EECS 442 – Computer vision

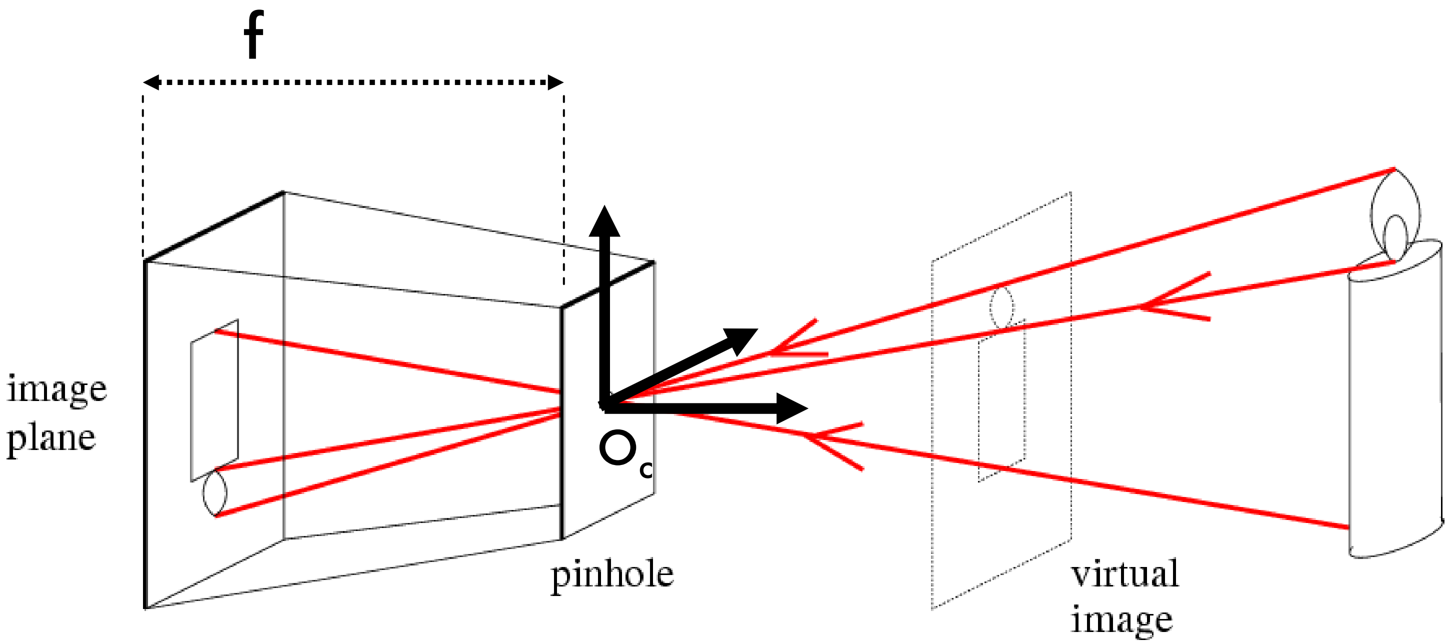
Camera Calibration

- Review camera parameters
- Camera calibration problem
- Example

Reading: [FP] Chapter 3
[HZ] Chapter 7

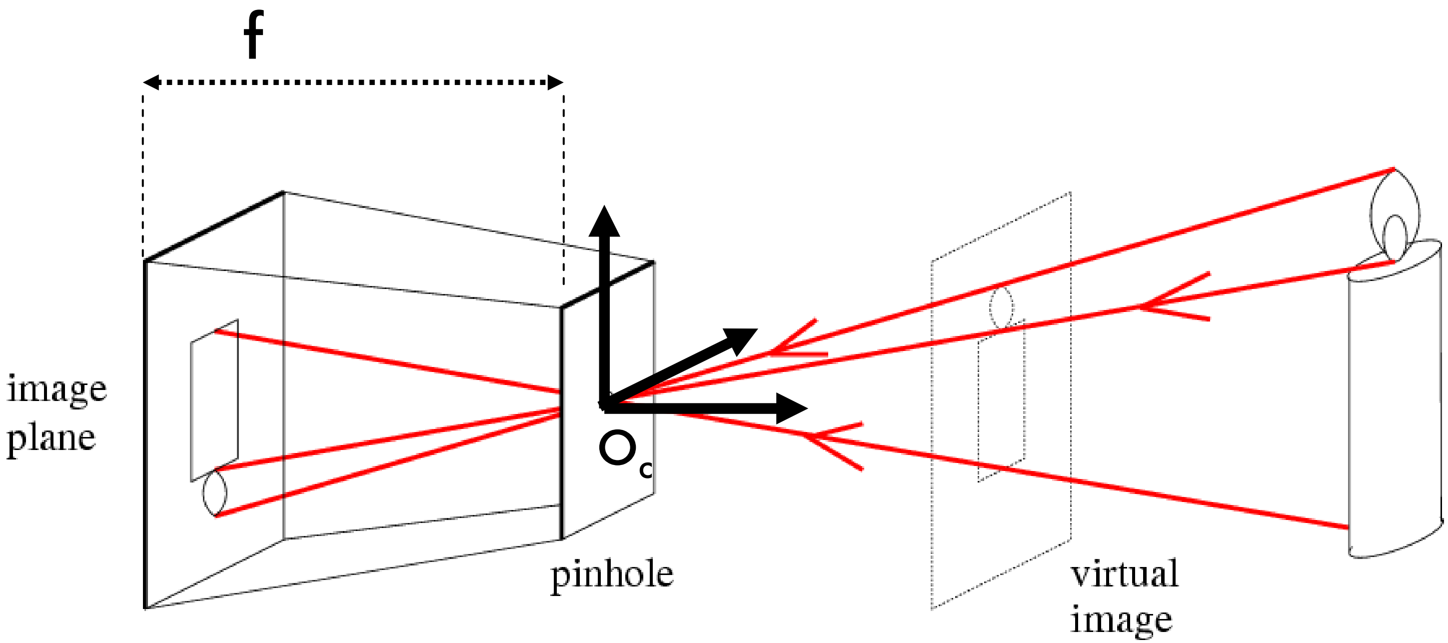
Some slides in this lecture are courtesy to Profs. J. Ponce & F-F Li

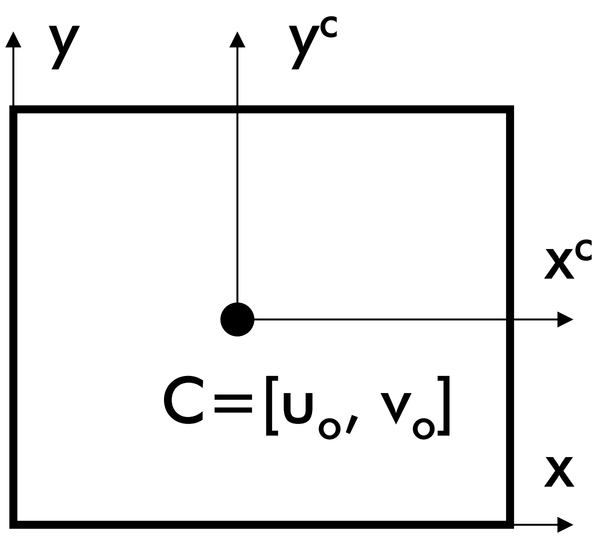
Projective camera



f = focal length

Projective camera

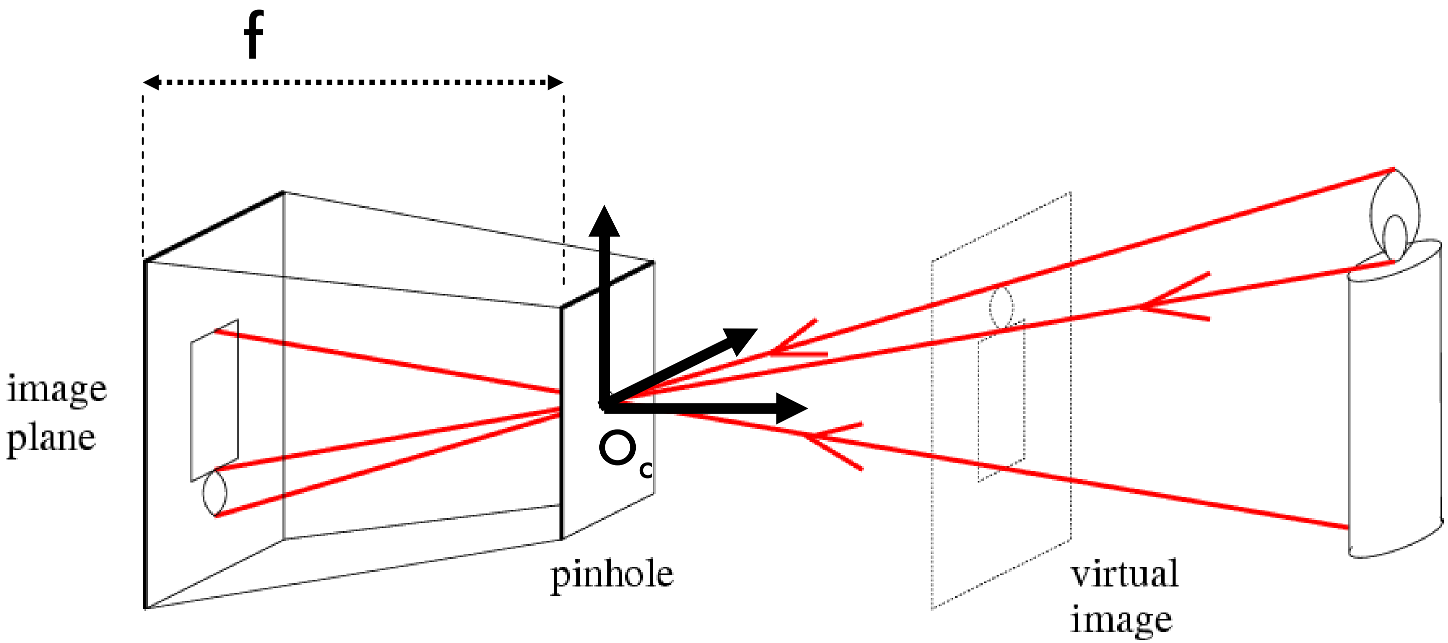




f = focal length

u_o, v_o = offset

Projective camera



Units: k,l [pixel/m]

f [m]

Non-square pixels

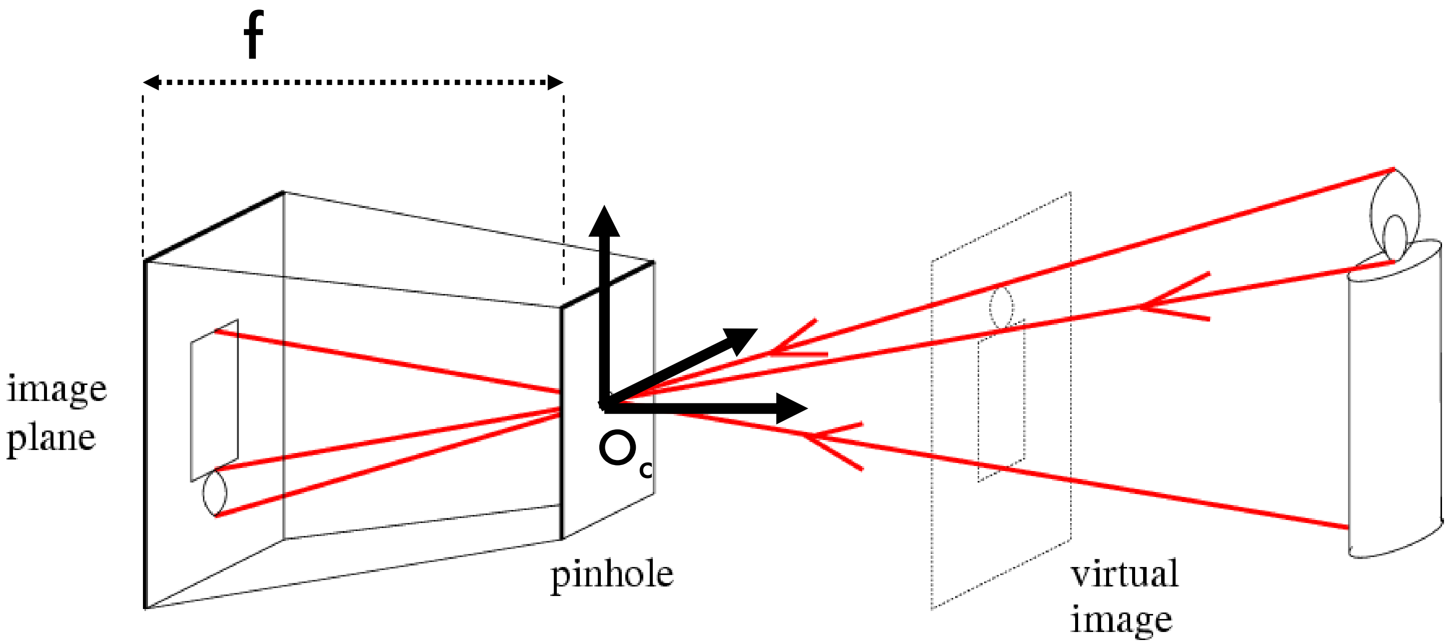
α, β [pixel]

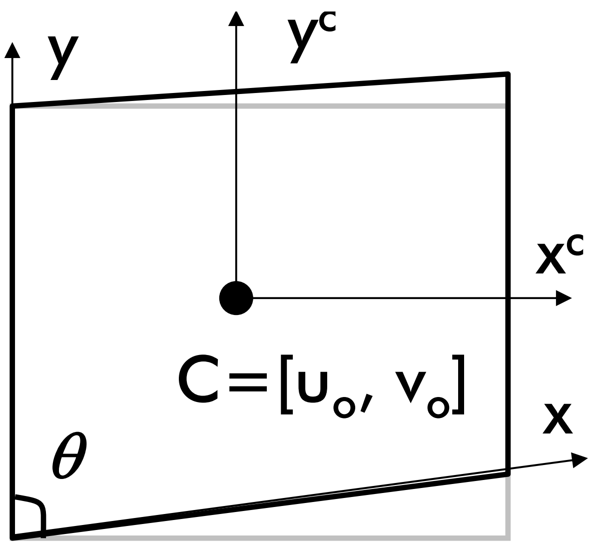
f = focal length

u_o, v_o = offset

α, β $\rightarrow$ non-square pixels

Projective camera





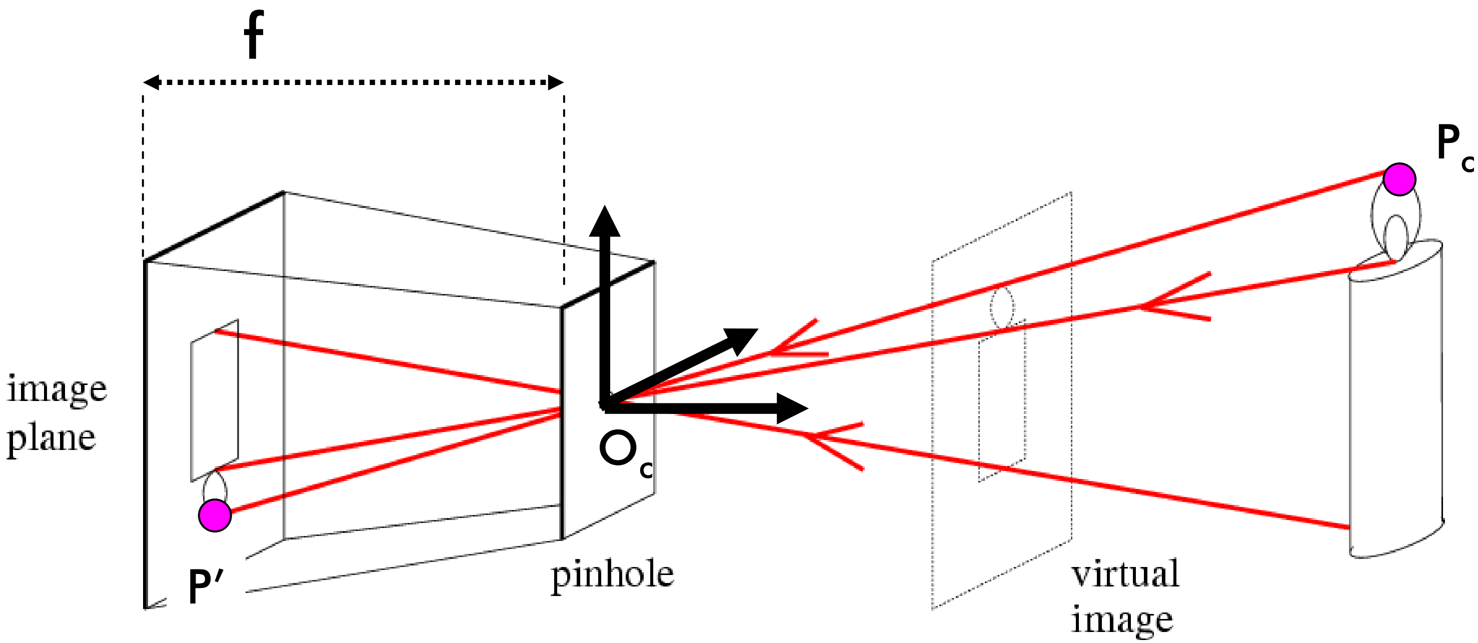
f = focal length

u_o, v_o = offset

$\alpha, \beta \rightarrow$ non-square pixels

θ = skewness

Projective camera



$$
P' = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_o & 0 \\ 0 & \dfrac{\beta}{\sin\theta} & v_o & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
$$

f = focal length

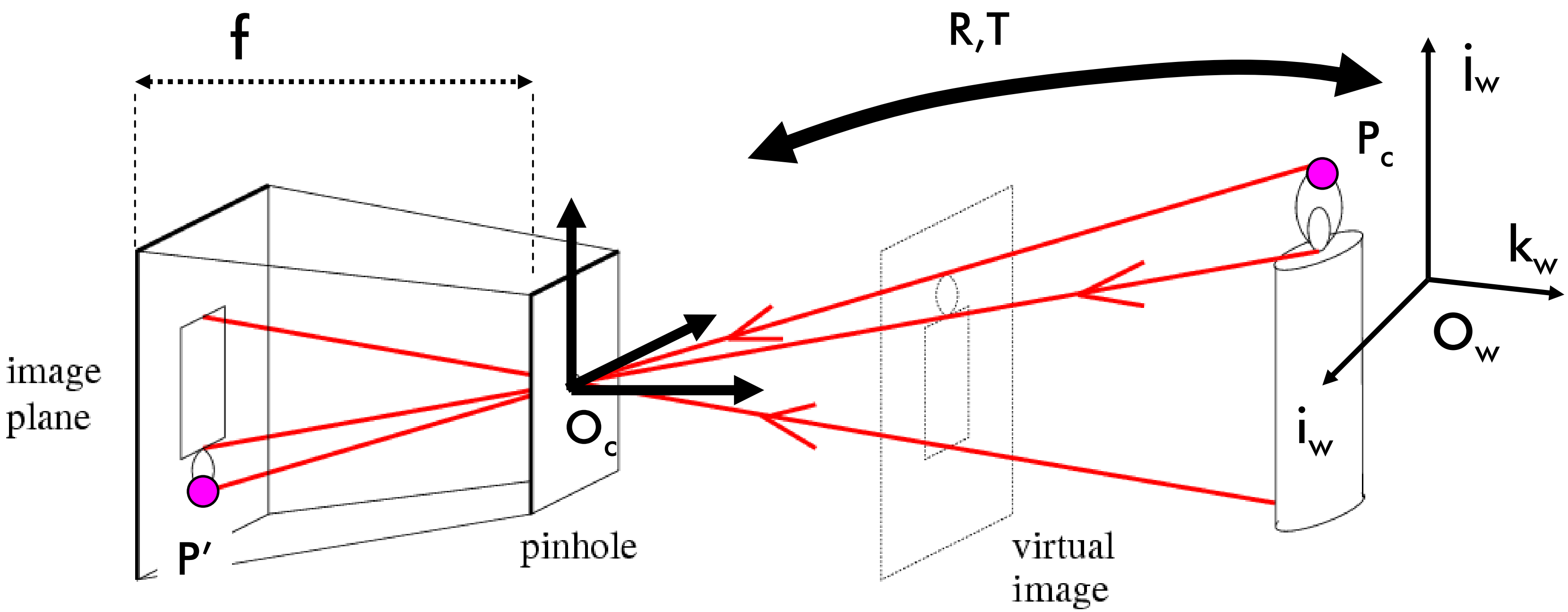
u_o, v_o = offset

$\alpha, \beta \rightarrow$ non-square pixels

θ = skewness

K has 5 degrees of freedom!

Projective camera



$$P_c = \begin{bmatrix} R & T \end{bmatrix} P_w$$

f = focal length

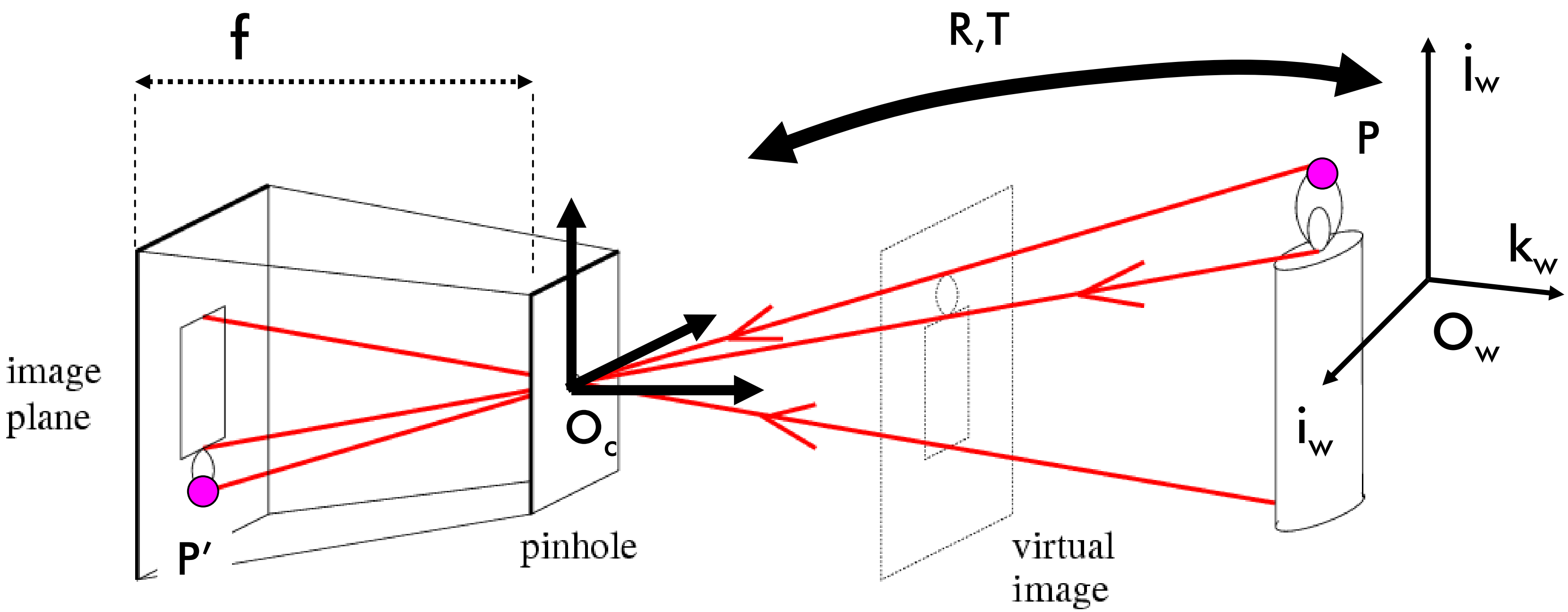
u_o, v_o = offset

$\alpha,\ \beta\ \rightarrow$ non-square pixels

θ = skewness

R,T = rotation, translation

Projective camera



$$P' = M\,P_w$$

$$= \underbrace{K}_{\text{Internal parameters}}\underbrace{[R \quad T]}_{\text{External parameters}} P_w$$

f = focal length

u_o, v_o = offset

α, β $\rightarrow$ non-square pixels

θ = skewness

R,T = rotation, translation

Goal of calibration

Estimate intrinsic and extrinsic parameters from 1 or multiple images

$$P' = M\,P_w \quad = \boxed{K}\boxed{\begin{bmatrix} R & T \end{bmatrix}}P_w$$

Internal parameters

External parameters

Goal of calibration

Estimate intrinsic and extrinsic parameters from 1 or multiple images

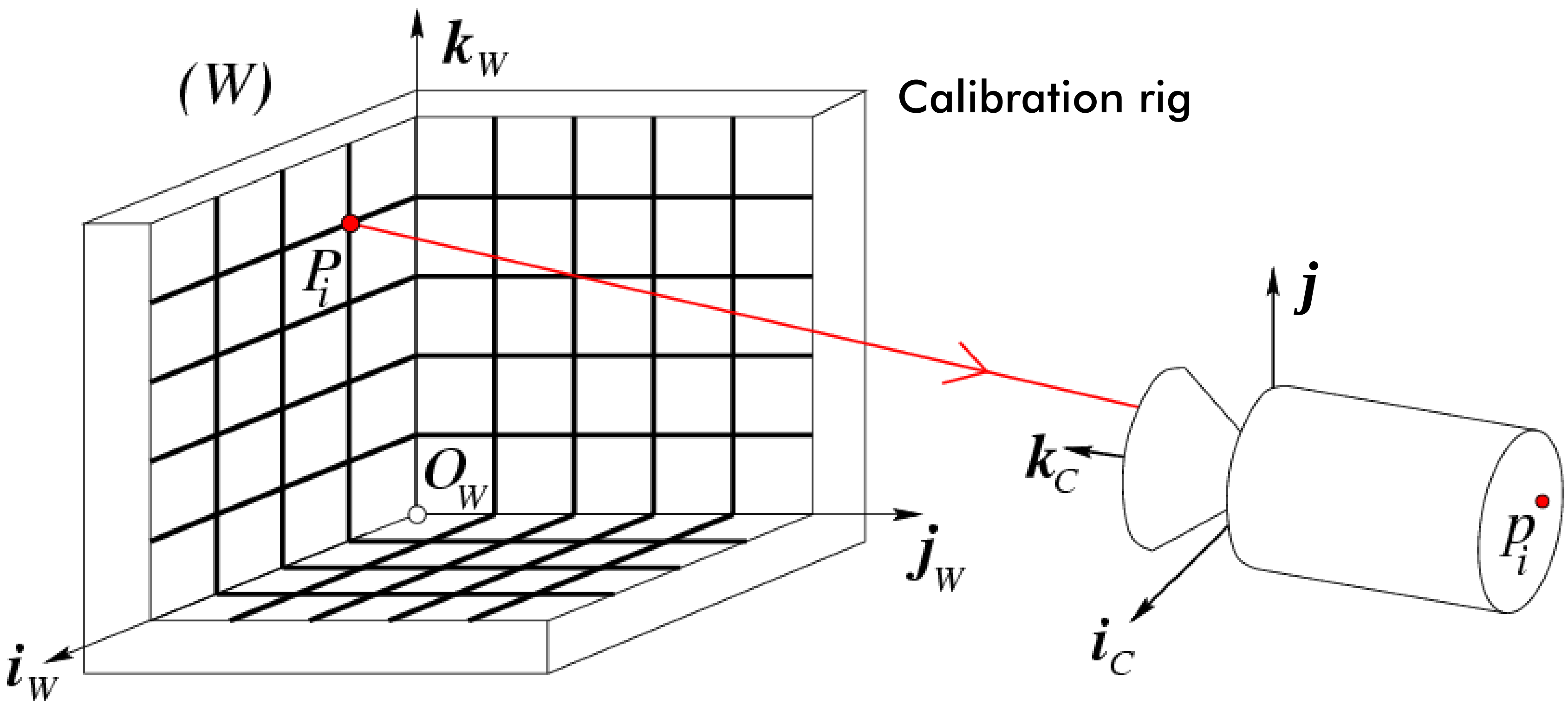
$$P' = M\,P_w \quad = K\begin{bmatrix} R & T \end{bmatrix} P_w$$

$$\boxed{\mathcal{M} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix}}$$

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_o \\ 0 & \dfrac{\beta}{\sin\theta} & v_o \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix} \qquad T = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Note: To simplify notation let $P = P_w$

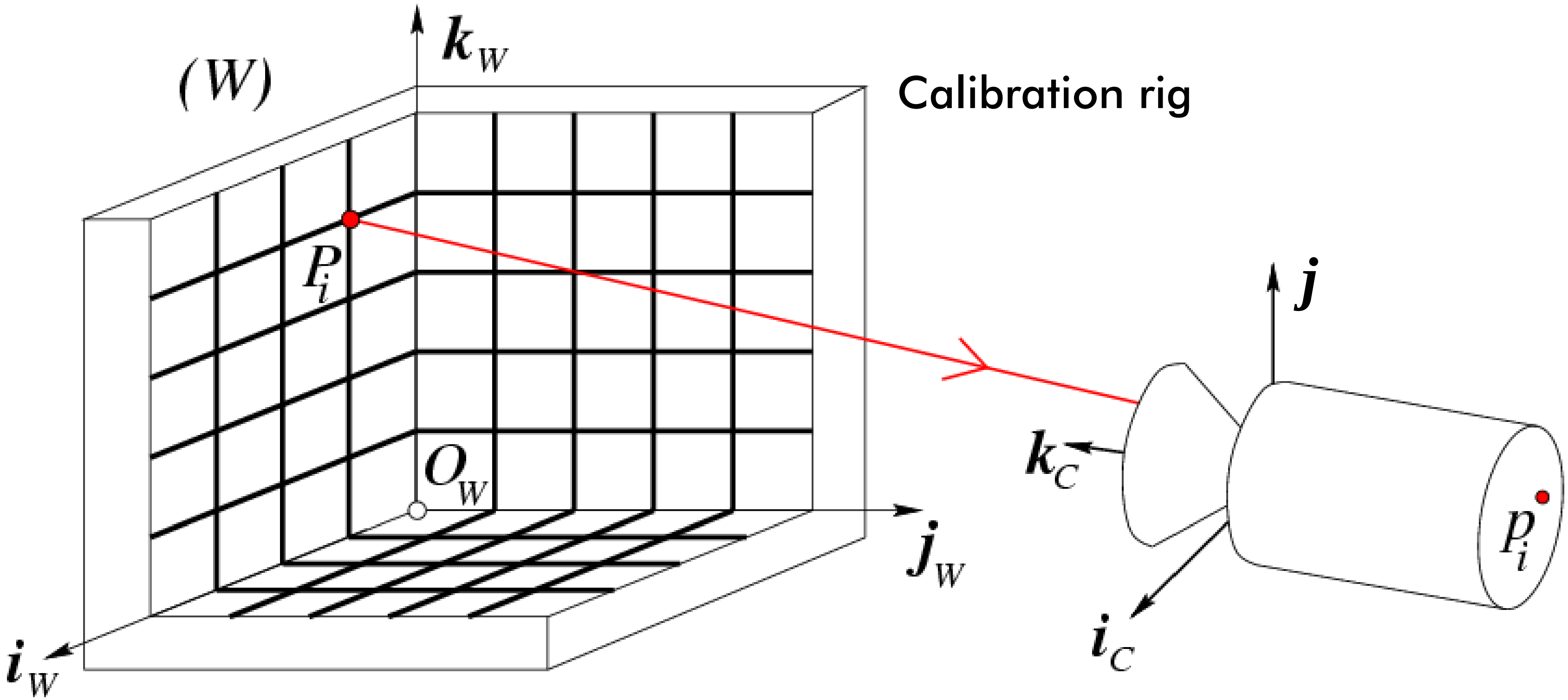
Calibration Problem



- $P_1 \ldots P_n$ with known positions in $[O_w, i_w, j_w, k_w]$
- $p_1, \ldots p_n$ known positions in the image

Goal: compute intrinsic and extrinsic parameters

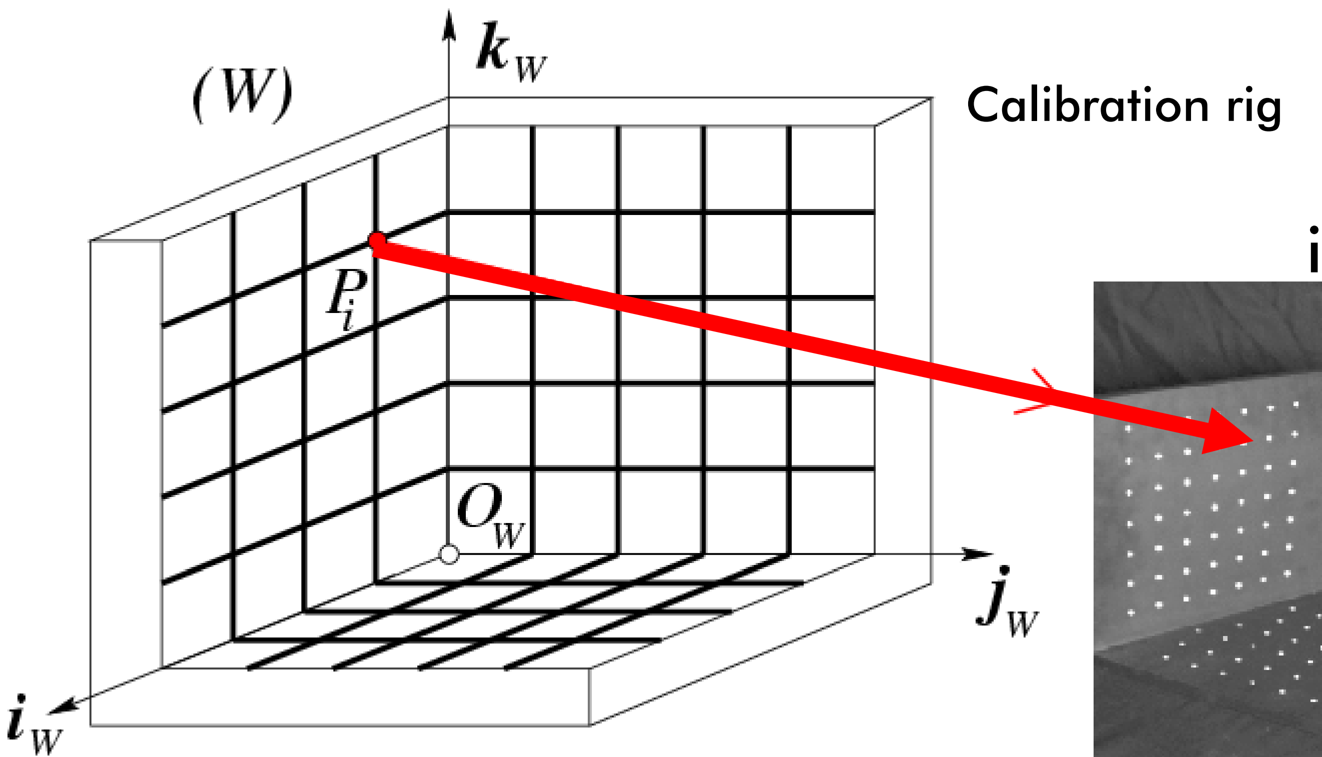
Calibration Problem

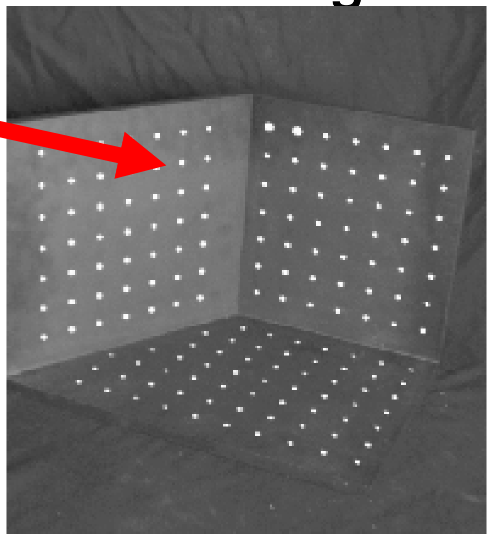


How many correspondences do we need?

•P has 11 unknown • We need 11 equations • 6 correspondences would do it

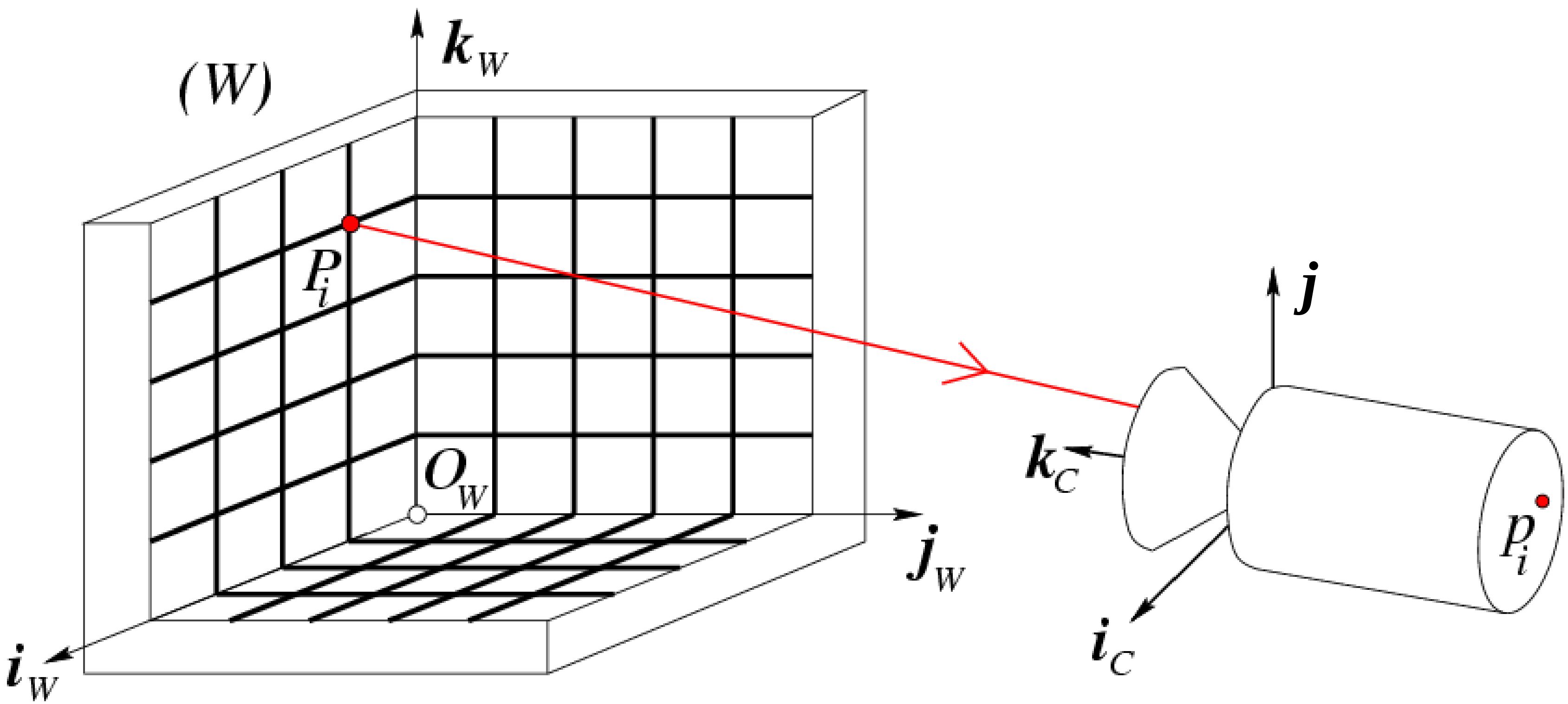
Calibration Problem





In practice: user may need to look at the image and select the n>=6 correspondences

Calibration Problem



$$P_i \rightarrow M\,P_i \rightarrow p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{m}_1\,P_i}{\mathbf{m}_3\,P_i} \\ \dfrac{\mathbf{m}_2\,P_i}{\mathbf{m}_3\,P_i} \end{bmatrix} \qquad M = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

in pixels

Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \dfrac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix}$$

$$u_i = \frac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \rightarrow u_i(\mathbf{m}_3 P_i) = \mathbf{m}_1 P_i$$

$$v_i = \frac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \rightarrow v_i(\mathbf{m}_3 P_i) = \mathbf{m}_2 P_i$$

$$\rightarrow \begin{pmatrix} \mathbf{m}_1 - u_i \mathbf{m}_3 \\ \mathbf{m}_2 - v_i \mathbf{m}_3 \end{pmatrix} P_i = 0$$

Calibration Problem

$$\left\{ \begin{array}{l} \begin{pmatrix} \mathbf{m}_1 - u_1\mathbf{m}_3 \\ \mathbf{m}_2 - v_1\mathbf{m}_3 \end{pmatrix} P_1 = 0 \\ \quad\vdots \\ \begin{pmatrix} \mathbf{m}_1 - u_i\mathbf{m}_3 \\ \mathbf{m}_2 - v_i\mathbf{m}_3 \end{pmatrix} P_i = 0 \\ \quad\vdots \\ \begin{pmatrix} \mathbf{m}_1 - u_n\mathbf{m}_3 \\ \mathbf{m}_2 - v_n\mathbf{m}_3 \end{pmatrix} P_n = 0 \end{array} \right.$$

Block Matrix Multiplication

$$\mathrm{A} = \begin{bmatrix} \mathrm{A}_{11} & \mathrm{A}_{12} \\ \mathrm{A}_{21} & \mathrm{A}_{22} \end{bmatrix} \qquad \mathrm{B} = \begin{bmatrix} \mathrm{B}_{11} & \mathrm{B}_{12} \\ \mathrm{B}_{21} & \mathrm{B}_{22} \end{bmatrix}$$

What is *AB* ?

$$AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

Calibration Problem

$$
\begin{cases}
\quad\vdots \\
\begin{pmatrix} \mathbf{m}_1 - u_i \mathbf{m}_3 \\ \mathbf{m}_2 - v_i \mathbf{m}_3 \end{pmatrix} P_i = 0 \\
\quad\vdots \\
\begin{pmatrix} \mathbf{m}_1 - u_n \mathbf{m}_3 \\ \mathbf{m}_2 - v_n \mathbf{m}_3 \end{pmatrix} P_n = 0
\end{cases}
\longrightarrow
\boxed{\overbrace{\mathcal{P}}^{\text{known}}\,\overbrace{\boldsymbol{m}}^{\text{unknown}} = 0}
$$

Homogenous linear system

$$
\mathcal{P} \stackrel{\text{def}}{=} \begin{pmatrix}
\boldsymbol{P}_1^T & \mathbf{0}^T & -u_1 \overbrace{\boldsymbol{P}_1^T}^{1\text{x}4} \\
\mathbf{0}^T & \boldsymbol{P}_1^T & -v_1 \boldsymbol{P}_1^T \\
\dots & \dots & \dots \\
\boldsymbol{P}_n^T & \mathbf{0}^T & -u_n \boldsymbol{P}_n^T \\
\mathbf{0}^T & \boldsymbol{P}_n^T & -v_n \boldsymbol{P}_n^T
\end{pmatrix}_{2\text{n x }12}
\qquad
\boldsymbol{m} \stackrel{\text{def}}{=} \begin{pmatrix}
\overbrace{\mathbf{m}_1^T}^{4\text{x}1} \\
\mathbf{m}_2^T \\
\mathbf{m}_3^T
\end{pmatrix}_{12\text{x}1}
$$

Homogeneous M x N Linear Systems

M=number of equations
N=number of unknown

$$A \, x = 0$$

M x N

Square system (M=N):

- if Det(A) $\neq$ 0,
  unique solution: 0

$$A \, x = 0$$

Rectangular system

- 0 is always a solution
- non-zero solution if det(A) $\neq 0$
- noisy measurements

  Minimize $|Ax|^2$

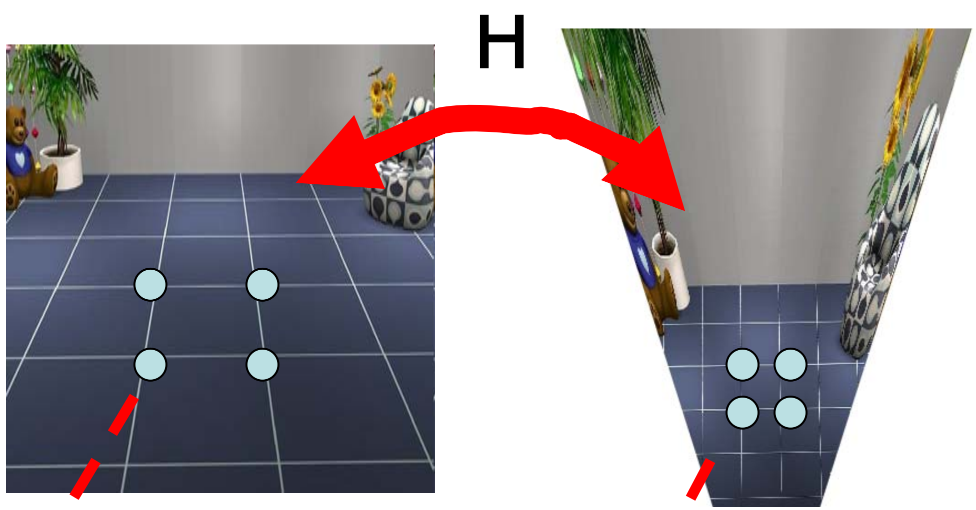
  under the constraint $|x|^2 = 1$

Calibration Problem

$$\mathcal{P}\boldsymbol{m} = 0 \ .$$

How do we solve this homogenous linear system?

DLT algorithm (Direct Linear Transformation)



x_i

x'_i

$$x'_i = H x_i$$

$$\underbrace{A_i}_{\text{Function of measurements}} \overbrace{\mathbf{h}}^{\text{unknown}} = 0$$

$$x'_i \times H x_i = 0 \longrightarrow A_i \mathbf{h} = 0$$

General Calibration Problem

$$\boxed{\mathcal{P}}\boldsymbol{m} = 0$$

$$\boxed{U_{2n\times 12}\; D_{12\times 12}\; V^{T}_{12\times 12}}$$

Last column of V gives $\boldsymbol{m}$

$$M\,P_i \rightarrow p_i$$

Extracting camera parameters

$$\mathcal{M} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix} = \mathrm{K}\begin{bmatrix} \mathrm{R} & \mathrm{T} \end{bmatrix}$$

$$\underbrace{\qquad}_{\rho \mathrm{A}} \qquad \underbrace{\qquad}_{\rho \mathbf{b}}$$

$$\mathrm{A} = \begin{bmatrix} \mathbf{a}_1^{\mathrm{T}} \\ \mathbf{a}_2^{\mathrm{T}} \\ \mathbf{a}_3^{\mathrm{T}} \end{bmatrix} \qquad \mathrm{b} = \begin{bmatrix} \mathrm{b}_1 \\ \mathrm{b}_2 \\ \mathrm{b}_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\rho = \frac{\pm 1}{|\mathbf{a}_3|} \qquad \mathrm{u}_{\mathrm{o}} = \rho^2 (\mathbf{a}_1 \cdot \mathbf{a}_2)$$

$$\mathrm{v}_{\mathrm{o}} = \rho^2 (\mathbf{a}_2 \cdot \mathbf{a}_3)$$

$$\cos\theta = \frac{(\mathbf{a}_1 \times \mathbf{a}_3) \cdot (\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_1 \times \mathbf{a}_3| \cdot |\mathbf{a}_2 \times \mathbf{a}_3|}$$

Theorem (Faugeras, 1993)

Let $\mathcal{M} = (\mathcal{A} \quad \boldsymbol{b})$ be a 3×4 matrix and let $\boldsymbol{a}_i^T$ $(i = 1, 2, 3)$ denote the rows of the matrix $\mathcal{A}$ formed by the three leftmost columns of $\mathcal{M}$.

- A necessary and sufficient condition for $\mathcal{M}$ to be a perspective projection matrix is that $\mathrm{Det}(\mathcal{A}) \neq 0$.
- A necessary and sufficient condition for $\mathcal{M}$ to be a zero-skew perspective projection matrix is that $\mathrm{Det}(\mathcal{A}) \neq 0$ and

$$(\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3) = 0.$$

- A necessary and sufficient condition for $\mathcal{M}$ to be a perspective projection matrix with zero skew and unit aspect-ratio is that $\mathrm{Det}(\mathcal{A}) \neq 0$ and

$$\begin{cases} (\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3) = 0, \\ (\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_1 \times \boldsymbol{a}_3) = (\boldsymbol{a}_2 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3). \end{cases}$$

Extracting camera parameters

$$\mathcal{M} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix} = \mathrm{K}\begin{bmatrix} \mathrm{R} & \mathrm{T} \end{bmatrix}$$

$$\rho\, \mathrm{A} \qquad \rho\, \mathbf{b}$$

$$\mathrm{A} = \begin{bmatrix} \mathbf{a}_1^{\mathrm{T}} \\ \mathbf{a}_2^{\mathrm{T}} \\ \mathbf{a}_3^{\mathrm{T}} \end{bmatrix} \qquad \mathrm{b} = \begin{bmatrix} \mathrm{b}_1 \\ \mathrm{b}_2 \\ \mathrm{b}_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\alpha = \rho^2 \left| \mathbf{a}_1 \times \mathbf{a}_3 \right| \sin\theta$$

$$\beta = \rho^2 \left| \mathbf{a}_2 \times \mathbf{a}_3 \right| \sin\theta$$

$\rightarrow$ **f**

Extracting camera parameters

$$\mathcal{M} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix} = \mathrm{K}\begin{bmatrix} \mathrm{R} & \mathrm{T} \end{bmatrix}$$

$$\rho\, \mathrm{A} \qquad \rho\, \mathbf{b}$$

$$\mathrm{A} = \begin{bmatrix} \mathbf{a}_1^{\mathrm{T}} \\ \mathbf{a}_2^{\mathrm{T}} \\ \mathbf{a}_3^{\mathrm{T}} \end{bmatrix} \qquad \mathrm{b} = \begin{bmatrix} \mathrm{b}_1 \\ \mathrm{b}_2 \\ \mathrm{b}_3 \end{bmatrix}$$

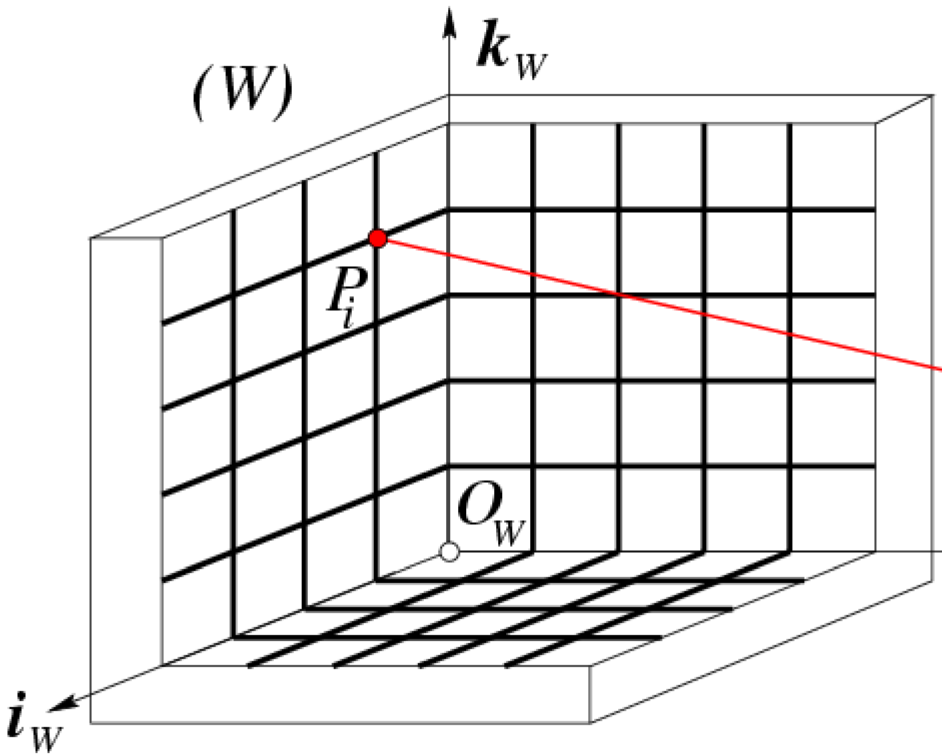
Estimated values

Extrinsic

$$\mathbf{r}_1 = \frac{(\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_2 \times \mathbf{a}_3|} \qquad \mathbf{r}_3 = \frac{\pm 1}{|\mathbf{a}_3|}$$

$$\mathbf{r}_2 = \mathbf{r}_3 \times \mathbf{r}_1 \qquad \mathrm{T} = \rho\, \mathrm{K}^{-1}\mathbf{b}$$

Degenerate cases

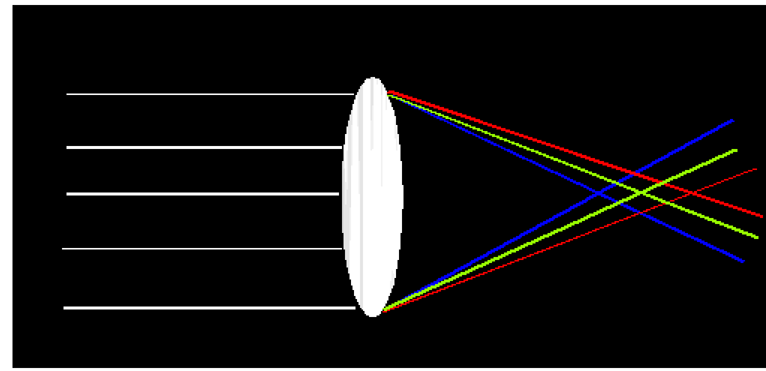


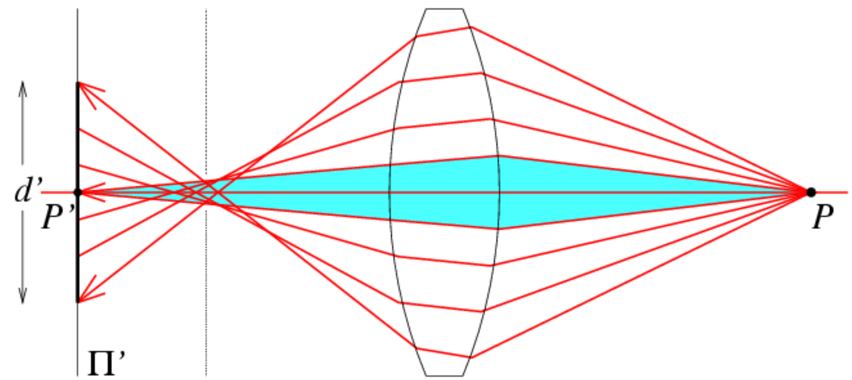
$$\begin{array}{c}
\boldsymbol{i} \\
\mathcal{P} \stackrel{\text{def}}{=} \begin{pmatrix} \boldsymbol{P}_1^T & \boldsymbol{0}^T & -u_1\boldsymbol{P}_1^T \\ \boldsymbol{0}^T & \boldsymbol{P}_1^T & -v_1\boldsymbol{P}_1^T \\ \dots & \dots & \dots \\ \boldsymbol{P}_n^T & \boldsymbol{0}^T & -u_n\boldsymbol{P}_n^T \\ \boldsymbol{0}^T & \boldsymbol{P}_n^T & -v_n\boldsymbol{P}_n^T \end{pmatrix}
\end{array}$$

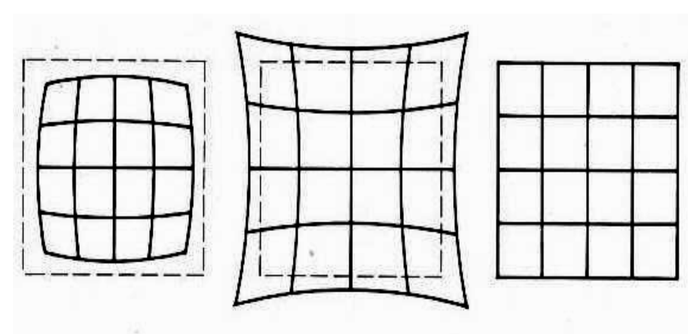
- P_i's cannot lie on the same plane!
- Points cannot lie on the intersection curve of two quadric surfaces

Taking lens distortions into account

- Chromatic Aberration
- Spherical aberration
- Radial Distortion

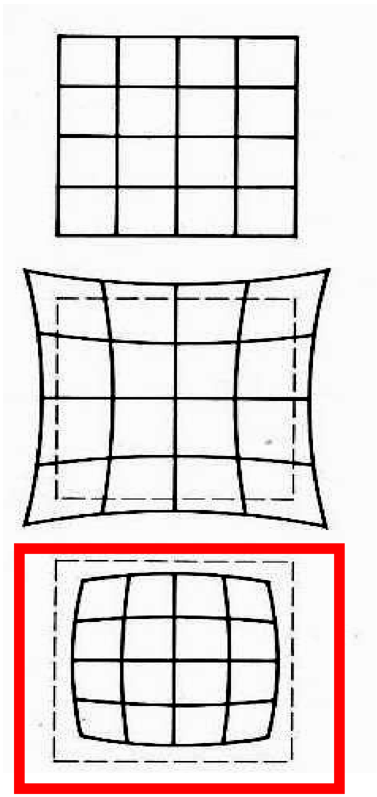






Radial Distortion

- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens

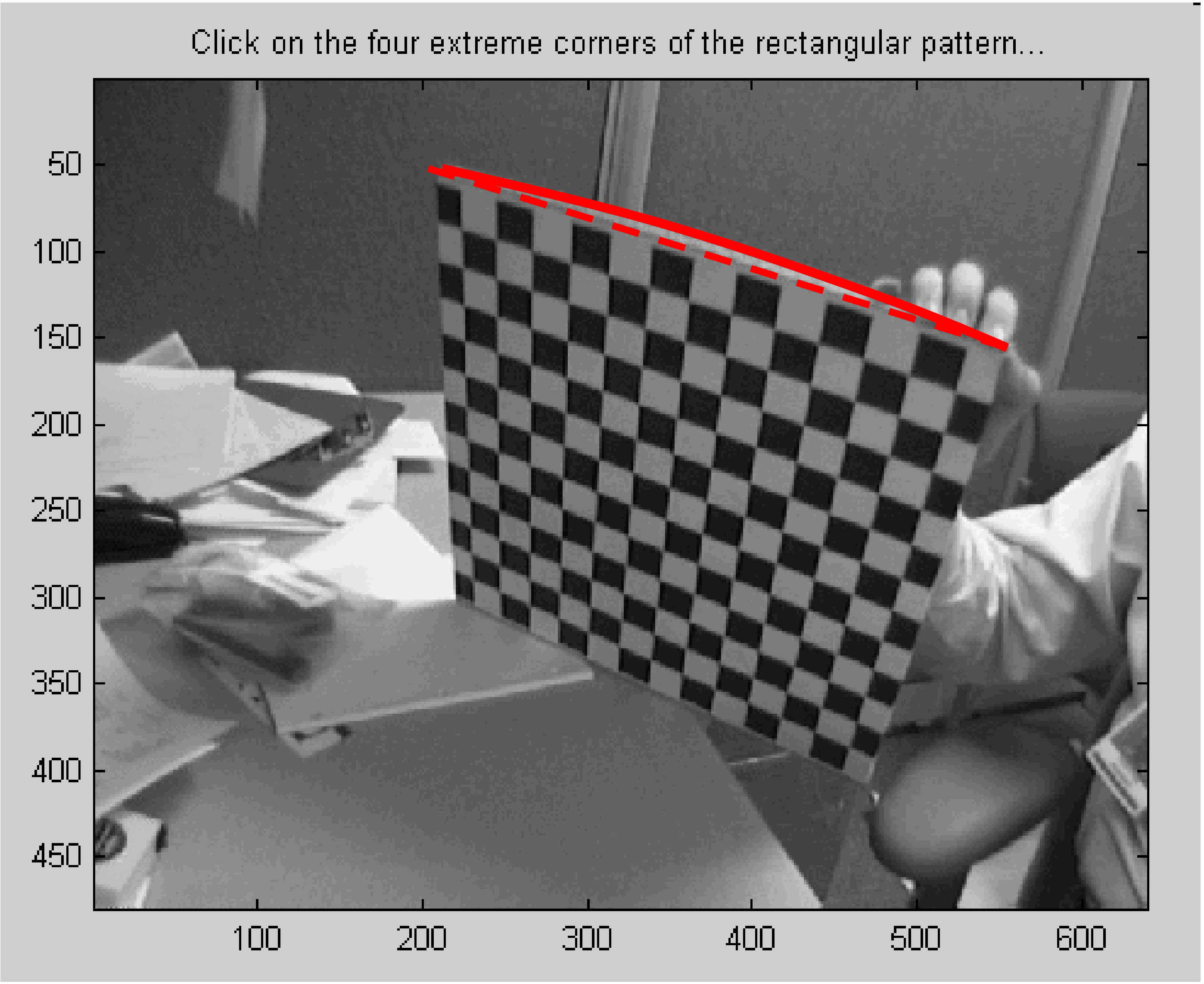


No distortion

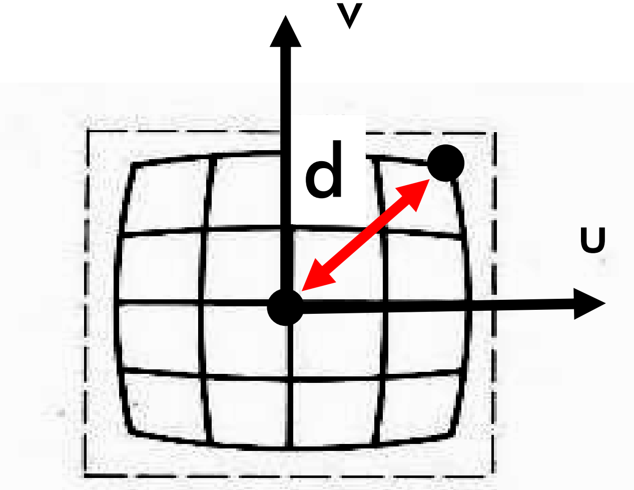
Pin cushion

Barrel

Radial Distortion



Radial Distortion



$$d^2 = a\,u^2 + b\,v^2 + c\,u\,v$$

$$\begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} M\ P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i$$

$$\lambda = 1 \pm \underbrace{\sum_{p=1}^{3} \overset{\text{Distortion coefficient}}{\boxed{\kappa_p}}\, d^{2p}}_{\text{Polynomial function}}$$

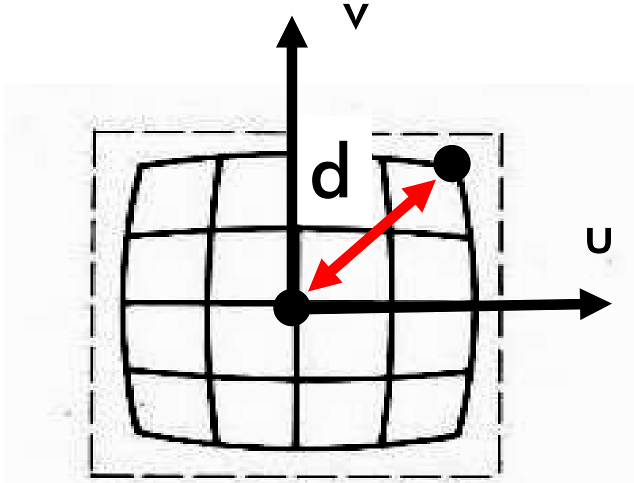
Radial Distortion

Estimating m_1 and m_2…

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \dfrac{\mathbf{m}_1 \, P_i}{\mathbf{m}_3 \, P_i} \\ \dfrac{\mathbf{m}_2 \, P_i}{\mathbf{m}_3 \, P_i} \end{bmatrix}$$

How to do that?

Hint:



$$\frac{u_i}{v_i} = \text{slope}$$

Radial Distortion

Estimating m_1 and m_2…

$$\mathrm{p}_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \dfrac{\mathbf{m}_1 \, P_i}{\mathbf{m}_3 \, P_i} \\ \dfrac{\mathbf{m}_2 \, P_i}{\mathbf{m}_3 \, P_i} \end{bmatrix}$$

How to do that?

$$\begin{cases} v(\mathbf{m}_1 \, P_1) - u(\mathbf{m}_2 \, P_1) = 0 \\ v(\mathbf{m}_1 \, P_i) - u(\mathbf{m}_2 \, P_i) = 0 \\ \vdots \\ v(\mathbf{m}_1 \, P_n) - u(\mathbf{m}_2 \, P_n) = 0 \end{cases}$$

$$Q \, \mathbf{n} = 0 \qquad \mathbf{n} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \end{bmatrix}$$

Tsai technique [87]

Radial Distortion

Estimating m_1 and m_2…

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \dfrac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \dfrac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix}$$

How to do that?

$\mathbf{m}_3$ is non linear function of $\mathbf{m}_1$, $\mathbf{m}_2$, λ

Radial Distortion

$$\underbrace{\begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} M}_{Q} P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i$$

$$Q = \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \end{bmatrix}$$

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 \, P_i}{\mathbf{q}_3 \, P_i} \\ \dfrac{\mathbf{q}_2 \, P_i}{\mathbf{q}_3 \, P_i} \end{bmatrix} \rightarrow \overbrace{\begin{cases} u_i \mathbf{q}_3 \, P_i = \mathbf{q}_1 \, P_i \\ v_i \mathbf{q}_3 \, P_i = \mathbf{q}_2 \, P \end{cases}}^{\text{Non-linear system of equations}}$$

General Calibration Problem

$$X = f(P)$$

f( ) is nonlinear

measurement

parameter

-Newton Method
-Levenberg-Marquardt Algorithm

- Iterative, starts from initial solution
- May be slow if initial solution far from real solution
- Estimated solution may be function of the initial solution
- Newton requires the computation of J, H
- Levenberg-Marquardt doesn't require the computation of H

General Calibration Problem

$$X = f(P)$$

f( ) is nonlinear

measurement

parameter

A possible algorithm

1. Solve linear part of the system to find approximated solution
2. Use this solution as initial condition for the full system
3. Solve full system using Newton or L.M.

General Calibration Problem

$$X = f(P)$$

f( ) is nonlinear

measurement

parameter

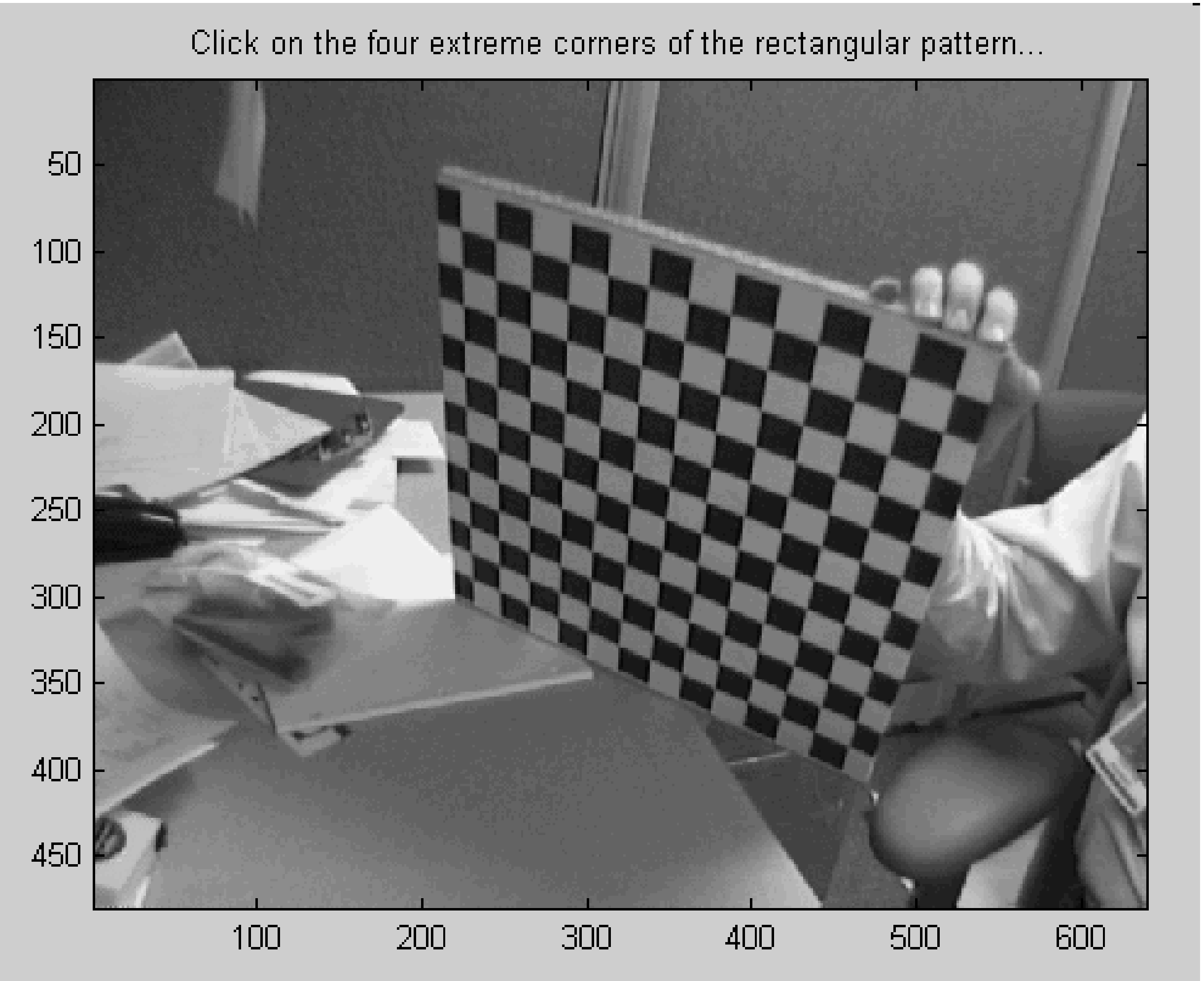
Typical assumptions:

- zero-skew, square pixel
- u_o, v_o = known center of the image
- no distortion

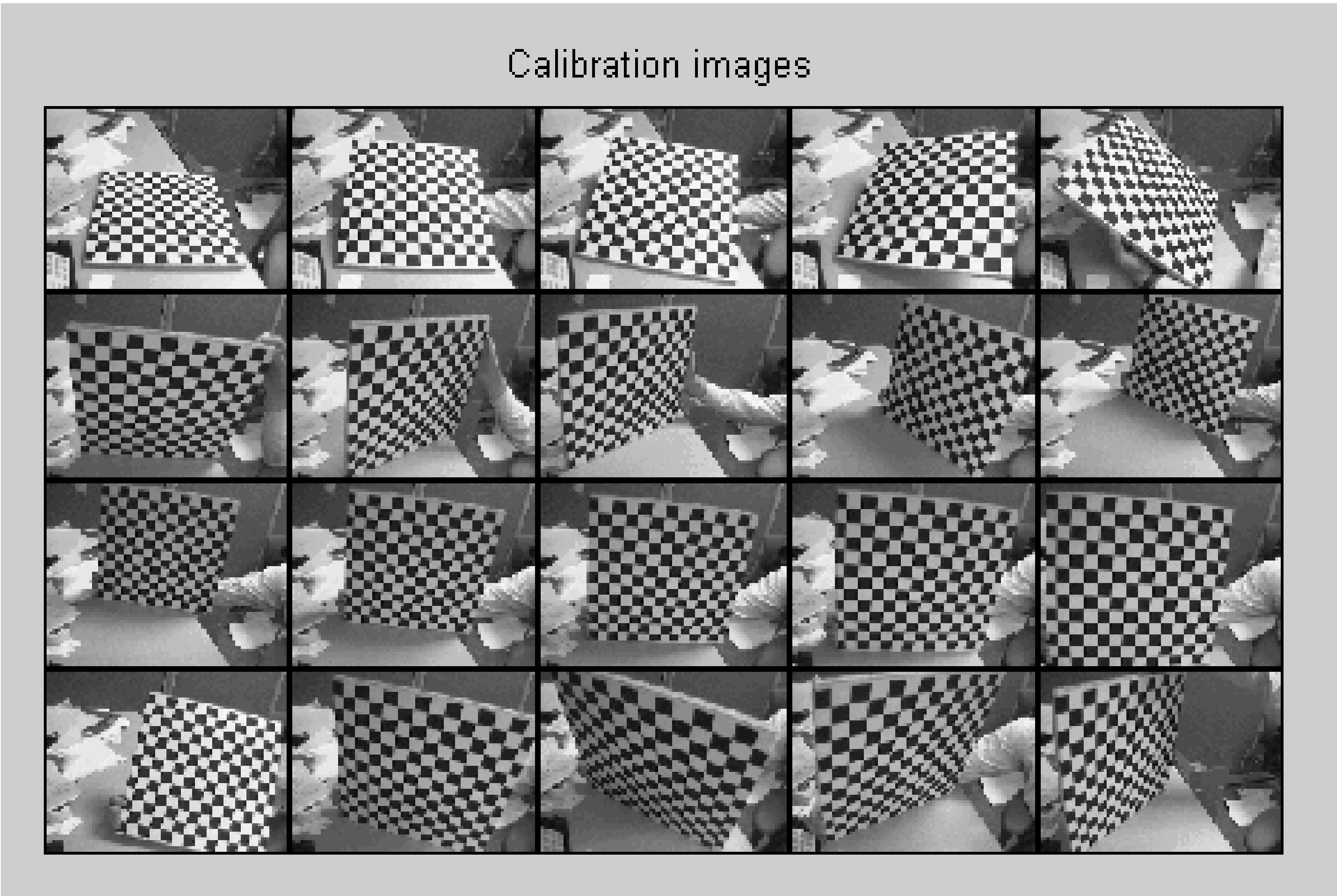
Just estimate f and R, T

Calibration Procedure

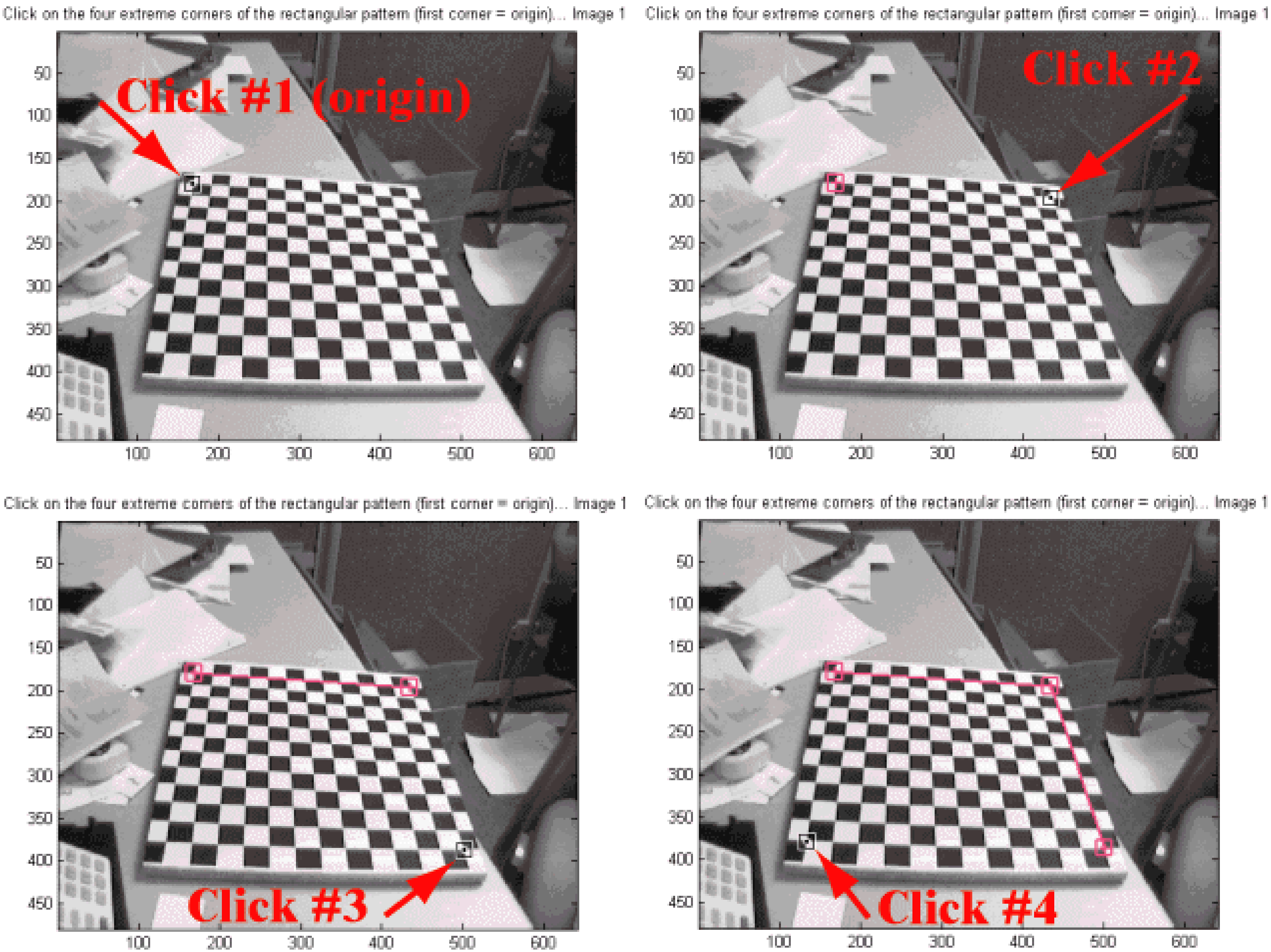
Camera Calibration Toolbox for Matlab
J. Bouguet – [1998-2000]



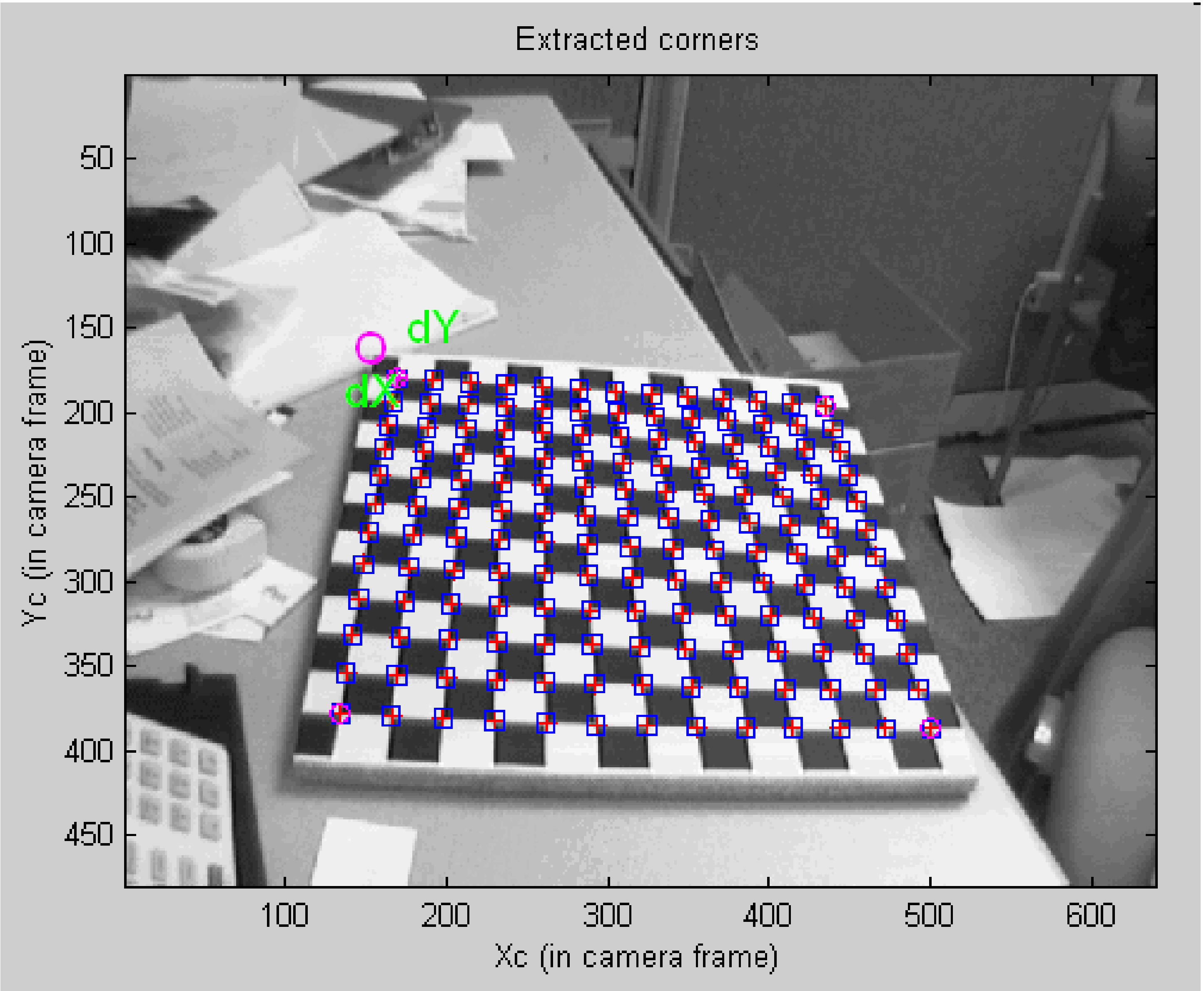
Calibration Procedure



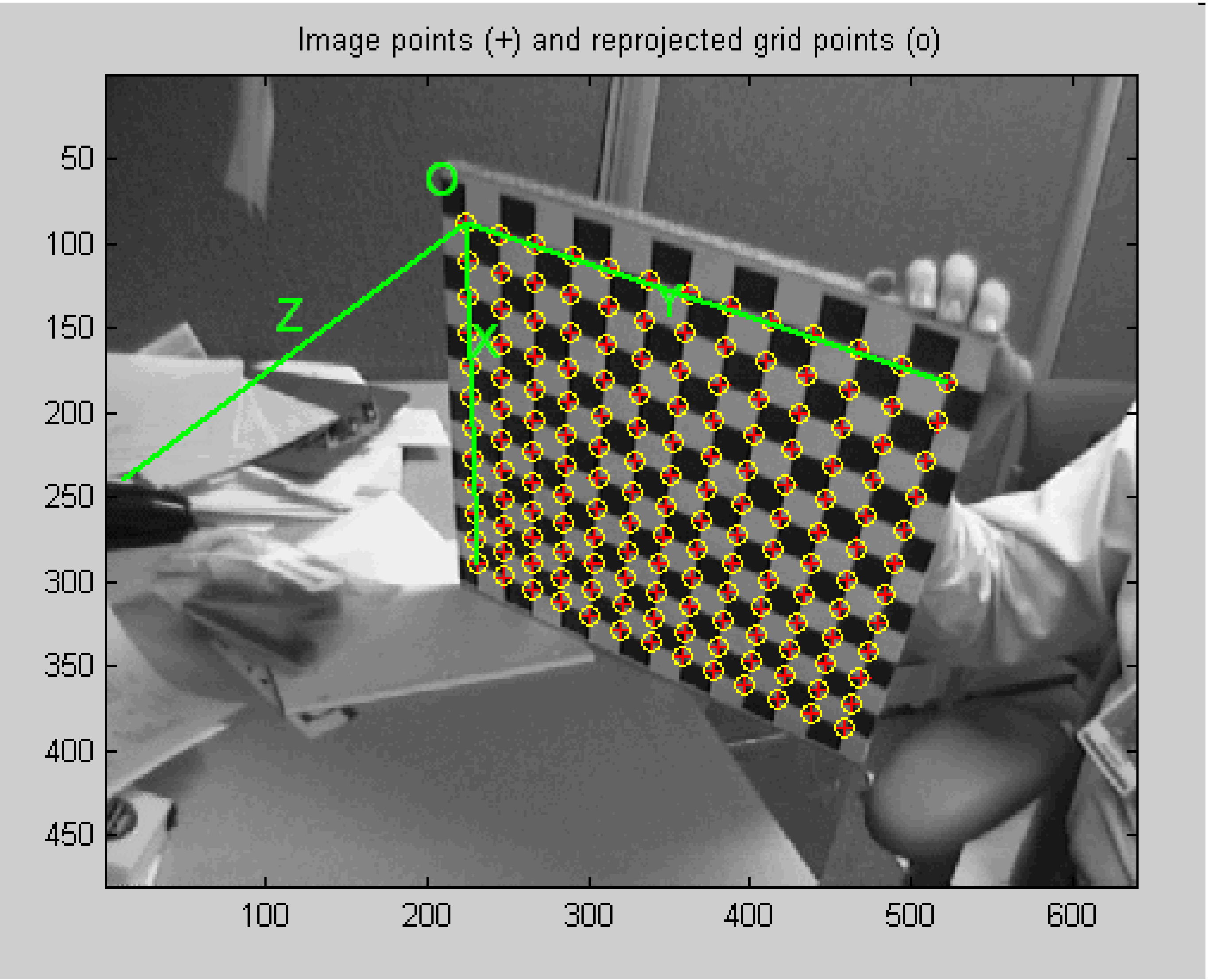
Calibration Procedure



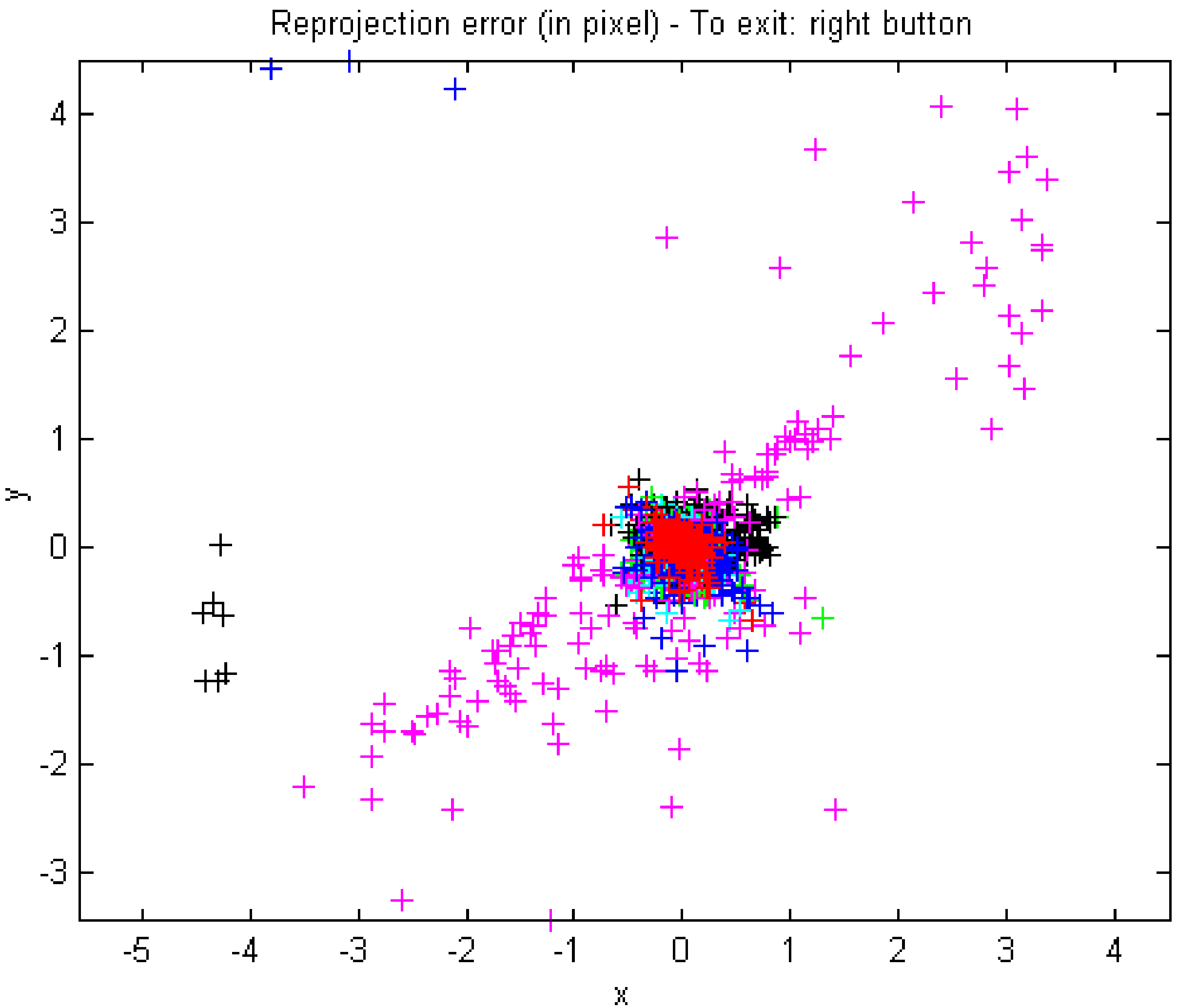
Calibration Procedure



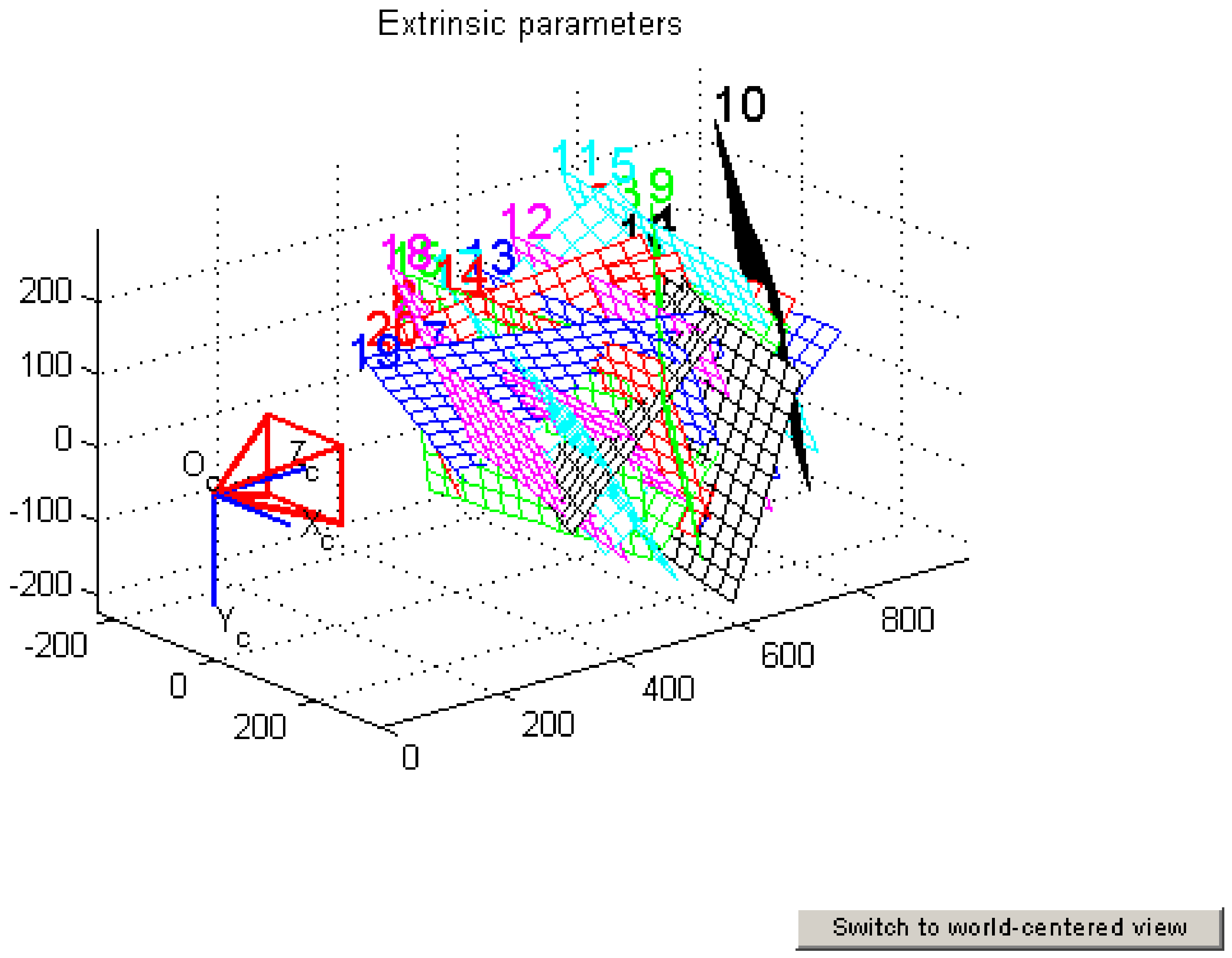
Calibration Procedure



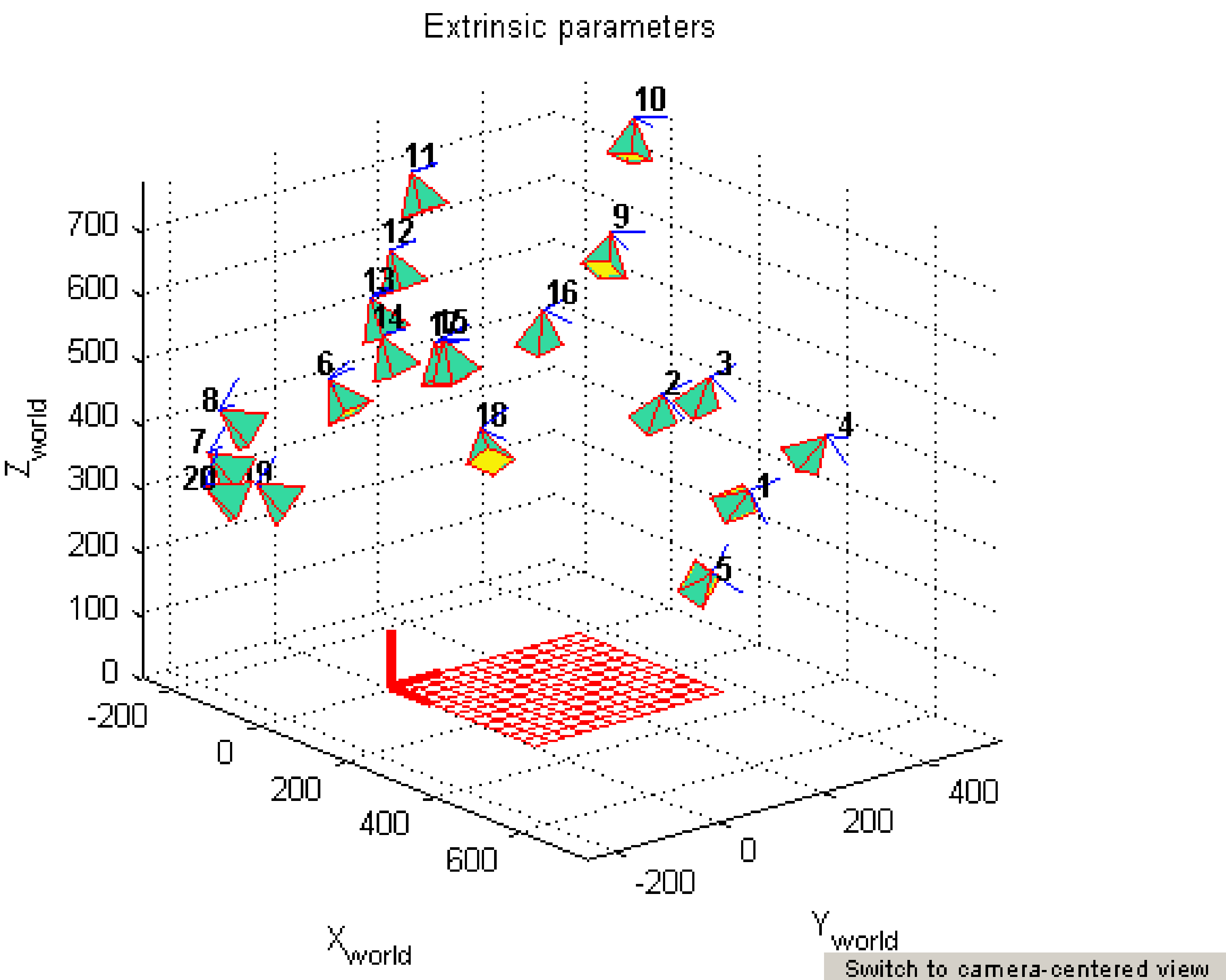
Calibration Procedure



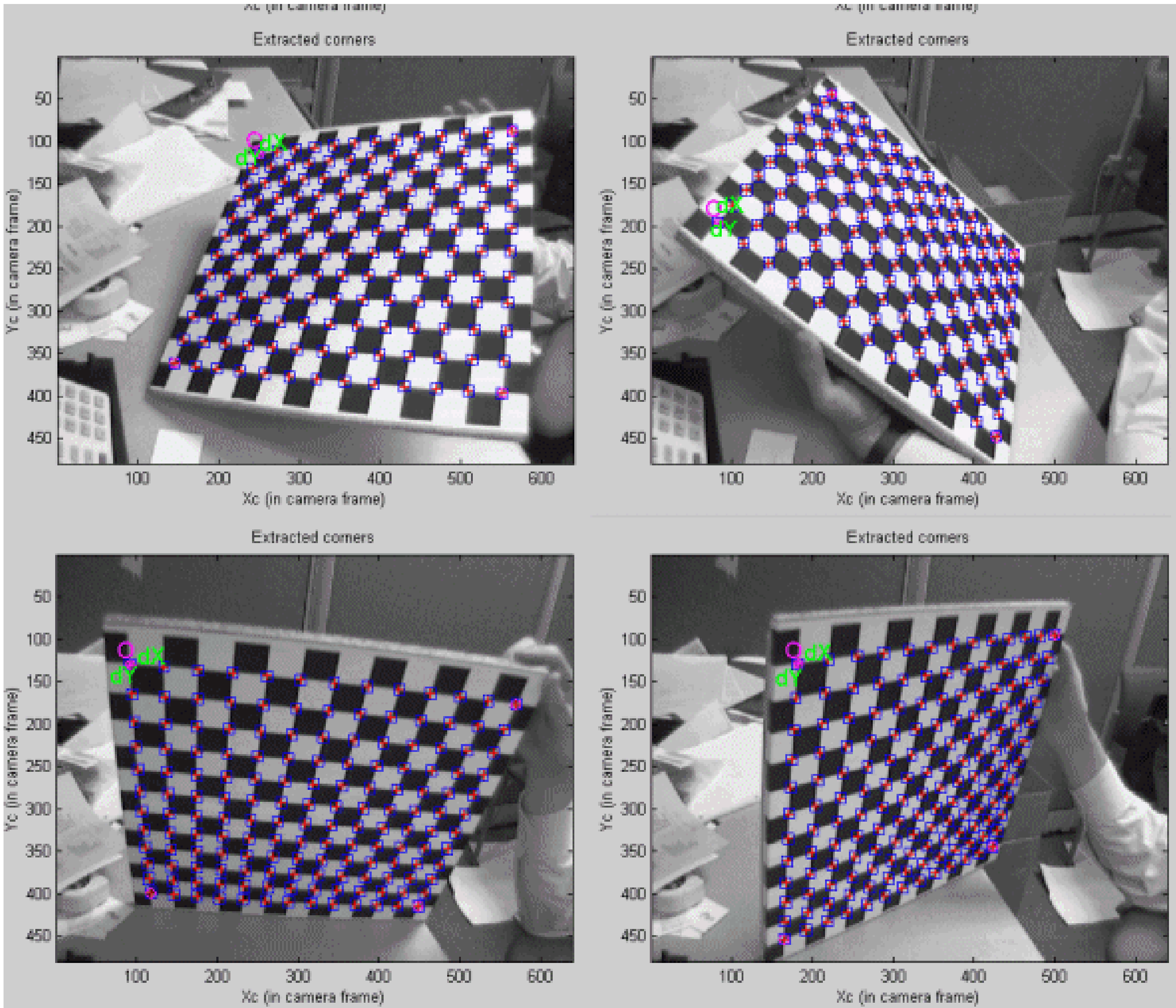
Calibration Procedure



Calibration Procedure



Calibration Procedure



Next lecture

- Single view reconstruction

- Homework will be online tomorrow night (syllabus page)