Quasilinear Control Theory

Performance Analysis and Design in Feedback Systems with Nonlinear Actuators and Sensors

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September 9, 2009

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Foreword

Purpose: This volume is devoted to the study of feedback control of the so-called *linear plant/nonlinear instrumentation* (LPNI) systems. Such systems appear naturally in situations where the plant can be viewed as linear but the instrumentation, i.e., actuators and sensors, can not. For instance, when a feedback system operates efficiently and maintains the plant close to a desired operating point, the plant may be linearized, but the instrumentation may not, because to counteract large perturbations or to track large reference signals, the actuator may saturate and the nonlinearities in sensors, e.g., quantization and dead zones, may be activated.

The problems of stability and oscillations in LPNI systems have been studied for a long time. Indeed, the theory of absolute stability [.-.] and harmonic balance method [.-.] are among the best known topics of control theory. More recent literature also addressed LPNI scenarios, largely from the point of view of stability [.,.] and anti-windup [.,.]. However, the problems of performance analysis and design, e.g., reference tracking and disturbance rejection, have not been investigated in sufficient details. This volume is intended to contribute to this end by providing methods for designing *linear controllers* that ensure the desired *performance* of closed loop LPNI systems.

The methods developed in this work are similar to the usual linear system techniques, e.g., root locus, LQR, LQG, etc., modified appropriately and coupled with additional equations that account for the instrumentation nonlinearities. Therefore, we refer to these methods as quasilinear and to the resulting area of control as *quasilinear control systems*.

This volume is intended as a supplementary textbook for the standard courses on Linear or Nonlinear Systems and Control and for self-study by practicing engineers involved in the analysis and design of control systems with nonlinear instrumentation.

Problems addressed: Consider the SISO system shown in Figure 0.1, where $P(s)$ and $C(s)$ are the transfer functions of the plant and the controller, and $f(\cdot)$, $g(\cdot)$ are static nonlinearities characterizing the actuator and the sensor, while r, d, u, y and y_m are the reference, disturbance, control, plant output, and sensor output, respectively. In the framework of this system and its MIMO generalizations, this volume considers the following problems:

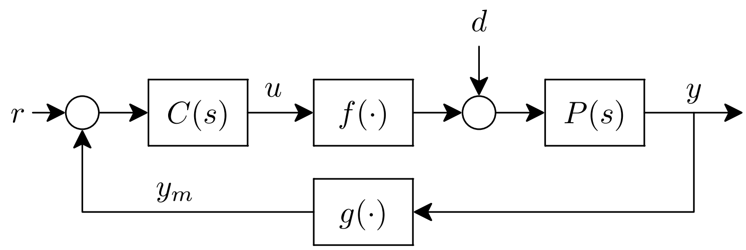

Figure 0.1: Linear plant/nonlinear instrumentation control system

P1. *Performance analysis:* Given $P(s)$, $C(s)$, $f(\cdot)$ and $g(\cdot)$ quantify the quality of reference tracking and disturbance rejection.

P2. *Narrow-sense design:* Given $P(s)$, $f(\cdot)$ and $g(\cdot)$, design a controller $C(s)$ so that the quality of reference tracking and disturbance rejection meets specifications.

P3. *Wide-sense design:* Given $P(s)$, design a controller $C(s)$ and select instrumentation $f(\cdot)$ and $g(\cdot)$ so that reference tracking and disturbance rejection meet specifications.

P4. *Partial performance recovery:* Let $C_\ell(s)$ be a controller, which is designed under the assumption that the actuator and the sensor are linear and which meets reference tracking and disturbance rejection specifications. Given $C_\ell(s)$, select $f(\cdot)$ and $g(\cdot)$ so that the performance degradation is guaranteed to be less than a given bound.

P5. *Complete performance recovery:* Given $f(\cdot)$ and $g(\cdot)$, modify $C_\ell(s)$ so that performance degradation does not take place.

This volume provides conditions under which solutions of these problems exist and equations and algorithms that can be used to calculate these solutions.

Nonlinearities considered: We consider actuators and sensors characterized by arbitrary piece-wise continuous single-valued scalar functions. For example, we address saturating actuators,

$$f(u) = \text{sat}_\alpha(u) = \begin{cases} \alpha, & u > +\alpha \\ u, & -\alpha \le u \le \alpha \\ -\alpha, & u < -\alpha, \end{cases} \tag{0.1}$$

quantized sensors,

$$\begin{aligned} g(y) = \tfrac{\Delta}{2} \sum_{k=1}^{m} [&\text{sgn}(2y + \Delta(2k-1)) \times \\ &\text{sgn}(2y - \Delta(2k-1))], \end{aligned} \tag{0.2}$$

where Δ is the quantization interval, and sensors with a deadzone of width Δ,

$$g(y) = \begin{cases} y - \frac{\Delta}{2}, & y > +\frac{\Delta}{2} \\ 0, & -\frac{\Delta}{2} \leq y \leq \frac{\Delta}{2} \\ y + \frac{\Delta}{2}, & u < -\frac{\Delta}{2}. \end{cases} \tag{0.3}$$

The methods developed here are *modular* in the sense that they can be modified to account for any instrumentation nonlinearity just by replacing the general function representing the nonlinearity by a specific one corresponding to the actuator or sensor in question.

