EECS 442 – Computer vision

Shape from reflections

Special topic
Recovering the shape of an object

Visual cues: texture • shading • contours • shadows
Recovering the shape of an object

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Number of observers: monocular • multiple views
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**Visual cues:** texture • shading • contours • shadows

**Number of observers:** monocular • multiple views

**Active lighting:** laser stripes • structured lighting patterns • shadows

A taxonomy
Recovering the shape of an object

**Visual cues:** texture • shading • contours

**Number of observers:** monocular • multiple views

**Active lighting:** laser stripes • structured lighting patterns • shadows

**Limitation:** Assumptions on surface reflectance function
Lambertian surfaces

Non Lambertian surfaces
Recovering the shape of an object

**Visual cues:** texture • shading • contours

**Number of observers:** monocular • multiple views

**Active lighting:** laser stripes • structured lighting patterns • shadows

**Limitation:** Assumptions on surface reflectance function

A taxonomy

- Specular
- Transparent
Recovering the shape of an object

**Visual cues:** texture • shading • contours

**Number of observers:** monocular • multiple views

**Active lighting:** laser stripes • structured lighting patterns • shadows

**Limitations:**
- No intrinsic surface features: textures – shading...
- No clear contours
Recovering the shape of an object

Visual cues: texture • shading • contours

Number of observers: monocular • multiple views

Active lighting: laser stripes • structured lighting patterns • shadows

• Epipolar geometry doesn’t work

Stereo matching algorithms fail!
Recovering the shape of an object

**Visual cues:** texture • shading • contours

**Number of observers:** monocular • multiple views

**Active lighting:** laser stripes • structured lighting patterns • shadows

**Limitations:** Reflections are a nuisance!
A new perspective

Johann Erdmann Hummel -- *The Granite Bowl in the Berlin Lustgarten* [1831]

Use specular reflections as additional visual cue to estimate the surface’s shape

Savarese, et al. CVPR 01, IJCV 07
Recover local shape of mirror surface by measuring the deformation of a reflected scene.
A new perspective

Scene

Mirror surface

Image
A new perspective

Explicit mapping: local scene patch $\rightarrow$ reflected image patch

Goal: study this mapping

1. Smooth surface
2. Calibrated scene
Mirror surface

$P(t) = P_o + t \Delta P$

$\mathbf{f}(t) \in \mathbb{R}^3$

$E$ understood

$\mathbf{f}(t) \rightarrow \mathbf{f}_i(t) \in \mathbb{R}^2$

How to find $f(t)$?
A background note....

How to find $f_o$?

For spherical mirror: Alhazen’s problem

Recently Peter M. Neumann proved that the problem is insoluble using a "compass-and-straightedge" construction because the solution requires extraction of a cube root.

Alhazen was an Iranian Philosopher and Physicist. Through his extensive researches on optics, he has been considered as the father of modern Optics.

For spherical mirror:

- Alhazen’s problem
- Recently, Peter M. Neumann proved that the problem is insoluble using a "compass-and-straightedge" construction because the solution requires extraction of a cube root.
- Alhazen was an Iranian Philosopher and Physicist. Through his extensive researches on optics, he has been considered as the father of modern Optics.

$\text{How to find } f_o$?

$\text{For spherical mirror: Alhazen’s problem}$

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$\text{Alhazen was an Iranian Philosopher and Physicist. Through his extensive researches on optics, he has been considered as the father of modern Optics.}$
Mirror surface

- Assume $f_0$ is known - how about $\dot{f}(t), \ddot{f}(t), \ldots$ at $t_0$?

Why? $f(t) \approx f_0 + \dot{f}(t_0)(t - t_0) + \frac{1}{2} \ddot{f}(t_0)(t - t_0)^2 + ...$
Mirror surface

Image

\[ P(t) = P_0 + t \Delta P \]

GOAL: \( \dot{f}(t_o), \ddot{f}(t_o), \ldots \)

TOOLS:
- Lagrange Multiplier theorem
- Implicit Function Theorem
- Chen and Arvo, 2000
A new reference system \([u \ v \ w]\)

\[w = \text{Normal } N\]

\[v \perp \text{P-O-}f_o \text{ plane}\]

\[u = v \times w\]
Surface expanded in Monge form:

\[ w = \frac{1}{2} (au^2 + 2cuv + bv^2) + \frac{1}{3!} (eu^3 + fu^2v + guv^2 + hv^3) + \ldots \]
Surface expanded in Monge form:

\[ w = \frac{1}{2} (au^2 + 2cuv + bv^2) + \frac{1}{3!} (eu^3 + fu^2v + guv^2 + hv^3) + \ldots \]
\[ \gamma \]

\[ o B \left( f \right) = \gamma \]

Observation $c b, a, O, f_0$

Similarly, for:

\[ \frac{df_0(t)}{dt} = \frac{df}{dt} = \hat{f}(t_0), \ldots \]

Summary

- **Mapping**
  
  \[ f : t \in \mathbb{R} \rightarrow f_0 \in \mathbb{R}^3 \]

- **GOAL**: \( \frac{df_0(t)}{dt}, \frac{d^2f_0(t)}{dt^2}, \ldots \)

\( w = \text{Normal } N \)

\( v \perp \text{P-O-f}_0 \text{ plane} \)

\( u = v \times w \)

**Monge form**

\[
\begin{align*}
  w &= \frac{1}{2}(au^2 + 2cuv + bv^2) + \\
  &+ \frac{1}{3}(eu^3 + fu^2v + guv^2 + hv^3) + \ldots
\end{align*}
\]

\[ \text{Similarly, for:} \]

\[ \ddot{f}(t_0) = B^{-1} \gamma \]

\[ \{ f_0, O, a, b, c \} \]

\( \text{Local surface, scene geometry} \)
Mirror surface

\[
f : u, v \in \mathbb{R} \rightarrow f_i(u, v) \in \mathbb{R}^3
\]

\[
f_i(u, v) = f_{io} + \dot{f}_u u + \dot{f}_v v + \frac{1}{2} \ddot{f}_{uu} u^2 + \frac{1}{2} \ddot{f}_{vv} v^2 + f_{uv} uv + \ldots
\]

Local surface, scene geometry
Mirror surface

Conclusion:
image patch = F(scene patch, local shape)

... up to second order
Tangent as function of 2nd and 3rd order parameters

Monge form

\[ w = \frac{1}{2} \left( au^2 + 2cuv + bv^2 \right) + \]
\[ \frac{1}{3!} \left( eu^3 + fu^2v + guv^2 + hv^3 \right) + \ldots \]

Observation

\[ f_i(t_o) = A^{-1} \beta \]

\[ f_o, O, P_o, \Delta P \]

\[ f_o, O, a, b, c \]
Tangent as function of 2\textsuperscript{nd} and 3\textsuperscript{rd} order parameters

Monge form

\[ w = \frac{1}{2}(au^2 + 2cuv + bv^2) + \]
\[ \frac{1}{3!}(eu^3 + fu^2v + guv^2 + hv^3) + \ldots \]

\[ f_i(t_o) = A^{-1}\beta \]

\[ f_o, O, P_o, \Delta P \]

Observation

\[ f_o, O, a, b, c \]
Theorem: Convex paraboloids are always non-singular
Singular configurations

\[ \dot{f}_i(t) = A^{-1} \beta(t) \]
Inverse problem:

From the mapping: reflected image patch $\rightarrow$ local shape

Measurements $\rightarrow$ Unknown
Known quantities:

- Scene
- Camera
Measured quantities:

• Point position Q
• Orientation of curves at Q
Measured quantities:

- Point position Q
- Orientation of curves at Q
- Scale – distribution of points in a neighborhood of Q

Equivalent to $\dot{f_i}(t), \ddot{f_i}(t), \ldots$ at Q
Local geometry we wish to recover:

- Position of the reflection point \( M \)
- Normal of the surface at \( M \)
- Curvature of the surface at \( M \)
- Third order surface parameters around \( M \)
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| • 1 point $Q_0$  
• Orientation along 3 lines at $Q_0$ | ✔               | ✔             | ✔             | up to 1 unknown          |                             |
| • 3 points    | ✔               | ✔             | ✔             |                  | ✔                          |
| • 5 points    | ✔               | ✔             | ✔             |                  | ✔                          |
### Surface Geometry Measurements

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| • 1 point $Q_o$  
• Orientation along 3 lines at $Q_o$ | ✓ | ✓ | ✓ | up to 1 unknown | |
| • 3 points    | ✓ | ✓ | ✓ | ✓ | ✓ |
| • 5 points    | ✓ | ✓ | ✓ | ✓ | ✓ |
We know:

- \( P_o \) (scene point)
- \( O_c \) (center of the camera)
- \( Q_o \) (measured reflect. point)

We don’t know:

- \( M_o \) (reflect. point)

But:

\[
M_o = S \hat{Q}_o
\]

line of sight constraint
\[ \mathbf{O}_c - \mathbf{M}_o = \mathbf{s} \mathbf{Q}_o \]

How about \( N \)?

Reflection constraints

- \( N \in \mathbf{O}_c - \mathbf{M}_o - \mathbf{P}_o \) plane

Summary

- Distance: \( s = \| \mathbf{O}_c - \mathbf{M}_o \| \)
- Line of sight constraint:
Reflection constraints

- $N \in \text{O}_c\text{-M}_o\text{-P}_o$ plane
- $\theta_i = \theta_r$

\[
N = F(s)
\]

Summary

- distance: $s = \|O_c - M_o\|
- line of sight constraint:
  \[
  M_o = s \hat{Q}_o
  \]
- The principal plane defined by $P_o, O_c, Q_o$

How about $N$?
CONCLUSION:
Surface position ($M_o$) and orientation ($N$) up to 1 unknown

Summary

- distance: $s = \|O_c - M_o\|

- line of sight constraint:

$$M_o = s\hat{Q}_o$$

- The principal plane defined by $P_o$, $O_c$, $Q_o$

- $N = F(s)$
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\[ P(t) = P_0 + t \Delta P \]

We need a further constraint!

\[ f_i(t_o) = A^{-1} \beta \]

Observation

\[ \hat{\beta}, \| f_i \|, \hat{\phi} \]

\[ M_o, O_c, P_o, \Delta P \]

\[ s, (unknown) \]

\[ w = \frac{1}{2}(au^2 + 2cuv + bv^2) + \ldots \]

\[ M_o = s\hat{Q}_o \]

\[ N = F(s) \]

\[ s = \| O_c - M_o \| \]

\[ 4 \text{ unknowns!} \]

\[ a, b, c \text{ (unknown)} \]
\[ s = \|O_c - M_o\| \]
\[ M_o = s\hat{Q}_o \]
\[ N = F(s) \]
\[ w = \frac{1}{2}(au^2 + 2cuv + bv^2) + \cdots \]
\[ F(\hat{\phi}, a, b, c, s) \]

\[ \hat{P}_1(t) = P_o + t\Delta P_1 \]
\[ \hat{P}_2(t) = P_o + t\Delta P_2 \]
\[ \hat{P}_N(t) = P_o + t\Delta P_N \]

\[ f_i^1 = A^{-1}\beta_1 \]
\[ f_i^2 = A^{-1}\beta_2 \]
\[ \vdots \]
\[ f_i^N = A^{-1}\beta_N \]

\[ H \begin{bmatrix} g \end{bmatrix}_{N \times 3} = 0 \]

\[ \det(H^TH) = \Psi'(s) \neq 0 \]

\[ a, b, c, s \]
Rank THEOREM: for $s = s^*$, rank($H$) = 2

$s^*$ = actual value of $s$

$H_{Nx3} \mathbf{g} = 0$

$\det(H^T H)_{s=s^*} = 0$

Summary

- $s = \|O_c - M_o\|$
- $M_o = s\hat{Q}_o$
- $N = F(s)$
- $w = \frac{1}{2}(au^2 + 2cuv + bv^2) + \cdots$
- $F(\hat{\phi}, a, b, c, s)$
$$\log_{10}\det(H^TH)$$

$$\det(H^TH)_{s=s^*} = 0 \rightarrow s^*$$
CONCLUSION:
Surface position ($M_o$) and orientation ($N$) can be computed using at least 3 orientations

Summary

- $s = \| O_c - M_o \|
- M_o = s\hat{Q}_o \quad N = F(s)
- w = \frac{1}{2}(au^2 + 2cuv + bv^2) + \cdots
- F(\phi, a, b, c, s)
- $H_{nx3} g = 0$
- $det(H^T H)_{s=s^*} = 0 \Rightarrow s^*$
• From the rank theorem
  \[ \text{rank}(H) = 2 \]

\[ \begin{align*}
  H_{3 \times 3} \mathbf{g} &= 0 \\
  \hat{\phi}_1, \hat{\phi}_2, \ldots, \hat{\phi}_N, s^* \\
  a,b,c, s^* \\
\end{align*} \]

- \[ H \mathbf{g} = 0 \]
- \[ a = a_o + r a_1 \]
- \[ b = b_o + r b_1 \]
- \[ c = c_o + r c_1 \]
Summary

- \( M_o = s\hat{Q}_o \)
- \( N = F(s) \)

\[
H g = 0
\]

Rank THEOREM:
for \( s = s^* \), rank(\( H \)) = 2

If \( s \neq s^* \), in general rank(\( H \)) = 3

\[
\text{det}(H^T H)\bigg|_{s=s^*} = 0 \Rightarrow s^*
\]

Theorem: There exists a family of 2nd order surface parameters for which the observed tangents are invariant w.r.t. surface curvature
Second order ambiguity

\[
\begin{align*}
1a + 1b &= 1c \\
1b + 1c &=
\end{align*}
\]

\[
\begin{align*}
a &= a_o + ra_1 \\
b &= b_o + rb_1 \\
c &= c_o + rc_1
\end{align*}
\]
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Some experimental results

(3 orientations case)
Mirror

Scene

0.2% error in position estimate
2% error in the curvature estimate
Experimental results

(2 orientations + scale case)
Conclusions

• Explicit local relationship between scene, shape and reflections

• Local surface shape can be recovered up to third order...

... if and only if

- the scene is calibrated,
- at least three (five) points are available in a neighborhood
Other path minimization approaches
Dense reconstruction of mirror surfaces

S. Rozenfeld et al. Dense mirroring surface recovery from 1D homographies and sparse Correspondences, CVPR 07

Such pairs are used to estimate a 1D homography
Dense reconstruction of mirror surfaces
Recover geometry of mixed specular-diffuse objects from two frames.

Specular flow

(1) Recover radius and location of the sphere
Further reading:

- Voxel Carving for Specular Surfaces - T. Bonfort and P. Sturm
- Shape from Distortion: 3D Range Scanning of Mirroring Objects - M. Tarini, H. Lensch, M. Gösle, HP. Seidel
- Reconstructing Curved Surfaces From Specular Reflection Patterns - M. Halstead, B. Barsky, S. Klein R. Mandell
- PDE Based Shape from Specularities - J. Solem, H. Aanæs, A. Heyden
- Acquiring a Complete 3D Model from Specular Motion - J.Y. Zheng and A. Murata
- Structured Highlight Inspection of Specular Surfaces - A.S. Sanderson, L.E. Weiss, S.K. Nayar
Next lecture: affine structure from motion