is shown, both theoretically and in applications, that each of these approaches may result in a substantial improvement of the *WIP* vs. *TP* behavior. However, to the best of our knowledge, no analytical expressions for the number of kanbans or the level of CONWIP necessary to achieve the desired *LT* in re-entrant lines with unreliable machines, have been obtained. By providing analytical expressions for open- and closed-loop job release policies, which ensure the desired lead time while maximizing *TP*, the current paper contributes to this end.

1.5 Abbreviations and notations

Abbreviations: BNWC – bottleneck workcenter, CC – characteristic curve, FBFS – First Buffer First Served, LBFS – Last Buffer First Served, CL – closed-loop, OL – open-loop, RM – release mechanism.

Notations: α – scaling ratio, e – machine efficiency, E – deterministic release, LT – lead time, lt – relative lead time, λ – breakdown rate, μ – repair rate, M – number of machines in BNWC, N – number of processing stages, ρ – relative workload, RI – release interval, RR – release rate, τ – machine cycle-time, T_{down} – machine downtime, T_i – time delay, T_{up} – machine uptime, TP – throughput, WIP – work-in-process.

1.6 Paper outline

The remainder of this paper is structured as follows: Section 2 defines formally the BNWC-based model of re-entrant lines and formulates the problems addressed. Sections 3-5 investigate CCs and control policies in synchronous re-entrant lines with identical exponential machines. Specifically, Section 3 investigates the position of sweet points and provides a comparative investigation of CCs under FBFS and LBFS dispatch policies; Sections 4 and 5 develop open- and closed-loop control techniques for job release to ensure the operation at the sweet point or, for that matter, at any desired point of CC. Section 6 addresses similar issues for the so-called release-asynchronous systems. Sections 7 and 8 extend the results to asynchronous re-entrant lines with non-identical exponential machines and to synchronous lines with non-exponential machines, respectively. The conclusions and topics for future work are provided in Section 9.

2 Modeling and Problems Addressed

Let, as before, LT_{BNWC} be the average lead time of the system in Figure 1.2, and LT the average lead time of the same system, but with T_i, $i = 0, 1, \ldots, N$, equals to 0. Then, in the steady state,

$$LT_{BNWC} = LT + \sum_{i=0}^{N} T_i. \tag{2.1}$$

Since T_i's affect LT_{BNWC} as an additive constant, throughout this paper we consider the system without the delays, evaluate its LT, and then specify LT_{BNWC} by (2.1).

Accordingly, we consider the system shown in Figure 1.2 and define its operation by the following assumptions:

(i) The system consists of:

- BNWC with M *producing machines*, $m_1, m_2, \ldots, m_M$;
- $N-1$ *re-entrant paths* with *in-process buffers*, $b_1, b_2, \ldots, b_{N-1}$, containing parts at various stages of their processing;
- The *input path*, consisting of the *release mechanism*, RM, and the *raw material buffer*, b_0. RM releases raw material into the system according to a *release policy*. RM may be modeled as a release machine, m_0, or as an once-per-interval release policy (e.g., once-per-hour or once-per-shift).

(ii) Each machine m_i, $i = 0, 1, \ldots, M$, is characterized by:

- The *reliability model*, i.e., continuous random variables that define its up- and downtime. If the up- and downtime distributions are exponential, i.e., defined by the *breakdown rate* λ_i and *repair rate* μ_i, $i = 0, 1, \ldots, M$, (in 1/min), the line is called *exponential*; otherwise, it is *non-exponential*. For the producing machines, λ_i and μ_i, $i = 1, 2, \ldots, M$, are fixed, for the release machine, λ_0 and μ_0 are design parameters and can be selected at will.
- The *cycle time*, τ_i (in min), i.e., the time necessary to process a part by machine m_i, $i = 0, 1, \ldots, M$. While τ_i, $i = 1, 2, \ldots, M$, are fixed, τ_0 can be chosen at will. When $\tau_i = \tau$, $i = 0, 1, \ldots, M$, the system is referred to as *synchronous*; when $\tau_i = \tau$, $i = 1, 2, \ldots, M$, and $\tau_0 \neq \tau$, the system is called *release-asynchronous*; when cycle times of some or all producing machines are different, the system is *asynchronous*.

(iii) Each buffer is of *infinite capacity*.

(iv) The assignment of machines $m_1, m_2, \ldots, m_M$ to process parts from buffers $b_0, b_1, \ldots, b_{N-1}$ is carried out according to a *dispatch policy*. This paper considers two dispatch policies:

- FBFS, whereby the priority is given to buffer b_i with the smallest i.
- LBFS, whereby the priority is given to buffer b_i with the largest i.

(v) Modeling assumptions:

- The *flow model* [27] is assumed, i.e., an infinitesimal quantity of a part, produced during an infinitesimal time interval, is transferred to and from the buffers.
- A machine is *starved*, if the buffer in front of it is empty; m_0 is never starved.
- Machine failures are *time-dependent* [27], i.e., a machine can be down even if it is starved.

Assumption (iii) is introduced to reflect the fact that in most re-entrant lines the buffer capacity is not hardware-limited. Assumption (iv) can be extended to other dispatch policies as well. Assumption (v) is introduced for technical reasons: it permits a precise formulation of the equations describing the systems at hand.

Let $T_{up,i}$ and $T_{down,i}$ denote the average up- and downtime of m_i, $i = 0, 1, \ldots, M$. Then the machine *efficiency* for any continuous reliability model is:

$$e_i := \frac{T_{up,i}}{T_{up,i} + T_{down,i}}, \; i = 0, 1, \ldots, M, \tag{2.2}$$

and its *throughput in isolation* (i.e., when the machine is not starved) is

$$TP_{isol,i} := \frac{T_{up,i}}{\tau_i (T_{up,i} + T_{down,i})}, \; i = 0, 1, \ldots, M. \tag{2.3}$$

Since for exponential machines, $T_{up,i} = \frac{1}{\lambda_i}$ and $T_{down,i} = \frac{1}{\mu_i}$, then

$$e_i = \frac{\mu_i}{\lambda_i + \mu_i} \text{ and } TP_{isol,i} = \frac{\mu_i}{\tau_i(\lambda_i + \mu_i)}, \; i = 0, 1, \ldots, M. \tag{2.4}$$

In the framework of model (i)-(v), this paper addresses the following problems:

- Develop a method for calculating characteristic curves and sweet points of re-entrant lines.
- Develop open-loop raw material release control policies, which ensure the desired lead time (in particular, sweet point operation) when the parameters of the machines are known precisely.
- Develop closed-loop release control policies, which ensure the desired lead time when the parameters of the machines are not known precisely.

We address these problems by considering several specific cases of increasing complexity and practical significance.

- First, we address the synchronous case with identical exponential producing machines (i.e., with $\lambda_i = \lambda$ and $\mu_i = \mu$, $i = 1, 2, \ldots, M$). This case allows for a detailed investigation of LT and various release policies, leading to insights useful for other cases as well.
- Then, we consider the release-asynchronous case with identical exponential producing machines. This case is a stepping stone to analysis of asynchronous systems.
- Asynchronous case with non-identical exponential producing machines. We analyze the systems of this type using lower- and upper-bounds derived based on the results of the previous two cases.
- Finally, we address the synchronous case with identical non-exponential producing machines. The asynchronous systems with non-exponential machines can be analyzed using the same approach as in the previous case, but it is not included here due to space limitations. Also, for the same reason, this and the previous cases are analyzed in fewer details than the first one.

The results obtained are described below.

3 Synchronous Case with Identical Exponential Producing Machines: LT Analysis

In this and two subsequent sections, we consider synchronous re-entrant lines defined by assumptions (i)-(v) with identical exponential producing machines and analyze their CCs, sweet points, and LT control issues. Initially, we assume that RM is represented by an exponential machine with $\tau_0 = \tau$, and then generalize the results based on deterministic, e.g., once-per-hour or once-per-shift, release.

3.1 Approach

The approach of this section is based on the results of [2], where the function $F(RR)$ (i.e., the CC) and its knee have been analyzed for synchronous serial lines with identical exponential machines. This analysis has been carried out in terms of two dimensionless quantities. The first one, referred to as the *relative lead time*, is given by

$$lt = \frac{LT}{M\tau}, \tag{3.1}$$

where M is the number of producing machines and τ is the machine cycle-time. Obviously, lt quantifies LT in units of its smallest possible value, i.e., $M\tau$. For instance, $lt = 5$ implies that LT is 5 times longer than the total processing time.

The second quantity, referred to as the *relative workload*, is defined as

$$\rho = \frac{e_0}{e}, \tag{3.2}$$

where e and e_0 are the efficiency of the producing and release machines, respectively. Note that e_0 can be interpreted as the probability to release a job during the machine cycle-time, τ.

In terms of these normalizations and under the assumption

$$e_0 < e \tag{3.3}$$

(to ensure the existence of a steady state), the following expression, which defines the CC, has been derived in [2]:

$$\widehat{lt}(\rho) = \frac{a}{1-\rho} + b, \tag{3.4}$$

where $\widehat{lt}$ is a sufficiently accurate estimate of lt (see [2] for details), and a and b are constants given by

$$\begin{aligned} a &= \frac{1-e}{M\tau}[T_{down,0} + (2M-1)T_{down}], \\ b &= 1 - \frac{1-e}{M\tau}T_{down,0}. \end{aligned} \tag{3.5}$$

The CC, specified by (3.4), (3.5), is shown in Figure 3.1 for the serial line with parameters indicated in the figure caption. Using the definition of the knee as a point of CC with the largest curvature, it is shown in [2] that the knee of $\widehat{lt}(\rho)$ satisfies the relationship:

$$\alpha \frac{\mathrm{d}(\widehat{lt})}{\mathrm{d}\rho}\bigg|_{\rho=\rho_{knee}} = 1 \tag{3.6}$$

or, taking into account (3.4),

$$\rho_{knee} = 1 - \sqrt{\alpha a}. \tag{3.7}$$

In (3.6) and (3.7), α is the scaling ratio defined by the operating regime of the system at hand. Specifically, if $\rho \in [\rho_{min}, 1)$ and $\widehat{lt} \in [1, \widehat{lt}_{max}]$, where ρ_{min} is the smallest relative load factor of interest and $\widehat{lt}_{max}$ is the largest acceptable relative lead time, then

$$\alpha = \frac{1-\rho_{min}}{\widehat{lt}_{max}}. \tag{3.8}$$

In Figure 3.1, $\alpha = 1/4000$ and the knee, defined by (3.7), is shown by the black dot (with the coordinates indicated). Thus, the position of the knee depends on the regime of system operation (i.e., α) and the parameter a of CC, while it is independent of b.

Unfortunately, an expression similar to (3.4) for re-entrant lines defined by assumptions (i)-(v) proved to be all but impossible to derive analytically. Therefore, we resort to an empirical/analytical approach, whereby the constants a and b are determined empirically and the validity of (3.4) for re-entrant lines is justified by simulations, while the position of the knee as well as the open- and closed-loop release control policies are quantified analytically.

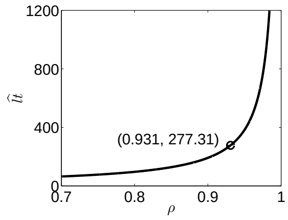


Figure 3.1: Characteristic curve and sweet point of serial line with $M = 10$, $\tau = 1$min, $e = 0.9$, $T_{down,0} = 10$min, $T_{down} = 100$min

Specifically, consider the system of Figure 1.2 and assume that $M < N$, i.e., the number of machines in the bottleneck workcenter is less than the number of processing stages (which is typically the case in practice). Then, the approach of this paper consists of the following:

(a) Introduce the relative lead time and relative workload for synchronous re-entrant lines with identical producing machines as

$$\begin{aligned} lt_r &= \frac{LT}{N\tau}, \\ \rho_r &= \frac{N}{M}\frac{e_0}{e}. \end{aligned} \tag{3.9}$$

To ensure existence of the steady state, assume $\rho_r < 1$, i.e.,

$$e_0 < \frac{Me}{N}. \tag{3.10}$$

(b) Identify empirically the two constants involved in expression (3.4). This is accomplished by evaluating lt either by simulations or by operating the system under two workload factors, say, ρ_{r1} and ρ_{r2}. Using (3.4), this leads to two equations with two unknowns. Solving these equations, we obtain:

$$\begin{aligned} a_r &= \frac{lt_r(\rho_{r1}) - lt_r(\rho_{r2})}{\rho_{r1} - \rho_{r2}}(1 - \rho_{r1})(1 - \rho_{r2}), \\ b_r &= \frac{lt_r(\rho_{r2})(1 - \rho_{r2}) - lt_r(\rho_{r1})(1 - \rho_{r1})}{\rho_{r1} - \rho_{r2}}, \end{aligned} \tag{3.11}$$

where $lt_r(\rho_{ri})$, is the empirically determined relative lead time under the relative load factor, ρ_{ri}, $i = 1, 2$. Having a_r and b_r, we postulate that CC for re-entrant lines under consideration is given by

$$\widehat{lt}_r(\rho_r) = \frac{a_r}{1-\rho_r} + b_r. \tag{3.12}$$

(c) Next, we justify that expression (3.12) holds for any $\rho_r \in [\rho_{r,min}, 1)$. This is accomplished by simulating a sufficiently large number of systems with randomly selected parameters (see Subsection 3.2 for details) and showing that lt_r obtained by simulations and by (3.12) are sufficiently close to each other. This constitutes the empirical part of the approach.

(d) The analytical part is based on (3.12). Specifically, as in (3.7), we quantify the position of the knee by

$$\rho_{r,knee} = 1 - \sqrt{\alpha a_r} \tag{3.13}$$

and develop open- and closed-loop job release policies that ensure the operation at the sweet point or at any other desired point of the characteristic curve.

3.2 Accuracy

The accuracy of $\widehat{lt}_r(\rho_r)$ is investigated in two steps. First, we construct 100 re-entrant lines obeying the BNWC-based model, identify empirically their constants a_r and b_r, and, thus, specify their characteristic curves (3.12). Second, we evaluate the accuracy of the resulting expressions by comparing them with the true characteristic curves, $lt_r(\rho_r)$, identified by simulations for $\rho_r \in [\rho_{r,min}, 1)$.

The 100 re-entrant lines satisfying assumptions (i)-(v) are constructed by selecting their parameters randomly and equiprobably from the following sets:

$$\begin{aligned} &M \in [2,5],\ N \in [M+1, 10],\ e \in [0.7, 0.99], \\ &T_{down,0} = T_{down} \in [1\text{min}, 10\text{min}], \end{aligned} \tag{3.14}$$

and, without loss of generality, $\tau = 1$min. For each of these lines, ρ_{r1} is selected randomly and equiprobably from the interval $\rho_{r1} \in [0.8, 0.97]$ and ρ_{r2} is assumed to be $0.9\rho_{r1}$. To evaluate $lt_r(\rho_{ri})$, $i = 1, 2$, we employ the following simulation procedure (also used in all subsequent sections): In addition to a warm-up period of $2,000,000$min, the simulation runs for $22,000,000$min, and 20 repetitions of this procedure are carried out. This procedure results in a 95% confidence interval of ±2.5% of lt_r. Using $lt_r(\rho_{r1})$ and $lt_r(\rho_{r2})$, thus evaluated, the constants a_r and b_r are calculated according to (3.11). Thus, the characteristic curve $\widehat{lt}_r(\rho_r)$ is specified analytically by (3.12) for all $\rho_{r1} \in [\rho_{r,min}, 1)$.

To investigate the accuracy of $\widehat{lt}_r(\rho_r)$, we simulate the 100 systems discussed above with

$$\rho_r \in P = \{0.8, 0.85, 0.9, 0.92, 0.93, 0.94, 0.95, 0.96, 0.97\}, \tag{3.15}$$

evaluate $lt_r(\rho_r)$, and quantify the relationship between $lt_r(\rho_r)$ and $\widehat{lt}_r(\rho_r)$ by

$$\epsilon = \max_{\rho_r \in P} \frac{|\widehat{lt}_r(\rho_r) - lt_r(\rho_r)|}{lt_r(\rho_r)} \times 100\%. \tag{3.16}$$

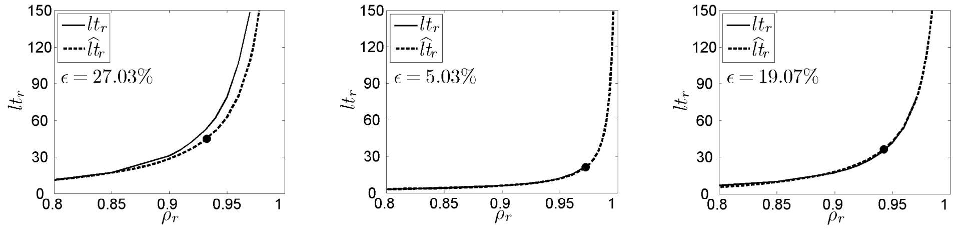
We carry out these analyses under both FBFS and LBFS dispatch policies. As a results, we obtain:

Observation 3.1 *For all* 100 *re-entrant lines analyzed,*

(a) under FBFS dispatch, the smallest and the largest ϵ*'s are* 1.45% *and* 33.10%*, respectively; the average error (over the* 100 *systems analyzed) is* 12.19%*;*

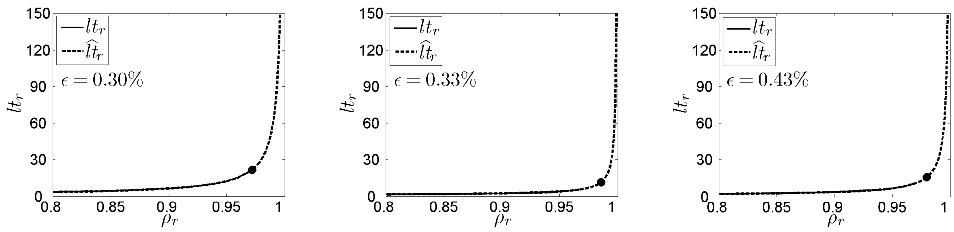
(b) under LBFS dispatch, the smallest and the largest ϵ*'s are* 0.05% *and* 1.93%*, respectively; the average error is* 0.60%.

Typical relationships between $\widehat{lt}_r(\rho_r)$ and $lt_r(\rho_r)$ are illustrated in Figures 3.2 and 3.3 for FBFS and LBFS, respectively, along with the position of the sweet point (calculated using (3.13) with $\alpha = 1/750$). Obviously, the accuracy of (3.12) for LBFS is very high, while for FBFS the errors in some cases are significant. However, recognizing that machine parameters on the factory floor are rarely known with accuracy better than ±5%, we conclude that the estimate (3.12) is acceptable for the lead time analysis and control.



(a) $\rho_{r1} = 0.8378$, $M = 2$, $N = 9$, $e = 0.7451$, $T_{down} = 5.54$min (b) $\rho_{r1} = 0.8964$, $M = 3$, $N = 4$, $e = 0.7163$, $T_{down} = 1.08$min (c) $\rho_{r1} = 0.9669$, $M = 5$, $N = 9$, $e = 0.9685$, $T_{down} = 5.85$min

Figure 3.2: Typical relationships between $\widehat{lt}_r(\rho_r)$ and $lt_r(\rho_r)$ under FBFS dispatch



(a) $\rho_{r1} = 0.8378$, $M = 2$, $N = 9$, $e = 0.7451$, $T_{down} = 5.54$min (b) $\rho_{r1} = 0.8964$, $M = 3$, $N = 4$, $e = 0.7163$, $T_{down} = 1.08$min (c) $\rho_{r1} = 0.9669$, $M = 5$, $N = 9$, $e = 0.9685$, $T_{down} = 5.85$min

Figure 3.3: Typical relationships between $\widehat{lt}_r(\rho_r)$ and $lt_r(\rho_r)$ under LBFS dispatch

3.3 Comparison of lead time under FBFS and LBFS dispatch

To illustrate the utility of (3.12) for analysis of re-entrant lines, we compare their behavior under two dispatch policies – FBFS and LBFS. Specifically, assume that

$$M = 3,\ N = 10,\ e = 0.9,\ T_{down,0} = T_{down} = 5\text{min},\ \tau = 1\text{min},\ \rho_{r1} = 0.9 \tag{3.17}$$

and evaluate the constants involved in (3.12). The results are:

- under FBFS, $a_r = 2.7526$, $b_r = -6.0343$;
- under LBFS, $a_r = 0.3807$, $b_r = 0.6366$.

The corresponding $\widehat{lt}_r(\rho_r)$'s calculated according to (3.12) are shown in Figure 3.4, along with the position of the knees (with $\alpha = 1/400$). Clearly, LBFS outperforms FBFS (since CC of the former is below CC of the latter). As one can see, operating both systems at the sweet points leads to $\widehat{TP}^{LBFS}$, which is 5.7% larger than $\widehat{TP}^{FBFS}$, and $\widehat{lt}_r^{LBFS}$, which is 52% smaller than $\widehat{lt}_r^{FBFS}$.

In addition, based on the analysis of 100 systems mentioned in Subsection 3.2, we obtain:

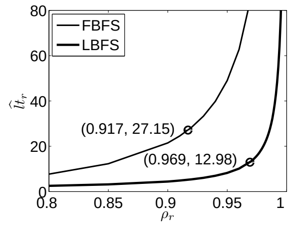


Figure 3.4: Characteristic curves of system (3.17) under FBFS and LBFS

Observation 3.2 *For all* 100 *re-entrant lines analyzed,*

(a) $a_r^{LBFS} < a_r^{FBFS}$*, i.e.,* $\rho_{r,knee}^{FBFS} < \rho_{r,knee}^{LBFS}$*; in other words,* $\widehat{TP}_{knee}^{LBFS} > \widehat{TP}_{knee}^{FBFS}$.

(b) $\frac{\widehat{lt}_r^{FBFS}(\rho_r)}{\widehat{lt}_r^{LBFS}(\rho_r)} \xrightarrow{\rho_r \to 1^-} N$*, i.e., for high loads,* $\widehat{lt}_r^{FBFS}$ *is N times larger than* $\widehat{lt}_r^{LBFS}$*, where N is the number of processing stages.*

(c) $\frac{\widehat{lt}_r^{FBFS}(\rho_r)}{\widehat{lt}_r^{LBFS}(\rho_r)} \xrightarrow{\rho_r \to 0^+} 1$*, i.e., for low loads, the lead times of the system with FBFS and LBFS are practically the same.*

While this subsection shows that LBFS outperforms FBFS, reference [8] arrived at the opposite conclusion. However, in [8], the perturbation considered was a catastrophic breakdown of all machines in the BNWC. Under this perturbation, [8] showed that FBFS leads to a faster recovery than LBFS. Thus, the relative advantages/disadvantages of each dispatch policy depend on the model of perturbations considered. Assuming that a statistical (e.g., exponential) reliability model is more prevalent in practice than catastrophic breakdowns, LBFS could be viewed as superior to FBFS.

4 Synchronous Case with Identical Exponential Producing Machines: LT Open-loop Control

In this section, we quantify the set of attainable lead times (i.e., feasible set) and derive formulas for random job release rates that ensure the desired feasible lead time, while maximizing the throughput. Then we extend this result to the deterministic job release.

4.1 Random job release

Expression (3.13) can be used to evaluate job release rate $\hat{e}_{0,knee}$, which ensures the operation at the sweet point. Indeed, combining (3.13) with the second expression of (3.9), we obtain:

$$\hat{e}_{0,knee} = \frac{Me}{N}(1 - \sqrt{\alpha a_r}). \tag{4.1}$$

This can be implemented as releasing a job once per machine cycle-time with probability $\hat{e}_{0,knee}$. The release rate to ensure operation at *any point* of CC is specified by:

Proposition 4.1 *Consider the re-entrant lines defined by assumptions (i)-(v) with $M < N$. Then, the set of feasible lead times, $\mathscr{F}_{\widehat{lt}}$, is given by*

$$\widehat{lt}_r > a_r + b_r. \tag{4.2}$$

For any feasible desired lead time, $lt_d \in \mathscr{F}_{\widehat{lt}}$, the corresponding release rate is given by

$$\hat{e}_0^*(lt_d) = \frac{Me}{N}\Big(1 - \frac{a_r}{lt_d - b_r}\Big). \tag{4.3}$$

With this release rate,

$$\widehat{TP}^* = \frac{\hat{e}_0^*}{\tau}, \ \widehat{WIP}^* = N\hat{e}_0^*\Big(\frac{Ma_r e}{Me - N\hat{e}_0^*} + b_r - 1\Big). \tag{4.4}$$

Proof: As it follows from (3.12), $\widehat{lt}_r(\rho_r)$ is an increasing function of ρ_r. Since $0 < \rho_r < 1$, this implies (4.2).

As for the optimal release rate, from (3.12) it follows that

$$\rho_r^*(lt_d) = 1 - \frac{a_r}{lt_d - b_r}, \tag{4.5}$$

which, using (3.9), leads to (4.3). As far as (4.4) is concerned, clearly, $\widehat{TP}^* = \frac{\hat{e}_0^*}{\tau}$, and, based on Little's law and (3.9), we obtain:

$$\begin{aligned}\widehat{WIP}^* &= \widehat{TP}^*(LT_d - N\tau) = \frac{\hat{e}_0^*}{\tau}(lt_d - 1)N\tau \\ &= \frac{\hat{e}_0^*}{\tau}\Big(\frac{a_r}{1-\rho_r^*} + b_r - 1\Big)N\tau \\ &= N\hat{e}_0^*\Big(\frac{Ma_re}{Me - N\hat{e}_0^*} + b_r - 1\Big).\end{aligned} \tag{4.6}$$

■

The behavior of $\hat{e}_0^*$ as a function of lt_d under both FBFS and LBFS is illustrated in Figure 4.1 for the re-entrant line (3.17), with black dots indicating $(\widehat{lt}_{r,knee}, \hat{e}_0^*(\widehat{lt}_{r,knee}))$. From this figure follows:

Observation 4.1 *Under both FBFS and LBFS, for $lt_d < \widehat{lt}_{r,knee}$, the optimal release rate $\hat{e}_0^*$* (and, therefore, $\widehat{TP}^*$) *is a rapidly increasing function of lt_d. For $lt_d > \widehat{lt}_{r,knee}$, $\hat{e}_0^*$ is practically constant.*

Thus, releasing raw material with the rate beyond the knee is not only unnecessary (since $\widehat{TP}^*$ is practically a constant), but detrimental as well (since $\widehat{WIP}^*$ grows almost linearly in accordance with $\widehat{WIP}^* = \widehat{TP}^*(LT_d - N\tau)$).

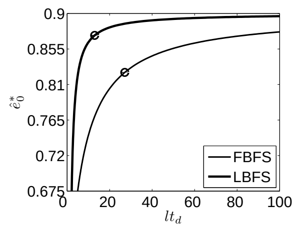


Figure 4.1: Release rate, $\hat{e}_0^*$, as a function of the desired lead time, lt_d

4.2 Deterministic job release

The random, once-per-cycle, job release may be inconvenient for practical implementation. Therefore, below we use the results of Subsection 4.1 to derive deterministic, e.g., once-per-hour or once-per-shift, release policies with guaranteed LT and insignificant losses of the throughput (as compared with once-per-cycle release).

Let $\hat{e}_0^*(lt_d)$ be the release rate calculated according to (4.3). Then, within a release interval, RI (in hours), the deterministic release, $\hat{E}_{RI}^*$ (jobs/release interval), is defined as:

$$\hat{E}_{RI}^* = \lfloor H\hat{e}_0^*(lt_d) \rfloor, \tag{4.7}$$

where $\lfloor x \rfloor$ denotes the largest integer not greater than x, and H is the number of cycles in a release interval, i.e., $H = \frac{60RI}{\tau}$.

While the release according to (4.7) results in the obvious inequality

$$\widehat{LT}(\hat{E}_{RI}^*) < \widehat{LT}(\hat{e}_0^*) + 60RI, \tag{4.8}$$

where $\widehat{LT}(\hat{E}_{RI}^*)$ and $\widehat{LT}(\hat{e}_0^*)$ are the lead times under (4.7) and (4.3), respectively, the losses of the throughput under deterministic release (4.7) are not obvious and must be evaluated. We carry out this evaluation by simulating re-entrant line (3.17) under both FBFS and LBFS. We ran the simulations with $\hat{e}_0^*$ and $\hat{E}_{RI}^*$ according to (4.3) and (4.7), respectively, and evaluated the resulting throughputs, TP_C and TP_{RI} (both in jobs/min), where the subscripts C and RI denote cycle and release interval, respectively. Based on these simulations, we quantified losses in TP by

$$TP_{loss} = \frac{TP_C - TP_{RI}}{TP_C} \times 100\%. \tag{4.9}$$

The results are shown in Tables 4.1 and 4.2 for $RI = 1$hour and for $RI = 8$hour shift, respectively. As one can see, under both FBFS and LBFS, throughput losses for once-per-hour release are significant (due to relatively small $\hat{E}_{RI}^*$), while for once-per-shift release, the losses are negligible. Thus, while the deterministic release may increase the lead time (in accordance with (4.8)), it leads to practically no throughput losses if the release interval is sufficiently large.

Actually, the production loss can be analytically quantified. Since $TP_C = \frac{\hat{e}_0^*}{\tau}$ and $TP_{RI} = \frac{\hat{E}_{RI}^*}{60RI}$,

Table 4.1: Throughput losses due to once-per-hour release

(a) FBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	TP_C	TP_{RI}	TP_{loss} (%)
6	0.2082	12	0.2083	0.2000	3.97
8	0.2170	13	0.2171	0.2167	0.18
20	0.2415	14	0.2414	0.2333	3.36
50	0.2567	15	0.2567	0.2500	2.61
100	0.2630	15	0.2630	0.2500	4.95

(b) LBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	TP_C	TP_{RI}	TP_{loss} (%)
2.4	0.2117	12	0.2117	0.2000	5.54
3.2	0.2299	13	0.2299	0.2167	5.75
8	0.2560	15	0.2560	0.2500	2.35
20	0.2647	15	0.2647	0.2500	5.55
40	0.2674	16	0.2674	0.2667	0.27

Table 4.2: Throughput losses due to once-per-shift release

(a) FBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	TP_C	TP_{RI}	TP_{loss} (%)
6	0.2082	99	0.2083	0.2063	0.97
8	0.2170	104	0.2171	0.2167	0.18
20	0.2415	115	0.2414	0.2396	0.77
50	0.2567	123	0.2567	0.2563	0.18
100	0.2630	126	0.2630	0.2625	0.20

(b) LBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	TP_C	TP_{RI}	TP_{loss} (%)
2.4	0.2117	101	0.2117	0.2104	0.63
3.2	0.2299	110	0.2299	0.2292	0.31
8	0.2560	122	0.2560	0.2542	0.72
20	0.2647	127	0.2647	0.2646	0.04
40	0.2674	128	0.2674	0.2667	0.27

based on (4.9) and taking into account (4.7), the production loss can be calculated by

$$TP_{loss} = \left(1 - \frac{\left\lfloor \frac{60\hat{e}_0^*}{\tau} RI \right\rfloor}{\frac{60\hat{e}_0^*}{\tau} RI}\right) \times 100\%. \tag{4.10}$$

In terms of (4.10), we can prove that the production loss for once-per-shift release is always not larger than that for once-per-hour release. To accomplish this, we establish the following:

Proposition 4.2 *Assume $RI_1 > 0$ and $RI_2 = D \cdot RI_1$, where D is a positive integer constant. Then, for all $x > 0$,*

$$\frac{\lfloor RI_1 \cdot x \rfloor}{RI_1} \leqslant \frac{\lfloor RI_2 \cdot x \rfloor}{RI_2}. \tag{4.11}$$

Proof of Proposition 4.2: Let

$$y = RI_1 \cdot x - \lfloor RI_1 \cdot x \rfloor. \tag{4.12}$$

Then we have $0 \leqslant y < 1$. Thus,

$$\begin{aligned}
&\frac{\lfloor RI_2 \cdot x \rfloor}{RI_2} - \frac{\lfloor RI_1 \cdot x \rfloor}{RI_1} \\
=&\frac{\lfloor RI_2 \cdot x \rfloor - D\lfloor RI_1 \cdot x \rfloor}{RI_2} \\
=&\frac{\lfloor D \cdot RI_1 \cdot x \rfloor - D\lfloor RI_1 \cdot x \rfloor}{RI_2} \\
=&\frac{\lfloor D\lfloor RI_1 \cdot x \rfloor + Dy \rfloor - D\lfloor RI_1 \cdot x \rfloor}{RI_2} \\
=&\frac{\lfloor Dy \rfloor}{RI_2} \geqslant 0.
\end{aligned} \tag{4.13}$$

In other words, (4.11) holds. ■

Based on Proposition 4.2, it is easy to draw the conclusion that the production loss for once-per-shift release is always not larger than that for once-per-hour release, which can be observed in Tables 4.1 and 4.2.

5 Synchronous Case with Identical Exponential Producing Machines: LT Closed-loop Control

5.1 Scenario

The previous section provides methods for estimating job release rates that ensure the desired lead time, if the parameters of the machines are known precisely. In practice, however, this is seldom the case – the machine parameters (e.g., their efficiencies or up- and downtimes) are known only nominally, and their real values may vary. In this situation, the above methods may result in lead times dramatically different from the expected ones. For instance, if the real machine efficiency, e_{real}, is lower than the nominal one, e_{nom}, and the desired lead time, lt_d, is sufficiently large, it may happen that

$$\hat{e}_0^*(lt_d) > \frac{M}{N} \max_{1 \leqslant i \leqslant M} e_{real,i}, \tag{5.1}$$

resulting in an arbitrarily large lead time.

To prevent this situation, feedback control may be used to throttle the job release if the work-in-process in the systems exceeds a certain limit. A number of such control strategies can be utilized. Here, we investigate the one proposed for serial lines in [2], which is simple enough for factory floor implementations. Specifically, we consider a relay-type release policy based on the real-time total work-in-process, WIP_{total}: if at the end of the release interval, RI, the WIP_{total} is below $WIP_{nominal}$, the release takes place; otherwise it does not. In Subsection 5.2 below we formally introduce this control law and in Subsection 5.3 investigate its performance using simulations.

5.2 Control law

Consider a re-entrant line defined by assumptions (i)-(v) with the nominal breakdown and repair rates λ and μ. Let LT_d be the desired lead time (in min). Based on this information, calculate $\hat{e}_0^*$ and $\hat{E}_{RI}^*$ using (4.3) and (4.7), respectively. Also, calculate the nominal total work-in-process using Little's law: since $\widehat{TP}^*$ is given by the first formula in (4.4), and the total waiting time in all buffers is $LT_d - N\tau$, we obtain:

$$\widehat{WIP}_{nominal} = \frac{\hat{e}_0^*}{\tau}(LT_d - N\tau). \tag{5.2}$$

Using these data, introduce the following control law:

$$E(s+1) = \begin{cases} \hat{E}_{RI}^*, & \text{if } WIP_{total}(s) \leqslant \widehat{WIP}_{nominal}, \\ 0, & \text{otherwise}, \end{cases} \tag{5.3}$$

where $s = 0, 1, \ldots,$ is the index of the release interval; $E(s+1)$ is the number of jobs released at the beginning of release interval $s+1$; and $WIP_{total}(s)$ is the real-time total work-in-process in the system at the end of release interval s.

5.3 Performance evaluation

To evaluate the performance of feedback law (5.3), we use the re-entrant line (3.17) as the nominal one and form a real one by increasing or decreasing machine up- and downtimes randomly and equiprobably within ±50% of their nominal values. One realization of the systems, thus constructed, is as follows:

$$e = [0.9154, 0.7678, 0.8708],\ \boldsymbol{T_{down}} = [3.02, 7.15, 4.09]\text{min}. \tag{5.4}$$

We simulate this system with and without feedback control (5.3) under both FBFS and LBFS. The desired lead time, lt_d, for FBFS is selected 10 times larger than that for LBFS (due to Observation 3.2). Based on these simulations, we evaluate the lead times in open- and closed-loop cases (denoted as lt_{OL} and lt_{CL}, respectively). The results are shown in Tables 5.1 and 5.2. Similar results have been obtained for other realizations of real lines, corresponding to the nominal line (3.17). Based on these results, we formulate:

Observation 5.1 *Under both FBFS and LBFS, closed-loop job release according to (5.3) ensures a bounded lt, while the open-loop release (4.7) may result in lt being unbounded.*

Table 5.1: Lead time under control law (5.3) with once-per-hour release

(a) FBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
30	0.2494	14	7.09	7.09	0.2333	0.2333
50	0.2567	15	15.57	14.09	0.2500	0.2497
80	0.2614	15	15.53	15.28	0.2500	0.2500
100	0.2630	15	15.52	15.45	0.2500	0.2500
200	0.2664	15	15.53	15.53	0.2500	0.2500
300	0.2676	16	∞	288.77	0	0.2553
400	0.2682	16	∞	394.55	0	0.2553

(b) LBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
3	0.2265	13	3.59	3.59	0.2167	0.2167
5	0.2464	14	3.86	3.86	0.2333	0.2333
8	0.2560	15	4.87	4.82	0.2500	0.2499
10	0.2590	15	4.87	4.86	0.2500	0.2500
20	0.2647	15	4.87	4.87	0.2500	0.2500
30	0.2665	15	4.87	4.87	0.2500	0.2500
40	0.2674	16	∞	41.72	0.2554	0.2554

Table 5.2: Lead time under control law (5.3) with once-per-shift release

(a) FBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
30	0.2494	119	47.94	47.16	0.2479	0.2455
50	0.2567	123	∞	61.26	0	0.2399
80	0.2614	125	∞	80.53	0	0.2351
100	0.2630	126	∞	91.82	0	0.2347
200	0.2664	127	∞	144.68	0	0.2454
300	0.2676	128	∞	197.23	0	0.2505
400	0.2682	128	∞	250.04	0	0.2543

(b) LBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
3	0.2265	108	22.17	22.17	0.2250	0.2250
5	0.2464	118	24.27	24.26	0.2458	0.2456
8	0.2560	122	29.25	26.51	0.2542	0.2502
10	0.2590	124	∞	28.90	0.2554	0.2433
20	0.2647	127	∞	35.00	0.2554	0.2445
30	0.2665	127	∞	40.27	0.2554	0.2509
40	0.2674	128	∞	45.72	0.2554	0.2540

6 Release-asynchronous Case with Identical Exponential Producing Machines: *LT* Analysis and Control

In this section we analyze release-asynchronous lines, i.e., re-entrant lines similar to those considered in Sections 3-5, but with $\tau_0 \neq \tau$. This analysis is used for investigating a more general case – asynchronous lines – addressed in Section 7.

6.1 *LT* analysis

The approach to *LT* analysis in release-asynchronous lines is similar to that of the synchronous case. The main difference is that the second expression in normalization (3.9), i.e., the relative load factor, is defined as

$$\rho_r^{ras} = \frac{N}{M}\frac{TP_{isol,0}}{TP_{isol}} \tag{6.1}$$

and to ensure existence of a steady state it is assumed that

$$TP_{isol,0} < \frac{M}{N} TP_{isol}, \tag{6.2}$$

where $TP_{isol,0}$ and TP_{isol} are the throughputs of the release and producing machines in isolation, respectively. In terms of normalization (6.1), the CC is defined similar to (3.12), i.e.,

$$\widehat{lt}_r(\rho_r^{ras}) = \frac{a_r}{1 - \rho_r^{ras}} + b_r, \tag{6.3}$$

where a_r and b_r are given in (3.11) (with ρ_r^{ras} instead of ρ_r). Then the position of the knee can be specified by

$$\rho_{r,knee}^{ras} = 1 - \sqrt{\alpha a_r}, \tag{6.4}$$

where, as before, α is a scaling factor.

The accuracy of CC (6.3) has been evaluated by simulating the 100 lines used in Section 3, but with additional parameters τ_0 and τ selected randomly and equiprobably from the interval $[0.8\text{min}, 1.2\text{min}]$. The simulation procedure and quantification of the error remain the same as in Section 3. As a result, we obtained:

Observation 6.1 *For all* 100 *re-entrant lines analyzed,*

(a) under FBFS dispatch, the smallest and the largest ϵ's are 0.90% *and* 29.08%*, respectively; the average error (over the* 100 *systems analyzed) is* 11.03%*;*

(b) under LBFS dispatch, the smallest and the largest ϵ's are 0.07% *and* 1.60%*, respectively; the average error is* 0.56%*.*

Thus, the accuracy of CC (6.3) is similar to that of (3.12).

6.2 *LT* open- and closed-loop control

The equations for open- and closed-loop control of *LT* in release-asynchronous lines are generalizations of those obtained in Sections 4 and 5 for the synchronous case. Specifically, for open-loop control:

- The once-per-cycle release rate that ensures the sweet point operation is

$$\hat{e}_{0,knee} = \frac{M\tau_0 e}{N\tau}(1 - \sqrt{\alpha a_r}). \tag{6.5}$$

- The once-per-cycle release rate that ensures the desired lead time, $lt_d \in \mathscr{F}_{\widehat{lt}}$, can be calculated using

$$\hat{e}_0^*(lt_d) = \frac{M\tau_0 e}{N\tau}\Big(1 - \frac{a_r}{lt_d - b_r}\Big). \tag{6.6}$$

  With this release rate,

$$\widehat{TP}^* = \frac{\hat{e}_0^*}{\tau_0},\ \widehat{WIP}^* = \frac{N\tau\hat{e}_0^*}{\tau_0}\Big(\frac{Ma_r\tau_0 e}{M\tau_0 e - N\tau\hat{e}_0^*} + b_r - 1\Big). \tag{6.7}$$

- The release per release interval, RI (min), is given by

$$\hat{E}_{RI}^* = \lfloor H\hat{e}_0^*(lt_d)\rfloor, \tag{6.8}$$

  where $H = \frac{60RI}{\tau_0}$.

For the closed-loop case, the control law remains the same as in (5.3), but with $\hat{E}_{RI}^*$ defined by (6.8) and $\widehat{WIP}_{nominal}$ given by

$$\widehat{WIP}_{nominal} = \frac{\hat{e}_0^*}{\tau_0}(LT_d - N\tau). \tag{6.9}$$

These relationships are used in Section 7 for control of asynchronous re-entrant lines.

7 Asynchronous Case with Non-identical Exponential Producing Machines: LT Analysis and Control

7.1 Approach

While the first normalization of (3.9) does not seem to be possible to generalize to the asynchronous case (since the total processing time would depend on the machine assignment, which may be different for different parts), the second normalization of (3.9) can be extended to the asynchronous case as

$$\rho_{ra} = \frac{(N)(TP_{isol,0})}{\sum_{i=1}^{M} TP_{isol,i}}, \tag{7.1}$$

where the subscript "a" stands for "asynchronous" and $TP_{isol,i}$, denotes the isolation throughput of machine m_i, $i = 0, 1, \ldots, M$.

Based on (7.1) and similarly with the synchronous case, it is possible to postulate the expression for LT estimate as

$$\widehat{LT} = \frac{A_r}{1 - \rho_{ra}} + B_r, \tag{7.2}$$

(where constants A_r and B_r can be evaluated using equations similar to (3.11)) and show (by simulations) that $\widehat{LT}$ provides a relatively precise estimate for LT. However, we do not pursue this approach here because it does not lead to an analytical expression for $\widehat{WIP}_{nominal}$ necessary for the closed-loop control (5.2). Therefore, we develop a different approach – based on *auxiliary lines*. Specifically, we introduce two auxiliary release-asynchronous lines, which provide a lower- and upper-bounds of LT in the asynchronous case and use them for analysis and control of LT as described in Section 6. This development is carried out below.

7.2 Auxiliary lines

Consider the BNWC-based model of asynchronous re-entrant line with non-identical exponential producing machines defined by assumptions (i)-(v). To ensure the existence of a steady state, assume

$$TP_{isol,0} < \frac{M}{N} \frac{\min\limits_{1 \leqslant i \leqslant M} e_i}{\max\limits_{1 \leqslant i \leqslant M} \tau_i} \tag{7.3}$$

and introduce two auxiliary release-asynchronous re-entrant lines with identical exponential producing machines defined as follows:

$$\underline{\tau} = \min_{1 \leqslant i \leqslant M} \tau_i, \ \underline{e} = \max_{1 \leqslant i \leqslant M} e_i, \ \underline{\mu} = \max_{1 \leqslant i \leqslant M} \mu_i \tag{7.4}$$

and

$$\overline{\tau} = \max_{1\leqslant i\leqslant M} \tau_i,\ \overline{e} = \min_{1\leqslant i\leqslant M} e_i,\ \overline{\mu} = \min_{1\leqslant i\leqslant M} \mu_i. \tag{7.5}$$

The parameters of the release machine (i.e., τ_0, e_0, and $T_{down,0}$) of these auxiliary lines remain the same as in the original asynchronous line.

The lead time of the original line and its auxiliary lines as a function of e_0 can be evaluated by simulations; we denote these lead times as $LT(e_0)$, $\underline{LT}(e_0)$, and $\overline{LT}(e_0)$, respectively. To investigate the relationship among $LT(e_0)$, $\underline{LT}(e_0)$, and $\overline{LT}(e_0)$, we constructed 100 asynchronous re-entrant lines with the parameters selected randomly and equiprobably from the following sets:

$$\begin{aligned}
&M \in [2,5],\ N \in [M+1,10],\ \tau_i \in [0.8\text{min}, 1.2\text{min}],\ i = 1,2,\ldots,M,\ \tau_0 = \min_{1\leqslant i\leqslant M} \tau_i,\\
&e_i \in [0.7, 0.99],\ i = 1,2,\ldots,M,\ e_0 \in [0.8\frac{M}{N}\frac{\min_{1\leqslant i\leqslant M} e_i}{\max_{1\leqslant i\leqslant M} \tau_i}, 0.97\frac{M}{N}\frac{\min_{1\leqslant i\leqslant M} e_i}{\max_{1\leqslant i\leqslant M} \tau_i}],\\
&T_{down,0} \in [1\text{min}, 10\text{min}],\ T_{down,i} \in [T_{down,0}, 1.1T_{down,0}],\ i = 1,2,\ldots,M.
\end{aligned} \tag{7.6}$$

For each of these lines, we formed auxiliary lines according to (7.4) and (7.5) and simulated the resulting 300 lines under both FBFS and LBFS. As a result, we obtain:

Observation 7.1 *For all* 100 *re-entrant lines and their auxiliary lines analyzed under both FBFS and LBFS,*

$$\underline{LT}(e_0) \leqslant LT(e_0) \leqslant \overline{LT}(e_0). \tag{7.7}$$

Thus, the release-asynchronous lines with identical machines (7.4) and (7.5) indeed provide lower- and upper-bounds of LT in the original asynchronous line. These bounds can be used to investigate the issues of CCs, sweet points, and control for asynchronous lines with non-identical exponential machines. Below, we address one of these issues – the closed-loop control.

7.3 *LT* closed-loop control

The approach to closed-loop control of asynchronous lines is based on the following:

- For a given asynchronous re-entrant line with non-identical exponential machines, construct the lower-bound release-asynchronous line with identical producing machines (using (7.4)).

- For this lower-bound line and the desired LT_d, calculate $\hat{e}_0^*$, $\hat{E}_{RI}^*$, and $\widehat{WIP}_{nominal}$ (using (6.6), (6.8), and (6.9), respectively).

- Use these $\hat{E}_{RI}^*$ and $\widehat{WIP}_{nominal}$ in the feedback control law (5.3).

The resulting performance has been investigated by simulations. As an example, consider the asynchronous line defined by the following parameters:

$$\begin{aligned} &M = 5,\ N = 10,\ e_0 = 0.2876,\ \boldsymbol{e} = [0.7989, 0.8549, 0.9109, 0.7897, 0.9432], \\ &T_{down,0} = 4.33\text{min},\ \boldsymbol{T_{down}} = [4.64, 4.57, 4.53, 4.63, 4.60]\text{min}, \\ &\tau_0 = 0.9159\text{min}, \boldsymbol{\tau} = [1.0580, 1.1272, 1.0641, 0.9368, 0.9159]\text{min}. \end{aligned} \tag{7.8}$$

Carrying out the above calculations, we obtain the release-asynchronous line with the parameters

$$\underline{\tau} = 0.9159\text{min},\ \underline{e} = 0.9432,\ \underline{T}_{down} = 4.53\text{min}. \tag{7.9}$$

We simulated system (7.8) with and without feedback control law (5.3) under both FBFS and LBFS for various LT_d. The results, shown in Tables 7.1 and 7.2, indicate that (5.3) is effective in maintaining bounded LT_{CL}, even when the open-loop control results in LT_{OL} being unbounded.

Table 7.1: Lead time of asynchronous line (7.8) under open- and closed-loop control with once-per-hour release

(a) FBFS

LT_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	LT_{OL}	LT_{CL}	TP_{OL}	TP_{CL}
200	0.4479	29	∞	174.64	0	0.3888
400	0.4596	30	∞	308.03	0	0.4217
600	0.4636	30	∞	578.42	0	0.4242
800	0.4656	30	∞	820.97	0	0.4242

(b) LBFS

LT_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	LT_{OL}	LT_{CL}	TP_{OL}	TP_{CL}
20	0.3814	24	39.21	39.09	0.4000	0.3987
40	0.4373	28	∞	58.54	0.4242	0.4041
60	0.4504	29	∞	73.46	0.4242	0.4237
80	0.4563	29	∞	97.37	0.4242	0.4242

Table 7.2: Lead time of asynchronous line (7.8) under open- and closed-loop control with once-per-shift release

(a) FBFS

LT_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	LT_{OL}	LT_{CL}	TP_{OL}	TP_{CL}
200	0.4479	234	∞	534.06	0	0.2438
400	0.4596	240	∞	547.50	0	0.2500
600	0.4636	242	∞	853.92	0	0.3361
800	0.4656	244	∞	858.39	0	0.3375

(b) LBFS

LT_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	LT_{OL}	LT_{CL}	TP_{OL}	TP_{CL}
20	0.3814	199	246.18	245.31	0.4146	0.4016
40	0.4373	229	∞	280.28	0.4242	0.2393
60	0.4504	236	∞	290.76	0.4242	0.2514
80	0.4563	239	∞	316.85	0.4242	0.2988

8 Synchronous Case with Identical Non-exponential Producing Machines: LT Analysis and Control

In this section, we show by simulations that:

- CC expression (3.12) holds for re-entrant lines with identical non-exponential producing machines;
- these curves are practically independent of the up- and downtime distributions involved, as long as their coefficients of variation are the same;
- the feedback control law (5.3) can be used to ensure a bounded LT for non-exponential re-entrant lines as well.

8.1 LT analysis

The approach to LT analysis considered here is similar to that of Section 3. Specifically, we use the re-entrant lines analyzed in Section 3, but with the reliability models being either Weibull, or gamma, or log-normal and with the coefficient of variation (CV) taking values on $\{0.01, 0.1, 0.25, 0.5, 0.75, 1\}$. (We consider $CV \leqslant 1$ since, as it is indicated in [28], manufacturing equipment on the factory floor typically has CV's limited to $(0, 1]$; it is shown in [29] that this is due to the fact that the machine breakdown and repair rates are increasing functions of operating time.) Three examples of such lines are as follows:

$$\begin{aligned}
&\text{Line 1: } M = 3,\ N = 6,\ e = 0.9386,\ e_0 = 0.4505,\ T_{down,0} = T_{down} = 7.72\text{min},\ \tau = 1\text{min},\\
&\text{Line 2: } M = 2,\ N = 9,\ e = 0.8489,\ e_0 = 0.1709,\ T_{down,0} = T_{down} = 3.74\text{min},\ \tau = 1\text{min},\\
&\text{Line 3: } M = 2,\ N = 3,\ e = 0.7427,\ e_0 = 0.3989,\ T_{down,0} = T_{down} = 2.49\text{min},\ \tau = 1\text{min}.
\end{aligned} \tag{8.1}$$

Using (3.11), for each of these lines, we calculated the constants a_r and b_r involved in (3.12). The results are shown in Tables 8.1-8.3. Based on these a_r and b_r, we have calculated CCs shown in Figures 8.1-8.6. From these results we conclude:

Observation 8.1 *For synchronous re-entrant lines with machines obeying Weibull, gamma, and log-normal reliability models, the constants a_r and b_r are practically independent of the reliability model and, therefore, CCs are practically the same for all reliability models considered with any fixed CV.*

We hypothesize, that this conclusion holds for all machines with reliability models characterized by unimodal probability density functions. Note, however, that the issue of CCs in systems with multi-modal pdf's of up- and downtime remains open.

8.2 LT closed-loop control

In this subsection, we demonstrate the closed-loop control of LT in non-exponential re-entrant lines. As an example, we use the same nominal and real systems as in Subsection 5.3 for exponential case. Specifically, we assume that the nominal re-entrant line is given by (3.17) and the real one by (5.4). Since, as it is shown in the previous subsection, machine reliability models do not have much effect on the systems performance, we assume that the machines (3.17) and (5.4) have up- and downtimes distributed according to Weibull distribution. As before, we assume that CV takes value on $\{0.01, 0.1, 0.25, 0.5, 0.75, 1\}$.

As for the control law, we assume that it is given by (5.3) with $\hat{E}^*_{RI}$ and $\widehat{WIP}_{nominal}$ calculated, using the nominal line (3.17), according to (4.7) and (5.2).

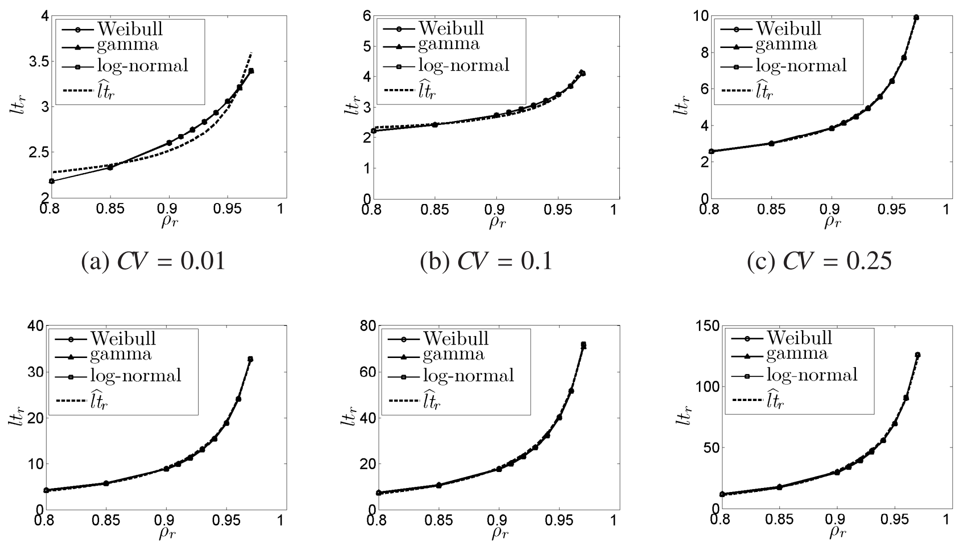
Table 8.1: Constants a_r and b_r for Line 1

(a) FBFS

CV	Reliability model					
	Weibull		gamma		log-normal	
	a_r	b_r	a_r	b_r	a_r	b_r
0.01	0.0465	2.0471	0.0463	2.0473	0.0468	2.0453
0.1	0.0678	1.9938	0.0675	1.9937	0.0675	1.9934
0.25	0.2575	1.2871	0.2583	1.2549	0.2580	1.2465
0.5	1.0020	−0.9454	1.0053	−1.0358	1.0012	−1.1169
0.75	2.2616	−4.3963	2.2504	−4.3515	2.2436	−4.7065
1	3.9905	−8.9382	3.9905	−8.9382	4.0184	−10.0319

(b) LBFS

CV	Reliability model					
	Weibull		gamma		log-normal	
	a_r	b_r	a_r	b_r	a_r	b_r
0.01	0.0113	1.4504	0.0113	1.4504	0.0113	1.4501
0.1	0.0150	1.4442	0.0150	1.4441	0.0150	1.4440
0.25	0.0480	1.3465	0.0480	1.3388	0.0479	1.3370
0.5	0.1785	1.0844	0.1788	1.0586	0.1778	1.0361
0.75	0.4001	0.7393	0.3981	0.7345	0.3959	0.6340
1	0.7050	0.3273	0.7050	0.3273	0.7074	0.0394



(a) *CV* = 0.01 (b) *CV* = 0.1 (c) *CV* = 0.25

(d) *CV* = 0.5 (e) *CV* = 0.75 (f) *CV* = 1

Figure 8.1: Relationships between $\widehat{lt}_r(\rho_r)$ and $lt_r(\rho_r)$ under FBFS dispatch (Line 1)

Simulating system (5.4) with Weibull machines without and with feedback release, we evaluate lt_{OL} and lt_{CL}. The results are shown in Tables 8.4-8.15. Based on these results, we arrive at the same

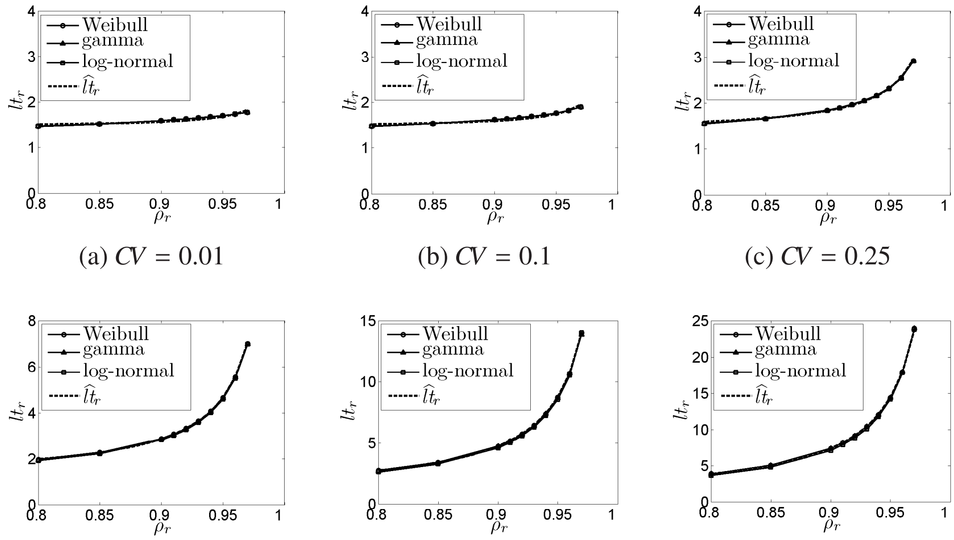
Table 8.2: Constants a_r and b_r for Line 2

(a) FBFS

CV	Reliability model					
	Weibull		gamma		log-normal	
	a_r	b_r	a_r	b_r	a_r	b_r
0.01	0.0306	1.2812	0.0307	1.2815	0.0308	1.2806
0.1	0.0414	1.2451	0.0410	1.2461	0.0410	1.2463
0.25	0.1541	0.8692	0.1547	0.8592	0.1540	0.8598
0.5	0.6177	−0.4439	0.6182	−0.4608	0.6147	−0.4761
0.75	1.3939	−2.5036	1.3958	−2.5400	1.3931	−2.6326
1	2.4873	−5.3954	2.4873	−5.3954	2.4685	−5.5514

(b) LBFS

CV	Reliability model					
	Weibull		gamma		log-normal	
	a_r	b_r	a_r	b_r	a_r	b_r
0.01	0.0084	1.1534	0.0084	1.1537	0.0085	1.1534
0.1	0.0100	1.1516	0.0100	1.1516	0.0099	1.1516
0.25	0.0259	1.1156	0.0259	1.1132	0.0258	1.1131
0.5	0.0935	1.0015	0.0933	0.9949	0.0926	0.9886
0.75	0.2079	0.8515	0.2080	0.8406	0.2065	0.8105
1	0.3691	0.6512	0.3691	0.6512	0.3639	0.5851



(a) CV = 0.01 (b) CV = 0.1 (c) CV = 0.25

(d) CV = 0.5 (e) CV = 0.75 (f) CV = 1

Figure 8.2: Relationships between $\widehat{lt}_r(\rho_r)$ and $lt_r(\rho_r)$ under LBFS dispatch (Line 1)

conclusion as in Subsection 5.3: The feedback raw material release according to (5.3) maintains finite lead time, while the open-loop release may result in lt being unbounded.

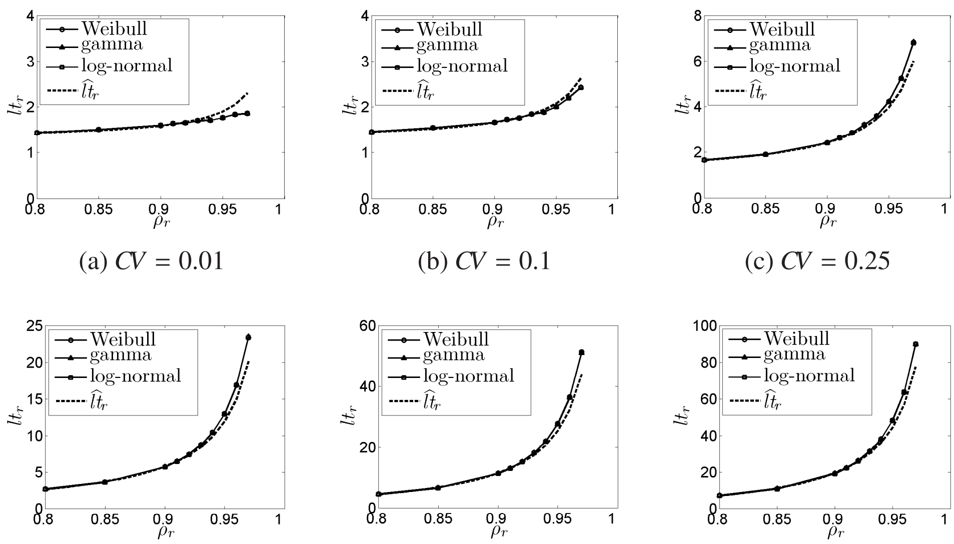
Table 8.3: Constants a_r and b_r for Line 3

(a) FBFS

CV	Reliability model					
	Weibull		gamma		log-normal	
	a_r	b_r	a_r	b_r	a_r	b_r
0.01	0.0867	1.3274	0.0868	1.3271	0.0872	1.3247
0.1	0.0901	1.3244	0.0895	1.3261	0.0894	1.3265
0.25	0.1368	1.2415	0.1351	1.2405	0.1344	1.2414
0.5	0.3897	0.8128	0.3872	0.7963	0.3795	0.7958
0.75	0.8414	0.1550	0.8417	0.1250	0.8182	0.0945
1	1.4836	−0.7392	1.4836	−0.7392	1.4279	−0.8219

(b) LBFS

CV	Reliability model					
	Weibull		gamma		log-normal	
	a_r	b_r	a_r	b_r	a_r	b_r
0.01	0.0353	1.1745	0.0353	1.1743	0.0356	1.1730
0.1	0.0368	1.1735	0.0366	1.1741	0.0365	1.1742
0.25	0.0533	1.1484	0.0527	1.1478	0.0524	1.1480
0.5	0.1418	1.0152	0.1406	1.0082	0.1377	1.0068
0.75	0.3009	0.8163	0.3007	0.8033	0.2915	0.7868
1	0.5278	0.5500	0.5278	0.5500	0.5059	0.5047

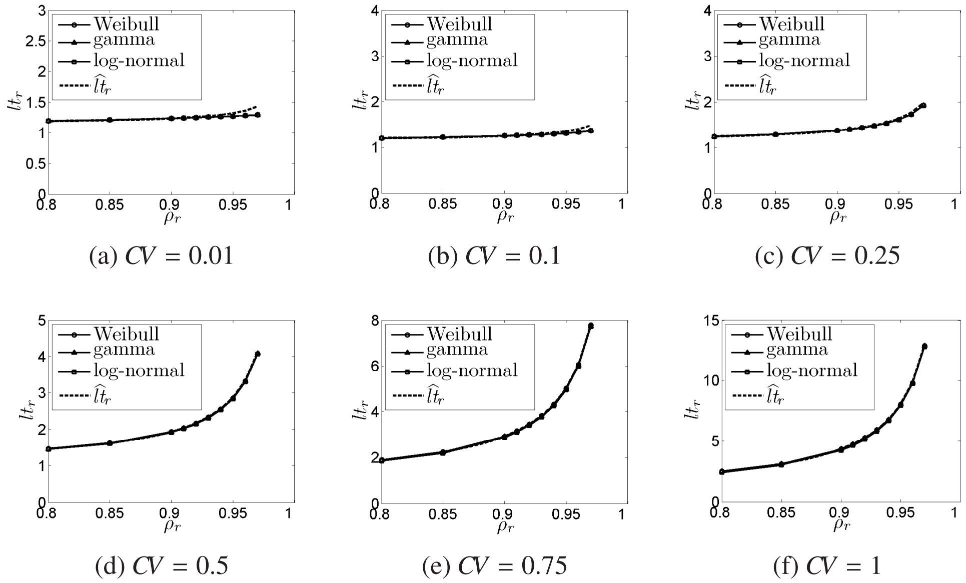


(a) *CV* = 0.01 (b) *CV* = 0.1 (c) *CV* = 0.25

(d) *CV* = 0.5 (e) *CV* = 0.75 (f) *CV* = 1

Figure 8.3: Relationships between $\widehat{lt}_r(\rho_r)$ and $lt_r(\rho_r)$ under FBFS dispatch (Line 2)

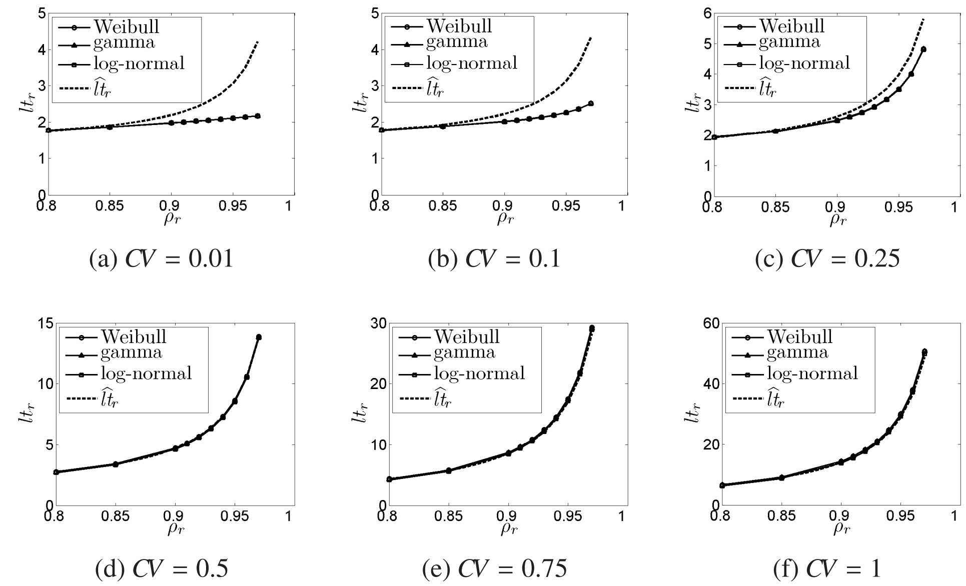
Thus, the results of this section indicate that non-exponential re-entrant lines can be analyzed and controlled using the same techniques as those for the exponential case.



(a) CV = 0.01 (b) CV = 0.1 (c) CV = 0.25

(d) CV = 0.5 (e) CV = 0.75 (f) CV = 1

Figure 8.4: Relationships between $\widehat{lt}_r(\rho_r)$ and $lt_r(\rho_r)$ under LBFS dispatch (Line 2)



(a) CV = 0.01 (b) CV = 0.1 (c) CV = 0.25

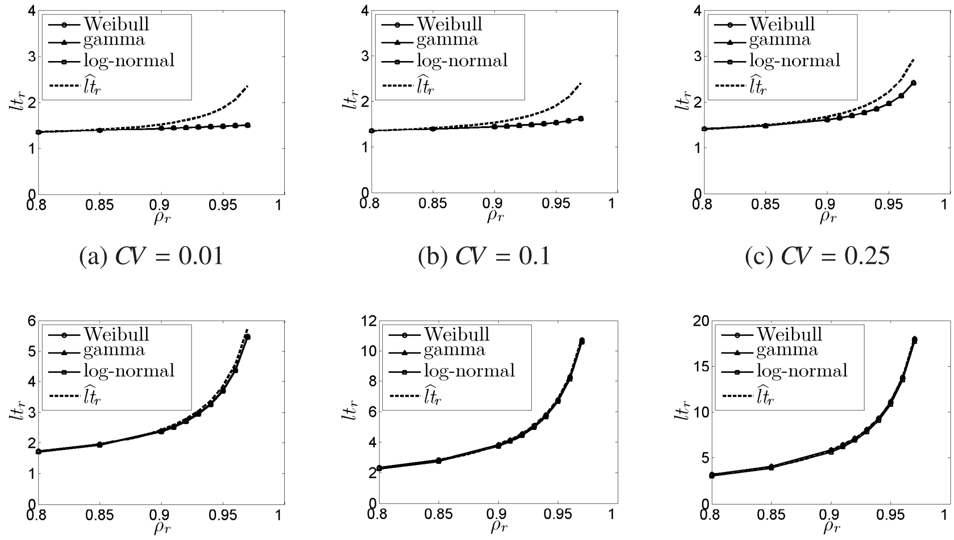
(d) CV = 0.5 (e) CV = 0.75 (f) CV = 1

Figure 8.5: Relationships between $\widehat{lt}_r(\rho_r)$ and $lt_r(\rho_r)$ under FBFS dispatch (Line 3)

9 Conclusions and Future Work

Results reported in this paper lead to the following conclusions concerning re-entrant lines described by the bottleneck workcenter model:

- The characteristic curves (CCs) of re-entrant lines with exponential machines can be quantified analytically by expression (3.12) for the synchronous case and (6.3) for the release-asynchronous case. These expressions contain two constants, which are evaluated empiri-



(a) $CV = 0.01$ (b) $CV = 0.1$ (c) $CV = 0.25$

(d) $CV = 0.5$ (e) $CV = 0.75$ (f) $CV = 1$

Figure 8.6: Relationships between $\widehat{lt}_r(\rho_r)$ and $lt_r(\rho_r)$ under LBFS dispatch (Line 3)

Table 8.4: Lead time of non-exponential re-entrant lines under control law (5.3) with once-per-hour release ($CV = 0.01$)

(a) FBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
12	0.2690	16	∞	12.07	0	0.2226
24	0.2695	16	∞	18.24	0	0.2415
48	0.2698	16	∞	30.74	0	0.2534
96	0.2699	16	∞	77.32	0	0.2554

(b) LBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
1.2	0.2065	12	3.36	3.36	0.2000	0.2000
2.4	0.2679	16	∞	4.81	0.2554	0.2290
4.8	0.2693	16	∞	6.08	0.2554	0.2506
9.6	0.2697	16	∞	10.21	0.2554	0.2554

cally. While for the asynchronous case similar expressions have not been derived, the lower- and upper-bounds, introduced in the paper, can be used to quantify their CCs.

- Using the expressions (or bounds) for CCs, the positions of the sweet points are quantified by (3.13) and (6.4).

- The job release rates, which ensure system operation at the sweet point, are defined by expressions (4.1) and (6.5). Similar expressions for operating at desired point of CC (and,

Table 8.5: Lead time of non-exponential re-entrant lines under control law (5.3) with once-per-shift release ($CV = 0.01$)

(a) FBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
12	0.2690	129	∞	48.97	0	0.1344
24	0.2695	129	∞	55.58	0	0.1564
48	0.2698	129	∞	68.24	0	0.1905
96	0.2699	129	∞	93.41	0	0.2199

(b) LBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
1.2	0.2065	99	20.39	20.39	0.2062	0.2062
2.4	0.2679	128	∞	26.06	0.2554	0.1333
4.8	0.2693	129	∞	27.51	0.2554	0.1792
9.6	0.2697	129	∞	30.02	0.2554	0.2150

Table 8.6: Lead time of non-exponential re-entrant lines under control law (5.3) with once-per-hour release ($CV = 0.1$)

(a) FBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
13	0.2688	16	∞	12.51	0	0.2246
26	0.2694	16	∞	19.38	0	0.2435
52	0.2697	16	∞	33.12	0	0.2545
104	0.2699	16	∞	85.75	0	0.2554

(b) LBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
1.3	0.2484	14	3.75	3.75	0.2333	0.2333
2.6	0.2679	16	∞	4.90	0.2554	0.2328
5.2	0.2692	16	∞	6.28	0.2554	0.2521
10.4	0.2697	16	∞	11.05	0.2554	0.2554

thus, ensuring any feasible lead time) are (4.3) and (6.6). These release rates assume that the parameters of the machines are known precisely.

- If the machine parameters are not known precisely, feedback control law (5.3) can be used to enforce the system behavior close to the desired.

- All methods of lead time analysis and control developed for exponential models can be used for re-entrant lines with Weibull, gamma, and log-normal machine reliability.

Table 8.7: Lead time of non-exponential re-entrant lines under control law (5.3) with once-per-shift release ($CV = 0.1$)

(a) FBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
13	0.2688	129	∞	48.97	0	0.1344
26	0.2694	129	∞	59.41	0	0.1713
52	0.2697	129	∞	73.37	0	0.2005
104	0.2699	129	∞	99.00	0	0.2239

(b) LBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
1.3	0.2484	119	24.30	24.30	0.2479	0.2479
2.6	0.2679	128	∞	26.08	0.2554	0.1340
5.2	0.2692	129	∞	27.53	0.2554	0.1794
10.4	0.2697	129	∞	30.52	0.2554	0.2186

Table 8.8: Lead time of non-exponential re-entrant lines under control law (5.3) with once-per-hour release ($CV = 0.25$)

(a) FBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
14	0.2665	15	6.92	6.92	0.2500	0.2500
28	0.2683	16	∞	20.25	0	0.2451
56	0.2692	16	∞	35.37	0	0.2552
112	0.2696	16	∞	94.03	0	0.2554

(b) LBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
1.4	0.2431	14	3.75	3.75	0.2333	0.2333
2.8	0.2655	15	3.98	3.98	0.2500	0.2500
5.6	0.2683	16	∞	6.47	0.2554	0.2532
11.2	0.2693	16	∞	11.88	0.2554	0.2554

- For non-exponential machines, the variability of LT as a function of job release rate is decreasing with the coefficient of variation of up- and downtimes. Thus, the importance of sweet point operation becomes less prominent when the variability of up- and downtimes is small (e.g., $CV \leqslant 0.1$).

A number of issues related to the topic of this paper, however, remain open. These include:

- Investigation of CCs, sweet points, and release control policies in re-entrant lines with mul-

Table 8.9: Lead time of non-exponential re-entrant lines under control law (5.3) with once-per-shift release ($CV = 0.25$)

(a) FBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
14	0.2665	127	∞	50.01	0	0.1434
28	0.2683	128	∞	58.83	0	0.1769
56	0.2692	129	∞	73.11	0	0.2003
112	0.2696	129	∞	100.98	0	0.2251

(b) LBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
1.4	0.2431	116	23.72	23.72	0.2417	0.2417
2.8	0.2655	127	∞	26.45	0.2554	0.1644
5.6	0.2683	128	∞	28.04	0.2554	0.1977
11.2	0.2693	129	∞	31.07	0.2554	0.2227

Table 8.10: Lead time of non-exponential re-entrant lines under control law (5.3) with once-per-hour release ($CV = 0.5$)

(a) FBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
15	0.2582	15	8.37	8.16	0.2500	0.2496
30	0.2640	15	8.37	8.36	0.2500	0.2500
60	0.2670	16	∞	38.38	0	0.2554
120	0.2685	16	∞	101.68	0	0.2554

(b) LBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
1.5	0.2170	13	3.56	3.56	0.2167	0.2167
3	0.2568	15	4.13	4.13	0.2500	0.2500
6	0.2647	15	4.13	4.13	0.2500	0.2500
12	0.2676	16	∞	12.62	0.2554	0.2554

tiple bottleneck workcenters.

- Investigation of re-entrant lines under dispatch policies other than FBFS and LBFS.
- Investigation of CCs in re-entrant lines with batch processing equipment.
- Investigation of re-entrant lines producing multiple job-types.
- Further investigation of a possibility of analytical evaluation of the two constants involved

Table 8.11: Lead time of non-exponential re-entrant lines under control law (5.3) with once-per-shift release ($CV = 0.5$)

(a) FBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
15	0.2582	123	∞	50.39	0	0.2211
30	0.2640	126	∞	58.95	0	0.1952
60	0.2670	128	∞	73.89	0	0.2072
120	0.2685	128	∞	105.38	0	0.2310

(b) LBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
1.5	0.2170	104	21.37	21.37	0.2167	0.2167
3	0.2568	123	∞	25.69	0.2554	0.2309
6	0.2647	127	∞	27.99	0.2554	0.2070
12	0.2676	128	∞	31.26	0.2554	0.2286

Table 8.12: Lead time of non-exponential re-entrant lines under control law (5.3) with once-per-hour release ($CV = 0.75$)

(a) FBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
30	0.2575	15	11.15	10.52	0.2500	0.2497
60	0.2635	15	11.16	11.14	0.2500	0.2500
120	0.2667	15	11.14	11.14	0.2500	0.2500
240	0.2683	16	∞	227.13	0	0.2554

(b) LBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
3	0.2431	14	3.80	3.80	0.2333	0.2333
6	0.2587	15	4.43	4.42	0.2500	0.2500
12	0.2648	15	4.43	4.43	0.2500	0.2500
24	0.2675	16	∞	25.10	0.2554	0.2554

in the expressions (3.12) and (6.3).

Solution of these problems will result in a relatively complete theory of lead time analysis and control in re-entrant lines.

Table 8.13: Lead time of non-exponential re-entrant lines under control law (5.3) with once-per-shift release ($CV = 0.75$)

(a) FBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
30	0.2575	123	∞	54.97	0	0.2334
60	0.2635	126	∞	73.00	0	0.2209
120	0.2667	127	∞	104.44	0	0.2349
240	0.2683	128	∞	167.41	0	0.2469

(b) LBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
3	0.2431	116	23.73	23.73	0.2417	0.2416
6	0.2587	124	∞	27.24	0.2554	0.2339
12	0.2648	127	∞	30.99	0.2554	0.2326
24	0.2675	128	∞	37.46	0.2554	0.2462

Table 8.14: Lead time of non-exponential re-entrant lines under control law (5.3) with once-per-hour release ($CV = 1$)

(a) FBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
50	0.2569	15	15.16	13.85	0.2500	0.2497
100	0.2631	15	15.18	15.12	0.2500	0.2500
200	0.2664	15	15.14	15.14	0.2500	0.2500
400	0.2682	16	∞	394.35	0	0.2554

(b) LBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
5	0.2464	14	3.86	3.86	0.2333	0.2333
10	0.2590	15	4.87	4.85	0.2500	0.2500
20	0.2647	15	4.86	4.86	0.2500	0.2500
40	0.2674	16	∞	41.72	0.2554	0.2554

References

[1] S. Biller, S. M. Meerkov, and C.-B. Yan, "Raw material release rates to ensure desired production lead time in Bernoulli serial lines," *International Journal of Production Research*, vol. 51, no. 14, pp. 4349–4364, 2013.

[2] S. M. Meerkov and C.-B. Yan, "Production lead time in serial lines: Evaluation, analysis, and control," *IEEE Transactions on Automation Science and Engineering*, vol. 13, no. 2, pp.

Table 8.15: Lead time of non-exponential re-entrant lines under control law (5.3) with once-per-shift release ($CV = 1$)

(a) FBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
50	0.2569	123	∞	61.22	0	0.2401
100	0.2631	126	∞	91.85	0	0.2347
200	0.2664	127	∞	144.56	0	0.2454
400	0.2682	128	∞	250.16	0	0.2543

(b) LBFS

lt_d	$\hat{e}_0^*$	$\hat{E}_{RI}^*$	lt_{OL}	lt_{CL}	TP_{OL}	TP_{CL}
5	0.2464	118	24.27	24.26	0.2458	0.2456
10	0.2590	124	∞	28.91	0.2554	0.2433
20	0.2647	127	∞	34.99	0.2554	0.2445
40	0.2674	128	∞	45.70	0.2554	0.2540

663–675, 2016.

[3] ——, “Cellular production lines with asymptotically reliable Bernoulli machines: Lead time analysis and control,” *IEEE Transactions on Automation Science and Engineering*, vol. 12, no. 2, pp. 764–768, 2015.

[4] O. Rose, “WIP evolution of a semiconductor factory after a bottleneck workcenter breakdown,” in *1998 Winter Simulation Conference*, 1998, pp. 997–1003.

[5] ——, “Estimation of the cycle time distribution of a wafer fab by a simple simulation model,” in *The International Conference on Semiconductor Manufacturing Operational Modeling and Simulation*, 1999, pp. 133–138.

[6] ——, “Why do simple wafer fab models fail in certain scenarios,” in *2000 Winter Simulation Conference*, 2000, pp. 1481–1490.

[7] ——, “Improving the accuracy of simple simulation models for complex production systems,” in *2007 INFORMS Simulation Society Research Workshop*, 2007, pp. 5–8.

[8] C.-B. Yan, M. Hassoun, and S. M. Meerkov, "Equilibria, stability, and transients in re-entrant lines under FBFS and LBFS dispatch and constant release," *IEEE Transactions on Semiconductor Manufacturing*, vol. 25, no. 2, pp. 211–229, 2012.

[9] J. W. Fowler, S. Park, G. T. Mackulak, and D. L. Shunk, "Efficient cycle time-throughput curve generation using a fixed sample size procedure," *International Journal of Production Research*, vol. 39, no. 12, pp. 2595–2613, 2001.

[10] L. Mönch, J. W. Fowler, and S. J. Mason, *Production Planning and Control for Semiconductor Wafer Fabrication Facilities: Modeling, Analysis, and Systems*. New York: Springer, 2013, vol. 52.

[11] R. C. H. Cheng and J. P. C. Kleijnen, "Improved design of queueing simulation experiments with highly heteroscedastic responses," *Operations Research*, vol. 47, no. 5, pp. 762–777, 1999.

[12] F. Yang, B. Ankenman, and B. L. Nelson, "Efficient generation of cycle time-throughput curves through simulation and metamodeling," *Naval Research Logistics*, vol. 54, no. 1, pp. 78–93, 2007.

[13] F. Yang, "Neural network metamodeling for cycle time-throughput profiles in manufacturing," *European Journal of Operational Research*, vol. 205, no. 1, pp. 172–185, 2010.

[14] F. Yang, J. Liu, B. L. Nelson, B. E. Ankenman, and M. Tongarlak, "Metamodeling for cycle time-throughput-product mix surfaces using progressive model fitting," *Production Planning and Control*, vol. 22, no. 1, pp. 50–68, 2011.

[15] J. G. Shanthikumar, S. Ding, and M. T. Zhang, "Queueing theory for semiconductor manufacturing systems: A survey and open problems," *IEEE Transactions on Automation Science and Engineering*, vol. 4, no. 4, pp. 513–522, 2007.

[16] M. Spearman, "An analytic congestion model for closed production systems with IFR processing times," *Management Science*, vol. 37, no. 8, pp. 1015–1029, 1991.

[17] W. J. Hopp and M. L. Spearman, *Factory Physics*, 3rd ed. New York: McGraw-Hill, 2008.

[18] W. J. Hopp, M. L. Spearman, S. Chayet, K. Donohue, and E. S. Gel, “Using an optimized queueing network model to support wafer fab design,” *IIE Transactions*, vol. 34, no. 2, pp. 119–130, 2002.

[19] J. W. Fowler, G. L. Hogg, and S. J. Mason, “Workload control in the semiconductor industry,” *Production Planning and Control*, vol. 13, no. 7, pp. 568–578, 2002.

[20] A. I. Sivakumar, C. Qi, H. K. Tan, and S. K. Lieu, “Job release control in semiconductor wafer fabrication,” *Industrial Engineeering & Management Systems*, vol. 2, no. 2, pp. 169–177, 2003.

[21] S. Kalisch, R. Ringel, and J. Weigang, “Managing WIP and cycle time with the help of loop control,” in *2008 Winter Simulation Conference*, 2008, pp. 2298–2304.

[22] B. Lee, Y. H. Lee, T. Yang, and J. Ignisio, “A due-date based production control policy using WIP balance for implementation in semiconductor fabrications,” *International Journal of Production Research*, vol. 46, no. 20, pp. 5515–5529, 2008.

[23] C. Qi, A. I. Sivakumar, and S. B. Gershwin, “An efficient new job release control methodology,” *International Journal of Production Research*, vol. 47, no. 3, pp. 703–731, 2009.

[24] T. Yang, C.-H. Hsieh, and B.-Y. Cheng, “Lean-pull strategy in a re-entrant manufacturing environment: A pilot study for TFT-LCD array manufacturing,” *International Journal of Production Research*, vol. 49, no. 6, pp. 1511–1529, 2011.

[25] C. K. Eng and L. K. Sin, “CONWIP based control of a semiconductor end of line assembly,” *Procedia Engineering*, vol. 53, pp. 607–615, 2013.

[26] Z. B. Chen, X. W. Pan, L. Li, Q. J. Chen, and W. S. Xu, “A new release control policy (WRELM) for semiconductor wafer fabrication facilities,” in *11th IEEE International Conference on Networking, Sensing and Control*, 2014, pp. 64–68.

[27] J. Li and S. M. Meerkov, *Production Systems Engineering*. New York: Springer, 2009.

[28] R. R. Inman, "Empirical evaluation of exponential and independence assumptions in queueing models of manufacturing systems," *Production and Operations Management*, vol. 8, no. 4, pp. 409–432, 1999.

[29] J. Li and S. M. Meerkov, "On the coefficients of variation of uptime and downtime in manufacturing equipment," *Mathematical Problems in Engineering*, vol. 2005, no. 1, pp. 1–6, 2005.