Main difficulty: LPNI systems are described by relatively complex nonlinear differential equations. Unfortunately, these equations cannot be treated by the methods of modern nonlinear control theory [.-.] since they assume that the control signal enters the state space equations in an affine manner and, thus, saturation and other nonlinearities in actuators are excluded. Therefore, a different approach to treat LPNI control systems is necessary.

Approach: The approach of this volume is based on the method of *stochastic linearization*. This method was introduced in [.] and [.] and applied widely in various areas of engineering. A modern account of this method and its applications can be found in [.-.].

Stochastic linearization is applicable to dynamical systems with random parametric or exogenous excitations. In the scenario of LPNI control systems, it assumes that the references and the disturbances are random. While this assumption is typical for the case of disturbances, reference signals are often modeled as deterministic, e.g., step, ramp, etc. In some applications, however, references can more readily be viewed as random than as deterministic. Indeed, in the problem of hard disk drive the reference signal (i.e., read/write positions) is often modeled as a Gaussian random process [.]. Similarly, in the aircraft homing problem, target maneuvers, which play a role of a reference, are typically viewed as random [.]. Thus, to apply stochastic linearization, we assume throughout this volume that both references and disturbances are random. However, several results on tracking step references and the so-called slanted-step references are also obtained.

According to stochastic linearization, the static nonlinearities in the feedback loop are replaced by *equivalent* or *quasilinear* gains N_a and N_s (see Figure 2, where $\hat{u}$, $\hat{y}$ and $\hat{y}_m$ replace u, y and y_m). Unlike the usual Jacobian linearization, the resulting approximation is global, i.e., it approximates the original system not only for small but for large signals as well. The price to pay is that the gains, N_a and N_s depend not only on the nonlinearities $f(\cdot)$ and $g(\cdot)$, but also on all other elements of Figure 1, including the transfer functions and the exogenous signals, since, as it turns out, N_a and N_s are functions of the standard deviations, σ_u and σ_y, of u and y, respectively, i.e., $N_a = N_a(\sigma_u)$ and $N_s = N_s(\sigma_y)$. Therefore, we refer to the system of Figure 2 as *quasilinear control system*. Systems of this type are the main topic of study in this volume.

Thus, instead of assuming that a linear system represents the reality, as it is in linear control, we assume that a quasilinear system represents the reality and carry out control-theoretic developments, which parallel those of linear control theory, leading to what we call *quasilinear control* (QLC) *theory*.

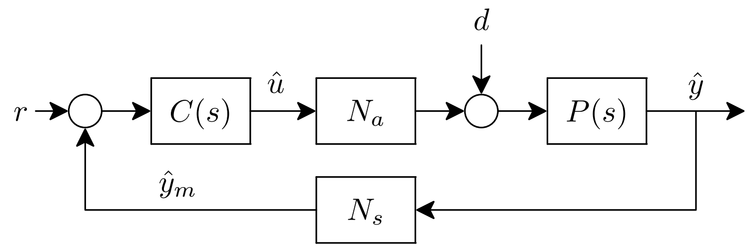

Figure 0.2: Quasilinear control system

The question of accuracy of stochastic linearization, i.e., the precision with which the system of Figure 0.2 approximates that of Figure 0.1, is clearly of importance. Unfortunately, no general results in this area are available. However, various numerical and analytical studies indicate that if the plant, $P(s)$, is low-pass filtering, the approximation is well within 10% in terms of the variances of y and $\hat{y}$ and u and $\hat{u}$ [.,.]. More details on stochastic linearization and its accuracy are included in Chapter 2 below. It should be noted that stochastic linearization is somewhat similar to the method of harmonic balance [.,.] with $N_a(\sigma_{\hat{u}})$ and $N_s(\sigma_{\hat{y}})$ playing the roles of describing functions.

Book organization: The book consists of ten chapters. Chapter 1 places LPNI systems and quasilinear control in the general field of control theory. Chapter 2 describes the method of stochastic linearization. Chapters 3 and 4 are devoted to analysis of quasilinear control systems from the point of view of reference tracking and disturbance rejection, respectively. Chapters 5 and 6 also address tracking and disturbance rejection problems, but from the point of view of design; both wide- and narrow-sense design problems are considered. Chapter 7 addresses the issues of performance recovery. Chapter 8 describes the *QLC Toolbox*, which is a set of user-friendly Matlab programs that implement the methods and algorithms developed in this book. Chapter 9 provides a summary fundamental facts of quaslinear control theory. Finally, Chapter 10 includes the proofs of all formal statements included in the book. Chapters 2-7 conclude with problems for homework assignments and annotated bibliography.

Intended audience, outcomes, and prerequisites: This volume is intended as a supplementary textbook for graduate and advanced undergraduate students, interested in linear and nonlinear control. It shows the students how the techniques, included in the standard control textbooks, (e.g., root locus, LQR/LQG, $\mathcal{H}_\infty$, etc.) can be extended to constructively analyze and design closed loop LPNI systems. In addition, practicing engineers might find it useful to learn the quasilinear methods, since, on one hand, they are not too different from those used in practice (i.e., PID controller design) and, on the other hand, allow for taking into account instrumentation nonlinearities at the initial stage of design, thus avoiding or reducing the need for re-design and simulations.

As an outcome, the readers will acquire rigorous and practical knowledge for designing control systems with ubiquitous nonlinearities in the feedback loop.

The only prerequisite for the material included in this supplementary textbook is a course on Linear Systems and Control. The material from Nonlinear Systems and Control may be desirable but by no means necessary.

Advice to instructors:

Acknowledgements: